



Notes on the Distribution of Estates in France and the United Kingdom

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I.—Notes on the Distribution of Estates in France and the United Kingdom. By H. C. STRUTT.

A RECENT publication by M. J. Séailles, in which particulars are given of the values and numbers of estates passing by deaths in France during the years 1903-04-05 and 1907, and classified according to the size of the estate, enables us to compare the distribution of estates in France with that in this country, the details of the latter being published in the yearly reports of the Commissioners of Inland Revenue.

The two sets of tables, as they stand, are not immediately comparable, as the *minimum* value of estates recorded in the French statistics is 1 franc, while the estates included in the Inland Revenue reports do not extend to those below 101*l*. By a rough calculation, however, the number and aggregate value of the French estates between 1 and 2,500 francs may be ascertained with sufficient accuracy for the purpose in hand, so that upon the removal of the figure so obtained from the French statistics, the figures for both countries can be made to cover the same range of values. The results are as follows :—

TABLE 1.—*Classification of the numbers of successions and the aggregate amounts according to categories of value. The figures are the arithmetical means of those relating to the years 1903-04-05 and 1907.*

FRANCE.					
Class.		Number in each class.	Percentage of total number.	Amount in each class.	Percentage of total amount.
£	£			£	
100 to	400	89,877	60·50	19,365,981	9·40
400 „	2,000	43,978	29·60	37,496,041	18·20
2,000 „	4,000	7,194	4·84	20,010,126	9·72
4,000 „	10,000 ...	4,632	3·12	28,856,271	14·00
10,000 „	20,000	1,601	1·08	22,587,889	10·96
20,000 „	40,000	765	·51	21,353,912	10·36
40,000 „	80,000	338	·23	19,096,000	9·27
80,000 „	200,000	131	·09	15,438,872	7·49
200,000 „	400,000	29	·02	8,325,896	4·04
400,000 „	2,000,000	9·25	·01	6,781,249	3·29
Above 2,000,000	1·75	—	6,747,329	3·27
		148,556	100·00	206,059,566	100·00

Classification of the numbers and aggregate amounts of estates according to categories of value. The figures are the arithmetical means of those relating to the years 1904-05-06 and 1907.

UNITED KINGDOM.

Class.		Number in each class.	Percentage of total number.	Amount in each class.	Percentage of total amount.
Exceeding £	Not exceeding £			£	
100 and	500 ...	33,961·75	52·17	9,947,533	3·56
500 "	1,000....	10,322·50	15·86	8,554,683	3·06
1,000 "	10,000....	16,822	25·84	61,550,267	22·02
10,000 "	25,000....	2,340·75	3·60	41,252,699	14·76
25,000 "	50,000....	907·75	1·40	35,363,725	12·65
50,000 "	75,000....	289·25	·44	19,437,164	6·96
75,000 "	100,000....	142·75	·22	13,093,382	4·69
100,000 "	150,000....	132·25	·20	16,949,494	6·06
150,000 "	250,000....	93·50	·14	20,324,934	7·27
250,000 "	500,000....	55·75	·09	20,718,998	7·41
500,000 "	1,000,000 ..	19·75	·03	14,963,037	5·35
Exceeding	1,000,000	6·50	·01	17,351,706	6·21
		65,094·50	100·00	279,507,622	100·00

A study of these figures reveals the fact that in both countries the inequality of distribution is very great.

In France we find at the beginning of the series, by referring to the class between 100*l.* and 400*l.*, that three-fifths of the total number of *propriétaires* who died left an aggregate property of less than one-tenth of the total passing, while the small number of 40 persons in the last three classes, forming practically a negligible fraction of the total, were possessed of a much larger aggregate amount. In England the inequality is still more striking, for the aggregate value of more than half the number of estates amounts to 3½ per cent. only of the total property passing, while 26 persons at the end of the series left nearly 12 per cent. of that total. Not to labour the point, the figures reveal conclusively that among estate owners themselves great inequality prevails in France, and much greater in the United Kingdom.

If we now bring these figures into comparison with the population of the two countries, the greater diffusion of property in France is shown in the most unmistakable manner. The population of the United Kingdom for the years under consideration may be roughly stated as 43,000,000, and that of France, as given by M. Séailles, as 38,000,000; nevertheless, we find that about 150,000 persons possessing property of at least 100*l.* die in the course of the year in France, as compared with 65,000 only in the United Kingdom. When, however, we consider the property involved we find that the 65,000 persons in this country possessed an aggregate of property amounting to 280,000,000*l.* against 206,000,000*l.* owned by nearly two and a half times the number in France.

A more complex matter, but one of great practical interest to statisticians, is the comparison of the multipliers for France and

England, the "multiplier" being defined as a figure by which the total amount of property left by deceased persons in a given year and country should be multiplied to obtain the total property in the hands of the living.

This problem is the more interesting as so many widely different figures have been proposed by statisticians at various times. The latest figure for England, advocated by Mr. Bernard Mallet in his Paper read before the Royal Statistical Society on 18th February, 1908, was 24; and as this was supported by carefully compiled statistical data it seems probable that this figure will hold its ground as far as this country is concerned.

TABLE 2.—*Numbers. Table showing under the several categories of age the number of living persons possessing estates (column 5), as inferred from the number of deceased persons leaving estates, the total number of deceased persons, and the living population.*

Categories of age. (x)	1 Mortality. (Number of deaths in the year.) (m_x)	2 Population. (Number of living persons). (n_x)	3 Quotient. ($\frac{n_x}{m_x}$)	4 Number of persons who died in the year leaving estates. (p_x)	5 Number of living persons possessing estates. ($\frac{p_x n_x}{m_x}$)	6 Ratio of column 4 to column 1. ($\frac{p_x}{m_x}$)
FRANCE.						
0 to 25	220,632	16,455,389	74.5	14,052	1,046,874	.063
25 ,, 40	69,632	8,454,624	121.4	31,942	3,877,758	.458
40 ,, 50	58,542	4,689,642	80.1	36,261	2,904,506	.619
50 ,, 60	76,692	3,965,684	51.7	56,707	2,931,752	.739
60 ,, 70	121,227	2,900,002	23.9	85,712	2,048,517	.707
70 ,, 80	149,119	1,518,381	10.2	89,560	913,512	.600
80 and over	84,196	350,055	4.1	42,076	172,511	.498
	780,040	38,333,777	49.13*	356,310	13,895,430	.4568*
ENGLAND.						
0 to 5	199,877	3,737,700	18.7	5	101	.00002
5 ,, 10	13,741	3,523,200	256.4	5	1,381	.0004
10 ,, 15	7,604	3,371,600	443.4	11	4,777	.0014
15 ,, 20	10,583	3,249,000	307.0	31	9,591	.0029
20 ,, 25	12,981	3,094,700	238.4	270	64,461	.0208
25 ,, 35	29,811	5,202,100	174.5	1,766	808,110	.0592
35 ,, 45	37,685	3,953,100	104.9	3,942	413,370	.1046
45 ,, 55	45,351	2,884,400	63.6	6,666	423,970	.1469
55 ,, 65	57,435	1,929,800	33.6	10,894	366,050	.1897
65 ,, 75	66,899	1,077,100	16.1	13,740	221,210	.2053
75 and over	64,187	436,480	6.8	14,084	95,780	.2194
	546,154	32,459,180	59.43*	51,414	1,908,801	.94135

* Not the total.

Statistics have, however, been published in France showing in categories of age the numbers of deceased persons who left estate during the year 1906; but, unlike the statistics at Mr. Mallet

disposal, no particulars have been given of the average *values* of the estates at the different ages nor of the aggregate value in each category.

Working on these details M. Séailles has arrived at the widely different figure of 39 for France, by reasoning, in some respects so closely analogous to that of Mr. Mallet that it may be well to examine briefly the facts on which they base their respective conclusions.

The statistical details for France are shown in Table 2; and the corresponding particulars used by Mr. Mallet, and classified in a precisely similar manner for England, are also appended. An algebraical notation has been included in the headings showing the relations *inter se* of the various functions of the age (x).

M. Séailles observes that "all the causes of increase of *fortune* are a function of age"—a proposition borne out by Mr. Mallet's figures both for 1905 and 1906, where the average estate passing by decrease increases up to the most advanced age at death. "Before 40 years of age," observes M. Séailles, "it is rare that a person has received the inheritances that would normally fall to him. At the same time possibilities of personal saving hardly exist before he has reached a certain age, often fairly advanced, to which his more ripened capacities and experience correspond, if commercial and industrial activities are under consideration; or, if an employment or office is involved, his promotion and the higher salary attained."

But when a still more advanced age is reached, M. Séailles considers that a contrary tendency begins to make itself felt, and savings diminish "par suite de l'action croissante des dotations, des partages anticipés et des viagers." Thus he is led to the conclusion that "*fortunes*" or "*estates*" increase with age up to a certain point and then decrease as the age further advances. In this latter respect the inference differs from the facts derived from the English experience presented by Mr. Mallet, for the estates increase in value with the ages up to the most advanced period of life (see Table 4, col. 3). Doubtless, however, conditions in France are very different from those in England, both as to the laws of inheritance and the customs regarding "dotalions, &c.," enumerated above. There seems, therefore, no reason to question the general correctness of M. Séailles's conclusion taken qualitatively. But in the entire absence of figures, it is conceivable that quantitative estimates of the results of the tendencies enumerated by him might give very diverse results. For instance, the tendency of successful businesses to grow after the proprietors have reached (say) 50 years of age might just be neutralised by the gifts *inter vivos* which it is customary to make in France at that period, in which case the average *fortunes* derived from business and left by deceased persons of that age and upwards would be stationary. In short, we can well imagine that with the same tendencies at work, but in different proportions, not only *fortunes* might remain stationary or decrease after 50, but they might actually increase, though much more slowly than at earlier ages.

M. Séailles is, however, not only sure of his inference that they decrease in the later years of life, but he supplements that inference

by another. He notices that the ratio of the number of deceased estate owners to the total number of deceased persons also increases with age until a certain period of life is reached, and then diminishes, as is shown in column 6 of Table 2 (French figures) given above, which indicates an increase up to an age somewhere between 50 and 60, and then a subsequent decrease.

Here then is M. Séailles's second inference. He says that it is at the same time logical and entirely probable that this variation in the ratio of the number of the propertied deceased to the total number of deceased should be the same as the variation in the average amounts of the *fortunes* left in each category of age. As the total value of estates left at all ages is quoted by M. Séailles as amounting to 5,351,000,000 francs it is quite easy, on this supposition, to arrive at the "*fortune totale*" and "*fortune moyenne*" for each category.

The results (including the consequent multiplier) are as follows:—

TABLE 3.—*France.*

Catégories d'âge.	Nombre de propriétaires vivants.	Fortune moyenne.	Fortune totale.
		frs.	frs.
0 à 25	1,046,874	1,550	1,622,654,700
25 „ 40	3,877,758	11,290	43,769,887,820
40 „ 50	2,904,506	15,260	44,322,761,560
50 „ 60	2,931,752	18,220	53,416,521,440
60 „ 70	2,048,517	17,500	35,869,047,500
70 „ 80	913,512	14,800	13,519,977,600
80 et au delà	172,511	12,270	2,116,709,970
	13,895,430	90,890	194,637,560,590
	$\frac{194,637,560,590}{5,351,000,000} = 36.37$ the "multiplier."		

By further sub-division of categories the multiplier is finally raised to 39.46.

Now it is a simple matter to test this conclusion by making the same inference and going through precisely the same calculations with the English figures presented by Mr. Mallet; and then testing the results with those based on actual statistics as given by him. This has been done, and the results, as a matter of fact, show that M. Séailles's bold inference is very close to the truth.

In other words while, so far as England is concerned, Mr. Mallet's calculations upon actual data produced a figure of 6,098,000,000*l.* for the property in the hands of living estate owners, an amount roughly twenty-four times the total of the estates annually passing at death, calculations based upon M. Séailles's hypothesis give 7,061,000,000*l.*, an amount roughly twenty-seven times the total annually passing at death.

We are thus, it would appear, bound to conclude that M. Séailles's hypothetical method is to a certain extent justified by results, and that it is at least probable that the figure, 39 or thereabouts, though perhaps a little too high, is not far removed from the truth.

Even, however, if we reduce it by 3 or 4 and take the multiplier as, say, 35, we are still faced with the enormous discrepancy between the respective multipliers for England and France, especially if we consider that both may (for the moment) be regarded as the measure of the movement of a unit of property from one person to another by reason of death. In spite of differences in institutions, in laws, and in customs, it is impossible to believe that across the Channel each franc of "fortune" created or inherited remains on an average in the possession of one person for thirty-five years, while on this side of the Channel each pound made or inherited to form an "estate" passes by death from one person to another in twenty-four years.

In order to explain this discrepancy, let us adopt the hypothesis that the English people have the habit of behaving in the same way as the French do, and trace the consequences. Glancing at the English figures, we will suppose that the 366,050 living persons between 55 and 65. (Table 2, col. 5) with an average estate of 3,954*l.* (Table 4, col. 3), give up, by way of the "dotations, &c.," referred to by M. Séailles, an average of 1,000*l.* out of their several estates to persons between the ages of 25 and 35; that those between 65 and 75, numbering 221,210, with an average estate of 5,423*l.*, similarly dispose of 2,000*l.* each to persons between the ages of 35 and 45; and that the 95,780 persons over 75 each yield 4,000*l.* to the class between 45 and 55. It will be quite obvious that this mere transfer would not in any way affect the total amount of the estates in the hands of the living, given as 6,098,000,000*l.*; nor would it affect the average actual length of time during which property is held by the same person, but it would very considerably alter the distribution among the age classes. This distribution, contrasted with that given by Mr. Mallet, would be as follows:—

TABLE 4.—*England.*

Age classes.	1 Mr. Mallet's figures showing the present distribution of property in age-groups of living persons.	2 The figures as altered by the supposed <i>donationes inter vivos.</i>	3 Average value of estate as given by Mr. Mallet.	4 Average value of estate as altered by the supposed <i>donationes inter vivos.</i>
	£	£		
0 to 5	37,400	37,400	400	400
5 ,, 10	13,946,000	13,946,000	10,878	10,878
10 ,, 15	3,400,900	3,400,900	697	697
15 ,, 20	6,474,700	6,474,700	680	680
20 ,, 25	53,383,000	53,383,000	829	829
25 ,, 35	335,340,000	701,390,000	1,088	2,276
35 ,, 45	735,750,000	1,178,170,000	1,779	2,850
45 ,, 55	1,591,100,000	1,974,220,000	3,753	4,656
55 ,, 65	1,447,600,000	1,081,550,000	3,954	2,954
65 ,, 75	1,199,700,000	757,280,000	5,423	3,423
75 and over....	711,180,000	328,060,000	7,426	3,426
	6,097,912,000	6,097,912,000		

The average values of the estates held by living persons at the various ages would also be considerably altered, as shown by a comparison of the figures in columns 3 and 4 of the table given above (Table 4). In short, estate owners between 25 and 55 would habitually possess estates much larger than at present, and those of 55 and upwards would be correspondingly impoverished. This supposed difference of custom would necessarily be revealed in the disclosure of the values of estates ascertained to be owned by deceased persons in any year for the purpose of the collection of estate duty. These altered values can be readily obtained by multiplying the mean values of estates in the several categories of age by the numbers dying within the year. Here then is a comparative statement, showing, first, the various total values of estates passing by death in the several age categories, as they actually were in 1906, and, secondly, these values as they would have been had the custom prevailed of making *donationes inter vivos* in the proportions supposed.

TABLE 5.—*England.*

Age classes.	Amounts of estates taxed in 1906 as passing by death.	Amounts corrected to accord with the foregoing supposition.	Results.
	£	£	
0 to 5	2,000	2,000	Unaltered
5 " 10	54,390	54,390	
10 " 15	7,670	7,670	
15 " 20	21,090	21,090	
20 " 25	223,920	223,920	
25 " 35	1,921,680	4,019,500	Greater
35 " 45	7,013,750	11,235,000	
45 " 55	25,017,500	31,000,000	
55 " 65	43,083,000	32,189,000	Less
65 " 75	74,514,000	48,034,000	
75 and upwards	104,586,000	48,250,000	
	256,445,000	175,037,000	The multiplier = 6098 (Table 4) 175 = 35 nearly

Thus we find that if in England it were the custom as, according to M. Séailles, it is in France, for old people to hand over to the younger generation large sums by way of gifts, so much property would escape revelation at death that the multiplier to be used in ascertaining the value of estates in the hands of the living would be increased from 24 to nearly 35. But it may particularly be noted that such a custom implies not the slightest alteration in the length of time during which each unit of property remains in the same hands. If 24 be the measure of the movement of property in the one case, it is equally so in the other, in spite of the multiplier running up to 35; because, although certain old people part with their property sooner under the influence of our imagined custom, they received it from others as much sooner when they were younger.

But in England a certain amount of property passes *inter vivos*,

and thus escapes revelation, though the ratio of such gifts to the property passing at death is believed to be relatively small. It follows from this that the figure 24 may be itself too high, regarded as a measure of the movement of property.

To sum up, the conclusion seems inevitable that in the ideal case of a country in which all property, either created or received, is retained up to death, and where, consequently, there are no *donationes inter vivos*, the multiplier is much more than is implied in the definition already given. It is not only a multiplier, but is also a real statistical or physical constant, and it measures in terms of years the actual average movement of property from one to another by reason of death; it also seems provisionally to be shown by Mr. Mallet that this constant does not exceed 24. But if from any causes, such as the customs referred to by M. Séailles, any considerable amount of property fails to be disclosed in the death duty statistics for a given year, the figure required, *quâ* multiplier, may be considerably increased, but will then cease to represent the number of years during which a unit of property is held by one person.

Although not falling strictly under the heading of these notes, it may not be out of place to call attention to the formula of distribution proposed by M. Séailles as being more general than that of Pareto.

The law of Pareto, it is well known, is of limited application only, and is apt to fail at both ends of a series treated. Mr. A. L. Bowley, for instance, found that the formula applied with fair accuracy to the classified statistics of estate duty, excepting where the estates were of very high value (see H. of C. paper 365, year 1906, p. 222); and it has been ascertained, on the other hand, that the law applies to the Prussian income-tax figures [Cd-2587, p. 5] with fair precision if it is not pushed to the numbers with very small incomes.

The equation itself reveals its own limitation for numbers with small incomes or values, as by taking x (the income) indefinitely small, N (the number of persons with incomes of and over x) becomes indefinitely large, and, as M. Séailles observes, may thus be made to exceed the whole population whose incomes or other values are considered. M. Séailles notices that when these classified returns are treated cumulatively so as to form for N the number of cases over the value x , the logarithms of N and x , when graphically treated, give a series of points forming a slightly bow-shaped curve, whose concavity faces the intersection of the axes, and tentatively concludes, from the parabolic form of the curve, that a more general formula embracing the lower values is to be found in the equation

$$(\log x)^2 = 2p \cdot \log N + q.$$

In this connection I may mention that in an unpublished note on a case of distribution of wealth, the basis of which consisted of a table, relating to the year 1715, of forfeited estates of varying values belonging to non-jurors, &c., Mr. Udny Yule was able to fit the figures given into an equation, $\eta = a + b\xi - c\xi^2$, where $\eta = \log y$ and $\xi = \log x$. The curve was not, however, like Pareto's, an

integral curve, but was formed from the successive figures included within small intervals without cumulation. Other values have also been fitted into curves of greater generality than that of Pareto.

It is thought, therefore, that the difficulty connected with this problem is not mathematical, but arises from our comparative ignorance of the distribution of wealth among the poorer classes, and it is worth while to consider a little more particularly what is the actual problem to be solved. We have Pareto's formula

$$N = \frac{C}{x^\alpha}, \quad (1)$$

where N is the number of persons with an income of or above x . Consequently, the equation

$$\frac{dN}{dx} \text{ or } N' = \frac{C\alpha}{x^{\alpha+1}} \quad (2)$$

gives the number of persons with the *exact* income x .

But the table of income corresponding to that of the numbers will also be implicated with these equations; for, if we multiply the last equation by x , thus,

$$N'x = I' = \frac{C\alpha}{x^\alpha}, \quad (3)$$

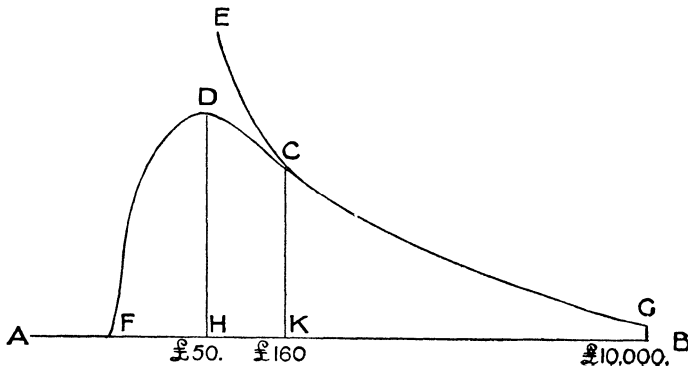
we obtain the aggregate income of the number of persons whose income is exactly x , and

$$\int I'dx = I = \frac{C\alpha}{(\alpha-1)x^{\alpha-1}} \quad (4)$$

gives the aggregate income of the number, N , of persons with incomes above x , and corresponds, as regards incomes, to equation (1) relating to numbers.

It follows, therefore, that any equation more general than Pareto's should be such as will include incomes (or whatever values are dealt with) as well as numbers. It may be noted, *en passant*, that all four equations are of the same form, viz., $y = \frac{C}{x^\alpha}$, and are of limited application only for the reason already stated.

Referring to equation (2), it is quite obvious that it forms part of a larger curve, which, if applied to incomes, may take the following form:—



The curve CG is supposed to answer to the equation $N' = \frac{C\alpha}{x^{\alpha+1}}$, and to be true of numbers and incomes, say, from 160*l.* to 10,000*l.* per annum, where the incomes are measured off along AB, and the numbers represented by verticals, such as CK and GB. It may also be supposed that were this curve continued to the left in accordance with the equation it would take the course indicated by CE, and would continue to rise indefinitely upwards from the axis AB, and thus be the graphic representation of the absurdity, already alluded to, of too large a number with inappreciable incomes. But there will most probably be a *maximum* number of persons at some small income. Suppose, then, we imagine that there are more persons earning, say, 50*l.* per annum, than there are persons earning 49*l.* or 51*l.*, or 48*l.* and 52*l.*, and so on. The ordinate DH would then represent that *maximum* number, and the curve would take the direction CDF, descending sharply along DF towards the line AB, because the incomes from the *minimum* income to 50*l.* would take up a very small portion only of AB, as compared with the length KB, which represents the large range of incomes from 160*l.* to 10,000*l.*

As far as this country is concerned it is even possible that there might be other *maxima* on the left of the line CK, owing to a combination of causes. We know that 1*l.* a week is a small wage for a man, but it is at present considered a fair one for a woman (say a seamstress or a typist), and a very large one for a boy. From the combination of these circumstances the curve at the left of DH might present various irregularities, not at present known.

Now if this total curve GCDF should possess an equation, capable of integration, the equation of the integral, reduced to logarithms, would, so M. Séailles suggests, be of the form

$$(\log x)^2 = 2p \cdot \log N + q.$$

But, speaking generally, it is precisely the facts relating to the unknown, or little known, portion of the curve on the left of the vertical CK, which this new equation is designed to embrace. It would not be true to say that no facts are known of the poorer classes, for in the Prussian figures the *minimum* income brought under review is as low as 45*l.* per annum, and it is well known that eminent statisticians and others have made many and careful observations as to the rates of wages, &c., amongst the poorer classes. But these facts are not known to us in the definite sense or organised form in which the incomes, or values of estates, of a higher class are known or can be inferred. It is, however, true that in the "classement" given by M. Séailles, the numbers and amounts of *fortunes* of as low a value as one franc are given, from which at first sight much might be inferred, but there is unfortunately every reason to regard the figures as very untrustworthy. M. Séailles himself lays considerable stress on the fraud and evasion that must necessarily occur in the valuation of estates in France; and, so far as England is concerned, it may well be supposed that the representatives of deceased persons take a very biased view

of the value of the property left, whenever it is subject to doubt. In the higher categories of the "classement," where the range of value is large, comparatively few *fortunes* would, in spite of such undervaluations, fall to lower categories, and none would be altogether excluded from classification; but in the lowest category many would fall entirely out of the table. Moreover, fraud and evasion are relatively easy in dealing with the simple estates of the poor, who are also much more liable to temptation.

These circumstances render it quite certain that the number of estates (118,555) in the category from 1 to 500 francs is very much understated, and that the number (104,225) in the next category (500 to 2,000 francs) most probably suffers from the same defect. This conclusion is confirmed by the fact that the calculated mean values of the *fortunes* are in excess of the respective averages of the minima and maxima of the two classes considered.

We seem therefore to be quite in the dark as to the real figures; but in the meantime the ingenious and luminous conjecture of M. Séailles, though requiring for its acceptance further information and experience, seems to be worth consideration and investigation by those mathematicians who are interested in the formation of statistical generalizations.

II.—On the Interpretation of Correlations between Indices or Ratios.

By G. UDNY YULE.

THE present note is concerned with the interpretation to be placed on the results obtained by Professor Pearson in his Paper "On a Form of Spurious Correlation which may arise when Indices are used in the Measurements of Organs" (*Proceedings of the Royal Society*, vol. lx, 1897, p. 489). As these results may not be generally familiar to the Fellows of this Society, I may perhaps summarise them here.

If r_{12} , r_{13} , &c., are the correlations between four organs, or, generally, four variables of any kind, and v_1 , v_2 , v_3 , v_4 their co-efficients of variation (the ratios of the standard-deviations to the means), the correlation ρ_{12} between the indices x_1/x_3 and x_2/x_4 is given approximately by

$$\rho_{12} = \frac{r_{12} \cdot v_1 v_2 - r_{14} \cdot v_1 v_4 - r_{23} \cdot v_2 v_3 + r_{34} \cdot v_3 v_4}{\sqrt{v_1^2 + v_3^2 - 2r_{13} \cdot v_1 v_3} \sqrt{v_2^2 + v_4^2 - 2r_{24} \cdot v_2 v_4}} \quad (1)$$

provided that deviations may be assumed to be small compared with the mean. Hence, if the organs or variables 3 and 4 are identical,

$$\rho_{12} = \frac{r_{12} \cdot v_1 v_2 - r_{13} \cdot v_1 v_3 - r_{23} \cdot v_2 v_3 + v_3^2}{\sqrt{v_1^2 + v_3^2 - 2r_{13} \cdot v_1 v_3} \sqrt{v_2^2 + v_3^2 - 2r_{23} \cdot v_2 v_3}} \quad (2)$$