Learning a REE in Open Economy

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Abstract

This paper analyzes the determinants of expectational coordination on the perfect foresight equilibrium of an open economy in the class of one-dimensional models where the price is determined by price expectations. In this class of models, we relate autarky expectational stability conditions to regional integration ones, showing that the degree of structural heterogeneity trades-off price stabilization and stabilizing price expectations. We provide an intuitive open economy interpretation to the elasticities condition obtained by Guesnerie [10]. Finally we argue that more traditional criteria to evaluate ex-ante the desirability of economic integration (net welfare gains) do not always advice integration between two expectationally stable economies.

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JELs: D84, C62, F15, E32.

(Preliminary)

1 Introduction

It is well known that price stabilization can be one of the potential benefits of economic integration, and economic openess in general. In this work we depart from a model where price stabilization is a consequence of economic integration. Nevertheless, the stability of price expectations is undermined, generating spurious price volatility and multiple equilibria. We depart from Guesnerie's [10] one-dimensional version of Muth's [17] model to explore the consequences of introducing a particular form of heterogeneity amenable to an open economy

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device in partial equilibrium[1]. Our *purpose* is to relate open economy expectational stability conditions to autarkic ones, adopting the eductive learning viewpoint[2].

In the class of models under study, infinitesimal producers have to take a production decision the ultimate consequences of which also depends on an agregate of the decisions taken by the rest of the agents[3]. There, Guesnerie [12] convincingly argues that although strategic substituabilities/complementarities determine the sign of agents' reactions to expectations, what is instrumental for eductive expectational coordination is the magnitude of these reactions. If new markets are available to the producers of a particular region, Guesnerie's [10] discussion of the favourable role of a high demand elasticity in expectational coordination suggests that producers' strategic uncertainty will be alleviated: own production decisions will be less sensible to own expectations on others' decisions. However, opening the home market to foreign producers will have the opposite effect, rendering production decisions more interdependent in a truly strategical sense: by rendering the crop relatively more abundant, the resulting price will increasingly depend on others' production decisions. Economic integration, by combining both effects, will result in an ambiguous total effect on producers' ability to coordinate on the new open economy equilibrium price.

Our first result states that the degree of expectational stability of the open economy equilibrium price lies between the autarkic expectational degrees of the most and the least stable regions in the linear class. Then, two autarkically stable regions cannot destabilize when integrating, nor two expectationally unstable regions can stabilize by mere integration. But a stable and an unstable regions can stabilize by integrating provided that the unstable economy is not 'too unstable' and that its demand elasticity is small relative to the integrated economy's. This result is qualified when differences in the maximal willigness to pay for the crop are allowed: two expectationally stable autarkic regions can destabilize after integration. Relative to the favourable role of a high demand elasticity in coordinating expectations, significant differences in the consumers' valuation for the crop make more likely a type of 'market disruption' phenomenon which, by excluding low valuation consumers, only exacerbates the producers' strategic uncertainty rendering own forecasts more dependent of others' forecasts. We show that this valuation disparity effect holds even in the non-linear case. In consequence, even if the new rationale for exogenous intervention identified by Guesnerie [10] was unnecessary at the autarky level, it becomes compelling after integration. Notice however that when a stable and a not 'too unstable' region integrate, the open economy equilibrium is likely to be

[1] As Evans and Honkapohja [7] (p.81) state that "*Learning is the adjustment mechanism whereby the economy is steered to the new equilibrium after a structural change*", the open economy device can also be interpreted as a structural change of the underlying autarkic economy.

[2] See Guesnerie [12] for an exposition of the eductive learning approach, applications to standard macroeconomic models and its relation to the adaptive approach.

[3] This basic framework encompasses the reduced form of standard macroeconomic models in their non-noisy versions, like the Lucas aggregate supply model or a simple version of the Cagan inflation model.

expectationally stable if consumers value the crop similarly, rendering pre-trade intervention (in the unstable region) unnecessary ex-post[4]. Our second result shows that this exogenous intervention is compelling for regions in the non-linear class which were autarkically expectationally stable, even if consumers value the crop similarly across regions so that no 'market disruption' phenomenon is at stake. This confirms the intuition that, after a structural change like economic integration, the higher the heterogeneity in producers' reactions the more difficult is to forecast accurately.

Our last result compares the expectational coordination criterion with a more traditional gains-from-trade criterion from an ex-ante viewpoint. Notice that the class of one-dimensional linear models describe a partial equilibrium framework where the open economy exercise considered always generates strictly positive welfare gains[5]. These gains are however larger, the higher the efficiency in producing the crop of the integrating partner. But as well, the higher the efficiency of the integrating partner, the higher the integration supply response to a given price change, and therefore, the lower the expectational stability of the global equilibrium price. The reason we adopt an ex-ante viewpoint (before effective integration takes place) is that the appropriate criterion would compute producers' welfare when the set of rationalizable expectations equilibria is not a singleton, which is beyond the scope of the present work[6].

The work proceeds as follows: In section 2, we describe the linear version of Guesnerie's [10] model, and his main results relevant to our work. The reader familiar with his work can start reading section 3, where we study the extension of the linear version of his model to open economy, allowing for differences in the maximal willigness to pay across regions. In section 4, we extend the results of section 3 to the non-linear class of integrating regions. In section 5, we compare the expectational coordination criterion to a more traditional one, which evaluates in welfare terms economic integration from an ex-ante viewpoint. Finally, in section 6 we conclude.

2 Preliminaries

If one is to recognize that economics is not a natural science because economic agents make forecasts that influence the time path of the system, it becomes crucial to understand how do economic agents form expectations. Faced with this problem, the modern macroeconomics literature has focused on how do

[4] There is then also a sense in which the eductive viewpoint offers new hope in overcoming some of the old arguments for coordination at the international level, as it provides conditions favouring expectational coordination in the absence of explicit coordinating institutions.

[5] From a classical normative point of view, the partial equilibrium framework is a particular case of a general equilibrium economy for which Dixit and Norman [5] showed the existence of ex-post transfers that leave everybody better off. However, the effective implementation of these transfers, from an eductive viewpoint, remains an open question because it is likely to modify the strategic behaviour of producers.

[6] Allen, Dutta and Polemarchakis (2002) address this problem in generic competitive exchange economies with countably many competitive equilibria.

economic agents 'learn'. A strand of the 'learning' literature views economic agents as statisticians who use sofisticated forecasting techniques to estimate the parameters of the law of motion governing the economic system, and on the same time taking into account that the use of these techniques shapes the motion itself. Stated otherwise, available information on the evolution of the economic system is at best incomplete even to the most sofisticated economic agent[7]. The question is then whether the estimated motion would (at least) asymptotically approximate the motion consistent with agents forming a rational expectation. This is called the 'adaptive approach to learning' (or evolutive learning) and has a long lasting tradition[8].

A different strand of the literature upon which we hinge here, is the 'eductive approach to learning'. This second approach admits that agents are rational and know the whole structure of the model describing the evolution of the system. Nevertheless, agents form expectations that need not coincide: Bernheim [2] and Pearce [18] show that rationality of the players and complete information of the game being played, even when they are 'common knowledge' (CK), do not imply the Nash equilibrium outcome but a different solution concept called a 'rationalizable equilibrium' ([9]). Guesnerie [10] applies the notion of rationalizability to a version of the standard Muthian model, to show that CK of rationality and of the model are not enough for them to always coordinate their expectations on the unique REE solution defined by Muth [17]. In this sense, since the definition of a REE requires expectational coordination[10], the eductive approach looks for structural conditions under which isolated independent agents' subjective expectations effectively coordinate in a REE.

In this section we present Guesnerie's [10] model, its linear version and his main results relevant to our work. The equilibrium concept will be a 'Rationalizable Expectations Equilibrium', as defined in Guesnerie's [10],[12] works. ([11])

[7] Manski [16] presents two serious reasons in support of the incomplete information workhorse assumption: empirical data captures the result of choices, and not the expectations of decision makers when confronted with choices. Second, one cannot expect to recover objective evidence on expectations because of the selection bias (logical unobservability of counterfactual outcomes). By these reasons, he supports data collection on expectations. Recent work by Evans and Honkapohja [7] along the lines of adaptive learning, solves the design of optimal monetary policies when observed data on private agents' expectations are incorporated in the policy maker's optimal monetary rule.

[8] Evans and Honkapohja [8] summarize this approach and its applications.

[9] Tan and Werlang [19] transform a non-cooperative game into a Bayesian decision problem where the uncertainty faced by a given agent is formed by the actions, priors over actions, priors over priors over actions, etc. of the other agents. They show that common knowledge of the actual strategies to be played is only necessary for players to play Nash strategies.

[10] Evans [6] asserts that a REE is in the class of Nash equilibria (in actions *and beliefs*).

[11] The contents of this section are from Guesnerie [10]. The reader familiar with its contents can skip this section.

2.1 The Model and the Equilibrium Concept

The model describes a two-period partial-competitive equilibrium of an agricultural commodity economy. A continuum of profit maximizing risk-neutral farmers $f \in [0,1]$ with a differentiable and strictly convex cost function $C(q,f)$ must decide the quantity q to be produced a period in advance on selling, given a predictable demand $D(p)$, assumed to be downward sloping $D'(p) < 0$ and resulting from the aggregation of a continuum of identical consumers indexed by c, $D(p) = \int D(p,c)dc$. The effective equilibrium price is unknown because it depends on what other farmers will decide to produce. Therefore, the supply of each producer will also depend on the probability distribution of the price, denoted $d\mu(p)$ ([12]). Since farmers are risk neutral, their production decisions will only depend on the expectation of the price $Ep = \int p d\mu(p)$:

$$S[p, d\mu(p), f] = (\partial_q C_f)^{-1} [p, d\mu(p)] \in \arg\max_q \int [pq - C(q,f)] \, d\mu(p)$$

Putting the Lebesgue measure on $[0,1]$, aggregate supply will be given by:

$$S[p, d\mu(p)] = \int S[p, d\mu(p), f] df$$

Under the above assumptions, the Rational Expectations Equilibrium (REE) price $\overline{p}$ of this model will be given by the equality of aggregate supply and aggregate demand in expectation, computed using $d\mu(\overline{p}, f) = d\mu(\overline{p}), \forall f$ (i.e. farmers form rational expectations):

$$\overline{p} = D^{-1}(S[\overline{p}, d\mu(\overline{p})])$$

Since there is no noise, the equilibrium $\overline{p}$ is a Perfect Foresight Equilibrium (PFE). Therefore, there exists a unique REE (PFE). Following Evans' [6] assertion according to which a REE is in the class of Nash equilibria in actions *and beliefs* (NE), $\overline{p}$ is also the unique NE[13].

Guesnerie [10], following Bernheim [2] and Pearce [18], builds upon the game-theoretic concept of 'rationalizability' to define the 'Rationalizable-Expectations Equilibria'. These are the limit of an iterative process which views the farmers' situation as a complete information normal-form game where the set of players is the set of farmers, and their strategies, the farmers' individual quantities of the

[12] Strictly speaking, the probability distribution should allow for subjective probabilities and therefore be written $d\mu(p,f)$. However, the only objective difference across farmers is the cost function which should not influence the individual expectation of the market price, i.e. a farmer with lower costs cannot be reasonably expected to have a more optimistic (or pessimistic) expectation on the prevailing market price.

[13] For an explicit formulation of this assertion in the class of models under consideration, see Desgranges and Gauthier [4].

crop $s_f \in \mathbf{S}_f, \forall f$ ([14]). Each farmer's payoff function is then his profit function:

$$\left\{D^{-1}\left(\int s_{f'} df'\right)\right\} s_f - C\left(s_f, f\right)$$

For each given profile of strategies of the other farmers $\left(s_{f'}\right)_{f' \in [0,1]}$, the best response of farmer f is the function that maximizes the above expression. The concept of a 'rationalizable solution' R exhausts the implications of individual rationality and common knowledge (CK) of rationality and of the model when considered as an iterative process taking place in 'mental time' τ (in each of the farmers' heads) following which non-best response strategies are progressively eliminated[15]. Where does this iterative process start? It starts at an initial restriction ($\tau = 0$) on the players' strategy sets called *anchorage assumption,* which is either naturally imbedded on the model at stark or exogenously given[16]. In either case, it is also CK. This iterative process of elimination of non-best responses will lead somewhere, defined by Pearce [18] and Bernheim [2] as a rationalizable solution R :

$$R = \left(s_{f'}\right)_{f'} \in \prod_{f'} \left(\cap_{\tau=0}^{\infty} \mathbf{S}(\tau, f')\right)$$

Whenever the sets of best response strategies $\mathbf{S}(\tau, f)$ shrink through 'mental time' τ to a singleton, farmers instantaneously coordinate on a unique (production) strategy. Because of the one-to-one correspondence between prices and quantities, that production decision will correspond to a price expectation. As market clearing is CK, that price expectation must clear the market, and therefore coincide with the *actual* equilibrium price. As that equilibrium price is the unique rationalizable solution, and because the Nash solution is always rationalizable, the equilibrium price must coincide with the Nash equilibrium of the normal-form game. However, when the sets of farmers' best responses do not collapse to a singleton, full coordination is not achieved. Although the Nash equilibrium will be included in, farmers equivalently consider each of the possible rationalizable strategies as an equilibrium production decision, corresponding each to an equilibrium price expectation[17].

[14] At this stage, it is important to understand that since the supply function is a one-to-one correspondence of the expected prevailing market price, as Guesnerie [10] points out (p.1258), the strategies *are also* the individual price expectations. For an exposition using price expectations, see Desgranges and Gauthier [4].

[15] Observe that a CK assumption is absolutely rational in a strategic context: when an individual recognizes that self-interest depends on others' actions, his conjectures on their likely behaviour are essential to the effective consecution of self intentions. The conjectures are the subjective expectations that each agent forms independently of others. But if one is to form conjectures about others' behaviour, it seems natural to recognize that others form conjectures as well in the same way as one does. Then the agent must conjecture about others' actions and conjectures. This process can go several steps further, triggered by the CK behavioural assumption.

[16] It is to be understood not as an exogenous intervention, but as a robustness test that any REE should pass for it to be 'implementable' through the iterative process of learning that is being described. See Guesnerie [12] for further details.

[17] It is important to stress that to compute the rationalizable equilibrium, the subjective

Guesnerie [10] otains structural conditions under which, *without assuming that farmers held rational expectations*, the Rationalizable Expectations Equilibrium of the farmers' normal-form game described above coincides with the REE (or NE). The unique Rationalizable Expectations Equilibrium is called by him a 'Strongly Rational Expectations Equilibrium' (SREE) or 'unique rationalizable'.

2.2 The Linear Specification

Consider the (non-noisy) linear version of the model presented above. The demand function for the crop is given by:

$$D(p) = \begin{cases} A - Bp \; if \; 0 \leq p \leq \frac{A}{B} \equiv p_0 \\ 0 \quad \text{otherwise} \end{cases}$$

and $C(q, f) = \frac{q^2}{2C_f}, f \in [0, 1]$ constitutes the farmers' cost function. Under this linear specification, the PFE price is given by[18]:

$$\overline{p} = \frac{A}{B + C} : C \equiv \int C_f df$$

The game that farmers play has a set of rationalizable strategies given by the limit of the iterative process of elimination of non-best responses from the strategy sets of farmers that we describe. The iteration is triggered by the CK of individual rationality and of the model, since the anchorage assumption is imbedded in the structure of the model: at virtual time $\tau = 0$ each farmer f recognizes that equilibrium prices cannot be negative nor larger than $p_0 \equiv \frac{A}{B}$ since $D(p_0) = 0$. Therefore each farmer deletes from his strategy set any quantity of the crop $s_f \geq S(p_0, f)$ defining the set $\mathbf{S}(0, f) = [0, S(p_0, f)], \forall f$. At $\tau = 1$ since each farmer knows that other farmers are rational as well, each farmer knows that other farmers $\forall f' \neq f$ will play strategies in their sets $\mathbf{S}(0, f')$. Therefore, total supply cannot be greater than $S(p_0) = \int S(p_0, f')df'$, which from the market clearing equation being common knowledge, each farmer deduces that the equilibrium price cannot be smaller than $p_1 = D^{-1}[S(p_0)]$ and proceeds to delete from his strategy set $\mathbf{S}(0, f)$ all these quantities that are smaller than $s_f \leq S(p_1, f)$. This defines the new set of strategies $\mathbf{S}(1, f) = [S(p_1, f), S(p_0, f)]$ for every farmer f. Now at $\tau = 2$ each farmer recognizes that the other farmers $\forall f' \neq f$ know what he knows, and therefore play also strategies in the set

price probability distribution and the cost function of every agent as well as market clearing are CK in the model considered. The work by Desgranges and Gauthier [4] makes clear the distinction between strategic uncertainty and model uncertainty in the linear one-dimensional version of Guesnerie [10] presented here: they show that whenever the CK assumption on farmers' subjective probability beliefs is violated, the success of the iterative process is compromised. Intuitively, when the subjective probability beliefs are not CK, farmers play an incomplete information game.

[18] It can be checked that with the encompassing definition of the demand function $D(p) = \max\{A - Bp, 0\}$, with $p_0 \equiv \min D^{-1}(0) = \frac{A}{B}$, the PFE price equals p_0 when total supply is zero.

$\mathbf{S}(1, f')$... and so on. This process leads each farmer to individually reproduce mentally the following sequence of (expected) prices $(p_\tau)_{\tau=0}^{\infty}$:

$$\begin{aligned}
p_1 &= D^{-1}\left[S(p_0)\right] = \frac{A}{B} - \frac{C}{B}p_0 \\
p_2 &= \frac{A}{B} - \frac{C}{B}p_1 = \frac{A}{B}\left[1 + \left(-\frac{C}{B}\right)\right] + \left(-\frac{C}{B}\right)^2 p_0 \\
&\cdots \\
p_\tau &= \frac{A}{B} - \frac{C}{B}p_{\tau-1} = \frac{A}{B}\left[\sum_{m=0}^{m=\tau-1}\left(-\frac{C}{B}\right)^m\right] + \left(-\frac{C}{B}\right)^\tau p_0
\end{aligned}$$

If this sequence has a limit, from the rationalizable solution concept, it must be the Nash equilibrium of the game $\overline{p}$. We reproduce Guesnerie's [10] proposition 1, which establishes conditions under which famers are able to coordinate on the PFE price $\overline{p}$. Under those conditions the equilibrium is a SREE:

Proposition 1 *(Guesnerie, [10]) (i) $B > C \iff \overline{p}$ is an SREE. (ii) $B \leq C \iff \overline{p}$ is not an SREE, and the set of rationalizable-expectations price equilibria comprises the segment* $[0, p_0]$

The conclusion of proposition 1 can be read as 'a low elasticity of aggregate supply (small C) and a high elasticity of demand (large B) favour expectational coordination from an eductive viewpoint'. Intuitively, it can be read also as 'producers' forecasts are more reliable the lower the sensibility of their decisions to others' forecasts'. Then under condition (i), the set of farmers' rationalizable strategies that are a rationalizable solution R of the farmers' game is:

$$R = (s_{f'})_{f'} \in \prod\nolimits_{f'} \left(\cap_{\tau=0}^{\infty}\mathbf{S}(\tau, f')\right) = \prod\nolimits_{f'} \mathbf{S}(\infty, f') = (S(\overline{p}, f'))_{f'}$$

If however condition (ii) is satisfied, then the price sequence $(p_\tau)_{\tau=0}^{\infty}$ does not have a limit and the set of farmers' rationalizable strategies that are a rationalizable solution R of the farmers' game is:

$$R = (s_{f'})_{f'} \in \prod\nolimits_{f'} \left(\cap_{\tau=0}^{\infty}\mathbf{S}(\tau, f')\right) = \prod\nolimits_{f'} \mathbf{S}(0, f') = \times_{f'} \left[0, S(p_0, f')\right]$$

In situations like (ii), Guesnerie [10] identifies the minimal set of conditions sufficient to achieve full coordination, calling them 'credible price restrictions' or 'exogenous price interventions', implemented by an exogenous third party.

In this particular example, the model definition imbeds the initial anchorage assumption. Furthermore, it is not 'close' to the equilibrium outcome. Then, under the (i) condition, the equilibrium price is 'Globally SR'. In general, when no such natural imbedding exists, the anchorage assumption is exogenously specified. When the model considered is non-linear, the anchorage assumption is settled 'close' to the equilibrium under scrutiny and the analysis is local (because there might exist multiple equilibria, which we assume locally determinate). Then, when the iterative process converges, the equilibrium is called

'Locally SR' or 'SR with respect to the CK anchorage assumption'. When the iterative process does not converge, the 'credible price restrictions' or 'exogenous price interventions' qualify the above definitions to be 'SR with respect to these restrictions'. For non-linear versions of the economy under study, the iterative process describing farmers' eductive learning can be characterized by the second iterate of the coweb function $\varphi(.) \equiv D^{-1}[S(.)]$, $\varphi^2(.) \equiv \varphi[\varphi(.)]$, conditional to the CK initial restriction[19], denoted $V(\overline{p})$:

Proposition 2 *(Guesnerie [10]):*
(i) If $|\varphi'(p)| < 1 \Leftrightarrow S'(p) < |D'[S(p)]|, \forall p$ and if there is a credible price restriction (floor or ceiling), then $\overline{p}$ is a SREE subject to the given price restriction.
(ii) If $|\varphi'(\overline{p})| < 1$, there is a credible price restriction (floor or ceiling) s.t. $\overline{p}$ is a SREE subject to the given price restriction.
(iii) If $|\varphi'(\overline{p})| > 1$, and if the graph of $\varphi^2(.)$ intersects transversely the 45-degree line more than once, then there is a credible price restriction (floor or ceiling) s.t.$[p_{c1}, p_{c2}]$ is the set of rationalizable-expectations equilibrium prices subject to the given price restriction, where $p_{c2} = \varphi(p_{c1}), \varphi^2(p_{ct}) = p_{ct}, t = 1, 2$ define cycles of order two of the coweb function ([20]),([21]).

The results in section 3 will provide examples of each of these cases.

3 Integration of Linear Autarkies

Most of the international trade literature concerns comparative statics excercises on the effect of changes in the production structure (factor endowments or production techniques) on the equilibrium outcome operated via the mobility of commodities or factors. Corollaries to these excercises are the consequences on factors and commodities prices of the comparative statics excercise under the same or alternative restrictions. However, they all necessitate of at least two commodities for the exchange channel to operate. In the class of agricultural economies considered, there is only a single homogenoeus crop produced at different costs depending on farmers' technologies. From the expectational stability viewpoint, the open economy device introduces heterogeneity in the autarkic economy, which according to Guesnerie's [12] general intuition (GI2), should undermine its expectational stability. A related way to understand the excercise is to assume that non-increasing returns to scale producers play a large oligopoly game with strategic substituabilities, the equilibrium of which is globally perturbed by the integration policy. The question would then be whether

[19] Subject to the condition that $\lim_{\tau\to\infty} (\varphi^2)^\tau (p_0) = \lim_{\tau\to\infty} \varphi^{2\tau}(p_0) = \overline{p}$, $p_0 \in V(\overline{p})$

[20] For a proof of the general statement which includes cases (ii) of proposition 1 and this case (iii), see Bernheim (1984), proposition 5.2., part (a).

[21] This is trivially true if $[p_{c1}, p_{c2}] \subset V(\overline{p})$. If however $V(\overline{p}) \subseteq [p_{c1}, p_{c2}]$, the learning dynamics will also converge to the set $[p_{c1}, p_{c2}]$, but, as discussed by Guesnerie [12], the CK anchorage assumption must then be understood not as a 'hypothetical' restriction, but as resulting from a non-enforceable 'exogenous price intervention'.

the dominance solvability of the autarkic equilibrium is robust to integration-related heterogeneity[22].

Although the answer will be related to the factors favouring coordination upon the integrating regions autarkic equilibrium (propositions 1,2 above), the answer is not immediate. From the comparative statics excercises of partial equilibrium, we know that aggregating demand curves results in a more elastic demand curve, which according to proposition 1, favours expectational coordination. However, by the same reason, aggregation of supply curves is detrimental to eductive coordination. Therefore, economic integration by simultaneously aggregating demand and supply autarkic schedules, does not necessarily undermine the coordinational ability of farmers. Actually, *mere replication of the Home economy will not affect its degree of expectational stability.*

To see it, consider the linear class of agricultural economies indexed by $n \in \mathbf{N} = \{1...N\}$ characterized by a set of risk neutral farmers $f_n \in [0,1]$ living in region n with strictly convex cost structures $C(s_{f_n}, f_n, n) = \frac{(s_{f_n})^2}{2C_{f_n}(n)}$ facing a (weakly) decreasing demand function $D_n(p) \equiv \int D_n(p, c_n)dc_n = \max\{A_n - B_n p, 0\}$ arising from a continuum of individual consumers living in that region[23]. Suppose that the N economies in the linear class are identical and decide to integrate (fix $n = n_0, \forall n$ and call economy n_0 the Home economy). The aggregate supply of such a global agricultural economy will be given by the sum of the aggregate supply functions of each of the N regions, $S(p) = \sum_{n=1}^{N} S_n(p) = N \int C_{f_{n_0}}(n_0)pdf_{n_0} = NS_{n_0}(p)$. So will the aggregate demand: $D(p) = \sum_{n=1}^{N} D_n(p) = ND_{n_0}(p)$. Substituting these definitions in proposition 1 above, we can immediately observe that the PFE-price is given by:

$$\overline{p} = \overline{p}_{n_0} = \frac{A_{n_0}}{B_{n_0} + C_{n_0}}$$

Then, the conditions under which farmers will be able to individually predict the PFE-price $\overline{p}$ coincide with those of proposition 1:

Proposition 3 *(i) $B_{n_0} > C_{n_0} \Longleftrightarrow \overline{p}$ is an SREE. (ii) $B_{n_0} \leq C_{n_0} \Longleftrightarrow \overline{p}$ is not an SREE, and the set of rationalizable-expectations price equilibria comprises the segment $[0, p_0]$.*

A perhaps more interesting result is that this proposition extends to the integration of N identical non-linear agricultural economies[24]. However, when considered in isolation, the effect of increasing the number of farmers facing a given aggregate demand curve is detrimental to the eductive stability of the equilibrium[25]. Consider our Home economy $n = n_0$. Suppose that in addition to the Home farmers, the farmers of the rest of the regions $\mathbf{N}\setminus\{n_0\}$ can also

[22] See Vives (1999) ch.4.4. for a synthetic dicussion of large Cournot markets.

[23] Throughout we assume that $A_n, B_n > 0$, $\forall n \in \mathbf{N}$. Notice that $p_0 \equiv \min D_n^{-1}(0) = \frac{A_n}{B_n}$.

[24] See the next section.

[25] As Vives [20] discusses for large Cournot games, the effect parallels adverse impact on dominance solvability of the equilibrium from increasing the number of producers without replicating the demand.

sell in the Home crop market. Denote by $C_\Sigma = C_{n_0} + \sum_{n \neq n_0} C_n$ the aggregate cost parameter characterizing the total supply of the crop. The PFE price is $\overline{p} = \frac{A_{n_0}}{B_{n_0}+C_\Sigma}$, which when:

Proposition 4 *(i) $B_{n_0} > C_\Sigma \Longleftrightarrow \overline{p}$ is an SREE. (ii) $B_{n_0} \leq C_\Sigma \Longleftrightarrow \overline{p}$ is not an SREE, and the set of rationalizable-expectations price equilibria comprises the segment $[0, p_0]$. (iii) Increasing the number of farmers is detrimental to expectational stability.*

Proof. Compute the limit $\lim_{\tau \to +\infty} p_\tau$ of the price sequence:

$$p_\tau = \frac{A_{n_0}}{B_{n_0}} \left[\frac{1 - \left(-\frac{C_\Sigma}{B_{n_0}}\right)^\tau}{1 - \left(-\frac{C_\Sigma}{B_{n_0}}\right)} \right] + \left(-\frac{C_\Sigma}{B_{n_0}}\right)^\tau p_0$$

Part (iii) follows trivially from the definition of C_Σ, (i) and noting that replicating the supply side of the Home economy makes $C_\Sigma = NC_{n_0}$. ■

Part (iii) states that the set of rationalizable solutions of the Home economy R_{n_0} will strictly include the set of rationalizable solutions of the global agricultural economy R of proposition 3: $R_{n_0} \supset R$. As the aggregation of supply curves increases the elasticity of the resulting aggregate supply schedule, each farmer's quantity choice becomes more sensible to other farmers' choices, rendering their predictions of the market clearing price less accurate. Therefore, *opening the Home market to Foreign competitors is destabilizing*, in the precise sense of producers' undermined ability to forecast the market clearing price[26].

Replication of the Home demand without replicating the supply side shows the beneficial role of the demand elasticity on expectational stability of the resulting PFE price, given now by $\overline{p} = \frac{NA_{n_0}}{NB_{n_0}+C_{n_0}}$. Then when:

Proposition 5 *(Guesnerie, [10]) (i) $NB_{n_0} > C_{n_0} \Longleftrightarrow \overline{p}$ is an SREE. (ii) $NB_{n_0} \leq C_{n_0} \Longleftrightarrow \overline{p}$ is not an SREE, and the set of rationalizable-expectations price equilibria comprises the segment $[0, p_0]$. (iii) Increasing the number of consumers favours stability.*

Proof. For parts (i),(ii) compute the limit $\lim_{\tau \to +\infty} p_\tau$ of the price sequence in the previous proposition after replacing $\left(-\frac{C_\Sigma}{B_{n_0}}\right)$ by $\left(-\frac{C_{n_0}}{NB_{n_0}}\right)$. Part (iii) follows from (i) and $NB_{n_0} > B_{n_0}$. ■

Intuitively, part (iii) states that as the number of consumers increases, the demand becomes more sensible to price changes, rendering the price equilibrium becomes more responsive to demand factors than to supply factors. It becomes then less sensible to farmers' production decisions, limiting the adverse effect of

[26] However, this proposition does not generalize to general non-linear schedules. See the next section for an example where increasing the number of producers stabilizes expectations.

the strategic component in producers' forecasts. Therefore, *opening new markets for the Home producers is stabilizing*, in the precise sense that producers' expectations become more reliable[27].

Notice that this proposition is not the exact analogue of proposition 4. However, defining $D(p) = \max\left\{\sum_n (A_n - B_n p), 0\right\} \equiv \max\left\{A_\Sigma - B_\Sigma p, 0\right\}$ *and imposing the additional condition* $\frac{A_n}{B_n} = \frac{A_{n'}}{B_{n'}}, \forall n, n' \in \mathbf{N}$ the exact analogue obtains. This additional condition imposes the equality of the maximal willignesses to pay for the crop across regions, i.e. some 'homogeneization' of consumers' valuation of the produced commodity. Its role on the expectational stability of the equilibrium price is the subject of the next two subsections.

3.1 From Global to Local Stability Conditions

In the class of linear economies considered, the anchorage assumption is imbedded in the model and it is unnecessary to specify it exogenously. Furthermore, the autarkic expectational stability of the PFE price is 'global' in the sense that the anchorage assumption is not 'close' to the equilibrium. The same is true for the PFE price of the integrated economy, provided that the consumers of different regions value the crop 'similarly', i.e. provided that consumers' maximal willigness to pay is identical across regions: $\frac{A_n}{B_n} = \frac{A_{n'}}{B_{n'}}, \forall n, n' \in \mathbf{N}$.

With the same notation as previously, we define the regional integration demand and supply by $D(p) = \max\left\{A_\Sigma - B_\Sigma p, 0\right\}$, $S(p) = C_\Sigma p$. The PFE price of the regional integration of N economies in the linear class is:

$$D(\overline{p}) = S(\overline{p}) \Longleftrightarrow \overline{p} = \frac{A_\Sigma}{B_\Sigma + C_\Sigma}$$

and will be expectationally stable when:

Proposition 6 *Suppose that* $\frac{A_n}{B_n} = \frac{A_{n'}}{B_{n'}}, \forall n, n' \in \mathbf{N}$. *Then: (i)* $B_\Sigma > C_\Sigma \Longleftrightarrow \overline{p}$ *is an SREE. (ii)* $B_\Sigma \leq C_\Sigma \Longleftrightarrow \overline{p}$ *is not an SREE, and the set of rationalizable-expectations price equilibria comprises the segment* $[0, p_0] : p_0 = \frac{A_\Sigma}{B_\Sigma}$. *(iii) The regional integration of* N *autarkically expectationally stable economies is expectationally stable, but the converse is false.*

Proof. Under the condition $\frac{A_n}{B_n} = \frac{A_{n'}}{B_{n'}} = \frac{A}{B}, \forall n, n' \in \mathbf{N}$ the anchorage assumption is given by $p_0 = \frac{A_\Sigma}{B_\Sigma} = \frac{A}{B}$. For parts (i),(ii) compute the limit $\lim\limits_{\tau \to +\infty} p_\tau$ of the price sequence in proposition 3 after replacing $\left(-\frac{C_\Sigma}{B_{n_0}}\right)$ by $\left(-\frac{C_\Sigma}{B_\Sigma}\right)$, and $\frac{A_{n_0}}{B_{n_0}}$ by $\frac{A_\Sigma}{B_\Sigma}$. To prove part (iii) notice that $D(p) = \sum_n D_n(p)$ implies that $D'(p) = \sum_n D'_n(p) \leq 0$ by $D'_n(p) \leq 0, \forall n$. Also, $S(p) = \sum_n S_n(p)$ implies that $S'(p) = \sum_n S'_n(p) > 0$ by $S'_n(p) > 0, \forall n$. The linearity of the regional demand and supply schedules implies that: $D'_n(p_1) = D'_n(p_2)$, $S'_n(p_1) = S'_n(p_2), \forall n$ and $\forall p_1, p_2 \in [0, p_0)$, $\forall p_1, p_2 \in [p_0, +\infty),$. From part (i) in proposition 2,

[27] This proposition does neither generalize to non-linear shedules. In the next section we give an example where increasing the number of consumers destabilizes expectations.

$|\varphi'(p)| = \left|\frac{S'(p)}{D'(p)}\right| < 1, \forall p$. Expanding the sums and using the linearity, we can rewrite it as $|\varphi'(p)| = \left|\sum_n \frac{D'_n(p)}{D'(p)} \frac{S'_n(p)}{D'_n(p)}\right| = |\sum_n \alpha_n \varphi'_n(p)| < 1, \forall p$. Since $\forall n, \alpha_n \geq 0$, $\sum_n \alpha_n = 1$, the regional integration expectational stability condition is a convex combination of the autarkic expectational stability conditions. Therefore:

$$\min_n |\varphi'_n(p)| \leq |\varphi'(p)| = \left|\sum_n \alpha_n \varphi'_n(p)\right| \leq \max_n |\varphi'_n(p)|$$

implies that if the autarkically most unstable region is expectationally stable, so must the regional integration be:

$$\max_n |\varphi'_n(p)| < 1 \Longrightarrow |\varphi'(p)| < 1$$

That the converse is not true follows trivially from the convex combination set of inequalities above. ■

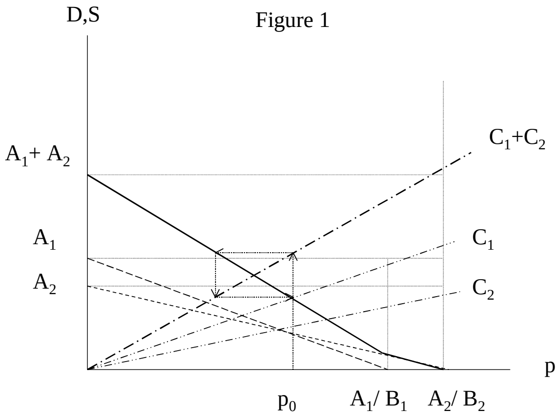
Intuitively, part (iii) states that it is not regional integration what undermines expectational coordination *per se,* but the integration with expectationally unstable regions. And even then, if the set of stable economies is sufficiently stable, *economic integration can favour expectational coordination.* This is a surprising conclusion in light of Guesnerie's [12] general intuition (GI2), following which heterogeneity is detrimental to expectational coordination[28].

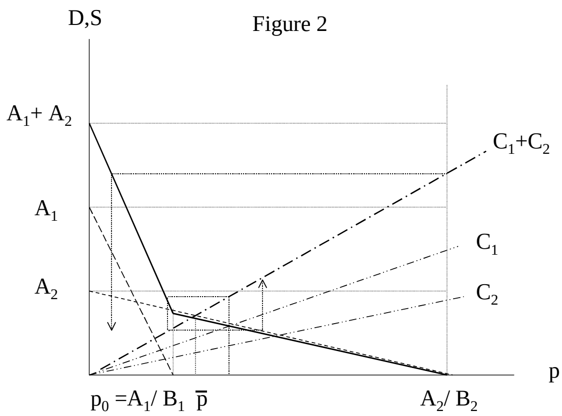
Actually, if we remove the condition imposing equal maximal willignesses to pay for the crop across regions, the regional integration demand becomes non-linear (piece-wise linear) and the results of propositon 1 above do not apply anymore. We have to resort to a local analysis of expectational stability, but as in this model the anchorage assumption is naturally imbedded in the definition and not necessarily 'close' to the PFE, we are in the class of situations described by Guesnerie [12], case I.2.(i).

To exemplify it, suppose that we applied the 'expectational stability test' of the above proposition ($\frac{C_\Sigma}{B_\Sigma} < 1$) to the regional integration of two economies $n = \{1, 2\}$ in the linear class $\mathbf{N}$, such that $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$. Two kinds of misleading

[28]Nevertheless, in the next section we will qualify this conclusion.

conclusions are likely to emerge, respectively depicted in figures 1 and 2 below:





In figure 1 the (global) 'expectational stability test' fails and nevertheless the PFE price is (locally) expectationally stable. Notice that in addition, the two regions are autarkically expectationally stable. In figure 2, the (global) 'expectational stability test' is passed, although the PFE price is (locally) expectationally unstable. What is even more striking is that both regions are expectationally stable in autarky[29]. In the next subsection we fully develop a two-region example and extend proposition 6 to the case where condition $\frac{A_n}{B_n} = \frac{A_{n'}}{B_{n'}}, \forall n, n' \in \mathbf{N}$ does not hold.

3.2 Structural Heterogeneity

In this subsection we consider the question of the eductive stability of the perfect foresight price when differences in the maximal willignesses to pay across regions in the linear class are allowed for[30]. These differences render piece-wise linear the coweb characterization of the eductive learning process (with respect to the autarkic coweb function, which is linear) with two main consequences: First, from the comparison of Guesnerie's [10] propositions 1 and 2, the necessity of an 'exogenous price intervention' is more stringent if expectational coordination is to be maintained at the global level. This is reminiscent of the traditional need to coordinate regional social planners at the open economy level to fullfill pre-trade national goals, and it can be then understood as a new 'rationale' justifying an exogenous intervention after integration[31]. This is because regional integration renders non-linear the pre-trade linear coweb characterization of the learning dynamics. Second, and in consequence, the study of its convergence

[29] But also more intuitive, in the sense that heterogeneity is detrimental to expectational coordination.

[30] We will assume throughout that the region with a relatively more elastic demand will have the lower maximal willigness to pay for the crop. This assumption can be dispensed with and the conclusions still hold.

[31] The precise instruments, or the study of their effective implementation, are left for a future work.

must be local, in the sense that the CK anchorage assumption must be settled 'close' to the PFE price. The problem is that the definition of the regionally integrated model already imbeds an anchorage assumption which is not 'close', leading in some cases to the type of inconsistencies aduced to in Guesnerie's [12] case I.2.(i). To give a precise content to these statements, we present a simple two-region integration exercise. Then we extend proposition 6 when differences in the maximal willignesses to pay exist.

3.2.1 A Robust Example

Consider the regional integration of two economies $n = \{1, 2\}$ in the linear class $\mathbf{N}$, such that $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$. Accordingly, and from the definition of regional demands, $p_0^n \equiv \min D_n^{-1}(0) = \frac{A_n}{B_n}, n = 1, 2$. Keeping the same notation, after integration farmers' demand will be $D(p) = \sum_n D_n(p)\mathbf{1}_{\{p \leq p_0^n\}}$, where $\mathbf{1}_{\{p \leq p_0^n\}}$ denotes the standard indicator function, taking value 1 only if the n-region consumers can afford to buy the crop at price p, and zero otherwise[32]. Then, the PFE price $\overline{p}$ will be given by[33]:

$$\overline{p} = \max \left\{ \frac{A_\Sigma}{B_\Sigma + C_\Sigma}, \frac{\inf\limits_n A_n}{\inf\limits_n B_n + C_\Sigma} \right\}$$

The PFE price is represented in figure 3 below as a function of the aggregate supply cost parameter C_Σ, $\overline{p}(C_\Sigma)$. We have parameterized the difference in the maximal willignesses to pay by $\overline{C}_\Sigma = A_2 \left[\frac{B_1}{A_1} - \frac{B_2}{A_2} \right]$. We can see that the PFE price changes for values of the aggregate supply cost parameter above and below $\overline{C}_\Sigma$. Values of C_Σ above $\overline{C}_\Sigma$ indicate that both regional markets will be served after integration, whereas values below indicate that only the highest valuation region will be served ($n = 2$, given our assumptions). The case where $\overline{C}_\Sigma = 0$ corresponds to the equality of maximal willignesses to pay of proposition 6 -only

[32] In this particular example, we can alternatively characterize the demand function as $D(p) = \max \left\{ A_\Sigma - B_\Sigma p, \inf\limits_n A_n - \left(\inf\limits_n B_n \right) p, 0 \right\}$.

[33] Recall that we assumed throughout $A_1 \geq ... \geq A_N > 0$ and $B_1 \geq ... \geq B_N > 0$. Then when $n = \{1, 2\}$, $\inf\limits_n A_n = A_2$ and $\inf\limits_n B_n = B_2$.

values above $\overline{C}_\Sigma$ are allowed-.

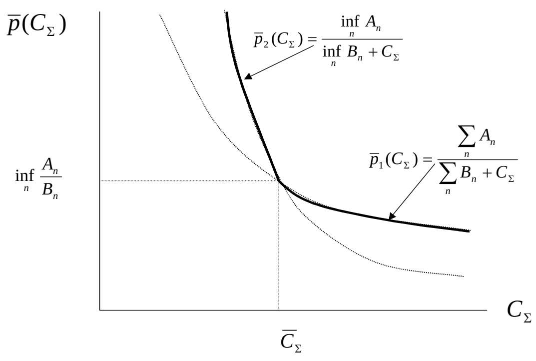


Figure 3

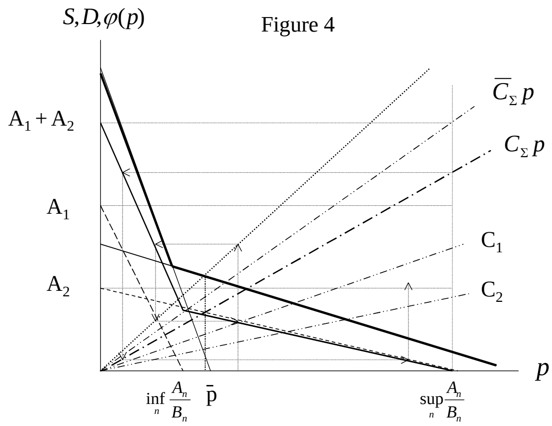


Figure 4

The learning dynamics of the regional integration PFE price are characterized by the coweb function $\varphi(p) \equiv D^{-1}[S(p)]$, adopting the following analytic form:

$$\begin{aligned} \varphi(p; C_\Sigma) &= \max\left\{\varphi_1(p; C_\Sigma \geq \overline{C}_\Sigma), \varphi_2(p; C_\Sigma \leq \overline{C}_\Sigma)\right\} \\ &= \begin{cases} \varphi_2(p; C_\Sigma \leq \overline{C}_\Sigma) & \text{if } p \leq p^i \\ \varphi_1(p; C_\Sigma \geq \overline{C}_\Sigma) & \text{if } p \geq p^i \end{cases} \end{aligned}$$

Where $\varphi_1(p; C_\Sigma \geq \overline{C}_\Sigma) = \frac{A_\Sigma}{B_\Sigma} - \frac{C_\Sigma}{B_\Sigma} p$ coincides with the linear coweb function of characterizing the learning dynamics when the condition $\frac{A_1}{B_1} = \frac{A_2}{B_2} \Longleftrightarrow \overline{C}_\Sigma = 0$ is satisfied, while $\varphi_2(p; C_\Sigma \leq \overline{C}_\Sigma) = \frac{\inf_n A_n}{\inf_n B_n} - \left(\frac{C_\Sigma}{\inf_n B_n}\right) p$ corresponds to the case in which $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$ *and* $C_\Sigma \leq \overline{C}_\Sigma$. Therefore, when $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$ *but* $C_\Sigma \geq \overline{C}_\Sigma$, the conclusions of proposition 6 apply even with different maximal valuations across regions. p^i is the price at which both functions $\varphi_1(.), \varphi_2(.)$ intersect ([34]). In figure 4 above, we have depicted the coweb function $\varphi(.)$ when $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$ *and* $C_\Sigma \leq \overline{C}_\Sigma$: then only region 2 consumers will be able to afford the consumption of the crop at the prevailing PFE price $\overline{p}$. Also notice that the conclusions of proposition 6 do not hold: the global 'expectational stability test' is satisfied, but the PFE price is (locally) expectationally unstable[35]. When the economy under study is non-linear, proposition 2 above provides conclusions on the basis of the second iterate of the coweb function $\varphi^2(.)$, given by[36]:

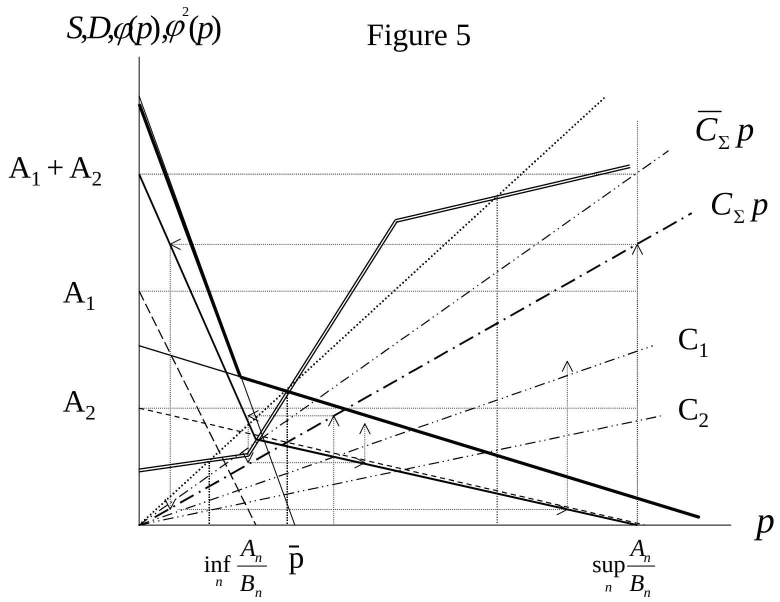
$$\varphi^2(p) = \begin{cases} (\varphi_1 \circ \varphi_2)(p) \ \text{ if } \ p \leq p^i_{\inf} \\ (\varphi_1 \circ \varphi_1)(p) \mathbf{1}_{\{p^i_{\inf} = p^i\}} + (\varphi_2 \circ \varphi_2)(p) \mathbf{1}_{\{p^i_{\inf} = p^{i\prime}\}} \ \text{ if } \ p \in \left(p^i_{\inf}, p^i_{\sup}\right) \\ (\varphi_2 \circ \varphi_1)(p) \ \text{ if } \ p \geq p^i_{\sup} \end{cases}$$

[34] For the derivation of the cowb function and the expression of the intersection price, see appendix 1.

[35] In appendix 1 it is shown that $\varphi'(.) \leq 0$ and that $\varphi(\overline{p}) = \overline{p}$. These are general properties of the coweb function in the class of economies under study.

[36] See appendix 2 for the derivation, and appendix 3 for its properties.

Where $p^{i\prime}$ denotes a second intersecting price[37] satisfying $\varphi_1\left[\varphi(p^{i\prime})\right] = \varphi_2\left[\varphi(p^{i\prime})\right]$. We define[38] $p^i_{\inf} = \max\left\{\min\left\{p^{i\prime}, p^i\right\}, p'_1\right\}$ and $p^i_{\sup} = \min\left\{\max\left\{p^{i\prime}, p^i\right\}, p_\infty\right\}$. Finally $\mathbf{1}_{\left\{p^i_{\inf}=p^i\right\}}$ takes value 1 if $p^i_{\inf} = p^i$ and 0 otherwise. To gain some intuition on its shape, figure 5 depicts the second iterate of the coweb function corresponding to the parameterization of figure 4:



Observe that the second iterate of the coweb function is piecewise linear, monotonically increasing, and with two nondiferentiability points corresponding respectively to the second and first intersecting prices $p^{i\prime}, p^i$. As well, it satisfies $\varphi^2(\overline{p}) = \varphi\left[\varphi(\overline{p})\right] = \varphi(\overline{p}) = \overline{p}$. Figure 5 illustrates the two main consequences aduced to: First, both regions were expectationally stable before integration (check figure 4). After integration, the resulting PFE price is 'expectationally unstable'. This is in line with the intuition that *heterogeneity is detrimental to expectational coordination*, qualifying the conclusions of proposition 6 above. Second, the imbedded anchorage assumption $p_0 = \frac{A_2}{B_2}$ is not 'local'. Conditional on that 'initial price restriction', the learning process converges but not to the PFE $\overline{p}$. It converges to the set $[p_{c1}, p_{c2}]$ of rationalizable-expectations equilibria, containing $\overline{p}$. If the 'local' approach would be adopted, the initial price restriction p_0 would have rather been settled in a neighbourhood of the PFE, $N_\epsilon(\overline{p}) = (\overline{p} - \epsilon, \overline{p} + \epsilon)$. Then the learning dynamics depicted in figure 5 would diverge, but not forever: the process stops at $[p_{c1}, p_{c2}]$. This provides an illustration of Guesnerie [12], case I.2.(i): whenever the type of inconsistency

[37] See appendix 2 for its derivation, explicit formulation and properties 1-5.

[38] See observation 2 of appendix 2 for the definitions of p'_1, p_∞ the interest of which is merely technical.

aduced to happens, pick p_0 outside the set of rationalizable prices $[p_{c1}, p_{c2}]$, but 'close' to it.

Then, the most salient result is:

Proposition 7 *Set* $\mathbf{N} = \{1,2\}$. *If* $C_\Sigma \geq \overline{C}_\Sigma$ *the results of proposition 6 extend to the case where* $\exists n, n' \in \mathbf{N} : \frac{A_n}{B_n} \neq \frac{A_{n'}}{B_{n'}}$. *If however* $C_\Sigma < \overline{C}_\Sigma$ *then even if both economies were autarkically expectationally stable, the global equilibrium price can end up being unstable.*

Proof. *(See the results in Table A4.1 in appendix 4 and the corresponding proofs)* ■

Intuitively, a large disparity in consumers' regional valuations renders farmers' forecasts increasingly unreliable because it renders a 'market disruption' phenomenon more likely: If as a result of regional integration the PFE price is 'too high', the consumers from the low-valuation region will be excluded ('market disruption') with the adverse net effect of a pure increase in the number of farmers' competitors, studied in proposition 4.

Assuming that both countries have identical cost structures, and that there is no regional disparity on consumers' valuation, the PFE price after integration will lie somewhere above the autarky PFE price of the low demand elasticity region, and below the high demand elasticity one. Then, producers in the low demand elasticity region expect profits to increase after integration, and conversely in the high demand elasticity one. If differences in the maximal willigness to pay exist, producers in the high demand elasticity region do not expect anymore the equilibrium price to necessarily decrease as a consequence of regional integration. This is because under our assumptions, the consumers in the low demand elasticity region can be willing to pay so much for the good that the PFE price prevailing after integration ends up above the autarky PFE price in the high demand elasticity region. Then producers in the high demand elasticity region do not necessarily expect anymore a reduction their profits after integration: they can either increase or decrease (and conversely in the low demand elasticity region). This additional uncertainty has an adverse impact on expectational coordination.

The next proposition generalizes this result to the regional integration of N economies in the linear class, such that $\exists n, n' \in \mathbf{N} : \frac{A_n}{B_n} \neq \frac{A_{n'}}{B_{n'}}$. From the discussion of the previous example, we adopt a 'local' approach of convergence of the learning dynamics. From proposition 2, the eductive stability condition depends on:

$$\begin{aligned}\varphi'(\overline{p}) &= \frac{\sum_n D_n'(\overline{p})}{\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p})}\left[\sum_n \frac{D_n'(\overline{p})}{\sum_n D_n'(\overline{p})}\varphi_n'(\overline{p})\right] \\ &= \frac{\sum_n D_n'(\overline{p})}{\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p})}\left[\sum_n \alpha_n^D \varphi_n'(\overline{p}_n)\right]\end{aligned}$$

The equality follows from linearity, $\varphi_n'(\overline{p}) = \varphi_n'(\overline{p}_n), \forall n$. The factor $\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p}) = \sum_n D_n'(\overline{p})\mathbf{1}_{\{\overline{p}\leq p_0^n\}}$ follows from the derivation of the definition of the regionally integrated demand function, summing the quantities demanded in each region at every possible value of the price, whenever the quantities are positive. Then:

Proposition 8 *For* $\mathbf{N} = \{1, ..., N\}$ *, if* $\exists n, n' \in \mathbf{N} : \frac{A_n}{B_n} \neq \frac{A_{n'}}{B_{n'}}$*, then the regional integration of autarkic expectationally stable economies can be expectationally unstable. It is more likely so, the larger the disparity in the willignesses to pay across regions.*

Proof. Since $\exists n, n' \in \mathbf{N} : \frac{A_n}{B_n} \neq \frac{A_{n'}}{B_{n'}}$, assume that there exists a region the consumers of which will not be able to afford the consumption of the crop at the prevailing PFE price $\overline{p}$, we have that:

$$\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p}) \geq \sum_n D_n'(\overline{p}) \Longrightarrow$$

$$\frac{\sum_n D_n'(\overline{p})}{\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p})} \left[\sum_n \alpha_n \varphi_n'(\overline{p}_n)\right] \leq \frac{\sum_n S_n'(\overline{p})}{\sum_n D_n'(\overline{p})} = \sum_n \alpha_n \varphi_n'(\overline{p}_n)$$

Taking absolute values on both sides:

$$|\varphi'(\overline{p})| \geq \left|\sum_n \alpha_n \varphi_n'(\overline{p}_n)\right|$$

So that when differences in the maximal willigness to pay for the crop exist (LHS), the PFE price is 'more unstable' than when they do not exist (RHS-proposition 6). But we can measure by how much, since:

$$|\varphi'(\overline{p})| < 1 \Longleftrightarrow \left|\sum_n \alpha_n \varphi_n'(\overline{p}_n)\right| < \frac{\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p})}{\sum_n D_n'(\overline{p})} \equiv \frac{1}{\varkappa_{\{n:\overline{p}\leq p_0^n\}}}$$

With $\varkappa_{\{n:\overline{p}\leq p_0^n\}} \geq 1$, taking value 1 when the integration equilibrium price $\overline{p}$ is low enough so that the consumers of all the integrating regions can afford to pay it (the situation in proposition 6): i.e. $\sum_{n:\overline{p}\leq p_0^n} D_n'(\overline{p}) = \sum_n D_n'(\overline{p})$. Then the conditions of proposition 6 are strengthened to:

$$\varkappa_{\{n:\overline{p}\leq p_0^n\}} \min_n |\varphi_n'(\overline{p}_n)| \leq |\varphi'(\overline{p})| \leq \varkappa_{\{n:\overline{p}\leq p_0^n\}} \max_n |\varphi_n'(\overline{p}_n)|$$

Meaning that even if all autarkic price equilibria are expectationally stable, so that $\max_n |\varphi_n'(\overline{p}_n)| < 1$, the PFE price $\overline{p}$ can fail to be so whenever:

$$\varkappa_{\{n:\overline{p}\leq p_0^n\}} > \frac{1}{\max_n |\varphi_n'(\overline{p}_n)|} \Longrightarrow |\varphi'(\overline{p})| > \max_n |\varphi_n'(\overline{p}_n)|$$

i.e. whenever there are sufficient economies the consumers of which cannot afford to pay the international price for the crop. The smaller the set of the economies in which consumers demand the crop at the international price $\{n : \overline{p} \leq p_0^n\}$, the smaller the elasticity of the integration aggregate demand, the larger the value of $\varkappa_{\{n:\overline{p}\leq p_0^n\}}$ above one, and the more likely it becomes the above inequality. ■

In the next section, we extend the conclusions obtained for the linear class of economies $\mathbf{N}$, to the non-linear class of economies $\mathbf{M}$.

4 Integration of Non-linear Autarkies

In this section, we explore the robustness of the conclusions of the previous section when the integrating regions are in the non-linear class $\mathbf{M} = \{1, ..., M\}$. The following assumption guarantees that the equilibria are unique (both autarkic and regionally integrated) and therefore (globally) determinate:

(A.1.) $\forall p \in [0, p_0^m)$, $D_m'(p) < 0, S_m'(p) > 0$; $p_0^m \equiv \min (D_m)^{-1}(0) > 0$, $S_m(0) = 0$, $D_m(.), S_m(.) \in C^1$, $\forall m \in \mathbf{M}$.

Notice that (A.1.) does not impose any condition on the second derivatives of the supply and demand schedules and that $S_m'(.) > 0$ implies that the underlying regional costs are convex. Under (A.1.), the unicity of the regionally integrated PFE price $\overline{p}$ obtains from the coweb function being decreasing and the boundary behaviour $\sum_m [D_m(0) - S_m(0)] > 0$, for a small $\varepsilon > 0$: $\sum_m \left[D_m\left(\underset{m}{\max} p_0^m - \varepsilon\right) - S_m\left(\underset{m}{\max} p_0^m - \varepsilon\right)\right] < 0$:

$$\varphi'(p) = \frac{\sum_m S_m'(p)}{\sum_{m:p\le p_0^m} D_m'(p)} < 0, \forall p \in [0, \underset{m}{\max} p_0^m - \varepsilon]$$

To avoid the type of inconsistencies discussed in the previous section, we adopt a 'local eductive viewpoint', choosing a CK initial price restriction 'close' to the PFE price (in a neighbourhood around it), $p_0 \in N_\epsilon(\overline{p}) = (\overline{p} - \epsilon, \overline{p} + \epsilon)$. Whenever the learning process converges to it, we will say that the equilibrium is (locally) strongly rational (LSR). Since $\overline{p}$ is locally determinate, applying the implicit function theorem to the market clearing equation $D(\overline{p}) = S(\overline{p})$, we obtain the following condition characterizing the learning dynamics:

$$\lim_{\tau\to\infty} (p_\tau - \overline{p}) = \left(\frac{S'(\overline{p})}{D'(\overline{p})}\right)^\tau (p_0 - \overline{p}) = 0 \Leftrightarrow |\varphi'(\overline{p})| = \left|\frac{S'(\overline{p})}{D'(\overline{p})}\right| < 1$$

According to the result (ii) of proposition 2 above. Since our purpose is to relate the conditions for the expectational stability of the regionally integrated equilibrium to the autarkic stability ones, we can expand it as:

$$\varphi'(\overline{p}) = \frac{\sum_m S_m'(\overline{p})}{\sum_{m:\overline{p}\le p_0^m} D_m'(\overline{p})} = \frac{\sum_m D_m'(\overline{p})}{\sum_{m:\overline{p}\le p_0^m} D_m'(\overline{p})} \sum_m \alpha_m \frac{D_m'(\overline{p}_m)}{D_m'(\overline{p})} \frac{S_m'(\overline{p})}{S_m'(\overline{p}_m)} \varphi_m'(\overline{p}_m)$$

Where the $\alpha_m \ge 0, \forall m : \sum_m \alpha_m = 1$ represent the relative (to the world) demand elasticities of each of the integrating economies evaluated at the PFE price $\overline{p}$, and $\overline{p}_m$ denotes the autarky PFE price of each region. The factor $\frac{\sum_m D_m'(\overline{p})}{\sum_{m:\overline{p}\le p_0^m} D_m'(\overline{p})} \equiv \varkappa_{\{m:\overline{p}\le p_0^m\}}$ has exactly the same interpretation as in the linear case: it accounts for differences in the maximal willignesses to pay across the integrating regions.

The first result extends proposition 3 to non-linear economies:

Proposition 9 *The expectational stability of the M-replica Home economy in the non-linear class* $\mathbf{M}$ *obtains under the same conditions it does in the Home non-linear economy.*

Proof. The regional integration equilibrium price $\overline{p}$ of M identical regions $m \in \mathbf{M}$, each with an identical autarkic equilibrium price $\overline{p}_m$, will satisfy:

$$\sum_m D_m(\overline{p}) = \sum_m S_m(\overline{p}) \Leftrightarrow MD_m(\overline{p}) = MS_m(\overline{p}) \Longrightarrow \overline{p} = \overline{p}_m$$

From:

$$\varphi'(\overline{p}) = \frac{S'(\overline{p})}{D'(\overline{p})} = \sum_m \frac{D'_m(\overline{p})}{D'(\overline{p})}\varphi'_m(\overline{p})$$

and because $D'(p) = \sum_{m:p \leq p_0^m} D'_m(p) = MD'_m(p)\mathbf{1}_{\{p \leq p_0^m\}}$ implies that $\frac{D'_m(\overline{p})}{D'(\overline{p})} = \frac{1}{M}$ since $\overline{p} \leq p_0^m$, we have $\varphi'(\overline{p}) = \sum_m \frac{1}{M}\varphi'_m(\overline{p}) = \varphi'_m(\overline{p})$. And by $\overline{p} = \overline{p}_m$, $\varphi'(\overline{p}) = \varphi'_m(\overline{p}_m)$. Therefore,

$$|\varphi'(\overline{p})| < 1 \Leftrightarrow |\varphi'_m(\overline{p}_m)| < 1 \Leftrightarrow S'_m(\overline{p}_m) < |D'_m(\overline{p}_m)|$$

and proposition 3 in the text is extended to the class of non-linear agricultural economies; so that, conditional to an initial price restriction $p_0 \in N_\epsilon(\overline{p})$, $\overline{p}$ is LSR if and only if $\overline{p}_m$ is also LSR. ∎

However, the conclusions of propositions 4 and 5 do not generally extend to the non-linear class. Figures 6 and 7 below illustrate, respectively, the reasons of such failures. In figure 6, as new producers enter the Home market, the equilibrium price decreases at a higher rate than entry does, because the elasticity of demand increases more than proportionately. Then, opening the Home market to Foreign producers may end up stabilizing expectations. The problem is connected to the convexity of demand, which in usual Cournot games, prevents the players' reaction functions from being downward sloping, or, players' strategies from being strategic substitutes.

Figure 6

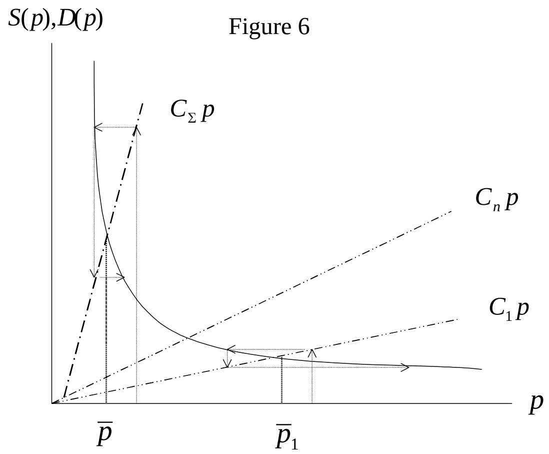
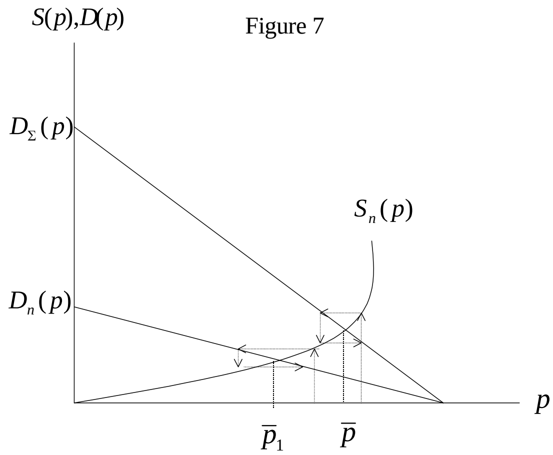


Figure 7



In figure 7, as the Home producers pregressively sell in Foreign markets, the effective demand they face increases and so will the equilibrium price. But if higher sales entail progressively lower marginal costs, the supply elasticity may considerably increase and the new price equilibrium becomes expectationally unstable. Then, new markets may actually destabilize expectations. The convexity of aggregate supply in picture 7 implies that marginal costs increase at a decreasing rate with units produced. This is rather counterintuitive in this model with no sunk costs. Rather, if we assume that marginal costs increase at an increasing rate (ex. because of the decreasing returns to some fixed factor in the short run) the supply locii will be concave and new markets stabilize producers' expectations.

Both propositions, 4 and 5, fail to generalize because the comparative statics excercise ultimately depends on the magnitude of the response of farmers' expectations to structural changes, like an increase in foreign competition, or having access to new markets. The magnitude is governed by the second derivative of the supply (and demand) schedule(s). Requiring (supply and) demand schedules to be concave and with equal marginal willignesses to pay allows a straightforward generalization to non-linear schedules of propositions 4,5.

Proposition 10 *Suppose that* $D''_m(.) < 0, S''_m(.) < 0, \forall m \in \mathbf{M}$. *Then: (i) Proposition 4 extends to the non-linear class, (ii) Proposition 5 extends to the non-linear class.*

Proof. Part (i): As only supply aggregates, $\alpha_m = \frac{D'_m(\overline{p})}{D'_m(\overline{p})} = 1$ and $(\overline{p}_m - \overline{p}) > 0$. Since aggregate demand is concave, this last fact implies that $\frac{D'_m(\overline{p}_m)}{D'_m(\overline{p})} > 1$. As well, the concavity of supply implies that $\frac{S'_m(\overline{p})}{S'_m(\overline{p}_m)} > 1$. Incorporating these observations in the above definition of the eductive stability condition of the integration equilibrium price, yields:

$$\varphi'(\overline{p}) = \sum_m \alpha_m \frac{D'_m(\overline{p}_m)}{D'_m(\overline{p})} \frac{S'_m(\overline{p})}{S'_m(\overline{p}_m)} \varphi'_m(\overline{p}_m) \leq \sum_m \varphi'_m(\overline{p}_m)$$

Because of the coweb functions being (weakly) decreasing and $\alpha_m \frac{D'_m(\overline{p}_m)}{D'_m(\overline{p})} \frac{S'_m(\overline{p})}{S'_m(\overline{p}_m)} > 1$. Taking absolute values on both sides of the inequality, and noticing that $\varphi'_m(.) < 0, \forall m$:

$$|\varphi'(\overline{p})| \geq \left| \sum_m \varphi'_m(\overline{p}_m) \right| = \sum_m |\varphi'_m(\overline{p}_m)| \geq |\varphi'_m(\overline{p}_m)|$$

So that the integration equilibrium price is more unstable than the original autarkic equilibrium.

Part (ii): As only demand aggregates, $(\overline{p}_m - \overline{p}) < 0, \forall m$. Since both aggregate demand and supply are concave, this implies that $\frac{D'_m(\overline{p}_m)}{D'_m(\overline{p})} < 1$ and $\frac{S'_m(\overline{p})}{S'_m(\overline{p}_m)} < 1$. Incorporating these observations in the above definition of the eductive stability condition of the integration equilibrium price, yields:

$$\varphi'(\overline{p}) = \alpha_m \frac{D'_m(\overline{p}_m)}{D'_m(\overline{p})} \frac{S'_m(\overline{p})}{S'_m(\overline{p}_m)} \varphi'_m(\overline{p}_m) \geq \varphi'_m(\overline{p}_m)$$

Because $\varphi_m'(.) < 0, \forall m$ and $\alpha_m \frac{D_m'(\overline{p}_m)}{D_m'(\overline{p})} \frac{S_m'(\overline{p})}{S_m'(\overline{p}_m)} < 1$, taking absolute values on both sides of the inequality:

$$|\varphi'(\overline{p})| \leq |\varphi_m'(\overline{p}_m)|$$

The inequality above states that the resulting integration equilibrium price is more stable than the original autarkic equilibrium. ■

Intuitively both propositions state that, *whenever both demand and supply schedules are (globally) concave, opening new markets favours expectational coordination whereas opening the Home market to Foreign competitors is detrimental.*

If we want to extend proposition 6 to the non-linear class of economies, notice that for non-linear economies, $\varphi_m'(\overline{p}) \neq \varphi_m'(\overline{p}_m)$. Rather,

$$\varphi_m'(\overline{p}) = \frac{D_m'(\overline{p}_m)}{D_m'(\overline{p})} \frac{S_m'(\overline{p})}{S_m'(\overline{p}_n)} \varphi_m'(\overline{p}_m)$$

Then, our main result states that:

Proposition 11 *If there are no differences in the maximal willignesses to pay across regions, then the regional integration of M autarkically expectationally stable economies* ***can result in an expectationally unstable PFE price.***

Proof. First, if there are no differences in the maximal willignesses to pay across regions, $\sum_{m:\overline{p}\leq p_0^m} D_m'(\overline{p}) = \sum_m D_m'(\overline{p})$ and $\varphi'(\overline{p}) = \sum_m \alpha_m \varphi_m'(\overline{p})$. Notice that we can expand $\varphi_m(.)$ as:

$$\varphi_m'(\overline{p}) = \varphi_m'(\overline{p}_m) + (\overline{p} - \overline{p}_m) \int_0^1 \varphi_m''[\overline{p}_m + \zeta(\overline{p} - \overline{p}_m)] \, d\zeta$$

Which plugged into $\varphi'(\overline{p})$ yields:

$$\varphi'(\overline{p}) = \sum_m \alpha_m \varphi_m'(\overline{p}_m) + \underbrace{\sum_m \alpha_m (\overline{p} - \overline{p}_m) \int_0^1 \varphi_m''[\overline{p}_m + \zeta(\overline{p} - \overline{p}_m)] \, d\zeta}_{\equiv R \gtreqless 0}$$

Then $\varphi'(\overline{p}) - R = \sum_m \alpha_m \varphi_m'(\overline{p}_m)$, which a convex combination of the autarkic stability conditions. Therefore, taking absolute values on both sides:

$$\min_m |\varphi_m'(\overline{p}_m)| \leq |\varphi'(\overline{p}) - R| \leq \max_m |\varphi_m'(\overline{p}_m)|$$

Using the property that $|\varphi'(\overline{p}) - R| \geq ||\varphi'(\overline{p})| - |R||$ and adding $+|R|$ to both sides of the second inequality in the above expression, we obtain:

$$|\varphi'(\overline{p})| = ||\varphi'(\overline{p})| - |R| + |R|| \leq ||\varphi'(\overline{p})| - |R|| + |R| \leq \max_m |\varphi_m'(\overline{p}_m)| + |R|$$

Reachinng the desired conclusion, for even if $\max_m |\varphi_m'(\overline{p}_m)| < 1$, so that all autarkic integrating economies are expectationally stable, the regional integration of them need not even without differences in the maximal willignesses to pay across regions. Finally, notice that for economies in the linear class $\mathbf{N}$ of proposition 6, $R = 0$ so that this proposition extends the results obtained there. But as well, notice that even in the non-linear case it can happen that $R = 0$, as it is the case when the integrating economies are identical (proposition 10 above). ■

This result is striking because it does not need any of the standard single-crossing assumptions which are typical of these comparative statics exercises. Intuitively, it states that *although regional integration stabilizes autarky prices across regions, it can destabilize producers' expectations*, rendering more compelling the necessity of an 'exogenous price intervention' than it was in the autarkic regime. Notice that the non-linear class of economies allows one to reconcile the results of proposition 6 with Guesnerie's [12] general intuition (GI2), following which, heterogeneity is detrimental to expectational coordination.

There remains to show that in the class of economies considered, regional integration actually 'stabilizes' autarky equilibrium prices across regions:

Lemma 12 $\overline{p} \in \left[\min_m \overline{p}_m, \max_m \overline{p}_m\right]$

Proof. By contradiction, suppose that $\overline{p} > \max_m \overline{p}_m$. Call $\overline{m} \in \mathbf{M}$ the region which autarkic equilibrium is $\max_m \overline{p}_m \equiv \overline{p}_{\overline{m}}$. By (A.1.) and $\overline{p} > \overline{p}_{\overline{m}}$, $S_{\overline{m}}(\overline{p}) > S_{\overline{m}}(\overline{p}_{\overline{m}}) = D_{\overline{m}}(\overline{p}_{\overline{m}}) > D_{\overline{m}}(\overline{p})$. As $\overline{p} > \overline{p}_{\overline{m}} \equiv \max_m \overline{p}_m$ we have that $\overline{p} > \overline{p}_m, \forall m \neq \overline{m}$ and by (A.1.), $S_m(\overline{p}) > S_m(\overline{p}_m) = D_m(\overline{p}_m) > D_m(\overline{p}), \forall m \neq \overline{m}$. Summing over all economies, $\sum_m S_m(\overline{p}) = S(\overline{p}) > D(\overline{p}) = \sum_m D_m(\overline{p})$, a contradiction. Assuming that that $\overline{p} < \min_m \overline{p}_m$ and denoting by $\underline{m} \in \mathbf{M}$ the region the autarkic equilibrium of which is $\min_m \overline{p}_m$, by reversing the inequalities in the preceding reasoning we similarly reach a contradiction. ■

For the sake of completeness, we let the reader remark that when differences in the maximal willignesses to pay are allowed across economies in the non-linear class $\mathbf{M}$, the PFE price of the regional integration is more difficult to learn than when they are absent. The proof follows the steps of proposition 9 and is immediate once we notice that:

$$\varphi'(\overline{p}) = \varkappa_{\left\{m:\overline{p} \leq p_0^m\right\}} \sum_m \alpha_m \varphi'_m(\overline{p}) : \varkappa_{\left\{m:\overline{p} \leq p_0^m\right\}} \geq 1$$

5 Coordination and Welfare

An important rationale motivating open economy excercises are welfare considerations. In this section we will study this more traditional rationale for opening our partial equilibrium economies and relate it to the coordinational considerations studied in the previous sections. However, the nature of the exercise is necessarily from an ex-ante viewpoint (before integration takes place): Suppose that a given economy is considering with which country to integrate among those in a given class. A possible evaluation criterion is welfare, disregarding coordinational issues. Another evaluation criterion is expectational coordination. If we compare the reccomendations of both, do they coincide? This is the precise question we answer in this section.

Consider the linear class of economies where farmers face the same aggregate demand function, $D_n(p) = D(p), \forall n$, but differ in their cost structures across regions. The integrated economy will be more efficient than the autarkic ones if we measure efficiency by the net change in the Marshallian aggregate surplus (net producers' profits plus net consumers' surplus) and this change is positive[39]. The increase in welfare from integration for a given n region is then defined by[40]:

$$\begin{aligned} \Delta W_n &\equiv W_n^* - \overline{W}_n = \Delta CS_n - \Delta \Pi_n \\ &= \int_{p^*}^{\overline{p}_n} D_n(p)dp - \int_{p^*}^{\overline{p}_n} S_n(p)dp \end{aligned}$$

It can be seen that a conflict exists between the consumers and the producers of each of the integrating economies. The economy with the relatively more performant producers[41] ($\max_i C_i$) experiences an increase in profits ($\Delta\Pi_n > 0$) from selling abroad part of their production at a price p^* higher than the autarkic one $\overline{p}_n$. This increase in the price damages the consumers living in that region, who see their consumer surplus eroded relative to the autarkic situation, $\Delta CS_n < 0$. The converse happens in the region with the least performant producers ($\min_i C_i$). But, the aggregate surplus increases after integration in each of the integrating economies[42]:

$$\begin{aligned} \Delta W_n &= \int_{p^*}^{\overline{p}_n} [A - Bp]\, dp - \int_{p^*}^{\overline{p}_n} [C_n p]\, dp \\ &= \left[\frac{Ap}{2} \left\{ 2 - \frac{B + C_n}{A} p \right\} \right]_{p^*}^{\overline{p}_n} = \frac{B + C_n}{2} \left[\overline{p}_n - p^* \right]^2 > 0, \forall n \end{aligned}$$

Because of this fact, we can assume that national (internal lump-sum) transfer schemes exist that are able to (more than) ex-post compensate the adversely affected party. This is always possible in this partial equilibrium framework, and everybody is made strictly better off after integration[43].

[39] Since there are no general equilibrium effects (because there is no trade as only one product is considered), two economies in the class considered here have an incentive to integrate when appropriate redistributional schemes are implemented. For a more detailed discussion, see Mas-Colell et al. [15], section 10.E.

[40] We slightly change notation relative to the previous sections: now p^* (instead of $\overline{p}$) denotes the integration equilibrium price, while $\overline{p}_n$ still denotes the autarkic equilibrium price of region n.

[41] Note that with the specified cost structures, $\partial_{C_f} C(q_f, f) = -\left(\frac{q_f}{\sqrt{2} C_f} \right)^2 < 0$. Therefore, lower values of cost parameter C_f correspond to higher production costs, and to a relatively less performant production technique.

[42] Where $C_\Sigma = C_n + \sum_{i \neq n} C_i$ denotes the parameter of the total cost function in the integrated economy $C_\Sigma(q)$.

[43] To simplify, consider an aggregate transfer scheme $\tau_n = \Delta CS_n + (1-t)\Delta W_n : t \in (0, 1)$

From this ex-ante welfare evaluation criterion, a given economy in the linear class would ideally choose an integration partner with which the increase in the net aggregate surplus is maximized. Region H must decide with which of the two region types (F or A) would it integrate, assuming that the producers in region F are more performant than producers in the H region, and those in region A are less performant:

$$+\infty > C_F > C_H > C_A > 0$$

Call the resulting integrated equilibrium prices p^*_{H+F} and p^*_{H+A}. From the analytic expression of the net welfare gains in region n, ΔW_n, we can see that

$$0 \in \arg \sup_{C_A : C_A \leq C_H} \Delta W_H^{H+A} = \frac{B + C_H}{2} \left[\overline{p}_H - p^*_{H+A}(C_A) \right]^2$$

Because, given the autarky price in the home region $\overline{p}_H$, the largest possible value of the integrated economy equilibrium price p^*_{H+A} is obtained when the less performant among the abroad regions (A) is selected, i.e. as $C_A \longrightarrow 0$. Since the home region (H) is more efficient, autarky prices are going to be lower: $p^*_{H+A} - \overline{p}_H > 0$. As this difference is maximal whenever $C_A \longrightarrow 0$, denote by $\overline{\varepsilon}$ its maximum value:

$$\overline{\omega} = \lim_{C_A \longrightarrow 0} p^*_{H+A}(C_A) - \overline{p}_H = \frac{A}{B + \frac{C_H}{2}} - \frac{A}{B + C_H} > 0$$

Now, we look, among the foreign economies (F) that are more efficient than the home region, whether there is one that allows the home region to attain this same level of welfare $\Delta W_H^{H+A}(\overline{\omega}) = \frac{B+C_H}{2} \left[\overline{\omega}\right]^2$. Since the foreign economy (F) is more efficient, the integrated economy equilibrium price will be lower than the autarky equilibrium price at home (H), $\overline{p}_H - p^*_{H+F} > 0$. Then our problem can be stated formally as:

$$\exists C_F : \overline{p}_H - p^*_{H+F}(C_F) > \overline{\omega}$$

To prove this statement, we are going to proceed as follows: first, we are going to show that there exists a foreign region with a cost function parameter $\overline{C}_F$ such that the home economy reaches the level of welfare $\Delta W_H^{H+A}(\overline{\omega})$. Then, we are going to show that there is a set of more performant foreign regions, the integration of home with which yields strictly larger welfare gains. Finally, we show that as the home region becomes more efficient, it also becomes increasingly difficult to find such an F-region, in the sense that the 'measure' of the set of F-regions the integration with which yields larger expected welfare gains for the H-region, becomes close to zero.

between producers and consumers, which are the two parties in conflict. Then: $\Delta CS_n = \int_{p^*}^{\overline{p}_n} D_n(p)dp + \tau_n = (1-t)\Delta W_n > 0$ and $\Delta \Pi_n = \int_{p^*}^{\overline{p}_n} S_n(p)dp - \tau_n = t\Delta W_n > 0$. Nevertheless, individual lump-sum transfer schemes could have been implemented in the way proposed by Dixit and Norman [5] (sec. 3.2.), as this partial equilibrium economy can be considered as a particular case of the general equilibrium economy they consider.

First, $\overline{C}_F = C_H\left(2+\frac{C_H}{B}\right)$ satisfies the equation[44]:

$$\overline{p}_H - p^*_{H+F}(\overline{C}_F) = \varpi \Longrightarrow \Delta W_H^{H+F}(\varpi) = \Delta W_H^{H+A}(\varpi)$$

Second, geometrically notice that $\overline{C}_F + C_H = \tan\overline{\theta}_{F+H}$. Provided that the home economy has an aggregate cost parameter strictly bounded from above, $C_H < +\infty$, and that it is not 'too expectationally unstable', $\frac{C_H}{B} < M < +\infty$, then $\overline{C}_F + C_H = C_H\left(3+\frac{C_H}{B}\right)$ will also be strictly bounded above: $\arctan C_H\left(3+\frac{C_H}{B}\right) = \overline{\theta}_{F+H} < \frac{\pi}{2} = \arctan(+\infty)$. By continuity, there will exist a $\delta > 0 : \overline{\theta}_{F+H} < \overline{\theta}_{F+H} + \delta < \frac{\pi}{2}$ which will correspond to a foreign region with an aggregate cost parameter $C_{\overline{F}} < +\infty : C_{\overline{F}} + C_H = \tan\left[\overline{\theta}_{F+H} + \delta\right] = C_H\left(3+\frac{C_H}{B}\right) + \Delta C$ and that will generate a strictly larger welfare gain for the home economy:

$$\Delta W_H^{H+\overline{F}} > \Delta W_H^{H+F}(\varpi) \Longleftrightarrow \Delta C > 0$$

Which is true by construction. Therefore, home integration with an F-region characterized by an aggregate cost parameter $C_{\overline{F}}$ displays strictly larger welfare gains than with the best possible integration partner in the set of A-regions.

Finally, under the just stated conditions, there exists an infinity of foreign regions (F) that satisfy this condition, but the size of the set becomes smaller the more efficient the home region is, i.e. the larger the value of the parameter C_H. If we put a uniform probability measure on $\left[0, \frac{\pi}{2}\right]$ we can interpret the expression

$$1 - \mu\left[\overline{\theta}_{F+H}\right] = \int_{\overline{\theta}_{F+H}}^{\frac{\pi}{2}} \frac{2}{\pi} dv = 1 - \frac{\arctan C_H\left(3+\frac{C_H}{B}\right)}{\frac{\pi}{2}}$$

as the likelihood of finding one F-region the integration with which provides higher welfare gains for the home region than integration with the best candidate in the set of A-regions. Then, from $\lim\limits_{C_H \to +\infty}\left\{1 - \mu\left[\overline{\theta}_{F+H}\right]\right\} = 0$ we conclude that the more efficient the home region is, the lower the probability of finding an F-region the integration with which will yield the same welfare gains for home than integration with the best candidate A-region (all relatively less performant). Alternatively, the more performant the home region is, the smaller the size of the set of those F-regions the integration with which provides home with higher welfare gains than integration with the best candidate in the set of A-regions.

[44] For the class of linear economies considered with identical aggregate demand, the F-region with a value of the aggregate cost parameter $\overline{C}_F$ that satisfies the above equation, must equivalently satisfy the condition:

$$\frac{B+\overline{C}_F}{B+C_H} = \left.\frac{B+C_H}{B+C_A}\right|_{C_A=0}$$

Whenever this condition is respected, the welfare gains for the home region from integrating with a more efficient (F) or with a less efficient (A) economy are the same, for economies in the linear class considered.

Now, the important point to be noted about this ex-ante welfare evaluation of the potential partner with which to integrate is that, the less performant the integrating partner is (the smaller the value of the aggregate cost parameter C), the easier the coordination upon the perfect foresight equilibrium of the integrated economy. And conversely. For a strictly finite value of C_H, we also see that the likelihood of finding such an F-region integration partner decreases with the 'degree of expectational instability' of the home region, as measured by $\frac{C_H}{B}$. This can be seen immediately from the fact that:

$$\partial_B \left\{1 - \mu\left[\overline{\theta}_{F+H}(B)\right]\right\} = -\frac{2}{\pi}\partial_B\overline{\theta}_{F+H}(B) = \frac{\frac{2}{\pi}\left(\frac{C_H}{B}\right)^2}{1 + C_H^2\left(3 + \frac{C_H}{B}\right)^2} > 0$$

Therefore, the higher the degree of expectational stability of the home economy (the higher the value of B, the lower the value of $\frac{C_H}{B}$), the higher the likelihood of finding an economy in the F-class the integration with which yields strictly larger welfare gains than integration with an economy in the A-class. Recall that regions in the A-class are those expectationally more stable than the home economy because they face the same aggregate demand (and therefore, the same value of the elasticity of demand B) but operate with higher costs: $\frac{1}{C_H} < \frac{1}{C_A}, \forall q$. Stated otherwise, if the purpose of Home economic integration is to maximize the welfare gains, relatively more performant regions will be preferred (F-regions will be preferred to A-regions), and government restrictions will be most likely called for to coordinate upon the equilibrium price of the resulting integrated economy. However, if the objective of Home economic integration is expectational coordination, relatively less performant regions will be preferred (A-regions will be preferred to F-regions). This is a surprising conclusion, the robustness of which remains to be ascertained[45].

A remark is in order. When the perfect foresight equilibrium is not the unique rationalizable expectations equilibrium, the aggregate surplus need not be the appropriate evaluation criterion in welfare terms. The reason is that it is based on the difference in welfare terms between the two Nash equilibrium prices (autarkic and integration) disregarding whether they can be educed or not. A more appropriate criterion in welfare terms would necessitate of computing producers' welfare when the set of rationalizable expectations equilibria is not a singleton, which is beyond the scope of the present paper.

6 Conclusion

Research since the 70s has devoted increasing effort to the assumptions justifying the implementation of a REE. Learning constitutes the current paradigm providing different justifications for the REE solution. In the class of one-dimensional models where the price is determined by price expectations, we have related the eductive learning conditions leading producers' expectations

[45] What seems crucial for the argument to extend to non-linear schedules is the existence of a finite maximal willigness to pay for the good.

to coordinate on an open economy REE to autarkic conditions, performing a comparative statics exercise. We have given an open economy interpretation to the elasticities conditions identified by Guesnerie [10], and shown that although expectational stability is essentially unaffected[46] by economic integration for regions in the linear class, it is undermined for regions in the non-linear class. Therefore, there is a sense in which even if economic integration promotes price stabilization, it also destabilizes price expectations. An important consequence of this fact is that an 'exogenous price intervention' becomes more compelling than it was under autarky. Intuitively, heterogeneity in producers' reactions to (structural) changes interplay with expectational heterogeneity, rendering coordination more difficult -Guesnerie [12]-. This same heterogeneity explains the existence of positive gains-to-trade in the class of models under consideration, advicing regions to integrate.

Comparing the expectational coordination objective to a more traditional 'welfare gain' consideration, we have shown that the maximal welfare gain for an autarkic expectationally stable economy is attained via integration with a region that the expectational coordination criterion at the global level will disapprove. However we also pointed out that this conclusion sidesteps the issue of evaluating equilibrium outcomes in welfare terms when the expectational stability test fails. In this sense, the application of continuous random selections over the set of rationalizable expectations equilibria, along the lines of Allen et al. [1], seems a necessary step in making progress through a meaningful gains-to-trade evaluation criterion meeting the learning justification requirements.

Although a natural extension would be to study whether the conditions for the eductive stability of the equilibrium in an open economy can be related to the basic theorems of international trade, a first difficulty stems in recognizing that most of such trade theorems concern comparative statics questions in a general equilibrium set up. Yet, most of the conclusions on the eductive stability literature relate to partial equilibrium economies[47].

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[46] Proposition 6 shows that without differences in the maximal willignesses to pay for the crop across regions in the linear class, the degree of expectational stability at the open economy equilibrium was some 'average' of the autarkic stability degrees.

[47] General equilibrium applications of the eductive viewpoint are scarce. Some of them are Guesnerie [11], Guesnerie and Hens [13] and Ghosal [9]. Calvo and Guesnerie [3] provide a brief introduction.

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Appendix 1

Derivation and properties of the coweb function

Derivation of the coweb function $\varphi(p;C_\Sigma)=\max\{\varphi_1(p;C_\Sigma),\varphi_2(p;C_\Sigma)\}$ for the agricultural economy $n=1+2$, with:

$$\begin{aligned}\varphi_1(p;C_\Sigma) &= \frac{\sum_n A_n}{\sum_n B_n}-\left(\frac{C_\Sigma}{\sum_n B_n}\right)p\\ \varphi_2(p;C_\Sigma) &= \frac{\inf\limits_n A_n}{\inf\limits_n B_n}-\left(\frac{C_\Sigma}{\inf\limits_n B_n}\right)p\end{aligned}$$

1) $\varphi(.)$ is a continuous function:

$\varphi(.)$ is a continuous function since it is composed by two linear functions which always intersect in the price domain. Denote by p^i the price function at which both linear functions intersect, i.e. $\varphi_1(p^i;C_\Sigma)=\varphi_2(p^i;C_\Sigma)$. It will be equal to:

$$p^i(C_\Sigma)=\frac{\inf\limits_n B_n\sum_n A_n-\inf\limits_n A_n\sum_n B_n}{\left(\inf\limits_n B_n-\sum_n B_n\right)C_\Sigma}=\frac{A_1\overline{C}_\Sigma}{B_1C_\Sigma}$$

With $\overline{C}_\Sigma\equiv A_2\left[\frac{B_1}{A_1}-\frac{B_2}{A_2}\right]$. The linear functions defining the intersecting price are well-defined, mapping $\varphi_1(p;C_\Sigma):\left[0,\frac{\sum_n A_n}{C_\Sigma}\right]\rightarrow\left[0,\frac{\sum_n A_n}{\sum_n B_n}\right]$ and $\varphi_2(p;C_\Sigma):\left[0,\frac{\inf\limits_n A_n}{C_\Sigma}\right]\rightarrow\left[0,\frac{\inf\limits_n A_n}{\inf\limits_n B_n}\right]$. To see that they always intersect, observe that $\varphi_2(0;C_\Sigma)-\varphi_1(0;C_\Sigma)=\frac{\inf\limits_n A_n}{\inf\limits_n B_n}-\frac{\sum_n A_n}{\sum_n B_n}=\frac{A_2}{B_2}-\frac{\sum_n A_n}{\sum_n B_n}=\frac{B_1}{B_1+B_2}\left[\frac{A_2}{B_2}-\frac{A_1}{B_1}\right]>0$ and that $\varphi_2(\frac{\inf\limits_n A_n}{C_\Sigma};C_\Sigma)-\varphi_1(\frac{\inf\limits_n A_n}{C_\Sigma};C_\Sigma)=0-\frac{\sum_n A_n}{\sum_n B_n}+\frac{C_\Sigma}{\sum_n B_n}\frac{\inf\limits_n A_n}{C_\Sigma}<0$. Since both are linear, both are continuous.

2) The coweb function is a maximum:

First observe that:

$$\begin{aligned}\varphi(\overline{p};\overline{C}_\Sigma) &= \max\left\{\varphi_1(\overline{p};\overline{C}_\Sigma),\varphi_2(\overline{p};\overline{C}_\Sigma)\right\}\\ &= \max\left\{\overline{p}(\overline{C}_\Sigma),\overline{p}(\overline{C}_\Sigma)\right\}\\ &= \overline{p}(\overline{C}_\Sigma)=\frac{A_1}{B_1}=p^i(\overline{C}_\Sigma)\end{aligned}$$

Meaning that there exists a value of the aggregate cost parameter for which the price equilibrium coincides with the common intesecting price, and therefore will be a point in the range of the coweb function.

Now consider w.l.o.g. a $C_\Sigma = C'_\Sigma > \overline{C}_\Sigma$, and denote by $\overline{p}'$ the corresponding perfect foresight price equilibrium. By definition, $\varphi(\overline{p}'; C'_\Sigma) = \overline{p}'$. Since $C'_\Sigma > \overline{C}_\Sigma$ then $\varphi_1(\overline{p}'; C'_\Sigma) = \overline{p}'$ whereas $\varphi_2(\overline{p}'; C'_\Sigma) \neq \overline{p}'$. If the function is a maximum, it must be the case that $\varphi_2(\overline{p}'; C'_\Sigma) < \overline{p}' = \varphi_1(\overline{p}'; C'_\Sigma)$.

Proof: It will be the case if the price at which both linear functions intersect is smaller than the equilibrium price, i.e. if $p^i(C'_\Sigma) < \overline{p}'$. Suppose that the opposite is true. Then:

$$\begin{aligned}
\frac{A_1\overline{C}_\Sigma}{B_1 C'_\Sigma} &= p^i(C'_\Sigma) > \overline{p}' = \frac{\sum_n A_n}{\sum_n B_n + C'_\Sigma} \Longleftrightarrow \\
A_2\frac{B_1}{A_1} - B_2 &= \overline{C}_\Sigma > \frac{B_1}{A_1}\frac{C'_\Sigma \sum_n A_n}{\sum_n B_n + C'_\Sigma} = \frac{C'_\Sigma\left(B_1 + A_2\frac{B_1}{A_1}\right)}{\sum_n B_n + C'_\Sigma} \Longleftrightarrow \\
A_2\frac{B_1}{A_1}\sum_n B_n - \left(\sum_n B_n + C'_\Sigma\right)B_2 &= \overline{C}_\Sigma \sum_n B_n - C'_\Sigma B_2 > C'_\Sigma B_1 \Longleftrightarrow \overline{C}_\Sigma > C'_\Sigma
\end{aligned}$$

A contradiction. Therefore, the coweb function is a maximum.

Using this fact and the intersecting price $p^i(C_\Sigma)$, we can also write the coweb function as follows:

$$\begin{aligned}
\varphi(p; C_\Sigma) &= \max\{\varphi_1(p; C_\Sigma), \varphi_2(p; C_\Sigma)\} \\
&= \begin{cases} \varphi_2(p; C_\Sigma) & \text{if } p \leq p^i(C_\Sigma) \\ \varphi_1(p; C_\Sigma) & \text{if } p \geq p^i(C_\Sigma) \end{cases}
\end{aligned}$$

A fact that follows from the observing that $\varphi_2(0; C_\Sigma) = \frac{\inf_n A_n}{\inf_n B_n} > \varphi_1(0; C_\Sigma) = \frac{\sum_n A_n}{\sum_n B_n}$ and that $|\partial_p \varphi_2(p; C_\Sigma)| = \left|-\frac{C_\Sigma}{\inf_n B_n}\right| > |\partial_p \varphi_1(p; C_\Sigma)| = \left|-\frac{C_\Sigma}{\sum_n B_n}\right|, \forall p \in [0, p_\infty)$ with $p_\infty \equiv (\varphi_1)^{-1}(0; C_\Sigma)$.

3) The coweb function is (weakly) decreasing:

We can conclude that the coweb function is decreasing in its price domain $\partial_p \varphi(p; C_\Sigma) < 0$ from the fact that it is the maximum of two strictly decreasing linear functions $\partial_p \varphi_n(p; C_\Sigma) < 0, n = 1, 2$. However $\varphi(p; C_\Sigma)$ is not C^1 because it is a max function with a non-differentiability point at the intersecting price $p^i(C_\Sigma)$.

4) Domain and Range of the coweb function:

Finally observe that since the coweb function maps prices into prices with domain and range given by $\varphi(p; C_\Sigma) : \left[0, \max\left\{(\varphi_1)^{-1}(0), (\varphi_2)^{-1}(0)\right\}\right] \to [0, \max\{\varphi_1(0; C_\Sigma), \varphi_2(0; C_\Sigma)\}]$ with $\max\left\{(\varphi_1)^{-1}(0), (\varphi_2)^{-1}(0)\right\} = (\varphi_1)^{-1}(0) = \frac{\sum_n A_n}{C_\Sigma}$ and $\max\{\varphi_1(0; C_\Sigma), \varphi_2(0; C_\Sigma)\} = \varphi_2(0; C_\Sigma) = \frac{\inf_n A_n}{\inf_n B_n}$.

Using that $\varphi(p;C_\Sigma)$ is a decreasing function and the intersecting price $p^i(C_\Sigma)$ to know where the equilibrium price is, i.e.

$$\begin{aligned}
\varphi_1(p^i;C_\Sigma) &= \varphi_2(p^i;C_\Sigma) > p^i(C_\Sigma) \Longrightarrow p^i(C_\Sigma) < \varphi(\overline{p};C_\Sigma) = \varphi_1(\overline{p};C_\Sigma) = \overline{p} \\
\varphi_1(p^i;C_\Sigma) &= \varphi_2(p^i;C_\Sigma) < p^i(C_\Sigma) \Longrightarrow p^i(C_\Sigma) > \varphi(\overline{p};C_\Sigma) = \varphi_2(\overline{p};C_\Sigma) = \overline{p} \\
\varphi_1(p^i;C_\Sigma) &= \varphi_2(p^i;C_\Sigma) = p^i(C_\Sigma) \Longrightarrow p^i(C_\Sigma) = \varphi(\overline{p};C_\Sigma) = \varphi_1(\overline{p};C_\Sigma) = \varphi_2(\overline{p};C_\Sigma) = \overline{p}
\end{aligned}$$

This observation will be useful in the study of the second iterate of the coweb function, in appendix 2. This concludes the description of the properties of the coweb function combining the results of Guesnerie's (1992) Lemma 1 and the particular shape of the function considered.

Appendix 2

Derivation of $\varphi^2(.)$.
Derivation of the second iterate of the coweb function $\varphi^2(p)$:

$$\begin{aligned}
\varphi^2(p) &= \varphi(\varphi(p)) \\
&= \max\left\{\varphi_1\left[\varphi(p)\right], \varphi_2\left[\varphi(p)\right]\right\} \\
&= \max\left\{\varphi_1\left[\max\left\{\varphi_1(p), \varphi_2(p)\right\}\right], \varphi_2\left[\max\left\{\varphi_1(p), \varphi_2(p)\right\}\right]\right\} \\
&= \max\left\{\min\left\{\left(\varphi_1 \circ \varphi_2\right)(p), \left(\varphi_1 \circ \varphi_1\right)(p)\right\}, \min\left\{\left(\varphi_2 \circ \varphi_2\right)(p), \left(\varphi_2 \circ \varphi_1\right)(p)\right\}\right\}
\end{aligned}$$

Where the first equalities follow by definition, and the last one follows from the downward slopingness of the functions $\varphi_n(.)$ so that $\varphi_n\left[\max\left\{\varphi_1(p), \varphi_2(p)\right\}\right] = \min\left\{\varphi_n\left(\varphi_2(p)\right), \varphi_n\left(\varphi_1(p)\right)\right\}$ for $n = 1, 2$. The different linear functions composing its definition are given by:

$$\begin{aligned}
\left(\varphi_1 \circ \varphi_2\right)(p) &= \frac{\sum_n A_n}{\sum_n B_n} + \frac{\inf\limits_n A_n}{\inf\limits_n B_n}\left(-\frac{C_\Sigma}{\sum_n B_n}\right) + \left(-\frac{C_\Sigma}{\sum_n B_n}\right)\left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right)p \\
\left(\varphi_2 \circ \varphi_1\right)(p) &= \frac{\inf\limits_n A_n}{\inf\limits_n B_n} + \frac{\sum_n A_n}{\sum_n B_n}\left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right) + \left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right)\left(-\frac{C_\Sigma}{\sum_n B_n}\right)p \\
\left(\varphi_1 \circ \varphi_1\right)(p) &= \frac{\sum_n A_n}{\sum_n B_n} + \frac{\sum_n A_n}{\sum_n B_n}\left(-\frac{C_\Sigma}{\sum_n B_n}\right) + \left(-\frac{C_\Sigma}{\sum_n B_n}\right)^2 p \\
\left(\varphi_2 \circ \varphi_2\right)(p) &= \frac{\inf\limits_n A_n}{\inf\limits_n B_n} + \frac{\inf\limits_n A_n}{\inf\limits_n B_n}\left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right) + \left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right)^2 p
\end{aligned}$$

Before proceeding, we can study the functions that compose $\varphi^2(p)$, which are $\varphi_1\left[\varphi(p)\right]$, and $\varphi_2\left[\varphi(p)\right]$. These two functions are well defined since they are the composition of a linear function and a continuous maximum fuction, mapping $(\varphi_1 \circ \varphi)(p) : \left[0, (\varphi_1)^{-1}(0)\right] \xrightarrow{\varphi} \left[0, \varphi_2(0;C_\Sigma)\right] \xrightarrow{\varphi_1} \left[\varphi_1\left[\varphi_2(0;C_\Sigma)\right], \varphi_1(0)\right]$ and $\left(\varphi_2 \circ \varphi\right)(p) : \left[0, (\varphi_1)^{-1}(0)\right] \xrightarrow{\varphi} \left[0, \varphi_2(0;C_\Sigma)\right] \xrightarrow{\varphi_2} \left[\varphi_2\left[\varphi_2(0;C_\Sigma)\right], \varphi_2(0)\right]$, since $\max\left\{(\varphi_1)^{-1}(0),(\varphi_2)^{-1}(0)\right\}=(\varphi_1)^{-1}(0),\ \max\{\varphi_1(0;C_\Sigma),\varphi_2(0;C_\Sigma)\}=$ $\varphi_2(0;C_\Sigma)$ and $(\varphi_n)'(.)<0,\forall n$. Therefore, the second iterate of the coweb function maps $\varphi^2(p):\left[0,(\varphi_1)^{-1}(0)\right]\overset{\max\{\varphi_1\circ\varphi,\varphi_2\circ\varphi\}}{\rightarrow}\left[\max\left\{\varphi_1\left[\varphi_2(0;C_\Sigma)\right],\varphi_2\left[\varphi_2(0;C_\Sigma)\right]\right\},\varphi_2(0)\right].$ Where $\max\left\{\varphi_1\left[\varphi_2(0;C_\Sigma)\right],\varphi_2\left[\varphi_2(0;C_\Sigma)\right]\right\}=\varphi_1\left[\varphi_2(0;C_\Sigma)\right].$

Derivation of $p^{i\prime}$ and properties characterizing the function $\varphi^2(.)$

There is a (second) intesecting price, denoted $p^{i\prime}(C_\Sigma)$, that characterizes the second iterate of the coweb function satisfying:

$$\exists p^{i\prime}:(\varphi_1\circ\varphi)(p^{i\prime})=(\varphi_2\circ\varphi)(p^{i\prime})=\varphi^2(p^{i\prime})$$

And with the **properties** that:

1) It characterizes the second iterate of the coweb function (toghether with p^i) as a piecewise linear function:

$$\varphi^2(p)=\begin{cases}(\varphi_1\circ\varphi)(p) & \text{if } \varphi(p)\geq\varphi(p^{i\prime})\Longleftrightarrow p\leq p^{i\prime}\\ (\varphi_2\circ\varphi)(p) & \text{if } \varphi(p)\leq\varphi(p^{i\prime})\Longleftrightarrow p\geq p^{i\prime}\end{cases}$$

Using the property $|\partial_p\varphi_2(p;C_\Sigma)|>|\partial_p\varphi_1(p;C_\Sigma)|$ and $\varphi'(.)<0$. And as we showed in appendix 1 for the coweb function, this intersecting price $p^{i\prime}$ also satisfies by definition and by $\varphi'(.)<0$ that if:

$$\begin{aligned}(\varphi_1\circ\varphi)(p^{i\prime}) &= (\varphi_2\circ\varphi)(p^{i\prime})=\varphi^2(p^{i\prime})>\varphi(p^{i\prime})\Longrightarrow\varphi(\overline{p})=\varphi_1(\overline{p})>\varphi(p^{i\prime})\Longleftrightarrow\overline{p}<p^{i\prime}\\ (\varphi_1\circ\varphi)(p^{i\prime}) &= (\varphi_2\circ\varphi)(p^{i\prime})=\varphi^2(p^{i\prime})<\varphi(p^{i\prime})\Longrightarrow\varphi(\overline{p})=\varphi_2(\overline{p})<\varphi(p^{i\prime})\Longleftrightarrow\overline{p}>p^{i\prime}\\ (\varphi_1\circ\varphi)(p^{i\prime}) &= (\varphi_2\circ\varphi)(p^{i\prime})=\varphi^2(p^{i\prime})=\varphi(p^{i\prime})\Longrightarrow\varphi(\overline{p})=\varphi_1(\overline{p})=\varphi_2(\overline{p})=\varphi(p^{i\prime})\Longleftrightarrow\overline{p}=p^{i\prime}\end{aligned}$$

2) Explicit expression for $p^{i\prime}$:

To obtain an explicit expression for $p^{i\prime}(C_\Sigma)$ we can solve the equation $(\varphi_1\circ\varphi)(p^{i\prime})=(\varphi_2\circ\varphi)(p^{i\prime})$ yielding:

$$C_\Sigma\varphi(p^{i\prime})=\frac{A_1}{B_1}\overline{C}_\Sigma\Longleftrightarrow\varphi(p^{i\prime})=p^i$$

Where $p^i\equiv p^i(C_\Sigma)=\frac{A_1}{B_1}\frac{\overline{C}_\Sigma}{C_\Sigma}$ is the (first) intersecting price we derived in appendix 1. Given that if:

$$\begin{aligned}\varphi(p^i) &\leq p^i\Longrightarrow\varphi(p)=\varphi_2(p),\forall p\leq p^i\\ \varphi(p^i) &\geq p^i\Longrightarrow\varphi(p)=\varphi_1(p),\forall p\geq p^i\end{aligned}$$

and that by $\varphi'(.)<0$ we have:

$$\begin{aligned}\varphi(p^{i\prime}) &= p^i\geq\varphi(p^i)\Longleftrightarrow p^{i\prime}\leq p^i\\ \varphi(p^{i\prime}) &= p^i\leq\varphi(p^i)\Longleftrightarrow p^{i\prime}\geq p^i\end{aligned}$$

we can conclude from both that:

$$\begin{array}{llll} \text{if } p^{i\prime} & \leq & p^i \Longrightarrow \varphi(p^{i\prime}) = \varphi_2(p^{i\prime}) = p^i \Longrightarrow p_2^{i\prime} = \varphi_2^{-1}(p^i) \\ \text{if } p^{i\prime} & \geq & p^i \Longrightarrow \varphi(p^{i\prime}) = \varphi_1(p^{i\prime}) = p^i \Longrightarrow p_1^{i\prime} = \varphi_1^{-1}(p^i) \end{array}$$

With explicit formulas:

$$\begin{aligned} p_1^{i\prime}(C_\Sigma) &= \frac{A_1 + A_2}{C_\Sigma} - \left(\frac{B_1 + B_2}{C_\Sigma}\right) p^i(C_\Sigma) \\ p_2^{i\prime}(C_\Sigma) &= \frac{A_2}{C_\Sigma} - \left(\frac{B_2}{C_\Sigma}\right) p^i(C_\Sigma) \end{aligned}$$

So that the explicit expression for $p^{i\prime}(C_\Sigma)$ is:

$$p^{i\prime}(C_\Sigma) = \max\left\{p_1^{i\prime}(C_\Sigma), p_2^{i\prime}(C_\Sigma), 0\right\}$$

Proof: Because $\forall C_\Sigma \leq (\geq)\overline{C}_\Sigma$ we have that $p^i(C_\Sigma) \geq (\leq) p^{i\prime}(C_\Sigma)$ and by the argument above, $p^{i\prime}(C_\Sigma) = p_2^{i\prime}(C_\Sigma) \geq p_1^{i\prime}(C_\Sigma)$ ($p^{i\prime}(C_\Sigma) = p_1^{i\prime}(C_\Sigma) \geq p_2^{i\prime}(C_\Sigma)$). (i) To see that $\forall C_\Sigma \leq (\geq)\overline{C}_\Sigma$ we have that $p_1^{i\prime}(C_\Sigma) \leq (\geq) p_2^{i\prime}(C_\Sigma)$ we remark that the only value of C_Σ at which both functions $p_2^{i\prime}(C_\Sigma), p_1^{i\prime}(C_\Sigma)$ intersect is $C_\Sigma = \overline{C}_\Sigma$ so that $p_2^{i\prime}(\overline{C}_\Sigma) = p_1^{i\prime}(\overline{C}_\Sigma)$. At that point we also have:

$$\partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma) = -\frac{1}{\overline{C}_\Sigma}\frac{A_1}{B_1}\left[1 - \frac{B_2}{\overline{C}_\Sigma}\right]$$

And:

$$\partial_{C_\Sigma} p_1^{i\prime}(\overline{C}_\Sigma) = -\frac{1}{\overline{C}_\Sigma}\frac{A_1}{B_1}\left[1 - \frac{B_1 + B_2}{\overline{C}_\Sigma}\right] = \partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma) + \frac{A_1}{\left(\overline{C}_\Sigma\right)^2}$$

So that at the unique point at which both functions intersect, we have $\partial_{C_\Sigma} p_1^{i\prime}(\overline{C}_\Sigma) > \partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma)$ concluding that $\forall C_\Sigma \leq (\geq)\overline{C}_\Sigma$ we have that $p_1^{i\prime}(C_\Sigma) \leq (\geq) p_2^{i\prime}(C_\Sigma)$. (ii) Now, it is also true that the only value of C_Σ at which each of the functions $p_2^{i\prime}(C_\Sigma), p_1^{i\prime}(C_\Sigma)$ intersects with $p^i(C_\Sigma)$ is $C_\Sigma = \overline{C}_\Sigma$. Therefore $p_2^{i\prime}(\overline{C}_\Sigma) = p_1^{i\prime}(\overline{C}_\Sigma) = p^i(\overline{C}_\Sigma)$. Since at that point it is also true that $\partial_{C_\Sigma} p^i(\overline{C}_\Sigma) = -\frac{1}{\overline{C}_\Sigma}\frac{A_1}{B_1}$, by (i) it is true that $\partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma) = \partial_{C_\Sigma} p^i(\overline{C}_\Sigma) + \frac{A_1 B_2}{B_1\left(\overline{C}_\Sigma\right)^2}$ and consequently $\partial_{C_\Sigma} p_1^{i\prime}(\overline{C}_\Sigma) > \partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma) > \partial_{C_\Sigma} p^i(\overline{C}_\Sigma)$. We can then conclude that $\forall C_\Sigma \leq (\geq)\overline{C}_\Sigma$ we have that $p^i(C_\Sigma) \geq (\leq) p^{i\prime}(C_\Sigma)$ as we wanted to show.

3) Non-monotonicity of $p^{i\prime}$:

Observe that:

$$\begin{aligned} \partial_{C_\Sigma} p_2^{i\prime}(\overline{C}_\Sigma) &> (\leq)0 \;\text{ if }\; \overline{C}_\Sigma < (\geq) B_2 \equiv C_\Sigma^2 \\ \partial_{C_\Sigma} p_1^{i\prime}(\overline{C}_\Sigma) &> 0 \;\text{ if }\; \overline{C}_\Sigma < B_2 + B_1 \equiv C_\Sigma^1 \end{aligned}$$

Where the last row follows from a fact that will be used below again, but that we prove here: Suppose by contradiction that $\overline{C}_\Sigma \geq C_\Sigma^1$. Equivalently,

$A_2\frac{B_1}{A_1} - B_2 \geq B_1 + B_2$ or $B_1\left[\frac{A_2}{A_1} - 1\right] \geq 2B_2$. Since $A_1 \geq A_2$, $\frac{A_2}{A_1} \leq 1$ and $0 \geq B_1\left[\frac{A_2}{A_1} - 1\right] \geq 2B_2$ violating the restriction $B_2 > 0$. Therefore $0 \leq \overline{C}_\Sigma < C^1_\Sigma$ and the slope of $p_1^{i\prime}$ at the point $C_\Sigma = \overline{C}_\Sigma$ can only be positive: The maximum of $p_1^{i\prime}$ will always be at a value of the aggregate cost larger than $\overline{C}_\Sigma$. The functional form of both functions displays a maximum in the aggregate cost domain $\mathbb{R}_{++}$.

Parameter values C^1_Σ, C^2_Σ will be used below to characterize the learning dynamics.

4) Value of $p^{i\prime}$ in the analytic case studied by Guesnerie (1992):

If $\frac{A_1}{B_1} = \frac{A_2}{B_2}$ then $\overline{C}_\Sigma = 0$ and $\varphi(p) = \varphi_1(p), \forall p$ and the intersecting price $p^i = 0$. In consequence, $p^{i\prime} = \varphi_1^{-1}(0) = \frac{A_1+A_2}{C_\Sigma} > 0$ and $\overline{p} = \overline{p}_1 \in \left[p^i, p^{i\prime}\right] = \left[0, p_1^{i\prime}\right]$ which corresponds to the analitical case studied by Guesnerie (1992).

5) Defines a non-empty interval to which the perfect foresight equilibrium price $\overline{p}$ always belongs:

Proof: Consider the (first) intesecting price $p^i(C_\Sigma)$. We are going to show that the perfect foresight equilibrium price $\overline{p}$ must be contained in a non-empty interval between the two intersecting prices p^i and $p^{i\prime}$. First suppose that:

$$\varphi^2(p^{i\prime}) > \varphi(p^{i\prime}) \Longrightarrow \varphi(\overline{p}) = \varphi_1(\overline{p}) > \varphi(p^{i\prime}) \Longleftrightarrow \overline{p} < p^{i\prime}$$

Now it can happen that $\overline{p} < p^i$ so that $\overline{p} \notin \left[p^i, p^{i'}\right]$. But if $\overline{p} < p^i$ from the definition of p^i we have that $\overline{p} = \varphi_2(\overline{p})$. A contradiction since $\overline{p} < p^{i\prime}$. Therefore, $\overline{p} > p^i$ and $\overline{p} \in \left[p^i, p^{i'}\right]$. Second suppose that:

$$\varphi^2(p^{i\prime}) < \varphi(p^{i\prime}) \Longrightarrow \varphi(\overline{p}) = \varphi_2(\overline{p}) < \varphi(p^{i\prime}) \Longleftrightarrow \overline{p} > p^{i\prime}$$

Which leads to $\overline{p} \in \left[p^{i'}, p^i\right]$ using the same reasoning. Finally, if:

$$\varphi^2(p^{i\prime}) = \varphi(p^{i\prime}) \Longrightarrow \varphi(\overline{p}) = \varphi_1(\overline{p}) = \varphi_2(\overline{p}) = \varphi(p^{i\prime}) \Longleftrightarrow \overline{p} = p^{i\prime}$$

Then it must be the case by the same argument, that $\overline{p} \in \left[p^{i'}, p^i\right] = \{\overline{p}\}$.

Using some of these properties we can rewrite $\varphi^2(p)$ alternatively as follows:

$$\begin{aligned}
\varphi^2(p) &= \max\left\{(\varphi_1 \circ \varphi)(p), (\varphi_2 \circ \varphi)(p)\right\} \\
&= \max\left\{\min\left\{(\varphi_1 \circ \varphi_2)(p), (\varphi_1 \circ \varphi_1)(p)\right\}, \min\left\{(\varphi_2 \circ \varphi_2)(p), (\varphi_2 \circ \varphi_1)(p)\right\}\right\} \\
&= \begin{cases} \min\left\{(\varphi_1 \circ \varphi_2)(p), (\varphi_1 \circ \varphi_1)(p)\right\} & \text{if } p \leq p^{i\prime} \\ \min\left\{(\varphi_2 \circ \varphi_2)(p), (\varphi_2 \circ \varphi_1)(p)\right\} & \text{if } p \geq p^{i\prime} \end{cases} \\
&= \begin{cases} \text{if } p^{i\prime} \geq p^i & \begin{cases} (\varphi_1 \circ \varphi_2)(p) & \text{if } p \leq p^i \\ (\varphi_1 \circ \varphi_1)(p) & \text{if } p^i < p < p^{i\prime} \\ (\varphi_2 \circ \varphi_1)(p) & \text{if } p \geq p^{i\prime} \end{cases} \\ \text{if } p^{i\prime} \leq p^i & \begin{cases} (\varphi_1 \circ \varphi_2)(p) & \text{if } p \leq p^{i\prime} \\ (\varphi_2 \circ \varphi_2)(p) & \text{if } p^{i\prime} < p < p^i \\ (\varphi_2 \circ \varphi_1)(p) & \text{if } p \geq p^i \end{cases} \end{cases}
\end{aligned}$$

Proof: To prove the fourth equality above, suppose that $p^{i\prime} \geq p^i$. Then for every price smaller than p^i, $p \leq p^i$, the definition of the intersecting price p^i implies that $\varphi_2(p) \geq \varphi_1(p)$, and since $(\varphi_1)'(.) < 0$ we have that $(\varphi_1 \circ \varphi_2)(p) \leq (\varphi_1 \circ \varphi_1)(p)$. Now since $\varphi^2(p) = \min\{(\varphi_1 \circ \varphi_2)(p), (\varphi_1 \circ \varphi_1)(p)\}$ if $p \leq p^{i\prime}$ from the third equality, it follows that $\varphi^2(p) = (\varphi_1 \circ \varphi_2)(p)$ for all prices $p \leq p^i$ which is the very first row behind the fourth equality in the case $p^{i\prime} \geq p^i$. For the second row, we have prices between both intersecting prices $p^i < p < p^{i\prime}$. By the same reasoning, $p \geq p^i \Longrightarrow \varphi_2(p) \leq \varphi_1(p)$ and using $(\varphi_1)'(.) < 0$ we have that $(\varphi_1 \circ \varphi_2)(p) \geq (\varphi_1 \circ \varphi_1)(p)$. Now since $\varphi^2(p) = \min\{(\varphi_1 \circ \varphi_2)(p), (\varphi_1 \circ \varphi_1)(p)\}$ if $p \leq p^{i\prime}$ from the third equality, it follows that $\varphi^2(p) = (\varphi_1 \circ \varphi_1)(p)$ for all prices $p^i < p < p^{i\prime}$. Finally, for the third row, prices satisfy $p \geq p^{i\prime}$ so that $\varphi^2(p) = \min\{(\varphi_2 \circ \varphi_2)(p), (\varphi_2 \circ \varphi_1)(p)\}$, but since $p^{i\prime} \geq p^i \Longrightarrow p \geq p^i$ and therefore $\varphi_2(p) \leq \varphi_1(p)$. By $(\varphi_2)'(.) < 0$ we have that $(\varphi_2 \circ \varphi_2)(p) \geq (\varphi_2 \circ \varphi_1)(p) \Longrightarrow \varphi^2(p) = \min\{(\varphi_2 \circ \varphi_2)(p), (\varphi_2 \circ \varphi_1)(p)\} = (\varphi_2 \circ \varphi_1)(p)$, which is what we wanted to show. To prove the case $p^{i\prime} \leq p^i$ the same argument applies.

Now, by defining $p^i_{\inf} = \max\left\{\min\left\{p^{i\prime}, p^i\right\}, p_1\right\}$ and $p^i_{\sup} = \min\left\{\max\left\{p^{i\prime}, p^i\right\}, p_\infty\right\}$, we can re-express in a more compact form $\varphi^2(p)$ as:

$$\varphi^2(p) = \left\{ \begin{array}{c} (\varphi_1 \circ \varphi_2)(p) \;\text{ if }\; p \leq p^i_{\inf} \\ (\varphi_1 \circ \varphi_1)(p)\mathbf{1}_{\{p^i_{\inf} = p^i\}} + (\varphi_2 \circ \varphi_2)(p)\mathbf{1}_{\{p^i_{\inf} = p^{i\prime}\}} \;\text{ if }\; p \in \left(p^i_{\inf}, p^i_{\sup}\right) \\ (\varphi_2 \circ \varphi_1)(p) \;\text{ if }\; p \geq p^i_{\sup} \end{array} \right.$$

Where the symbol $\mathbf{1}_{\{p^i_{\inf} = p^i\}}$ denotes the standard indicator function, taking value 1 if $p^i_{\inf} = p^i$ and 0 otherwise.

Observations:

1) Since it can both happen that $p^{i\prime} \leq p^i$ and $p^{i\prime} \geq p^i$ so that $p^{i\prime} = p^i$, the definition of $\varphi^2(p)$ is correct and specializes to:

$$\varphi^2(p) = (\varphi_1 \circ \varphi_2)(p) = (\varphi_2 \circ \varphi_1)(p), \forall p$$

Proof: To see it, observe that if $p^{i\prime} = p^i$ from the definitions of $p^i_{\inf}$ and $p^i_{\sup}$, we have that $p^i_{\inf} = p^i_{\sup} = p^i = p^{i\prime}$. Then:

$$\begin{aligned} \varphi^2(p) &= \left\{ \begin{array}{c} (\varphi_1 \circ \varphi_2)(p) \;\text{ if }\; p \leq p^i_{\inf} = p^i_{\sup} \\ (\varphi_1 \circ \varphi_1)(p)\mathbf{1}_{\{p^i_{\inf} = p^i\}} + (\varphi_2 \circ \varphi_2)(p)\mathbf{1}_{\{p^i_{\inf} = p^{i\prime}\}} \;\text{ if }\; p = \{\emptyset\} \\ (\varphi_2 \circ \varphi_1)(p) \;\text{ if }\; p \geq p^i_{\inf} = p^i_{\sup} \end{array} \right. \\ &= \left\{ \begin{array}{l} (\varphi_1 \circ \varphi_2)(p) \;\text{ if }\; p \leq p^i_{\inf} = p^i_{\sup} \\ (\varphi_2 \circ \varphi_1)(p) \;\text{ if }\; p \geq p^i_{\inf} = p^i_{\sup} \end{array} \right. \end{aligned}$$

Also since $\overline{p} \in \left[p^i_{\inf}, p^i_{\sup}\right] = \left\{p^i_{\inf} = p^i_{\sup}\right\}$, from the proof above we must have that $\overline{p} = p^i_{\inf} = p^i_{\sup} = p^{i\prime} = p^i$. Recalling from appendix 1 that $\overline{p} = p^i$ whenever $C_\Sigma = \overline{C}_\Sigma$ and using the above explicit formulas for $(\varphi_1 \circ \varphi_2)(p)$ and

$(\varphi_2 \circ \varphi_1)(p)$, observe that they have the same slope for all possible prices p in their respective domains. Since they are both linear with the same slope, they will coincide provided that their constant terms coincide:

$$\frac{\sum_n A_n}{\sum_n B_n} + \frac{\inf\limits_n A_n}{\inf\limits_n B_n}\left(-\frac{C_\Sigma}{\sum_n B_n}\right) \quad = \quad \frac{\inf\limits_n A_n}{\inf\limits_n B_n} + \frac{\sum_n A_n}{\sum_n B_n}\left(-\frac{C_\Sigma}{\inf\limits_n B_n}\right) \Longleftrightarrow$$

$$\Longleftrightarrow \quad C_\Sigma = A_2\left[\frac{B_1}{A_1} - \frac{B_2}{A_2}\right] \equiv \overline{C}_\Sigma$$

Stating that the constant terms in the definition of the functions $(\varphi_1 \circ \varphi_2)(p)$ and $(\varphi_2 \circ \varphi_1)(p)$ coincide whenever the aggregate cost parameter C_Σ takes the value $\overline{C}_\Sigma$ compatible with $\overline{p} = p^i$. Therefore, $\varphi^2(p) = (\varphi_1 \circ \varphi_2)(p) = (\varphi_2 \circ \varphi_1)(p), \forall p$ as we wanted to show.

2) The reader must notice that the price domain of the second iterate of the coweb function can exclude one of the limit prices $(p^i_{\text{inf}}, p^i_{\text{sup}})$ defining the interval inside which the price equilibrium is determinate. To this purpose, suppose first that $p^i_{\text{inf}} = p^i$ and $\varphi^2(p^i) < 0$. Then since $\varphi^2(.)$ is (weakly) increasing, $\exists p'_1 : \forall \varepsilon > 0, \varphi^2(p'_1 + \varepsilon) > 0 > \varphi^2(p^i)$. Then, we can enlarge the definition of p^i_{inf} to $p^i_{\text{inf}} = \max\left\{\min\left\{p^{i\prime}, p^i\right\}, p'_1\right\}$. Proceeding identically for p^i_{sup} we can redefine it as $p^i_{\text{sup}} = \min\left\{\max\left\{p^{i\prime}, p^i\right\}, p_\infty\right\}$ where p_∞ is the upper limit of the coweb function price domain, i.e. $p_\infty \equiv \varphi_1^{-1}(0) = \frac{\sum_n A_n}{C_\Sigma}$. If $\max\left\{p^{i\prime}, p^i\right\} = p^{i\prime}$ but $p^{i\prime} > p_0$, then we shall let $p^i_{\text{sup}} = p_\infty$ according to our enlarged definition. We will refer in the text to the enlarged definitions of p^i_{sup} and p^i_{inf} when appropriate.

Appendix 3

Properties of $\varphi^2(.)$

The properties of $\varphi^2(.)$ as given by Guesnerie (1992), are:

1) $\varphi^2(.)$ is (weakly) increasing:

It can immediately be observed from the explicit formulas of the different linear functions composing the second iterate of the coweb function, given at the very beginning of appendix 2. More generally, since $\partial\varphi^2(p) = \varphi'[\varphi(p)]\,\varphi'(p)$ and $\varphi'(.) < 0$, we must have $\partial\varphi^2(p) > 0$ (a.e.).

2) $\varphi^2(\overline{p}) = \overline{p}$

This is trivial from $\varphi(\overline{p}) = \overline{p}$ since $\varphi^2(\overline{p}) = \varphi[\varphi(\overline{p})] = \varphi[\overline{p}] = \overline{p}$.

3) The success of eductive learning can be assessed entirely from $\varphi^2(.)$ as long as there exists some CK (initial) information on prices (that starts the eductive game of guessing, i.e. $p \leq p_0$) and that $\lim\limits_{\tau\to\infty} \left(\varphi^2\right)^\tau (p_0) = \overline{p}$.

Appendix 4

We completely characterize the learning dynamics of section 4. The results are summarized in Table 1, and the definitions of the symbols immediately follow:

Table A4.1: Summary of Results of Proposition 8

C_Σ							
$>$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$\emptyset$
$=C_\Sigma^1$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$[0,p_0]$	$\emptyset$ $\emptyset$ $\emptyset$
$\geq$	$\{\overline{p}\}$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$ $[0,p_0]$ $[0,p_0]$	$\emptyset$
$=C_\Sigma^0$	$\{\overline{p}\}$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$	$\{\overline{p}\}\vert_P$ $[0,p_0]$ $[\overline{p}_{c1},\overline{p}_{c2}]$	$[\overline{p}_{c1},\overline{p}_{c2}]$	$\emptyset$
$>$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$ $\{\overline{p}\}$ $[\overline{p}_{c1},\overline{p}_{c2}]$	$[\overline{p}_{c1},\overline{p}_{c2}]$	$[\overline{p}_{c1},\overline{p}_{c2}]$	$\emptyset$
$=C_\Sigma^2$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$ $\{\overline{p}\}$ $[\overline{p}_{c1},\overline{p}_{c2}]\vert$	$[\overline{p}_{c1},\overline{p}_{c2}]\vert$	$[\overline{p}_{c1},\overline{p}_{c2}]\vert$	$[\overline{p}_{c1},\overline{p}_{c2}]\vert$	$\emptyset$
$>$	$\{\overline{p}\}$	$\{\overline{p}\}$ $\{\overline{p}\}$ $\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\{\overline{p}\}$	$\emptyset$
0	$\emptyset$ $\emptyset$ $\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$\begin{pmatrix} C_\Sigma > \overline{C}_\Sigma \\ C_\Sigma = \overline{C}_\Sigma \\ C_\Sigma < \overline{C}_\Sigma \end{pmatrix}$	0	$<$	$=C_\Sigma^2$	$<$	$=C_\Sigma^0$	$\leq$	$C_\Sigma^1 = \overline{C}_\Sigma$

The contents, following the results of proposition 2, indicate the set of rationalizable-expectations equilibria, where the exogenous price restriction p_0 is imbedded in the model (it is the maximum willigness to pay of the integrated economy demand):

-" $[0,p_0]$ " means that the set of rationalizable expectations equilibria usually contains the whole price domain $[0,p_0]$. As farmers learn nothing, $\overline{p}$ is not an SREE;

-" $[\overline{p}_{c1},\overline{p}_{c2}]$ " means that the set of rationalizable prices is the whole segment $[\overline{p}_{c1},\overline{p}_{c2}] \supset \overline{p}$, where $p_{c2} = \varphi(p_{c1}), \varphi^2(p_{ct}) = p_{ct}, t = 1,2$ define cycles of order two of the coweb function. For some parameterizations, the imbedded price restriction p_0 can belong to the set $[\overline{p}_{c1},\overline{p}_{c2}]$. Then, an exogenous price invertevention is called for restricting p_0 to be out of it: $p_0 \notin [\overline{p}_{c1},\overline{p}_{c2}]$, denoting such a requirement by " $[\overline{p}_{c1},\overline{p}_{c2}]\vert$ ", meaning '$[\overline{p}_{c1},\overline{p}_{c2}]$ is the set of rationalizable prices conditional to that price restriction'.

-" $\{\overline{p}\}$ " means that the only rationalizable-expectations price equilibrium is the PFE $\overline{p}$, and $\overline{p}$ is an SREE.

-" $\{\overline{p}\}|_P$ " means that the only rationalizable-expectations price equilibrium is the PFE $\overline{p}$ conditional to an exogenous price intervention restricting the natural one p_0 to be in the basin of attraction[48] of $\overline{p}$, $P(\overline{p}) = (\overline{p}_{c1}, \overline{p}_{c2}) \setminus \{\overline{p}\}$, and $\overline{p}$ is an 'SREE conditional to that price restriction'.

From the definitions of $p^{i\prime}, p^i$ and their properties[49], we know that ultimately, they depend on the value of the aggregate cost parameter C_Σ and in particular, on whether $C_\Sigma \gtreqless \overline{C}_\Sigma$. As the characterization of $\varphi^2(.)$ depends on $p^{i\prime}, p^i$ and they depend on C_Σ, the learning dynamics will ultimately depend on C_Σ. In principle, $C_\Sigma \in \mathbb{R}_{++}$. We are going to divide the C_Σ-parameter space in four regions according to the following definitions of C_Σ^0, C_Σ^1 and C_Σ^2 satisfying:

$$+\infty > C_\Sigma^1 \geq C_\Sigma^0 > C_\Sigma^2 > 0$$

With $C_\Sigma^1 \equiv B_1 + B_2$ characterizing the limit value of the aggregate cost parameter above which the PFE price $\overline{p}$ becomes eductively unstable; $C_\Sigma^0 \equiv B_2 \left[1 + \frac{A_1}{A_2}\right]$ would characterize the (global) eductive stability condition when there would be no difference in the maximal willigness to pay. Although it is not necessary nor even sufficient, it will play a role in the characterization of the learning dynamics, because when $\frac{A_2}{B_2} \geq \frac{A_1}{B_1}$, computing $\lim\limits_{\frac{A_2}{B_2} \to \frac{A_1}{B_1}} C_\Sigma^1 = C_\Sigma^0$ and the whole region $\left[C_\Sigma^0, C_\Sigma^1\right]$ collapses into that value $\left\{C_\Sigma^0\right\}$. Finally $C_\Sigma^2 \equiv B_2$ characterizes the limit value of the aggregate cost parameter below which the PFE price $\overline{p}$ prevailing under integration becomes eductively stable, i.e. if $C_\Sigma^2 > C_\Sigma > 0 \Longrightarrow \overline{p}$ globally a SREE, as it is the case in Guesnerie's [10] basic linear model. Since the difference in the maximal willignesses to pay is measured by the value of the aggregate cost parameter $\overline{C}_\Sigma$, its range of variation will also be constrained to the regions for C_Σ([50]). We allow the possibility that $0 = \overline{C}_\Sigma$ because it corresponds to the case studied in proposition 6.

Proof of the results in Table 1.

Now we are in a position to prove the results in Table 1. To do so, we are going to distinguish three broad cases, (those in brackets in the south-west corner of Table 1):

(*) Case $C_\Sigma > \overline{C}_\Sigma$ (We prove the results in the upper triangular matrix excluding the diagonal elements in the first bissectrix of the plane $(\overline{C}_\Sigma, C_\Sigma)$ defined by Table 1)

[48] The basin of attraction $P(\overline{p})$ of a given equilibrium price $\overline{p}$ is composed by the union of all the $p_0 \neq \overline{p}$ s.t. $\lim\limits_{\tau \longrightarrow +\infty} \varphi^\tau(p_0) = \overline{p}$.

[49] See appendices 1 and 2.

[50] With the exception introduced by property 3 of the (second) intersecting price $p^{i\prime}$ according to which $0 \leq \overline{C}_\Sigma < C_\Sigma^1$. Details are in appendix 2.

a) If $C_\Sigma > \overline{C}_\Sigma \Longrightarrow p^i < p^{i\prime} \equiv \varphi_1^{-1}(p^i)$ by property 3 of $p^{i\prime}$. Then, by the definitions of $p^i_{\text{inf}}, p^i_{\text{sup}}$ we have:

$$\begin{aligned} p^i_{\text{inf}} &= \max\left\{p^i, p_1'\right\} : p_1' \equiv (\varphi_1 \circ \varphi_1)^{-1}(0) = p_\infty \left[1 - \frac{B_1 + B_2}{C_\Sigma}\right] \\ p^i_{\text{sup}} &= \min\left\{p_1^{i\prime}, p_\infty\right\} : p_1^{i\prime} = p_\infty - \frac{B_1 + B_2}{C_\Sigma} p^i \end{aligned}$$

So that, since $\frac{B_1+B_2}{C_\Sigma} > 0$ and $p^i > 0$, we have $p_1^{i\prime} < p_\infty$ implying that $p^i_{\text{sup}} = p_1^{i\prime}$. On the other hand, the definition of p_1' implies that $p_1' > 0 \Longleftrightarrow 1 > \frac{B_1+B_2}{C_\Sigma}$ since $p_\infty > 0$ which is necessary for p_1' to be greater than $p^i > 0$ although not sufficient. The sufficient condition is that $C_\Sigma > \overline{C}_\Sigma + B_1 + B_2 \Longrightarrow p^i_{\text{inf}} = p_1'$ and that the second iterate of the coweb function is:

$$\varphi^2(p) = \begin{cases} (\varphi_1 \circ \varphi_1)(p), & \text{if } p \leq p_1^{i\prime} \\ (\varphi_2 \circ \varphi_1)(p), & \text{if } p \geq p_1^{i\prime} \end{cases}$$

And by $p_1' > 0 \Longleftrightarrow 1 > \frac{B_1+B_2}{C_\Sigma}$ we have that $\left(\varphi^2\right)'(\overline{p}) = (\varphi_1 \circ \varphi_1)'(\overline{p}) = \left(\frac{C_\Sigma}{B_1+B_2}\right)^2 > 1$ and in consequence $\overline{p}$ is not strongly rational in $[p_1', p_\infty]$, corresponding to the upper north-west corner of Table 1 (noticing that $B_1 + B_2 \equiv C_\Sigma^1$ so that $C_\Sigma > \overline{C}_\Sigma + C_\Sigma^1 > C_\Sigma^1$) but also applies to the whole first row (because $\overline{C}_\Sigma \geq 0$,.we have that $C_\Sigma > \overline{C}_\Sigma + C_\Sigma^1 \Longrightarrow C_\Sigma > C_\Sigma^1$). The set of rationalizable expectations equilibria is $[p_1', p_\infty]$. Q.E.D.

b) Now, considering the polar case, $\overline{C}_\Sigma < C_\Sigma \leq \overline{C}_\Sigma + C_\Sigma^1 \Longrightarrow p^i_{\text{inf}} = p^i$ and the definition of the second iterate of the coweb function becomes:

$$\varphi^2(p) = \begin{cases} (\varphi_1 \circ \varphi_2)(p), & \text{if } p \leq p^i \\ (\varphi_1 \circ \varphi_1)(p), & \text{if } p \in \left(p^i, p_1^{i\prime}\right) \\ (\varphi_2 \circ \varphi_1)(p), & \text{if } p \geq p_1^{i\prime} \end{cases}$$

Suppose then that $C_\Sigma = C_\Sigma^1$, implying no restriction on $\overline{C}_\Sigma$ by $C_\Sigma \leq \overline{C}_\Sigma + C_\Sigma^1$. Observe that $C_\Sigma = C_\Sigma^1 \Longleftrightarrow 1 = \frac{B_1+B_2}{C_\Sigma}$ and therefore $\left(\varphi^2\right)'(\overline{p}) = (\varphi_1 \circ \varphi_1)'(\overline{p}) = \left(\frac{C_\Sigma}{B_1+B_2}\right)^2 = 1, \forall p \in \left[p^i, p_1^{i\prime}\right] \supset \overline{p}$. Since $(\varphi_1 \circ \varphi_1)(p)$ is a linear function of p in $\left[p^i, p_1^{i\prime}\right]$, the whole segment is the set of rationalizable expectations equilibria. This segment is nonempty provided that $p_1^{i\prime} > p^i \Longleftrightarrow \frac{A_2}{B_2} < \frac{A_1}{B_1}\left[2 + \frac{B_1}{B_2}\right] \Longleftrightarrow \overline{C}_\Sigma < C_\Sigma^1$ which is always the case as showed above. Concerning the eductive stability of the segment of rationalizable expectations equilibria, observe that it depends on whether $(\varphi_1 \circ \varphi_2)^{-1}(0) \equiv p_1 \lesseqgtr 0$ and on $(\varphi_1 \circ \varphi_2)'(p) = \frac{C_\Sigma}{B_1+B_2}\frac{C_\Sigma}{B_2} \equiv \frac{C_\Sigma}{C_\Sigma^2}\frac{C_\Sigma}{C_\Sigma^1}$. We have that:

$$p_1 = \frac{C_\Sigma^2}{C_\Sigma}(p_0 - p_\infty) \underset{(\leq)}{>} 0 \Longleftrightarrow p_0 \underset{(\leq)}{>} p_\infty \Longleftrightarrow C_\Sigma \underset{(\leq)}{>} C_\Sigma^0$$

Since $C_\Sigma = C_\Sigma^1 > C_\Sigma^0 > C_\Sigma^2$ we must have $(\varphi_1 \circ \varphi_2)^{-1}(0) \equiv p_1 > 0$ and $(\varphi_1 \circ \varphi_2)'(p) = \left.\frac{C_\Sigma}{C_\Sigma^2}\frac{C_\Sigma}{C_\Sigma^1}\right|_{C_\Sigma = C_\Sigma^1} > 1$, for every price $p \in \left[p_1, p^i\right]$. But as well, by $(\varphi_1 \circ \varphi_2)'(p) = (\varphi_2 \circ \varphi_1)'(p) > 1$, $\forall p \in \left[p_1^{i\prime}, p_\infty\right]$. In consequence $\overline{p}$ is not strongly rational in $[p_1, p_\infty]$, the set of rationalizable expectations equilibria is a connected segment $\left[p^i, p_1^{i\prime}\right] \supset \overline{p}$. It also applies to the whole second row because we have not imposed any restriction on $\overline{C}_\Sigma$ except that $\overline{C}_\Sigma < C_\Sigma$,.which is always satisfied. Q.E.D.

Now suppose that $C_\Sigma < C_\Sigma^1$. We will distinguish two cases:

b.1.) $C_\Sigma^0 \leq C_\Sigma < C_\Sigma^1 \Longrightarrow \left(\varphi^2\right)'(\overline{p}) = (\varphi_1 \circ \varphi_1)'(\overline{p}) = \left(\frac{C_\Sigma}{C_\Sigma^1}\right)^2 < 1, \forall p \in \left[p^i, p_1^{i\prime}\right] \supset \overline{p}$, by the second inequality. This shows that $\overline{p}$ will be strongly rational on its basin of attraction, which will be non-empty since it will contain at least $\forall p \in \left[p^i, p_1^{i\prime}\right] \supset \overline{p}$.

We prove a preliminary result that we need to complete the study of the case b.1.): $(\varphi_1 \circ \varphi_1)(p^i) > p^i$. This is equivalent to

$$\frac{A_1 + A_2}{B_1 + B_2}\left[1 - \frac{C_\Sigma}{C_\Sigma^1}\right] + \frac{C_\Sigma}{C_\Sigma^1}\frac{C_\Sigma^2}{C_\Sigma^1}\left[\frac{A_2}{B_2} - \frac{A_1}{B_1}\right] > \frac{C_\Sigma^2}{C_\Sigma}\left[\frac{A_2}{B_2} - \frac{A_1}{B_1}\right] \Longleftrightarrow$$

$$\frac{A_1 + A_2}{B_1 + B_2} > \frac{A_1}{B_1}\frac{\overline{C}_\Sigma}{C_\Sigma}$$

But since $C_\Sigma > \overline{C}_\Sigma$ we have that $\frac{A_1}{B_1} > \frac{A_1}{B_1}\frac{\overline{C}_\Sigma}{C_\Sigma}$ whereas $\frac{A_1 + A_2}{B_1 + B_2} > \frac{A_1}{B_1}$ so that the above inequality is always true and $(\varphi_1 \circ \varphi_1)(p^i) > p^i$ which is what we wanted to show.

By definition of p^i and by the result just proved, we have $(\varphi_1 \circ \varphi_1)(p^i) = (\varphi_1 \circ \varphi_2)(p^i) > p^i$. By the first weak inequality of case b.1.), the definition of p_1 implies that $p_1 \geq 0 \Longleftrightarrow (\varphi_1 \circ \varphi_2)(p_1) = 0$. By Weierstrass' theorem, $\exists \overline{p}_{c1} \in \left[p_1, p^i\right] : (\varphi_1 \circ \varphi_2)(\overline{p}_{c1}) = \overline{p}_{c1}$. And since $\varphi^2(.)$ is C^1 in the domain $\left[p_1, p^i\right]$, by the mean value theorem, we must have $(\varphi_1 \circ \varphi_2)'(\overline{p}_{c1}) = \frac{(\varphi_1 \circ \varphi_2)(p^i) - (\varphi_1 \circ \varphi_2)(p_1)}{p^i - p_1} > 1$ so that $\overline{p}_{c1}$ is not eductively stable and therefore, not strongly rational.

Replicating the same reasoning on the price domain $\left[p_1^{i\prime}, p_\infty\right]$ of $\varphi^2(.)$, we have that $(\varphi_1 \circ \varphi_1)(p_1^{i\prime}) = (\varphi_2 \circ \varphi_1)(p_1^{i\prime}) < p_1^{i\prime}$ where the inequality follows from the fact that by property 5 of $p^{i\prime}$ $\left[p^i, p_1^{i\prime}\right] \supset \overline{p} \Longrightarrow p_1^{i\prime} \leq \overline{p}$ and by property 2 of $\varphi^2(.)$ we have $\overline{p} = \varphi^2(\overline{p})$, so that since $(\varphi_1 \circ \varphi_1)'(\overline{p}) < 1$ and $\left(\varphi^2\right)'(.)$ is of constant slope in $\left[p^i, p_1^{i\prime}\right]$, the inequality must be true. We also have that $(\varphi_2 \circ \varphi_1)(p_\infty) > p_\infty$, a fact that can be observed by direct computation since equivalent to

$$\frac{A_2}{C_\Sigma^2} - \frac{A_1 + A_2}{C_\Sigma^1}\frac{C_\Sigma}{C_\Sigma^2} + \frac{C_\Sigma}{C_\Sigma^1}\frac{C_\Sigma}{C_\Sigma^2}\frac{A_1 + A_2}{C_\Sigma} > \frac{A_1 + A_2}{C_\Sigma} \Longleftrightarrow C_\Sigma > C_\Sigma^0$$

Therefore by Weierstrass' theorem, $\exists \overline{p}_{c2} \in \left[p_1^{i\prime}, p_\infty\right] : (\varphi_2 \circ \varphi_1)(\overline{p}_{c2}) = \overline{p}_{c2}$. And since $\varphi^2(.)$ is C^1 in the domain $\left[p_1^{i\prime}, p_\infty\right]$, by the mean value theorem, we must have $(\varphi_2 \circ \varphi_1)'(\overline{p}_{c2}) = \frac{(\varphi_2\circ\varphi_1)(p_1^{i\prime})-(\varphi_2\circ\varphi_1)(p_\infty)}{p_1^{i\prime}-p_\infty} > 1$ so that $\overline{p}_{c2}$ is not eductively stable and therefore, not strongly rational. Finally, notice that $\varphi_1(\overline{p}_{c2}) = \overline{p}_{c1}$ and that $\varphi_2(\overline{p}_{c1}) = \overline{p}_{c2}$ so that $\{\overline{p}_{c1}, \overline{p}_{c2}\}$ form a cycle of period two.

To summarize the results of case b.1.): there exist three determinate equilibria $\{\overline{p}_{c1}, \overline{p}, \overline{p}_{c2}\}$ of which $\{\overline{p}_{c1}, \overline{p}_{c2}\}$ constitute respectively the lower and upper bounds of the connected segment of the rationalizable expectations equilibria $(\overline{p}_{c1}, \overline{p}_{c2}) \supset \overline{p}$ defining the basin of attraction of $\overline{p}$. When restricted to its basin of attraction, the perfect foresight equilibrium is strongly rational. This corresponds to the third and fourth rows of the upper triangular matrix in Table 1, excluding the first bissectrix elements and the first colum ones. The first column ones correspond to the case b.2.) because when $\overline{C}_\Sigma = 0$, $C_\Sigma^0 = C_\Sigma^1$ and case b.1.) collapses to case b.2.). Q.E.D.

b.2.) $C_\Sigma < C_\Sigma^0 \leq C_\Sigma^1$. By the first inequality and the definitions of p_1, p_∞ we have that $p_1 < 0 \Longrightarrow (\varphi_1 \circ \varphi_2)(0) > 0$ and that $(\varphi_2 \circ \varphi_1)(p_\infty) < p_\infty$ by the converse argument used in b.1.). It is still the case that $(\varphi_1 \circ \varphi_1)'(p) < 1$ for all $p \in \left[p^i, p_1^{i\prime}\right] \supset \overline{p}$. Now, even if $(\varphi_1 \circ \varphi_2)'(p) = (\varphi_2 \circ \varphi_1)'(p) \gtrless 1$ in their respective price domains $\left[0, p^i\right]$ and $\left[p_1^{i\prime}, p_\infty\right]$, $\overline{p}$ is the unique rationalizable expectations equilibrium and it is strongly rational in $[0, p_\infty]$ (globally). This corresponds to the results in the first column of table 1 ($\overline{C}_\Sigma = 0$) and to the fifth, sixth and seventh rows of the upper triangular matrix with respect to the first bissectrix, excluding the elements of the bissectrix. Q.E.D.

This completes the proof of the results corresponding to case $C_\Sigma > \overline{C}_\Sigma$. Q.E.D.

(**) Case $C_\Sigma < \overline{C}_\Sigma$ (We prove the results in the lower triangular matrix excluding the diagonal elements in the first bissectrix of the plane $(\overline{C}_\Sigma, C_\Sigma)$ defined by Table 1)

a) If $C_\Sigma < \overline{C}_\Sigma \Longrightarrow p^i > p^{i\prime} \equiv \varphi_2^{-1}(p^i)$ by property 3 of $p^{i\prime}$. Then, by the definitions of $p_{\inf}^i, p_{\sup}^i$ we have:

$$\begin{aligned} p_{\inf}^i &= \max\left\{p_2^{i\prime}, p_2^{\prime}, 0\right\} : p_2^{i\prime} = \frac{A_2}{C_\Sigma} - \frac{B_2}{C_\Sigma} p^i \\ p_{\sup}^i &= \min\left\{p^i, p_\infty\right\} : p^i = p_\infty - \frac{A_1}{C_\Sigma}\left[1 + \frac{B_2}{B_1}\right] < p_\infty \Longrightarrow p_{\sup}^i = p^i \end{aligned}$$

Where $p_2^{i\prime} = \frac{A_2}{C_\Sigma} - \frac{B_2}{C_\Sigma} p^i = p_2^{\prime} + \frac{A_1}{B_1}\left(\frac{B_2}{C_\Sigma}\right)^2 > p_2^{\prime}$ and $p_2^{\prime} \equiv (\varphi_2 \circ \varphi_2)^{-1}(0) = \frac{A_2}{C_\Sigma}\left[1 - \frac{B_2}{C_\Sigma}\right]$ implying that $p_{\inf}^i = \max\left\{p_2^{i\prime}, 0\right\}$.. From the definition of $p_2^{i\prime}$ we have $p_2^{i\prime} \leq 0 \Longleftrightarrow C_\Sigma \leq B_2\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right] < B_2$ by $\frac{A_1}{B_1} \leq \frac{A_2}{B_2}$, and by the definition of $p_{\inf}^i$ we have that $p_{\inf}^i = \max\left\{p_2^{i\prime}, 0\right\} = 0$ which on its turn implies that $(\varphi_2 \circ \varphi_2)(0) > 0$, by $C_\Sigma < B_2 \Longrightarrow p_2^{\prime} < 0$, and the second iterate of the coweb function becomes:

$$\varphi^2(p) = \begin{cases} (\varphi_2 \circ \varphi_2)(p), & \text{if } p < p^i \\ (\varphi_2 \circ \varphi_1)(p), & \text{if } p \geq p^i \end{cases}$$

Now, since $C_\Sigma < B_2 \equiv C_\Sigma^2$, we have that $(\varphi_2 \circ \varphi_2)'(p) = \left(\frac{C_\Sigma}{C_\Sigma^2}\right)^2 < 1$, $\forall p < p^i$ and by property 5 of $p^{i\prime}$ we know that $\overline{p} \in \left[p^{i\prime}, p^i\right]$ whenever $C_\Sigma < \overline{C}_\Sigma$, letting $p^{i\prime} = 0$. In consequence $(\varphi_2 \circ \varphi_2)'(\overline{p}) < 1$ so that $\overline{p}$ is eductively stable in $[0, p^i)$. That it is unique follows from the linearity (and therefore continuity) of $\varphi^2(.)$ in the price domain $[0, p^i)$, toghether with the facts $(\varphi_2 \circ \varphi_2)(0) > 0$ and $(\varphi_2 \circ \varphi_1)(p^i) = (\varphi_2 \circ \varphi_2)(p^i) < p^i$ that allow us to apply Weierstrass' theorem to $\varphi^2(.)$. To prove that $(\varphi_2 \circ \varphi_2)(p^i) < p^i$ we can use the explicit expression for $p^i = \frac{A_2}{C_\Sigma} - \frac{A_1}{B_1}\frac{B_2}{C_\Sigma}$ and operating we find:

$$\begin{aligned}\frac{A_2}{B_2}\left[1 - \frac{C_\Sigma}{B_2}\right] + \left(\frac{C_\Sigma}{B_2}\right)^2 p^i &< p^i \Longleftrightarrow \\ \frac{A_2}{B_2} - \frac{A_1}{B_1}\frac{C_\Sigma}{B_2} &< \frac{A_2}{C_\Sigma} - \frac{A_1}{B_1}\frac{B_2}{C_\Sigma} \Longleftrightarrow \\ C_\Sigma &< B_2\frac{A_2}{B_2}\frac{B_1}{A_1}\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right]\end{aligned}$$

Which, noticing that $B_2\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right] \leq B_2\frac{A_2}{B_2}\frac{B_1}{A_1}\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right]$ because $\frac{A_2}{B_2}\frac{B_1}{A_1} \geq 1$, and that we are considering values of the aggregate cost such that $C_\Sigma \leq B_2\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right]$, it is always the case. Finally, $\forall p \geq p^i$ we have that $(\varphi_2 \circ \varphi_1)'(p) = \left(\frac{C_\Sigma}{C_\Sigma^2}\frac{C_\Sigma}{C_\Sigma^1}\right) < 1$, by $C_\Sigma^1 > C_\Sigma^2 > C_\Sigma$. Therefore, there exists a unique rationalizable expectations equilibrium price $\overline{p}$ which is strongly rational in $[0, p_\infty]$ (globally).

If however $B_2\left[1 - \frac{A_1}{B_1}\frac{B_2}{A_2}\right] < C_\Sigma < B_2$ then $p_2^{i\prime} > 0 \Longrightarrow p_{\inf}^i = \max\left\{p_2^{i\prime}, 0\right\} = p_2^{i\prime}$ and the second iterate of the coweb function becomes:

$$\varphi^2(p) = \begin{cases} (\varphi_1 \circ \varphi_2)(p), & \text{if } p \leq p_2^{i\prime} \\ (\varphi_2 \circ \varphi_2)(p), & \text{if } p \in \left(p_2^{i\prime}, p^i\right) \\ (\varphi_2 \circ \varphi_1)(p), & \text{if } p \geq p^i \end{cases}$$

Since what was important for the existence and eductive stability of the equilibrium was that $C_\Sigma < B_2$ and by $(\varphi_2 \circ \varphi_1)'(p) = (\varphi_1 \circ \varphi_2)'(p) < 1$, the same conclusions follow for this enlarged definition of $\varphi^2(.)$. This case corresponds to the bottom row of the lower triangular matrix in table 1, excluding the first bissectrix diagonal terms.Q.E.D.

b) Suppose now that $\overline{C}_\Sigma > C_\Sigma = B_2$. Then from the above definition of p_2', $p_2' = 0$ and as well, $(\varphi_2 \circ \varphi_2)'(p) = 1$, for all $p \in \left(p_2^{i\prime}, p^i\right)$ implying that all the prices in this open interval will be rationalizable expectations prices. Whether they are eductively stable or not will depend on the slope of $(\varphi_2 \circ \varphi_1)'(p) = (\varphi_1 \circ \varphi_2)'(p)$ which is smaller than one because $C_\Sigma^1 > C_\Sigma^2 = C_\Sigma$. Therefore, the set of rationalizable expectations prices is the segment $\left[p_2^{i\prime}, p^i\right] \supset \overline{p}$, and $\overline{p}$ is locally strongly rational. Observe that $\forall p \in \left[p_2^{i\prime}, p^i\right] \setminus \{\overline{p}\}$ is an eductive cycle of period two. Since the reasoning is independent of the value of $\overline{C}_\Sigma$ as long as $\overline{C}_\Sigma > C_\Sigma$, the same is true for the whole before last bottom row of the lower triangular matrix excepting the elements in the firt bissectrix. Q.E.D.

c) The case $\overline{C}_\Sigma > C^0_\Sigma \geq C_\Sigma$ corresponds to the second and third rows of the lower triangular matrix of Table 1. Recalling that from the definition of p_1 from case (*) we have,

$$p_1 = \frac{C^2_\Sigma}{C_\Sigma}(p_0 - p_\infty) \underset{(\leq)}{>} 0 \Longleftrightarrow p_0 \underset{(\leq)}{>} p_\infty \Longleftrightarrow C_\Sigma \underset{(\leq)}{>} C^0_\Sigma$$

and that $(\varphi_2 \circ \varphi_2)'(p) = \left(\frac{C_\Sigma}{C^2_\Sigma}\right)^2 > 1$, for all $p \in \left(p^{i\prime}_2, p^i\right) \supset \overline{p}$. Therefore $\overline{p}$ will not be eductively stable in $\left(p^{i\prime}_2, p^i\right)$. Now since $\varphi^2(\overline{p}) = (\varphi_2 \circ \varphi_2)(\overline{p}) = \overline{p}$ by property 2 of the second iterate of the coweb function, and by property 5 of $p^{i\prime}$, $\overline{p} \in \left(p^{i\prime}_2, p^i\right)$ the linearity of $\varphi^2(.)$ in prices implies that $(\varphi_1 \circ \varphi_2)(p^{i\prime}_2) = (\varphi_2 \circ \varphi_2)(p^{i\prime}_2) < (\varphi_2 \circ \varphi_2)(\overline{p}) = \overline{p}$ which on its turn implies that $(\varphi_1 \circ \varphi_2)(p^{i\prime}_2) < p^{i\prime}_2$. Now the definition of p_1 states that for the case under consideration, $p_1 \leq 0 \Longrightarrow (\varphi_1 \circ \varphi_2)(0) \geq 0$. By property 1 of $\varphi^2(.)$, we have that $p^{i\prime}_2 > 0 \Longrightarrow (\varphi_1 \circ \varphi_2)(p^{i\prime}_2) > (\varphi_1 \circ \varphi_2)(0) \geq 0$. Since $\varphi^2(.)$ is linear, it is continuous and C^1 in the domain $\left[0, p^{i\prime}_2\right]$, and we can apply the mean value theorem for $p \in \left(0, p^{i\prime}_2\right)$, $(\varphi_1 \circ \varphi_2)'(p) = \frac{(\varphi_1 \circ \varphi_2)(p^{i\prime}_2) - (\varphi_1 \circ \varphi_2)(0)}{p^{i\prime}_2} < \frac{(\varphi_1 \circ \varphi_2)(p^{i\prime}_2)}{p^{i\prime}_2} < 1$ by the inequality $(\varphi_1 \circ \varphi_2)(p^{i\prime}_2) < p^{i\prime}_2$. By Weierstrass' theorem, we have that $\exists \overline{p}_{c1} \in \left[0, p^{i\prime}_2\right]$: $(\varphi_1 \circ \varphi_2)(\overline{p}_{c1}) = \overline{p}_{c1}$, and by the mean value theorem, $(\varphi_1 \circ \varphi_2)'(\overline{p}_{c1}) < 1$. By observing that $(\varphi_2 \circ \varphi_1)(p^i) = (\varphi_2 \circ \varphi_2)(p^i) > (\varphi_2 \circ \varphi_2)(\overline{p}) = \overline{p}$ and therefore $(\varphi_2 \circ \varphi_1)(p^i) > p^i$ by the parallel reasoning, but that now $(\varphi_2 \circ \varphi_1)(p_\infty) \leq p_\infty$, and using exactly the same argument, we can conclude that $\exists \overline{p}_{c2} \in \left[p^i, p_\infty\right]$: $(\varphi_2 \circ \varphi_1)(\overline{p}_{c2}) = \overline{p}_{c2}$, and by the mean value theorem, $(\varphi_2 \circ \varphi_1)'(\overline{p}_{c2}) < 1$. That $(\varphi_2 \circ \varphi_1)(p_\infty) \leq p_\infty$ was shown to be the case whenever $C^0_\Sigma \geq C_\Sigma$ in case (*) part b.1.). Therefore we can conclude that the perfect foresight equilibrium is locally strongly rational in $(\overline{p}_{c1}, \overline{p}_{c2})$, an interval formed by the two-period eductively stable cycle that now emerges, since it satisfies both $\varphi_1(\overline{p}_{c2}) = \overline{p}_{c1}$ and $\varphi_2(\overline{p}_{c1}) = \overline{p}_{c2}$. Whenever $C_\Sigma \longrightarrow C^0_\Sigma \equiv B_2\left[1 + \frac{A_1}{A_2}\right]$, we have that $\overline{p}_{c1} \longrightarrow 0$ and $\overline{p}_{c2} \longrightarrow p_\infty$. This completes the study of the results in the second and third rows of the lower diagonal matrix, excluding the elements in the first bissectrix.Q.E.D.

d) The case $C^1_\Sigma > \overline{C}_\Sigma > C_\Sigma > C^0_\Sigma$ corresponds to the first row of the lower triangular matrix, below the first bissectrix of Table 1. Relative to case c) above, the only thing that changes is that since $C_\Sigma > C^0_\Sigma \Longrightarrow p_1 > 0 \Longleftrightarrow (\varphi_1 \circ \varphi_2)(0) < 0$ but as well, $(\varphi_2 \circ \varphi_1)(p_\infty) > p_\infty$ and in consequence, there will be no intersection with the first bissectrix other than $\overline{p}$. It will be the unique rationalizable expectations equilibrium which will not be eductively stable in $[p_1, p_\infty] \setminus \{\overline{p}\}$. Therefore, $\overline{p}$ is not strongly rational in $[p_1, p_\infty]$. Q.E.D.

This completes the study of the case (**) $\overline{C}_\Sigma > C_\Sigma$. Q.E.D.

Finally the results included in the diagonal terms of the first bissectrix in Table 1, corresponding to $\overline{C}_\Sigma = C_\Sigma$, are easily proven recognizing that in such a case $p^i = p^{i\prime} = \overline{p}$ and that the second iterate of the coweb function is a linear function on its price domain equal to

$$\varphi^2(p) = (\varphi_1 \circ \varphi_2)(p) = (\varphi_2 \circ \varphi_1)(p), \forall p$$

A fact proved in the derivation of the second iterate of the coweb function, observation 1. Everything will here depend on whether $C_\Sigma \lesseqgtr C_\Sigma^0$ observing that the perfect foresight equilibrium price $\overline{p}$ will always be the unique rationalizable-expectations equilibrium (strongly rational in $[0, p_\infty]$ whenever $C_\Sigma < C_\Sigma^0$, and not strongly rational in $[p_1, p_\infty]$ whenever $C_\Sigma > C_\Sigma^0$) excepting when $C_\Sigma = C_\Sigma^0$. Then, a continuum of rationalizable-expectations equilibria exists, composed by the segment $[0, p_\infty]$, and therefore, a continuum of period-two cycles emerges. Q.E.D.

This completes the proof of the results in Table 1.