# The Origins of State Capacity

## Property Rights, Taxation, and Politics

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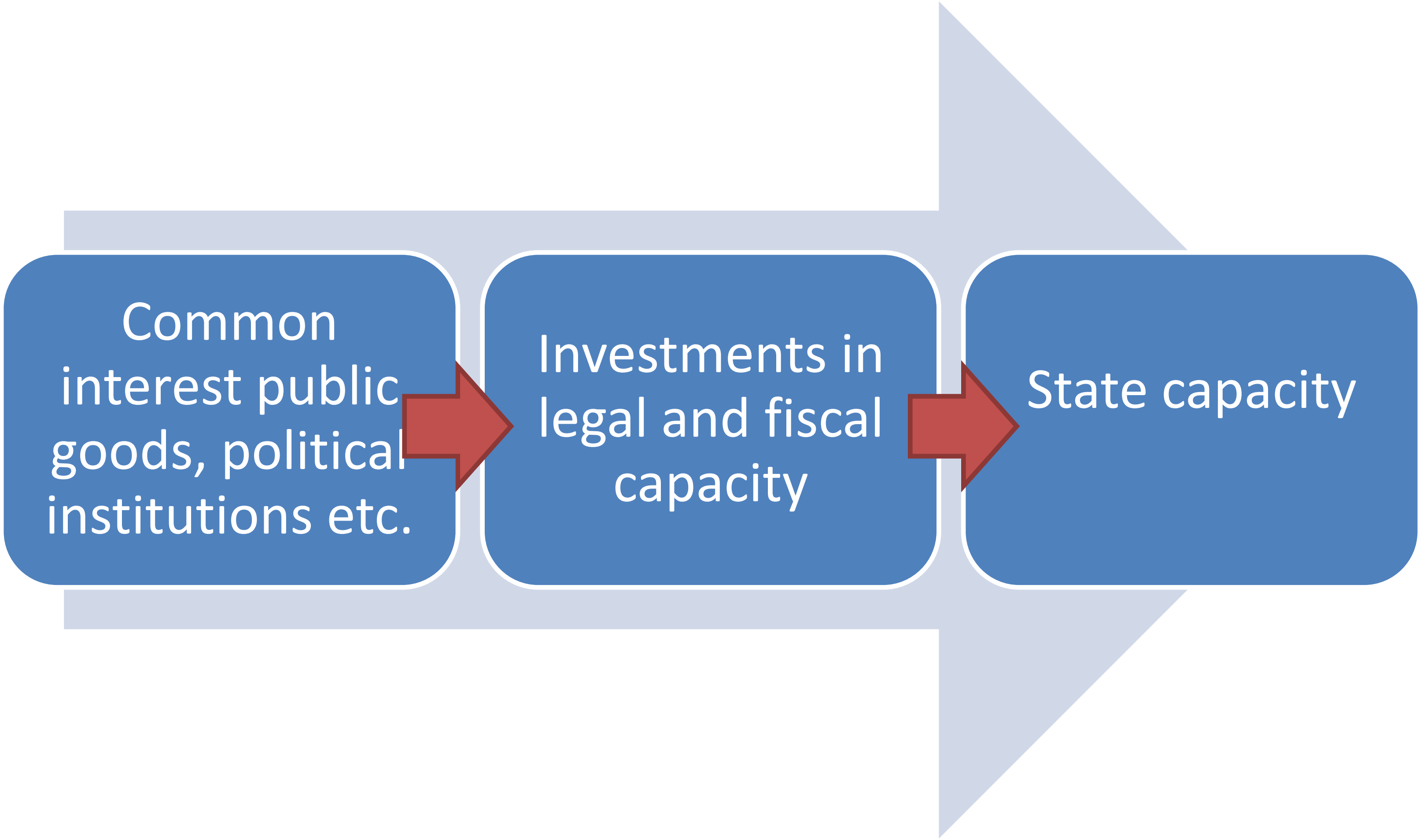
# Motivation

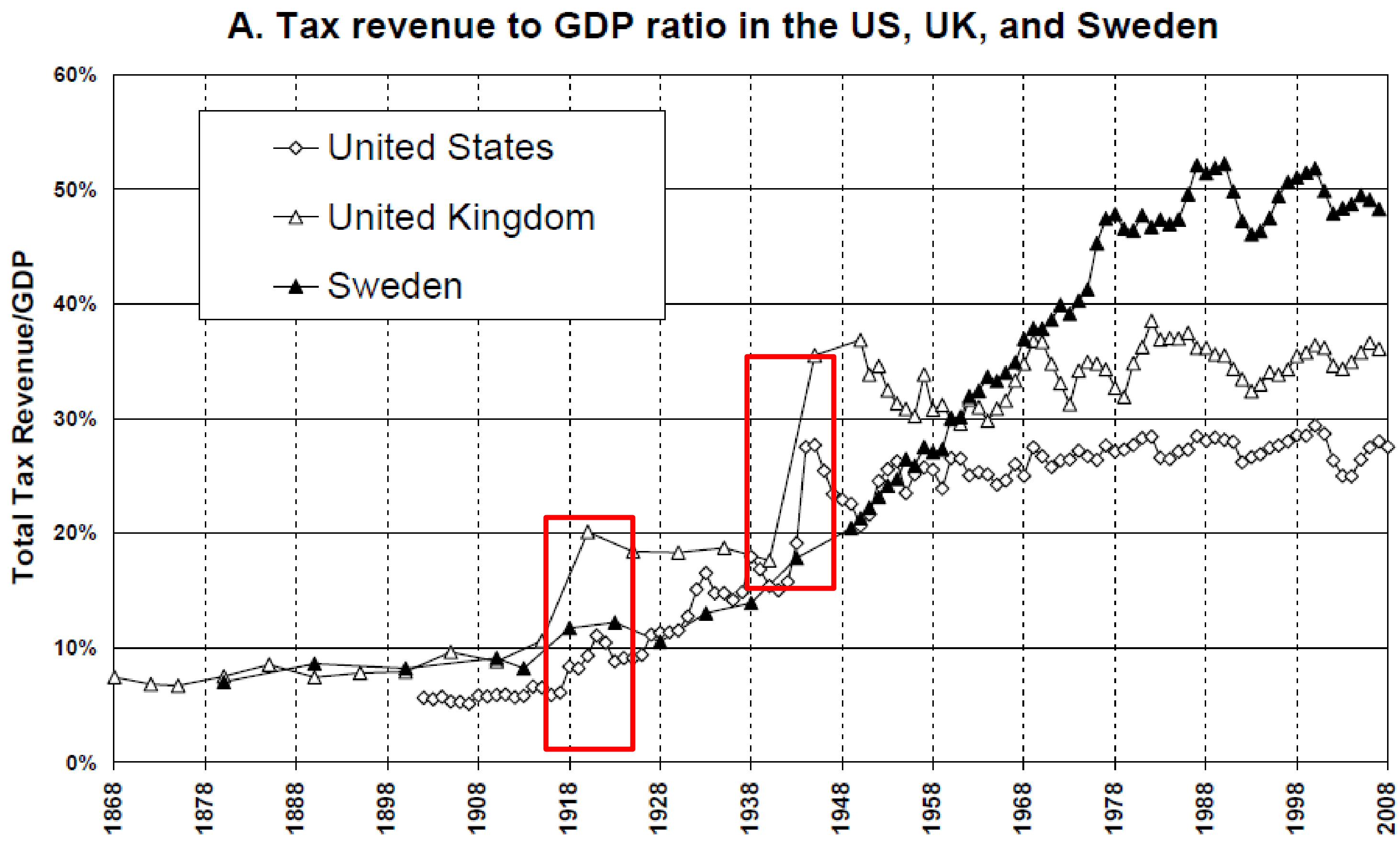
- What are the determinants of state capacity?
  - State capacity includes legal and fiscal capacity
    - Legal capacity: the state's ability to enforce contracts and property rights. In order words, just to make the market work well.
    - Fiscal capacity: the state's ability to raise revenue from taxes.

# Rich countries have higher state capacity

# Main Results

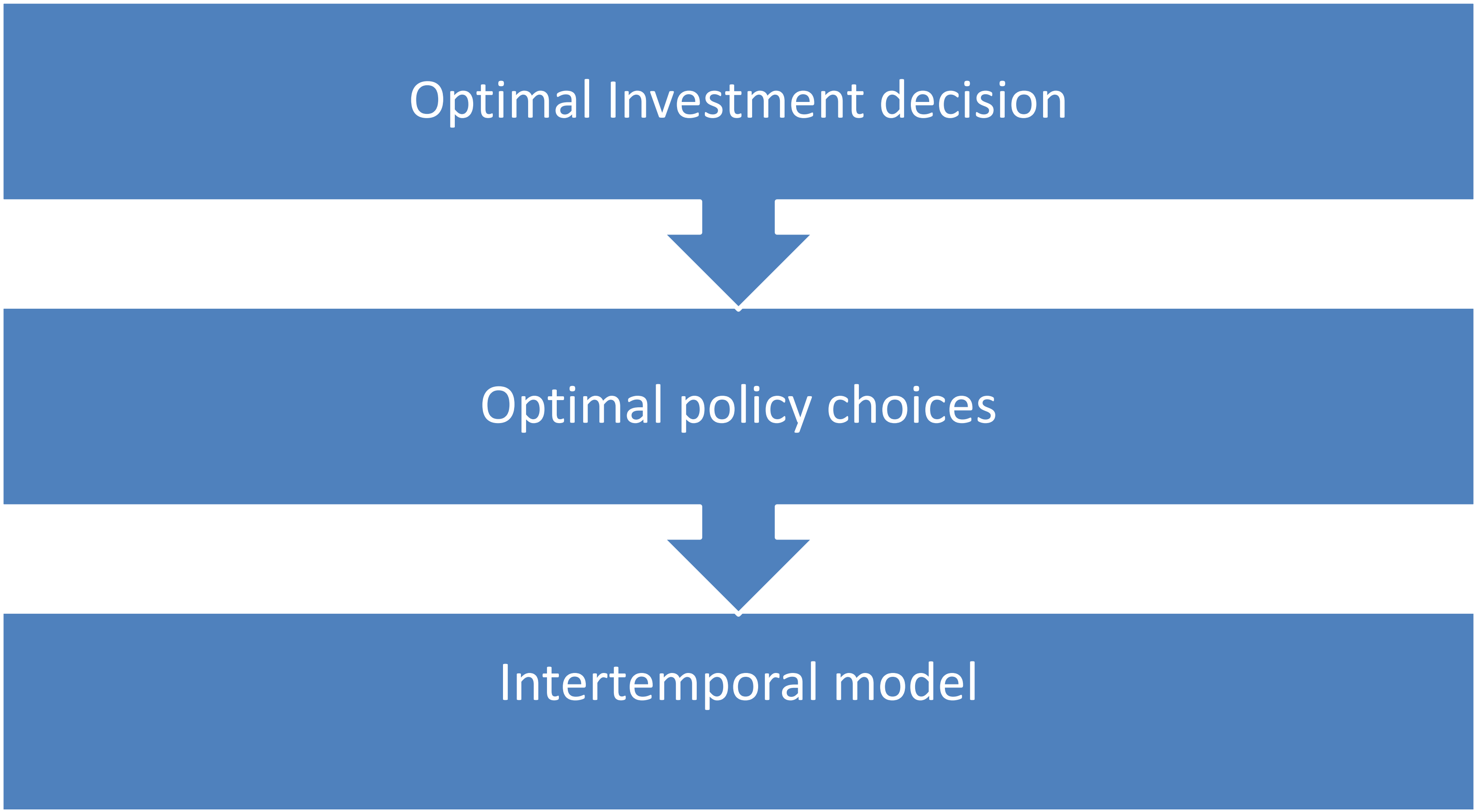


# state capacity evolved in response to war




Source: Kleven-Kreiner-Saez (2009)
Why can modern Governments Tax so much?

# Model


Optimal Investment decision
Optimal policy choices
Intertemporal model

# Model – Policy Choices

Objective function:

$$(8) \qquad \alpha_s G_s + \overline{\rho}\left(1 - t_s^J\right)\beta^J Y(p_s^J, \sigma^J, w^J) + \underline{\rho}\left(1 - t_s^K\right)\beta^K Y(p_s^K, \sigma^K, w^K),$$

Budget constraints:

$$(6) \qquad \sum_J t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) = G_1 + [L(\pi_2 - \pi_1) + F(\tau_2 - \tau_1)]$$

$$(7) \qquad \sum_J t_2^J \beta^J Y(p_2^J, \sigma^J, w^J) = G_2$$

Institutional constraints:

$$p_s^J \leq \pi_s,\ p_s^K \leq \pi_s,\ t_s^J \leq \tau_s \text{ and } t_s^K \leq \tau_s\ .$$

# Model – Policy Choices

- Solutions

$$p_s^J = p_s^J = \tau_S$$

$$\alpha_s \geq \bar{\rho} \longrightarrow t_s^J = t_s^K = \tau_s \qquad G_1 = \tau_1 Y_1 - L\left(\pi_2 - \pi_1\right) - F\left(\tau_2 - \tau_1\right) \qquad G_2 = \tau_2 Y_2$$

$$\alpha_2 < \bar{\rho} \qquad \bar{\rho} = \underline{\rho} = 1 \longrightarrow G_s = 0 \qquad t_2^J = t_2^K = 0 \qquad t_1^J = t_1^K = \hat{t}_1$$

$$\bar{\rho} > 1 > \underline{\rho} \longrightarrow G_s = 0 \quad t_1^K = \tau_1 \quad t_2^K = \tau_2$$

$$t_1^J = \frac{\left[L\left(\pi_2 - \pi_1\right) + F\left(\tau_2 - \tau_1\right)\right] - \tau_1 \beta^K Y\left(\pi_1, \sigma^K, \omega^K\right)}{\beta^J Y\left(\pi_1, \sigma^J, \omega^J\right)} \qquad t_2^J = -\frac{\tau_2 \beta^K Y\left(\pi_2, \sigma^K, \omega^K\right)}{\beta^J Y\left(\pi_2, \sigma^J, \omega^J\right)}$$

# Model – Investment in State Capacity

Expected payoff to group J who hold the power

$$W^J(\tau_2,\pi_2)=\gamma^J E\left\{w_J^J(\alpha_2,\tau_2,\pi_2)\right\}+\left(1-\gamma^J\right)E\left\{w_K^J(\alpha_2,\tau_2,\pi_2)\right\}$$

$$\begin{aligned} W^J(\tau_2,\pi_2) = & (1-\tau_2)[\overline{\rho}\beta^J Y(\pi_2,\sigma^J,\omega^J)+\underline{\rho}\beta^K Y(\pi_2,\sigma^K,\omega^K)] \\ & +\tau_2\{([1-H(\overline{\rho})]E(\alpha_2|\alpha_2\geq\overline{\rho}) \\ & +H(\overline{\rho})[\gamma^J\overline{\rho}+\left(1-\gamma^J\right)\underline{\rho}])[\beta^J Y(\pi_2,\sigma^J,\omega^J)+\beta^K Y(\pi_2,\sigma^K,\omega^K)]\}\,. \end{aligned} \tag{9}$$

- Maximizing:

$$W^J(\tau_2,\pi_2)-\lambda(\alpha_1)\left[L(\pi_2-\pi_1)+F(\tau_2-\tau_1)\right]$$

# FOCs

$$(12)\quad \left(\rho^J + \tau_2\lambda_2^J\right)\left(r_H - r_L\right)\Omega \le \lambda\left(\alpha_1\right)L_\pi\left(\pi_2 - \pi_1\right)$$
$$c.s.\pi_2 - \pi_1 \ge 0$$

$$(13)\quad \lambda_2^J\left[\left(1+\pi_2\right)\left(r_H - r_L\right)\Omega + r_L\left(\beta^J w^J + \beta^K w^K\right)\right] + \frac{r_L \cdot \left(\bar{\rho} - \underline{\rho}\right)\cdot \beta^J w^J \beta^K w^K \left(\sigma^J - \sigma^K\right)}{\Omega} \le \lambda\left(\alpha_1\right)F_\tau\left(\tau_2 - \tau_1\right)$$
$$c.s.\tau_2 - \tau_1 \ge 0.$$

$$(10)\quad \lambda_2^J = \left[1 - H\left(\bar{\rho}\right)\right]\left[E\left(\alpha_2 \middle| \alpha_2 \ge \bar{\rho}\right) - \bar{\rho} - \omega^J \cdot \left(\bar{\rho} - \underline{\rho}\right)\right] + H\left(\bar{\rho}\right)\left[\left(\gamma^J - \omega^J\right)\left(\bar{\rho} - \underline{\rho}\right)\right]$$

$$(11)\quad \rho^J = \underline{\rho} + \omega^J(\overline{\rho} - \underline{\rho}) .$$

Any factor that raises the value of the left hand side of both (12) and (13) will raise investments in both forms of state capacity.

# Propositions

- Proposition 4 : Higher wealth higher investment
- Proposition 5: Higher expected demand for public goods higher investment
- Proposition 6: Higher political stability higher investment $\frac{\partial \lambda_2^J}{\partial \gamma^J} = H(\overline{\rho})\left(\overline{\rho} - \underline{\rho}\right) \geq 0$
- Proposition 7: More representative more investment
- Proposition 8: Greater economic power of the ruling group higher investment in legal capacity and lower investment in fiscal capacity

# Implications for Economic Growth

$$\frac{Y_2 - Y_1}{Y_1} = \frac{(\pi_2 - \pi_1)(r_H - r_L)\Omega}{(1 + \pi_1)(r_H - r_L)\Omega + r_L \sum_J \beta^J \omega^J} .$$

- The growth rate is directly proportional to the investments in legal capacity.

# A look at the Data

Table 1: Economic and Political Determinants of Legal Capacity

| | (1)<br>Private Credit to GDP | (2)<br>Ease of Access to Credit<br>(country rank) | (3)<br>Investor Protection<br>(country rank) | (4)<br>Index of Government<br>Anti-diversion Policies |
|---|---|---|---|---|
| Incidence of External Conflict up to 1975 | 0.510***<br>(0.143) | 0.647**<br>(0.191) | 0.029<br>(0.209) | 0.576***<br>(0.170) |
| Incidence of Democracy up to 1975 | 0.953<br>(0.059) | 0.110<br>(0.267) | - 0.044<br>(0.078) | 0.126**<br>(0.050) |
| Incidence of Parliamentary Democracy up to 1975 | 0.001<br>(0.063) | 0.145<br>(0.114) | 0.339**<br>(0.137) | 0.112*<br>(0.061) |
| English Legal Origin | – 0.009<br>(0.033) | 0.068<br>(0.057) | 0.125**<br>(0.063) | – 0.007<br>(0.040) |
| Socialist Legal Origin | - | 0.098<br>(0.111) | 0.097<br>(0.115) | 0.010***<br>(0.035) |
| German Legal Origin | 0.406***<br>(0.120) | 0.295***<br>(0.064) | - 0.008<br>0.149) | 0.248***<br>(0.053) |
| Scandinavian Legal Origin | 0.112***<br>(0.041) | 0.204***<br>(0.067) | 0.087<br>(0.098) | 0.254***<br>(0.055) |
| Observations | 93 | 122 | 120 | 115 |
| R-squared | 0.524 | 0.334 | 0.256 | 0.596 |

Notes to Table: Robust standard errors in parentheses: * significant at 10%; ** significant at 5%; *** significant at 1%.
Socialist legal origin is dropped in column 1 due to Private Credit to GDP being missing for all countries in this category.

# A look at the Data

Table 2: Economic and Political Determinants of Fiscal Capacity

| | (1) One Minus Share of Trade Taxes in Total Taxes | (2) One Minus Share of Trade and Indirect Taxes in Total Taxes | (3) Share of Income Taxes in GDP | (4) Share of Taxes in GDP |
|---|---|---|---|---|
| Incidence of External Conflict up to 1975 | 0.762*** (0.250) | 0.598*** (0.241) | 0.579*** (0.220) | 0.555*** (0.162) |
| Incidence of Democracy up to 1975 | 0.143 (0.077) | – 0.078 (0.100) | 0.091 (0.059) | 0.088 (0.059) |
| Incidence of Parliamentary Democracy up to 1975 | 0.031 (0.083) | 0.122 (0.103) | 0.212*** (0.078) | 0.160** (0.068) |
| English Legal Origin | – 0.038 (0.058) | - 0.012 (0.061) | – 0.034 (0.043) | - 0.015 (0.042) |
| Socialist Legal Origin | 0.136** (0.058) | – 0.222*** (0.037) | – 0.109*** (0.065) | – 0.119 (0.031) |
| German Legal Origin | 0.175*** (0.052) | 0.196*** (0.090) | 0.171* (0.010) | 0.010*** (0.083) |
| Scandinavian Legal Origin | 0.189** (0.077) | 0.068** (0.084) | 0.258** (0.134) | 0.292*** (0.087) |
| Observations | 103 | 103 | 103 | 103 |
| R-squared | 0.356 | 0.305 | 0.600 | 0.576 |

Notes to Table: Robust standard errors in parentheses: * significant at 10%; ** significant at 5%; *** significant at 1%

Thank you!

Appendix

From (9) to (12).

$$\partial W^{J} / \partial \pi_z = (1-\tau_z)\bar{\rho}\beta^J\sigma^J w^J(r_H-r_L) + (1-\tau_z)\underline{\rho}\beta^K\sigma^K w^K(r_H-r_L) +$$

$$\tau_z\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z \geq \bar{\rho}) + H(\bar{\rho})[r^J\bar{\rho}+(1-r^J)\underline{\rho}]\} \cdot$$

$$\{[\beta^J\sigma^J w^J(r_H-r_L) + \beta^K\sigma^K w^K(r_H-r_L)]\}$$

$$= \Omega(r_H-r_K) \cdot \Big[\{(1-\tau_z)\bar{\rho} + \tau_z\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z \geq \bar{\rho}) + H(\bar{\rho})[r^J\bar{\rho}+(1-r^J)\underline{\rho}]\}\} \cdot W^J$$

$$+ \{(1-\tau_z)\underline{\rho} + \tau_z\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z \geq \bar{\rho}) + H(\bar{\rho})[r^J\bar{\rho}+(1-r^J)\underline{\rho}]\}\} \cdot W^K\Big]$$

$$= \Omega(r_H-r_K)\{\underline{\rho} + W^J(\bar{\rho}-\underline{\rho}) + \tau_z(\underline{\rho}W^J - \underline{\rho} - \bar{\rho}W) +$$

$$\tau_z\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z \geq \bar{\rho}) + H(\bar{\rho})[r^J\bar{\rho}+(1-r^J)\underline{\rho}]\}\}$$

$$= \Omega(r_H-r_K)\{\rho^J + \tau_z\lambda_z^J\}$$

where $\lambda_z^J = \{[1-H(\bar{\rho})][E(\alpha_z|\alpha_z \geq \bar{\rho}) + W^J(\bar{\rho}-\underline{\rho}) - \underline{\rho}] + H(\bar{\rho})(r^J - W^J)(\bar{\rho}-\underline{\rho})\}$

$$\rho^J = \underline{\rho} + W^J(\bar{\rho}-\underline{\rho})$$

$$W^J = \frac{\sigma^J w^J \beta^J}{\Omega} \qquad W^K = \frac{\sigma^K w^K \beta^K}{\Omega}$$

$$\Omega = \sigma^J w^J \beta^J + \sigma^K w^K \beta^K$$

From (9) to (13)

$$\partial W^J / \partial \tau_z = -\bar{\rho}\,\beta^J[\sigma^J(1+\pi_z)(r_H-r_L)+r_L]W^J - \underline{\rho}\,\beta^K[\sigma^K(1+\pi_z)(r_H-r_L)+r_L]W^K$$

$$+\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho})+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}]\}\cdot$$

$$\cdot\{\beta^J[\sigma^J(1+\pi_z)(r_H-r_L)+r_L]W^J+\beta^K[\sigma^K(1+\pi_z)(r_H-r_L)+r_L]W^K\}$$

$$=\Big\{\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho})+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}]-\bar{\rho}\}\cdot[W^J(1+\pi_z)(r_H-r_L)+r_L\beta^J W^J]\cdot$$

$$+\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho})+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}-\underline{\rho}\}\cdot[W^K(1+\pi_z)(r_H-r_L)+r_L\beta^K W^K]\Big\}\Omega$$

$$=\Big\{\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho}+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}]-(\bar{\rho}W^J+\underline{\rho}(1-W^J))\}(1+\pi_z)(r_H-r_L)$$

$$+\Big\{\{[1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho}+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}]-\bar{\rho}\}W^J\frac{r_L}{\sigma^J}\cdot$$

$$+\{(1-H(\bar{\rho})]E(\alpha_z|\alpha_z\geq\bar{\rho}+H(\bar{\rho})[\gamma^J\bar{\rho}+(1-\gamma^J)\underline{\rho}]-\underline{\rho}\}W^K\frac{r_L}{\sigma^K}\Big\}\Omega$$

If $\sigma^K=\sigma^J$

$$=\lambda_z^J[(1+\pi_z)(r_H-r_L)\Omega+r_L(\beta^J W^J+\beta^K W^K)]$$

where $\lambda_z^J=\{(1-H(\bar{\rho})][E(\alpha_z|\alpha_z\geq\bar{\rho})+W^J(\bar{\rho}-\underline{\rho})-\underline{\rho}]+H(\bar{\rho})(\gamma^J-W^J)(\bar{\rho}-\underline{\rho})\}$

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# Origins of State Capacity (Besley and Persson)

## Derivations of Formulas

### Deriving (4), p. 6, Besley and Persson

From (2) and (3)

Income of a high-return individual in group J

$$\left[ r_H + p_S^J (r_H - r_L) \right] w^J$$

Income of a low-return individual in group J

$$r_L w^J$$

$\sigma^J$: share of group J agents with high returns

⇒ Average income of an individual in group J:

$$\sigma^J \left[ r_H + p_S^J (r_H - r_L) \right] w^J + (1 - \sigma^J) r_L w^J$$

$$= \sigma^J \left[ r_H + p_S^J (r_H - r_L) - r_L \right] w^J + r_L w^J$$

$$= \left[ \sigma^J (1 + p_S^J)(r_H - r_L) + r_L \right] \cdot w^J$$

## Proof of Propositions (1) and (2) (p. 8 and p. 9)

The Incumbent Government has the following objective function

$$f(t_s^J, t_s^K, p_s^J, p_s^K, G_s)$$

$$= \alpha_s G_s + \bar{\rho}(1-t_s^J)\beta^J Y(p_s^J, \sigma^J, w^J) + \underline{\rho}(1-t_s^K)\beta^K Y(p_s^K, \sigma^K, w^K)$$

subject to the following constraints

$$p_s^J, p_s^K \leq \Pi_s, \quad t_s^J, t_s^K \leq \tau_s$$

$$\sum_J t_1^J \beta^J Y(p_s^J, \sigma^J, w^J) = G_1 + L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1)$$

$$\sum_J t_2^J \beta^J Y(p_s^J, \sigma^J, w^J) = G_2$$

Substitute $G_1 = \sum_J t_1^J \beta^J Y(p_s^J, \sigma^J, w^J) - L(\Pi_2 - \Pi_1) - F(\tau_2 - \tau_1)$ and $G_2$

into the objective function, we obtain:

$$f(t_s^J, t_s^K, p_s^J, p_s^K)$$

$$= \alpha_s \cdot \Big[ t_s^J \beta^J Y(p_s^J, \sigma^J, w^J) + t_s^K \beta^K Y(p_s^K, \sigma^K, w^K) - \underbrace{L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1)}_{\text{if } s=1} \Big]$$

$$+ \bar{\rho}(1-t_s^J)\beta^J Y(p_s^J, \sigma^J, w^J) + \underline{\rho}(1-t_s^K)\beta^K Y(p_s^K, \sigma^K, w^K)$$

$$= (\alpha_s - \bar{\rho})\, t_s^J \beta^J Y(p_s^J, \sigma^J, w^J) + (\alpha_s - \underline{\rho})\, t_s^K \beta^K Y(p_s^K, \sigma^K, w^K)$$

$$+ \bar{\rho}\beta^J Y(p_s^J, \sigma^J, w^J) + \underline{\rho}\beta^K Y(p_s^K, \sigma^K, w^K) - \underbrace{\alpha_s \big[L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1)\big]}_{\text{if } s=1}$$

with $Y(p_s^J, \sigma^J, w^J) = \left[\sigma^J (1 + p_s^J)(r_H - r_L) + r_L\right] w^J$

We maximize this (the last) expression of $f(t_s^J, t_s^K, p_s^J, p_s^K)$

subject to the following constraints $p_s^J \leq \Pi_s$, $p_s^K \leq \Pi_s$, $t_s^K \leq \tau_s$, $t_s^J \leq \tau_s$

Derived from the budget constraints. $\Leftarrow$

$$\begin{cases} -\sum_J t_1^J \beta^J Y(p_s^J, \sigma^J, w^J) \leq L(\Pi_2 - \Pi_1) - F(\tau_2 - \tau_1) \\ -\sum_J t_2^J \beta^J Y(p_s^J, \sigma^J, w^J) \leq 0 \end{cases}$$

Period 1 :

$$L_1 = f(t_1^J, t_1^K, p_1^J, p_1^K) - \lambda_1(p_1^J - \Pi_1) - \lambda_2(p_1^K - \Pi_1) - \lambda_3(t_1^J - \tau_1) - \lambda_4(t_1^K - \tau_1)$$
$$- \lambda_5\left[-\sum t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) + L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1)\right]$$

FOCs

$$\frac{\partial L_1}{\partial p_1^J} = \frac{\partial f}{\partial p_1^J} - \lambda_1 + \lambda_5 \cdot t_1^J \beta^J \frac{\partial Y(p_1^J, \sigma^J, w^J)}{\partial p_1^J}$$

$$= \left[(\alpha_1 - \bar{\rho})t_1^J + \bar{\rho}\right] \cdot \beta^J \cdot \frac{\partial Y(p_1^J, \sigma^J, w^J)}{\partial p_1^J} - \lambda_1 + \lambda_5 t_1^J \beta^J \frac{\partial Y(p_1^J, \sigma^J, w^J)}{\partial p_1^J}$$

$$= \left[(\alpha_1 - \bar{\rho} + \lambda_5)t_1^J + \bar{\rho}\right] \cdot \beta^J \cdot \sigma^J w^J (r_H - r_L) - \lambda_1$$
$$= \left[(\alpha_1 + \lambda_5)t_1^J + (1 - t_1^J)\bar{\rho}\right] \beta^J \sigma^J w^J (r_H - r_L) - \lambda_1 = 0$$

$$\frac{\partial L_1}{\partial p_1^K} = \left[(\alpha_1 - \underline{\rho} + \lambda_5)t_1^K + \underline{\rho}\right] \cdot \beta^K \sigma^K w^K (r_H - r_L) - \lambda_2$$
$$= \left[(\alpha_1 + \lambda_5)t_1^K + (1 - t_1^K)\underline{\rho}\right] \cdot \beta^K \sigma^K w^K (r_H - r_L) - \lambda_2 = 0$$

$$\frac{\partial L_1}{\partial t_1^J} = (\alpha_1 - \bar{\rho})\beta^J Y(p_1^J, \sigma^J, w^J) - \lambda_3 + \lambda_5 \cdot \beta^J Y(p_1^J, \sigma^J, w^J)$$

$$(\alpha_1 - \bar{\rho} + \lambda_5) \cdot \beta^J \left[\sigma^J (1 + p_1^J)(r_H - r_L) + r_L\right] w^J - \lambda_3 = 0$$

$$\frac{\partial L_1}{\partial t_1^K} = (\alpha_1 - \underline{\rho} + \lambda_5)\beta^K \left[\sigma^K (1 + p_1^K)(r_H - r_L) + r_L\right] w^K - \lambda_4 = 0$$

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \geq 0$

$$\lambda_1 (p_1^J - \Pi_1) = 0$$

$$\lambda_2 (p_1^K - \Pi_1) = 0$$

$$\lambda_3 (t_1^J - \tau_1) = 0$$

$$\lambda_4 (t_1^K - \tau_1) = 0$$

$$\lambda_5 \left[ -\sum t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) + L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1) \right] = 0$$

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Since the first term of $\frac{\partial L_1}{\partial p_1^J} > 0 \Rightarrow \lambda_1 > 0 \Rightarrow p_1^J - \Pi_1 = 0 \Rightarrow \boxed{p_1^J = \Pi_1}$

the first term of $\frac{\partial L_1}{\partial p_1^K} > 0 \Rightarrow \lambda_2 > 0 \Rightarrow p_1^K - \Pi_1 = 0 \Rightarrow \boxed{p_1^K = \Pi_1}$

(*) <u>If $\bar{\rho} = \underline{\rho} = 1$</u>

- If $\alpha_1 > 1$ $\Rightarrow$ the first term of $\frac{\partial L_1}{\partial t_1^J} > 0 \Rightarrow \lambda_3 > 0 \Rightarrow \boxed{t_1^J = \tau_1}$

  the first term of $\frac{\partial L_1}{\partial t_1^K} > 0 \Rightarrow \lambda_4 > 0 \Rightarrow \boxed{t_1^K = \tau_1}$

  $\Rightarrow \boxed{G_1 = \tau_1 \sum_J \beta^J Y(\Pi_1, \sigma^J, w^J) - L(\Pi_2 - \Pi_1) - F(\tau_2 - \tau_1)}$

- If $\alpha_1 < 1$ $\Rightarrow$ From the expression $\frac{\partial L_1}{\partial t_1^J}$, and $\lambda_3 \geq 0 \Rightarrow \alpha_1 - \bar{\rho} + \lambda_5 \geq 0$

  $\Rightarrow \lambda_5 > 0$

  $\Rightarrow \sum_J t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) = L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1) \Rightarrow \boxed{G_1 = 0}$

  $\Rightarrow$ Model predicts incumbent governments indifference to any $(t_1^J, t_2^J)$ satisfying the above equation. Authors <u>"assumed, without loss of generality"</u>, $t_1^J = t_2^J = \hat{t}_1$ (p.9)

- If $\alpha_1 = 1$ $\Rightarrow$ May have $\lambda_5 = \lambda_3 = \lambda_4 = 0 \Rightarrow t_1^J \leq \tau_1, t_1^K \leq \tau_1,$

  $\sum_J t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) \geq L(\Pi_2 - \Pi_1) + F(\tau_2 - \tau_1)$

(*) <u>If $\bar{\rho} > \underline{\rho}$ :</u>

- If $\alpha_1 > \bar{\rho}$ $\Rightarrow$ the 1st term of $\frac{\partial L_1}{\partial t_1^J} > 0 \Rightarrow \lambda_3 > 0 \Rightarrow \boxed{t_1^J = \tau_1}$

  the 1st term of $\frac{\partial L_1}{\partial t_1^K} > 0 \Rightarrow \lambda_4 > 0 \Rightarrow \boxed{t_1^K = \tau_1}$

$$\Rightarrow \boxed{G_1 = \tau_1 \sum_J \beta^J Y(\pi_1, \sigma^J, w^J) - L(\pi_2 - \pi_1) - F(\tau_2 - \tau_1)}$$

• If $\alpha_1 < \bar{\rho}$ : From the expression of $\frac{\partial L_1}{\partial t_1^J}$, and from $\lambda_3 \geq 0$

$$\Rightarrow \alpha_1 - \bar{\rho} + \lambda_5 \geq 0 \quad \Rightarrow \lambda_5 > 0$$

$$\Rightarrow \sum_J t_1^J \beta^J Y(\pi_1, \sigma^J, w^J) = L(\pi_2 - \pi_1) + F(\tau_2 - \tau_1)$$

$$\Rightarrow \boxed{G_1 = 0}$$

Also, since $\alpha_1 - \bar{\rho} + \lambda_5 \geq 0$

and $\alpha_1 - \underline{\rho} + \lambda_5 > \alpha_1 - \bar{\rho} + \lambda_5$ (because $\bar{\rho} > \underline{\rho}$)

$$\Rightarrow \alpha_1 - \underline{\rho} + \lambda_5 > 0$$

$\Rightarrow$ the first term of $\frac{\partial L_1}{\partial t_1^K} > 0 \Rightarrow \lambda_4 > 0$

$$\Rightarrow \boxed{t_1^K = \tau_1}$$

$$\Rightarrow \boxed{t_1^J = \frac{L(\pi_2 - \pi_1) + F(\tau_2 - \tau_1) - \tau_1 \beta^K Y(\pi_1, \sigma^K, w^K)}{\beta^J Y(\pi_1, \sigma^J, w^J)}}$$

• If $\alpha_1 = \bar{\rho}$ $\Rightarrow$ the first term of $\frac{\partial L_1}{\partial t_1^K} > 0 \Rightarrow \lambda_4 > 0 \Rightarrow \boxed{t_1^K = \tau_1}$

May have $\lambda_3 = \lambda_5 = 0$,

$G_1 \geq 0$

$$\sum_J t_1^J \beta^J Y(\pi_1, \sigma^J, w^J) \geq L(\pi_2 - \pi_1) + F(\tau_2 - \tau_1)$$

and $t_1^J \leq \tau_1$

## Period 2

Since the optimization problem of period 1 differs from that of period 2 only in the budget constraint

in period 1: $-\sum_J t_1^J \beta^J Y(p_1^J, \sigma^J, w^J) \leq -L(\Pi_2 - \Pi_1) - F(\tau_2 - \tau_1)$

in period 2: $-\sum_J t_2^J \beta^J Y(p_2^J, \sigma^J, w^J) \leq 0$

(p. 3 of this document)

Solutions of the period 2 optimization problem are (for comparison with period 1 solution, one could refer to p. 4-5 of this document)

(*) $p_2^J = p_2^K = \Pi_2$

(*) If $\bar{\rho} = \underline{\rho} = 1$

- If $\alpha_2 > 1 \Rightarrow t_2^J = \tau_2$; $t_2^K = \tau_2$; $G_2 = \tau_2 \sum_J \beta^J Y(\Pi_2, \sigma^J, w^J)$
- If $\alpha_2 < 1 \Rightarrow \sum_J t_2^J \beta^J Y(p_2^J, \sigma^J, w^J) = 0$, $G_2 = 0$
- If $\alpha_2 = 1$, again, may have $\lambda_5 = \lambda_3 = \lambda_4 = 0$
  $\sum_J t_2^J \beta^J Y(\Pi_2, \sigma^J, w^J) \geq 0$, $t_2^J \leq \tau_2$, $t_2^K \leq \tau_2$

(*) If $\bar{\rho} > \underline{\rho}$

- If $\alpha_2 > \bar{\rho}$, $t_2^J = \tau_2$; $t_2^K = \tau_2$; $G_2 = \tau_2 \cdot \sum_J \beta^J Y(\Pi_2, \sigma^J, w^J)$
- If $\alpha_2 < \bar{\rho}$, $G_2 = 0$; $t_2^K = \tau_2$; $t_2^J = -\dfrac{\tau_2 \cdot \beta^K Y(\Pi_2, \sigma^K, w^K)}{\beta^J Y(\Pi_2, \sigma^J, w^J)}$
- If $\alpha_2 = \bar{\rho}$, $t_2^K = \tau_2$, may have $\lambda_3 = \lambda_5 = 0$

  $\Rightarrow \sum_J t_2^J \beta^J Y(\Pi_2, \sigma^J, w^J) \geq 0$

  $t_2^J \leq \tau_2$

## Derivation of (9) from (15) and (16) [(15), (16) appear in Appendix, p. 21, 22]

Note that (15), (16) are derived from (8), and

that in (16), the authors made a small typo error,

the last term is written as $\tau_2(\underline{\rho}-\bar{\rho})\beta^J Y(\Pi_s, \sigma^K, w^K)$

it should be $\tau_2(\underline{\rho}-\bar{\rho})\beta^J Y(\Pi_s, \sigma^J, w^J)$

Deriving (9):

- Note that the 1st term in (15) = the 1st term in (16). Let them = M

  the 2nd term in (15) = the 2nd term in (16). Let them = N

  Let the 3rd term in (15) = P

  and the 3rd term in (16) = Q

- Then:

$$W^J(\tau_2, \Pi_2) = \gamma^J E\{W_J^J(\alpha_2, \tau_2, \Pi_2)\} + (1-\gamma^J)E\{W_K^J(\alpha_2, \tau_2, \Pi_2)\}$$

$$= \gamma^J\left[M + (1-H(\bar{\rho})).E(N) + H(\bar{\rho}).P\right]$$

$$+ (1-\gamma^J).\left[M + (1-H(\bar{\rho})).E(N) + H(\bar{\rho}).Q\right]$$

$$= M + \left[1-H(\bar{\rho})\right].E(N) + H(\bar{\rho}).\left[\gamma^J.P + (1-\gamma^J).Q\right]$$

$$= \bar{\rho}\beta^J Y(\Pi_2, \sigma^J, w^J) + \underline{\rho}\beta^K Y(\Pi_2, \sigma^K, w^K)$$

$$+ (1-H(\bar{\rho})).\tau_2.E\underbrace{\left[(\alpha_2-\bar{\rho}).\beta^J.Y(\Pi_2, \sigma^J, w^J) + (\alpha_2-\underline{\rho})\beta^K Y(\Pi_2, \sigma^K, w^K)\middle|\alpha_2 \geq \bar{\rho}\right]}_{(N)}$$

$$+ H(\bar{\rho}).\left[\gamma^J.\underbrace{\tau_2(\bar{\rho}-\underline{\rho})\beta^K Y(\Pi_2, \sigma^K, w^K)}_{(P)} + (1-\gamma^J).\underbrace{\tau_2.(\underline{\rho}-\bar{\rho})\beta^J Y(\Pi_2, \sigma^J, w^J)}_{(Q)}\right]$$

Let $J = \beta^J Y(\Pi_2, \sigma^J, w^J)$ and $K = \beta^K Y(\Pi_2, \sigma^K, w^K)$, then

$$w^J_{(\tau_2, \pi_2)} = \bar{\rho} J + \underline{\rho} K + (1 - H(\bar{\rho})) \cdot \tau_2 \cdot E\left[ (\alpha_2 - \bar{\rho}) J + (\alpha_2 - \underline{\rho}) K \middle| \alpha_2 \geq \bar{\rho} \right]$$
$$+ H(\bar{\rho}) \cdot \tau_2 \cdot \left[ \gamma^J (\bar{\rho} - \underline{\rho}) K + (1 - \gamma^J)(\underline{\rho} - \bar{\rho}) J \right]$$

$$= \bar{\rho} J + \underline{\rho} K + \tau_2 \cdot (1 - H(\bar{\rho})) \cdot E\left[ \alpha_2 (J + K) - (\bar{\rho} J + \underline{\rho} K) \middle| \alpha_2 \geq \bar{\rho} \right]$$
$$+ \tau_2 \cdot H(\bar{\rho}) \cdot \left[ \gamma^J \bar{\rho} K - \gamma^J \underline{\rho} K + (1 - \gamma^J) \underline{\rho} J - (1 - \gamma^J) \bar{\rho} J \right]$$

$$= \bar{\rho} J + \underline{\rho} K + \tau_2 (1 - H(\bar{\rho})) \left[ E(\alpha_2) \cdot (J + K) - (\bar{\rho} J + \underline{\rho} K) \middle| \alpha_2 \geq \bar{\rho} \right]$$
$$+ \tau_2 \cdot H(\bar{\rho}) \cdot \left[ -(\bar{\rho} J + \underline{\rho} K) + \gamma^J \bar{\rho} K + (1 - \gamma^J) \underline{\rho} K + (1 - \gamma^J) \underline{\rho} J + \gamma^J \bar{\rho} J \right]$$

$$= \bar{\rho} J + \underline{\rho} K + \tau_2 \cdot (1 - H(\bar{\rho})) \cdot \left[ E(\alpha_2) \cdot (J + K) - (\bar{\rho} J + \underline{\rho} K) \middle| \alpha_2 \geq \bar{\rho} \right]$$
$$+ \tau_2 \cdot H(\bar{\rho}) \cdot \left[ -(\bar{\rho} J + \underline{\rho} K) + \left( \gamma^J \bar{\rho} + (1 - \gamma^J) \underline{\rho} \right) \cdot (J + K) \right]$$

$$= \left[ \bar{\rho} J + \underline{\rho} K \right] \cdot \left[ 1 - \tau_2 (1 - H(\bar{\rho})) - \tau_2 \cdot H(\bar{\rho}) \right] + \tau_2 \cdot \left[ 1 - H(\bar{\rho}) \right] \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) \cdot (J + K)$$
$$+ \tau_2 \cdot H(\bar{\rho}) \cdot \left( \gamma^J \bar{\rho} + (1 - \gamma^J) \underline{\rho} \right) \cdot (J + K)$$

$$= \left[ \bar{\rho} J + \underline{\rho} K \right] \cdot \left[ 1 - \tau_2 \right] + \tau_2 \cdot \left[ (1 - H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot \left( \gamma^J \bar{\rho} + (1 - \gamma^J) \underline{\rho} \right) \right] \cdot$$
$$\cdot \left[ J + K \right]$$

$$= (1 - \tau_2) \cdot \left[ \bar{\rho} \beta^J Y(\pi_2, \sigma^J, w^J) + \underline{\rho} \beta^K Y(\pi_2, \sigma^K, w^K) \right]$$
$$+ \tau_2 \cdot \left[ (1 - H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot \left( \gamma^J \bar{\rho} + (1 - \gamma^J) \underline{\rho} \right) \right]$$
$$\cdot \left[ \beta^J Y(\pi_2, \sigma^J, w^J) + \beta^K Y(\pi_2, \sigma^K, w^K) \right]$$

Deriving (12) (p. 11, Besley & Persson)

Note: $Y(\pi_2, \sigma^J, w^J) = \left[\sigma^J (1+\pi_2)(r_H - r_L) + r_L\right] \cdot w^J$

$$W^J_{(\tau_2, \pi_2)} = (1-\tau_2) \cdot \left[\bar{\rho}\, \beta^J Y(\pi_2, \sigma^J, w^J) + \underline{\rho}\, \beta^K Y(\pi_2, \sigma^K, w^K)\right]$$
$$+ \tau_2 \cdot \left[(1-H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right]$$
$$\cdot \left[\beta^J Y(\pi_2, \sigma^J, w^J) + \beta^K Y(\pi_2, \sigma^K, w^K)\right]$$

$$\frac{\partial W^J(\tau_2, \pi_2)}{\partial \pi_2} = (1-\tau_2) \cdot \left[\bar{\rho}\, \beta^J \sigma^J (r_H - r_L) w^J + \underline{\rho}\, \beta^K \sigma^K (r_H - r_L) w^K\right]$$
$$+ \tau_2 \cdot \left[(1-H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right]$$
$$\cdot \left[\beta^J \sigma^J (r_H - r_L) w^J + \beta^K \sigma^K (r_H - r_L) w^K\right]$$

$$= (1-\tau_2)\left[\bar{\rho}\, \omega^J \Omega (r_H - r_L) + \underline{\rho}\, \omega^K \Omega (r_H - r_L)\right]$$
$$+ \tau_2 \cdot \left[(1-H(\bar{\rho})) E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho})(\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right] \cdot (r_H - r_L)(\omega^J \Omega + \omega^K \Omega)$$

($\omega^J, \omega^K, \Omega$ defined under (10) p. 11)

$$= (1-\tau_2)(r_H - r_L)\Omega\, (\bar{\rho}\, \omega^J + \underline{\rho}\, \omega^K)$$
$$+ \tau_2 \cdot \left[(1-H(\bar{\rho})) E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho})(\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right](r_H - r_L)\Omega$$

(since $\omega^J + \omega^K = 1$)

$$= (r_H - r_L)\Omega\, (\bar{\rho}\, \omega^J + \underline{\rho}\, \omega^K)$$
$$+ (r_H - r_L)\Omega\, \tau_2 \cdot \left[(1-H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho})(\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho}) - \bar{\rho}\, \omega^J - \underline{\rho}\, \omega^K\right]$$

Since $\bar{\rho}\, \omega^J + \underline{\rho}\, \omega^K = \bar{\rho}\, \omega^J + \underline{\rho}(1-\omega^J) = \underline{\rho} + (\bar{\rho} - \underline{\rho})\omega^J = \rho^J$

defined in formula (11), p. 11, Besley & Persson

$$\frac{\partial w^J(\tau_2, \pi_2)}{\partial \pi_2} = (r_H - r_L)\,\Omega\,\rho^J$$

$$+ (r_H - r_L)\,\Omega\,\tau_2 . \left[(1 - H(\bar{\rho})) . E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho})(\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right]$$

In the 2nd term,

$$(1 - H(\bar{\rho})) . E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) . (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J$$

$$= (1 - H(\bar{\rho})) . \left[E(\alpha_2 | \alpha_2 \geq \bar{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right] + H(\bar{\rho})\left[\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho} - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right]$$

$$= \left[1 - H(\bar{\rho})\right] . \left[E(\alpha_2 | \alpha_2 \geq \bar{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right] + H(\bar{\rho}) . \left[\gamma^J (\bar{\rho} - \underline{\rho}) - w^J (\bar{\rho} - \underline{\rho})\right]$$

$$= \left[1 - H(\bar{\rho})\right] . \left[E(\alpha_2 | \alpha_2 \geq \bar{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right] + H(\bar{\rho}) . (\gamma^J - w^J)(\bar{\rho} - \underline{\rho})$$

Therefore, $\dfrac{\partial w^J(\tau_2, \pi_2)}{\partial \pi_2} = (r_H - r_L)\,\Omega\,(\rho^J + \tau_2 \lambda_2^J)$

where $\lambda_2^J = \left[1 - H(\bar{\rho})\right]\left[E(\alpha_2 | \alpha_2 \geq \bar{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right] + H(\bar{\rho})(\gamma^J - w^J)(\bar{\rho} - \underline{\rho})$

Deriving (13) (p. 11, Besley & Persson)

Note : $Y(\Pi_2, \sigma^J, w^J) = \left[\sigma^J(1+\Pi_2)(r_H - r_L) + r_L\right] \cdot w^J$

$$W^J(\tau_2, \Pi_2) = (1-\tau_2) \cdot \left[\bar{\rho}\beta^J Y(\Pi_2, \sigma^J, w^J) + \underline{\rho}\beta^K Y(\Pi_2, \sigma^K, w^K)\right]$$
$$+ \tau_2 \cdot \left[(1-H(\bar{\rho})) \cdot E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right]$$
$$\cdot \left[\beta^J Y(\Pi_2, \sigma^J, w^J) + \beta^K Y(\Pi_2, \sigma^K, w^K)\right]$$

$$\frac{\partial W^J(\tau_2, \Pi_2)}{\partial \tau_2} = -\bar{\rho}\beta^J\left[\sigma^J(1+\Pi_2)(r_H - r_L) + r_L\right]w^J - \underline{\rho}\beta^K\left[\sigma^K(1+\Pi_2)(r_H-r_L)+r_L\right]w^K$$
$$+ \left[(1-H(\bar{\rho})) \cdot E(\alpha_2|\alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot (\gamma^J\bar{\rho} + (1-\gamma^J)\underline{\rho})\right]$$
$$\cdot \left[\beta^J\left[\sigma^J(1+\Pi_2)(r_H-r_L)+r_L\right]w^J + \beta^K\left[\sigma^K(1+\Pi_2)(r_H-r_L)+r_L\right]w^K\right]$$

Note from page 10 of this document (2nd page of the derivation of formula (12)) that

$$\lambda_2^J = \left[1-H(\bar{\rho})\right] \cdot \left[E(\alpha_2|\alpha_2 \geq \bar{\rho}) - \underline{\rho} - (\bar{\rho} - \underline{\rho})w^J\right] + H(\bar{\rho}) \cdot (\gamma^J - w^J)(\bar{\rho} - \underline{\rho})$$
$$= \left[1 - H(\bar{\rho})\right] \cdot E(\alpha_2|\alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) \cdot (\gamma^J\bar{\rho} + (1-\gamma^J)\underline{\rho}) - \left[\underline{\rho} + (\bar{\rho} - \underline{\rho})w^J\right]$$

(This appears below "in the 2nd term" also on p. 10 of this document.)

Since $\underline{\rho} + (\bar{\rho} - \underline{\rho})w^J = \bar{\rho}w^J + \underline{\rho}(1-w^J) = \bar{\rho}w^J + \underline{\rho}w^K$

($w^J + w^K = 1$ as explained on p. 11 of Besley & Persson's paper)

We have

$$\lambda_2^J = [1-H(\bar{\rho})] . E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho})(\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho}) - [\bar{\rho} \omega^J + \underline{\rho} \omega^K]$$

Thus

$$\frac{\partial W^J(\tau_2, \Pi_2)}{\partial \tau_2} = \left[(1 - H(\bar{\rho})) E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) . (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho})\right]$$
$$. \left[\beta^J [\sigma^J (1+\Pi_2)(r_H - r_L) + r_L] w^J + \beta^K [\sigma^K (1+\Pi_2)(r_H - r_L) + r_L] w^K\right]$$
$$- \bar{\rho} \beta^J [\sigma^J (1+\Pi_2)(r_H - r_L) + r_L] w^J - \underline{\rho} \beta^K [\sigma^K (1+\Pi_2)(r_H - r_L) + r_L] w^K$$

$$= \left[(1 - H(\bar{\rho})) . E(\alpha_2 | \alpha_2 \geq \bar{\rho}) + H(\bar{\rho}) . (\gamma^J \bar{\rho} + (1-\gamma^J)\underline{\rho}) - (\bar{\rho} \omega^J + \underline{\rho} \omega^K)\right]$$
$$. \left[\omega^J \Omega (1+\Pi_2)(r_H - r_L) + r_L \beta^J w^J + \omega^K \Omega (1+\Pi_2)(r_H - r_L) + r_L \beta^K w^K\right]$$
$$+ \left[\bar{\rho} \omega^J + \underline{\rho} \omega^K\right] . \left[\omega^J \Omega (1+\Pi_2)(r_H - r_L) + r_L \beta^J w^J + \omega^K \Omega (1+\Pi_2)(r_H - r_L) + r_L \beta^K w^K\right]$$
$$- \bar{\rho} \left[\omega^J \Omega (1+\Pi_2)(r_H - r_L) + \beta^J w^J r_L\right] - \underline{\rho} \left[\omega^K \Omega (1+\Pi_2)(r_H - r_L) + \beta^K w^K r_L\right]$$

(Since $\beta^J \sigma^J w^J = \omega^J \Omega$ and $\beta^K \sigma^K w^K = \omega^K \Omega$, p. 11, Besley & Persson)

$$= \lambda_2^J . \left[\Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K)\right]$$
$$+ \left[\bar{\rho} \omega^J + \underline{\rho} \omega^K\right] \left[\Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K)\right]$$
$$- \bar{\rho} \left[\omega^J \Omega (1+\Pi_2)(r_H - r_L) + \beta^J w^J r_L\right] - \underline{\rho} \left[\omega^K \Omega (1+\Pi_2)(r_H - r_L) + \beta^K w^K r_L\right]$$

(Since $\omega^J + \omega^K = 1$, p. 11, Besley and Persson)

$$= \lambda_2^J \left[ \Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K) \right]$$

$$+ r_L \cdot \left\{ \left[ \bar{\rho} w^J + \underline{\rho} w^K \right] . \left[ \beta^J w^J + \beta^K w^K \right] - \bar{\rho} \beta^J w^J - \underline{\rho} \beta^K w^K \right\}$$

(Since $\bar{\rho} w^J \Omega (1+\Pi_2)(r_H - r_L)$ is canceled out from the 2nd & 3rd term

$\underline{\rho} w^K \Omega (1+\Pi_2)(r_H - r_L)$ is canceled out from the 2nd & 4th term

in the last expression on the previous page, p. 12)

$$= \lambda_2^J \left[ \Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K) \right]$$

$$+ r_L \left\{ \left[ \bar{\rho} w^J + \underline{\rho} w^K \right] . \left[ \beta^J w^J + \beta^K w^K \right] - \bar{\rho} \left[ w^J + w^K \right] \beta^J w^J - \underline{\rho} \left[ w^J + w^K \right] \beta^K w^K \right\}$$

(Since $w^J + w^K = 1$, p. 11, Besley and Persson)

$$= \lambda_2^J \left[ \Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K) \right]$$

$$+ r_L \cdot \left\{ \bar{\rho} \left[ w^J \beta^K w^K - w^K \beta^J w^J \right] + \underline{\rho} \left[ w^K \beta^J w^J - w^J \beta^K w^K \right] \right\}$$

$$= \lambda_2^J \left[ \Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K) \right]$$

$$+ r_L . \left[ \bar{\rho} - \underline{\rho} \right] . \left[ w^J \beta^K w^K - w^K \beta^J w^J \right]$$

$$= \lambda_2^J \left[ \Omega (1+\Pi_2)(r_H - r_L) + r_L (\beta^J w^J + \beta^K w^K) \right]$$

$$+ r_L \left[ \bar{\rho} - \underline{\rho} \right] \cdot \frac{\beta^J w^J \beta^K w^K (\sigma^J - \sigma^K)}{\Omega}$$

(Since $w^J = \dfrac{\beta^J w^J \sigma^J}{\Omega}$ and $w^K = \dfrac{\beta^K w^K \sigma^K}{\Omega}$ )