Using elasticities to derive optimal income tax rates

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Tax and transfer policies
M2 PPD
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Introduction

- How much progressivity should there be in tax schedules? $\Longrightarrow$ equity-efficiency trade-off : redistribution vs incentives
- Optimal tax rate: Tax rate that collects the most revenue
- Original model : Mirrlees (1971)
- Saez’s goal: to clearly show that there is a simple link between optimal tax formulas and elasticities of earnings.

Plan

1. Optimal marginal tax rate for top incomes
2. General non-linear optimal tax rates for any tax bracket.
3. Numerical simulations of optimal tax schedules

1. HIGH INCOME OPTIMAL TAX RATES

Base specifications

- Maximisation of a utility function $u = u(c, z)$

Where $u_c = \frac{du}{dc} > 0$, $u_z = \frac{du}{dz} < 0$, ($z = wl$),

according to the constraint $c = z(1 - \tau) + R$

Where
- τ is the top marginal tax rate on
- *R* is virtual (non-labour) income : this is the post-tax income and individual would get if he supplied zero labour and was allowed to stay on the "virtual" linear schedule

For those who failed/skipped/forgot Micro 101…

- *Substitution effect :* If the price of a good increases relative to another, then people will consume relatively more of the other good.
- $\Longrightarrow$ If the tax rate goes up, leisure becomes more attractive because the ‘price’ paid for it (after-tax income forgone by not working) has fallen.
- *Income effect :* If total income is reduced, then people will cut back on the consumption of all goods that are not essential (i.e. normal goods).
- $\Longrightarrow$ If the tax rate goes up, I have less income, and therefore I ‘consume’ less leisure, i.e. I work more.

Elasticity concepts

- *Uncompensated elasticity* of earnings : $\zeta^u = \frac{dz}{z} / \frac{d(1-\tau)}{1-\tau}$: (*uncompensated,* because it does not compensate for a change in income)

- *Income effects* (= the marginal propensity to earn out of non-labour income): $\eta = (1-\tau)\frac{dz}{dR} \leq 0$, since leisure is assumed not to be an inferior good.

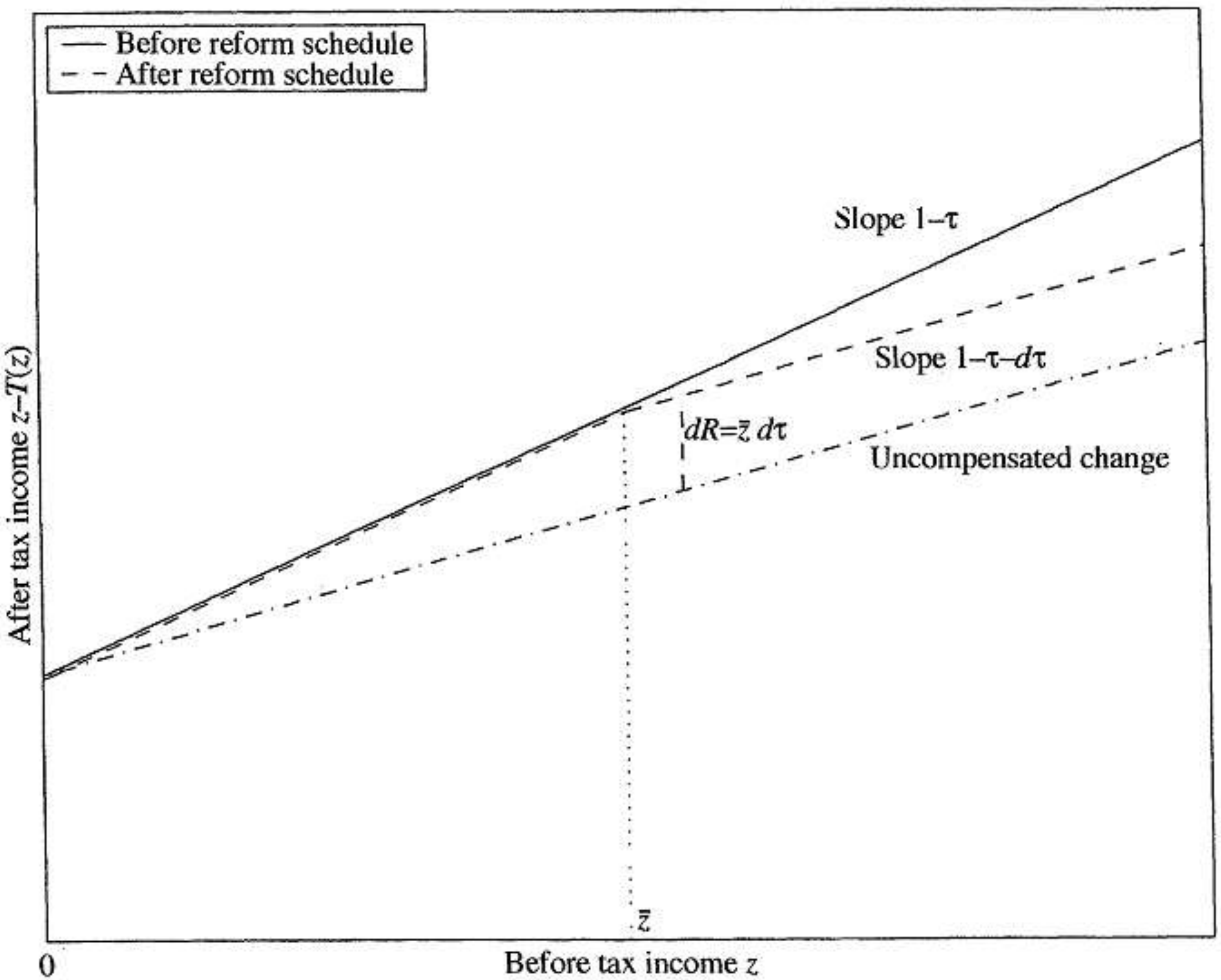
- *Compensated elasticity* of earnings : $\zeta^c = \frac{1-\tau}{z}\frac{dz}{d(1-\tau)}(u = cst)$: (purely substitution effects since it compensates for a change income)

- Slutsky equation: $\zeta^c = \zeta^u - \eta \geq 0$

Deriving the high income optimal tax rate

- Government sets top marginal rate τ for incomes above $\bar{z}$
- Population with income above $\bar{z}$ normalised to 1
- $h(z)$: density of earnings distribution at optimum tax regime
- Consider a small increase $d\tau$ in the top tax rate τ for incomes above $\bar{z}$

High income tax rate perturbation



Decomposing the change in total taxes paid

- Total taxes paid at income z above $\bar{z}$ = Marginal rate for incomes above $\bar{z}$ × Income above $\bar{z}$ + Total taxes paid at income $\bar{z}$
- $\Longrightarrow T(z) = \tau(z - \bar{z}) + T(\bar{z}\,)$
- $\Longrightarrow dT(z) = (z - \bar{z})d\tau + \tau dz$
- $\Longrightarrow \int_{\bar{z}}^{\infty} dT(z)h(z)dz = M + B$

- The total taxes paid therefore changes due to two things : a *mechanical effect* and *behavioural responses*

The mechanical effect

- *Mechanical effect*: The increase in tax receipts if there were no behavioural responses.

- Taxpayer with income $z > \bar{z}$ pays $(z - \bar{z})d\tau$ in additional taxes.

- Summing over population with $z > \bar{z}$, we have total mechanical effect on tax receipts: $M = (z_m - \bar{z})d\tau$

Behavioural responses

- As $z = z(1-\tau, R)$, therefore with total differential:
- $dz = -\frac{\partial z}{\partial(1-\tau)} d\tau + \frac{\partial z}{\partial R} dR$
- Let's express this in terms of *income effect* and *uncompensated elasticity :*
- $\eta = (1-\tau)\frac{dz}{dR} \Rightarrow \frac{\partial z}{\partial R} = \frac{\eta}{(1-\tau)}$
- $\zeta^u = \frac{dz}{z} / \frac{d(1-\tau)}{1-\tau} \Rightarrow -\frac{\partial z}{\partial(1-\tau)} = \frac{\zeta^u z}{1-\tau}$
- And as $dR = \bar{z} d\tau$ (overall increase in virtual income),
- Therefore: $dz = -(\zeta^u z - \eta \bar{z})\frac{d\tau}{1-\tau}$: reduction in individual *z's* earnings due to *behavioural responses*

Reduction in tax receipts due to behaviour responses

- As we saw, a reduction in earnings of *dz* implies a reduction in tax receipts of τdz, for one individual.
- This implies total that the total reduction in tax receipts is :

- $B = \int_{\bar{z}}^{\infty} -(\zeta^u z - \eta \bar{z}) \frac{\tau d\tau}{1-\tau} h(z) dz$

$$= -(\bar{\zeta}^u z_m - \bar{\eta} \bar{z}) \frac{\tau d\tau}{1 - \tau}$$

- Where $\bar{\zeta}^u$ is the weighted average of the uncompensated elasticity, and $\bar{\eta}$ the average income effect.

Obtaining the optimal tax rate

- Need to equalise the *revenue effect* (the sum of the mechanical effect and behavioural response) to the *welfare effect.*
- Compute *welfare effect* : Let $\bar{g}$ = Marginal social utility of money for top bracket tax payers divided by marginal value of public funds for government. Thus each additional dollar raised by government as a result of tax reduces on average social welfare of the top bracket by $\bar{g}$.
- Hence the total welfare loss due to tax reform is $\bar{g}$M.
- *Revenue effect = Welfare effect* ⇔ M+B = gM

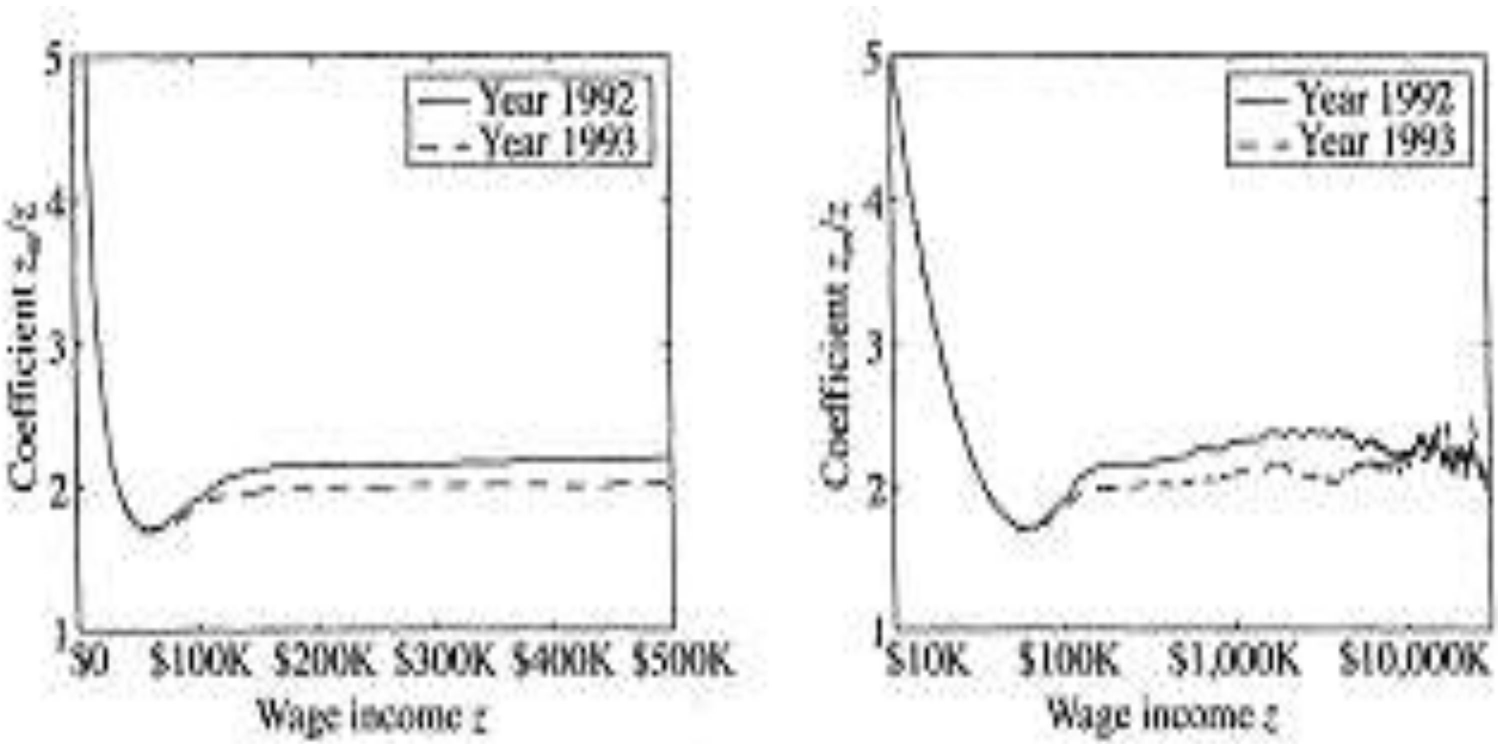
Interpretation

- Result: $\frac{\tau}{1-\tau} = \frac{(1-\bar{g})(z_m/\bar{z}-1)}{\bar{\zeta}^u z_m/\bar{z}-\bar{\eta}}$

- Decreasing function of $\bar{g}, \bar{\zeta}^u$, and increasing in $\bar{\eta}$.

- When $\bar{z}$ is close to the top, $z_m/\bar{z}$ tends to 1 $\Longrightarrow \tau$ tends to zero. This is because M is negligible compared to B near the top.

$z_m/\bar{z}$ for the U.S. in 1992/93 : Constant for high incomes $\Longrightarrow$ **Zero top result has no practical interest**



Pareto distributions

- Distributions with constant $z_m/\bar{z}$ ratio are exactly Pareto distributions.
- A Pareto distribution is such that: $Prob(Income > z) = (\bar{z}/z)^a$
- We have $E(Z) = z_m = \frac{a\bar{z}}{a-1} \Rightarrow \frac{z_m}{\bar{z}} = \frac{a}{a-1}$. For $z_m = 2, a = 2$.
- The higher *a*, the thinner is the tail of the income distribution

Rewriting the optimal marginal tax as a limiting tax for high incomes

- From $\frac{\tau}{1-\tau} = \frac{(1-\bar{g})(z_m/\bar{z}-1)}{\bar{\zeta}^u z_m/\bar{z}-\bar{\eta}}$:
- $\Rightarrow \bar{\tau} = \frac{1-\bar{g}}{1-\bar{g}+\bar{\zeta}^u+\bar{\zeta}^c(a-1)}$ with $\frac{z_m}{\bar{z}} = \frac{a}{a-1}$
- Decreasing function of *a* : thinner tail
- Role of elasticity effects vs income effects is visible
- $\bar{g} = 0, \bar{\zeta}^u = \bar{\zeta}^c$ gives the Laffer rate $\bar{\tau} = \frac{1}{1+\bar{\zeta}^c a}$.

Optimal tax rates for high earners (using asymptotic rate formula)

	Uncompensated elasticity = 0			Uncompensated elasticity = 0·2			Uncompensated elasticity = 0·5	
	Compensated elasticity			Compensated elasticity			Compensated elasticity	
	0·2	0·5	0·8	0·2	0·5	0·8	0·5	0·8
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Social marginal utility with infinite income g = 0								
Pareto parameter								
1·5	91	80	71	77	69	63	57	53
2	83	67	56	71	59	50	50	43
2·5	77	57	45	67	51	42	44	37
Panel B: Social marginal utility with infinite income g = 0·25								
Pareto parameter								
1·5	88	75	65	71	63	56	50	45
2	80	60	48	65	52	43	43	37
2·5	71	50	38	60	44	32	38	31

2.OPTIMAL NON-LINEAR INCOME TAX RATES FOR ANY TAX BRACKET

Initial specifications

- $H(z)$: Cumulated income distribution function i.e. the number of people with earnings below *z* (total population normalised to 1)
- $h(z)$: Density of the income distribution at *z*, i.e. the number of people earning *z*
- $\hat{h}(z)$: Virtual density : density of income distribution at *z* that would exist if the tax schedule were replaced by a linear tax schedule at *z*.
- $g(z)$: Social marginal value of consumption for taxpayers with income *z,* at optimum

Formula for optimal tax rate at level $\hat{z}$

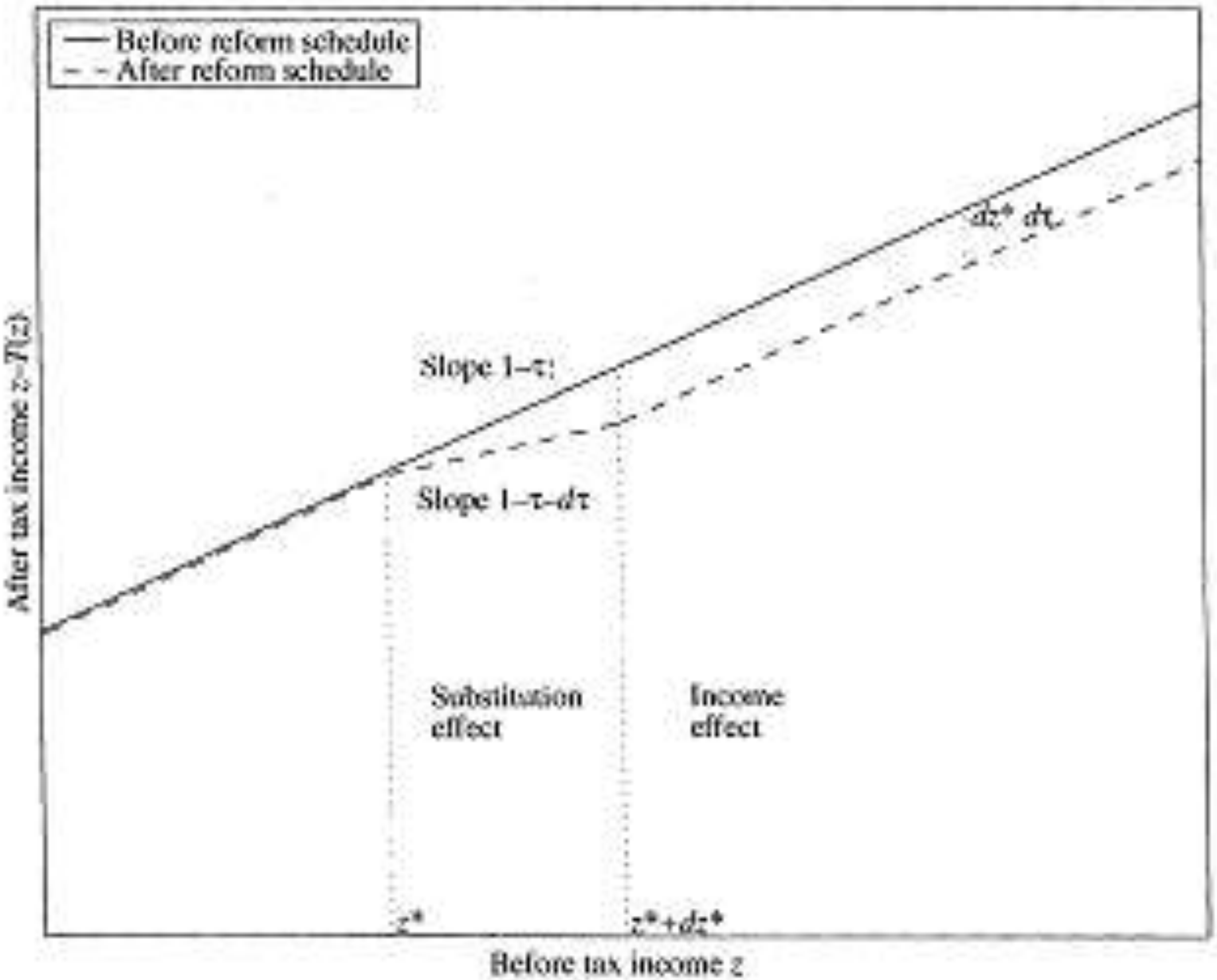
$$\frac{T'(\hat{z})}{1-T'(\hat{z})} =$$

$$\frac{1}{\zeta^{c}(\hat{z})} \times$$

$$\left(\frac{1-H(\hat{z})}{\hat{z}\widehat{h}(\hat{z})}\right) \times$$

$$\int_{\hat{z}}^{\infty} \big(1-g(z)\big) \exp\left[\int_{\hat{z}}^{z} \frac{1}{z'}\left(1-\frac{\zeta^{u}(z')}{\zeta^{c}(z')}\right) dz'\right] \frac{h(\hat{z})}{1-H(\hat{z})}\, dz$$

An increase in the marginal rate for $[\hat{z}, \hat{z} + d\hat{z}]$



Mechanical effect net of welfare loss

- Every taxpayer with income $z > \hat{z}$ pays $d\tau d\hat{z}$ additional taxes, which are valued $\big(1 - g(z)\big)d\tau d\hat{z}$ by the government.
- Therefore overall mechanical effect net of welfare loss is:
- $M = d\tau d\hat{z} \int_{\hat{z}}^{\infty} \big(1 - g(z)\big) h(z) dz$

Elasticity effect

- Two components:
- Direct *compensated elasticity effect* due to exogenous increase $d\tau$
- Indirect effect due to the shift of the taxpayer on the tax schedule by dz, inducing an endogenous additional change in marginal rates equal to $dT' = dT''dz$
- $dz = \zeta^c \hat{z} \frac{d\tau + dT'}{1 - T'}$.
- Using virtual density and summing:
- $\Rightarrow \; E = -\zeta^c \hat{z} \frac{T'}{1-T'} \widehat{h}(\hat{z}) d\tau d\hat{z}$

Income effect

- A taxpayer with income $z > \hat{z}$ pays $-dR = d\tau d\hat{z}$ additional taxes
- $\Longrightarrow$ Taxpayers above the bracket $[\hat{z}, \hat{z} + d\hat{z}]$ are induced to work more through *income effects*, which reinforce mechanical effect.
- Direct income effect $\eta\, dR/(1 - T')$
- Indirect elastic effect due to endogenous change in marginal rates $dT' = dT'' dz$
- $dz = -\zeta^c \hat{z} \frac{d\tau + dT'}{1 - T'} - \eta \frac{d\tau d\hat{z}}{1 - T\prime}$.
- Using virtual density and summing:
- $\Rightarrow I = \ d\tau d\hat{z} \int_{\hat{z}}^{\infty} -\eta \frac{T\prime}{1 - T\prime} \widehat{h}(z) dz$

Total effect of tax reform

- Revenue effect = Welfare effect therefore $M + E + I = 0$ giving differential equation:
- $\Rightarrow \frac{T\prime}{1-T\prime} = \frac{1}{\zeta^c}\left(\frac{1-H(\hat{z})}{\hat{z}\hat{h}(\hat{z})}\right)\left[\int_{\hat{z}}^{\infty}(1-g(z))\frac{h(z)}{1-H(\hat{z})}dz + \int_{\hat{z}}^{\infty} -\eta\frac{T\prime}{1-T\prime}\frac{\hat{h}(z)}{1-H(\hat{z})}dz\right]$
- By integration:
- $\frac{T\prime(\hat{z})}{1-T\prime(\hat{z})} = \frac{1}{\zeta^c(\hat{z})}\left(\frac{1-H(\hat{z})}{\hat{z}\hat{h}(\hat{z})}\right)\int_{\hat{z}}^{\infty}(1-g(z))\exp\left[\int_{\hat{z}}^{z}\frac{1}{z\prime}\left(1-\frac{\zeta^u(z')}{\zeta^c(z')}\right)dz'\right]\frac{h(\hat{z})}{1-H(\hat{z})}dz$

Formula for optimal tax rate at level $\hat{z}$

$$\frac{T'(\hat{z})}{1-T'(\hat{z})} =$$

$$\frac{1}{\zeta^{c}(\hat{z})} \times$$

$$\left(\frac{1-H(\hat{z})}{\hat{z}\widehat{h}(\hat{z})}\right) \times$$

$$\int_{\hat{z}}^{\infty} \big(1-g(z)\big) \exp\left[\int_{\hat{z}}^{z} \frac{1}{z'}\left(1-\frac{\zeta^{u}(z')}{\zeta^{c}(z')}\right) dz'\right] \frac{h(\hat{z})}{1-H(\hat{z})}\, dz$$

Interpretation

- Three elements determine optimal tax rates at $\hat{z}$:

- shape of the income distribution : $\left(\frac{1-H(\hat{z})}{\hat{z}\widehat{h}(\hat{z})}\right)$
- substitution/income effects : $\frac{1}{\zeta^c(\hat{z})}$ and $\exp\left[\int_{\hat{z}}^{z}\frac{1}{z\prime}\left(1-\frac{\zeta^u(z')}{\zeta^c(z')}\right)dz'\right]$
- and social marginal weights : $\left(1-g(z)\right)$

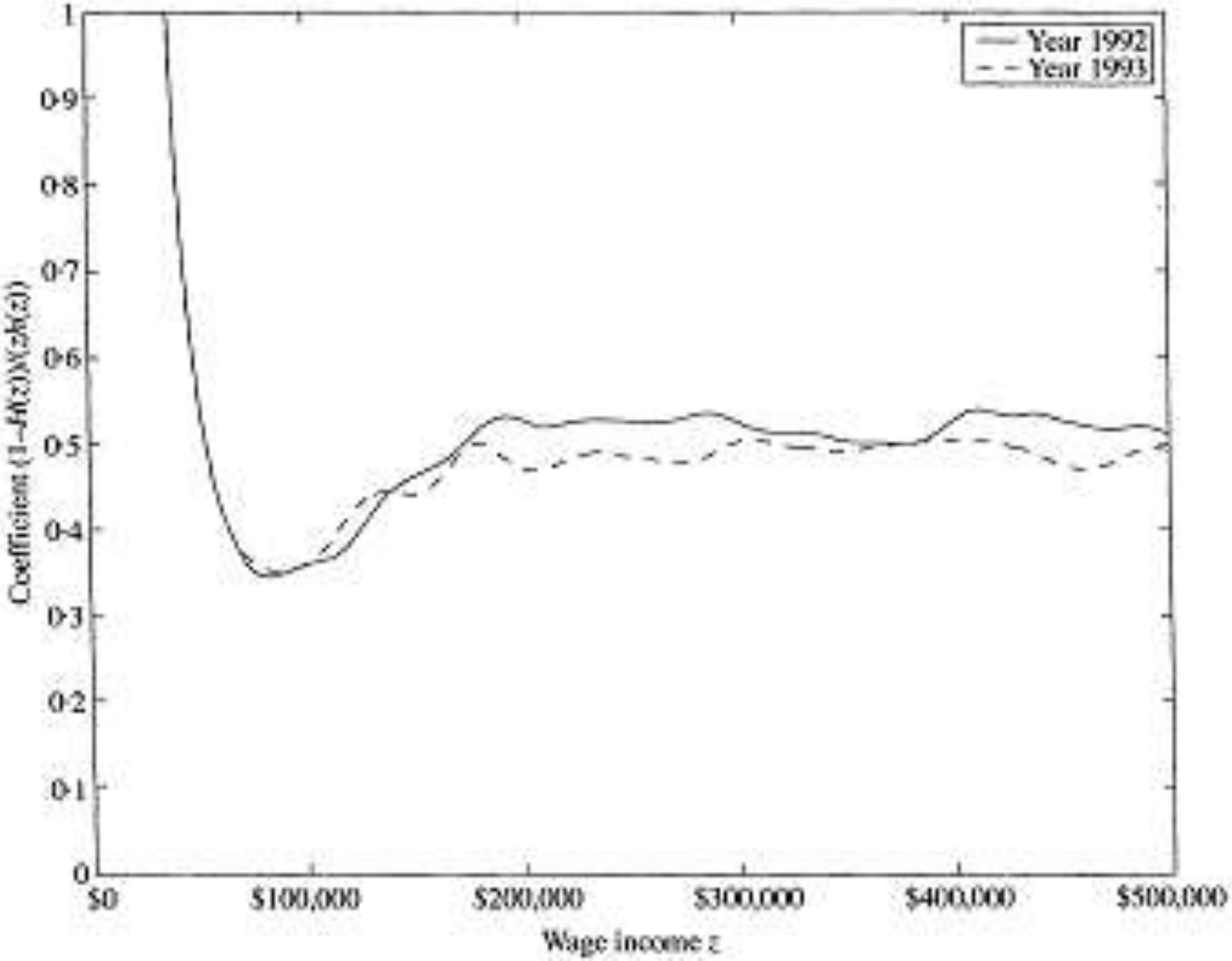
Shape of income distribution

- The shape of the income distribution: $\left(\frac{1-H(\hat{z})}{\hat{z}\widehat{h}(\hat{z})}\right)$
- The elastic distortion at $\hat{z}$ induced by marginal rate increase is proportional to income at that level times number of people at that level: $\hat{z}h(\hat{z})$.
- Gain in tax receipts is proportional to the number of people above $\hat{z}$: $1 - H(\hat{z})$
- $\Longrightarrow$ Government should apply high marginal rates at levels where the density of taxpayers is low compared to the number of taxpayers with higher income

Further explanation

- This is clearly the case at the bottom : $\hat{z}h(\hat{z})$ is close to 0 while $1 - H(\hat{z})$ is close to 1
- At the top, assuming a Pareto distribution of parameter *a*, $\frac{1-H(\hat{z})}{\hat{z}h(\hat{z})} = 1/a$
- For U.S., $a = 2 \Longrightarrow 1/a = 0.5$

Variations of $\frac{1-H(z)}{zh(z)}$ *across incomes*



Substitution and income effects

- Behavioural effects enter the formula in two ways:
- Compensated response from taxpayers (substitution effect) via compensated elasticity $\frac{1}{\zeta^c(\hat{z})}$
- Increase in the tax burden of taxpayers above $\hat{z}$ inducing them to work more (via exponential term which is larger than 1)

Social marginal welfare weights

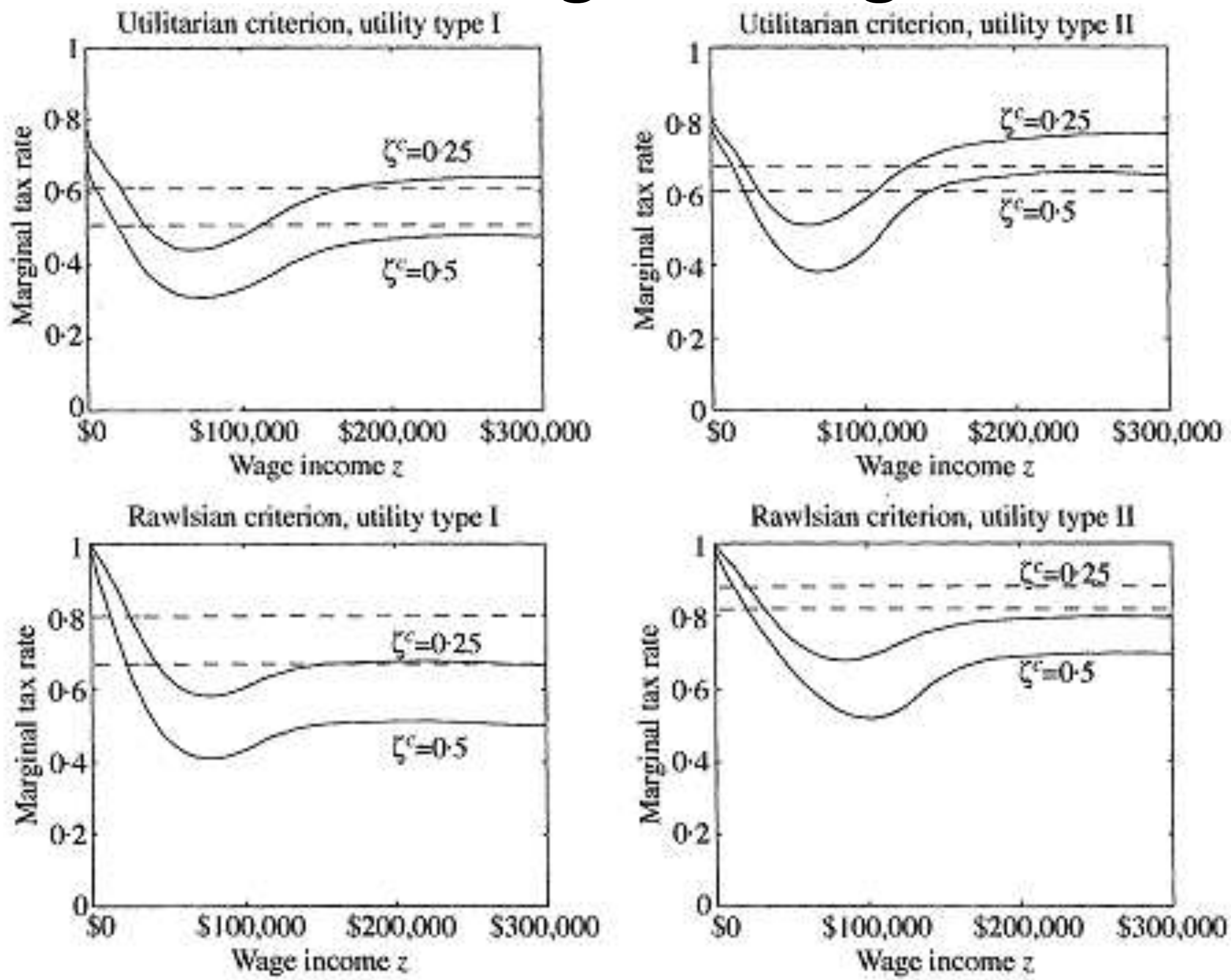
- Represented by the term $\left(1 - g(z)\right)$.
- $g(z)$: the relative value for the government of an additional dollar of consumption at income *z.*
- If $g(z)$ decreases with *z,* then the government has redistributive tastes.

3. NUMERICAL SIMULATIONS

Methodology

- Aim : To simulate the importance of substitution vs income effects and utilitarian vs Rawlsian social welfare weights
- Two utility functions:
- Type 1 : $u = \log\left(c - \frac{l^{1+k}}{1+k}\right)$, no income effects
- Type 2 : $u = \log(c) - \log\left(1 + \frac{l^{1+k}}{1+k}\right)$, with income effects.
- In both cases, constant compensated elasticity = 1/k
- Use of the skill distribution as exogenous measure of income distribution

Results : optimal non-linear & linear rates according to wage income



Results

- In all four cases optimal rates are U-shaped: close to actual tax schedules
- High rates for low *w* correspond to phasing-out of guaranteed income levels
- Income effects increase rates
- Higher compensated elasticity decreases rates
- Rawlsian criterion leads to higher rates, but difference between Rawlsian and utilitarian decreases for higher incomes

GENERAL CONCLUSIONS

- Elasticity estimates from the empirical literature suggest that top marginal rates should not be below 50% and can go as high as 80%.
- The elasticity method is fruitful as it precisely divides the individual impact of the shape of the income distribution, substitution and income effects, and redistributive tastes on the optimal marginal tax rate.