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Voting as Communicating

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This paper develops a model where voters trade-off two different motives when deciding how to vote: they care about current decision-making (they are "strategic"), but they also care about communicating their views about their most-preferred candidate so as to influence future elections, by influencing other voters' opinion and/or party positioning. In effect, voters in this model are intermediate between "strategic" and "sincere" voters of conventional models in elections with more than 2 candidates. This allows us to better investigate the relative efficiency of various electoral systems: our main conclusion is that since voting is used as a communication device electoral systems should be designed to facilitate efficient communication, *e.g.* by opting for 2-round systems rather than 1-round systems.

I. INTRODUCTION

In conventional calculus-of-voting models, voters care only about current decision-making, *i.e.* about the winner of the current election. This paper develops a theory of voting as communicating. That is, we propose a model where voters also use voting to communicate their views about their most-preferred policy in order to influence future decision-making processes, so that they explicitly trade-off these two objectives when deciding how to vote. There are at least three different channels through which "communicative voting" can influence future elections:

(1) voters expect mainstream parties to move towards their expressed views;
(2) voters want to learn the strength of each candidate so as to better coordinate their future votes;
(3) voters try to influence others' opinions by expressing their political beliefs.[1]

As a first step towards a more general theory, this paper chooses to concentrate primarily on the third channel in the context of a simple two-period voting model where voters use period-1 election results to reveal their information about the candidates and influence period-2 voting strategies.[2] We characterize the exact conditions under which the trade-off between being pivotal for today's elections and being pivotal by influencing tomorrow's elections results into communicative voting. Although this trade-off between infinitesimal probabilities of being pivotal can seem to rely on excessively sophisticated rationality, we argue that the predictions we obtain about communicative voting are very intuitive and reasonable.

We provide two applications to illustrate the usefulness of this approach. First, we show that even in elections with two candidates, where sincere voting is usually thought to be "straightforward", some voters may decide to vote for the candidate they prefer less so as to express their faith in a third, better candidate/policy. We show that this leads

1 This channel requires uncertain preferences and the possibility of learning from others' signals, unlike channel (2) which relies on a pure coordination problem and stable preferences.

2. See however Sections II and III below for discussion of how our results can be extended in order to deal with the first and second channels.

towards a systematic bias towards close margins in two-candidate elections. A good example of such a phenomenon is the Maastricht Treaty referendum held in France in September 1992. There is clear evidence that a large majority of the electorate preferred not to stop European integration, and indeed the first polls predicted a large margin for the "Yes" (around 60%), after which many pro-Europe voters shifted to the "No" to express their dissatisfaction with a treaty they viewed as too technocratic and that was going to pass anyway. Ten days before the referendum, some polls started predicting a victory of the "No", at which point some of these "communicative voters" turned back to the "Yes" to give Maastricht a 52–48 victory. This type of voting behaviour is also well documented in other contexts.[3]

Next and mostly, we apply our model to investigate the relative efficiency of various electoral systems in elections with more than two candidates. Very few existing models address this issue, and the explanation might well be the persistent lack of a proper model describing how voters behave when they face three or more candidates.[4] On the one hand, it is clear that the assumption of "sincere" voting cannot account for the fact that voters often choose not to vote for the candidate they prefer the most when the latter has no chance of winning.[5] On the other hand, the assumption of complete "strategic" voting is not fully satisfactory either, since it predicts that only two parties should get a positive number of votes.[6] The point is that real-world voters seem to behave in a way that is intermediate between "sincere" and "strategic" voters: they rationally and continuously trade-off their desire to communicate their most-preferred policy and their preference for the one of the top two contenders they dislike less, *e.g.* they "communicate" less when the race between the top two contenders is very close.[7] Our model accounts for this key regularity of observed voting patterns and allows for a rigorous analysis of their efficiency properties.

When we apply this model to electoral system design, our main conclusion is the following: since voting is used as a communication device, electoral systems should be

3. By using the 1987 British Election Study (a survey asking voters both their most-preferred party and the party they voted for), Franklin, Niemi and Whitten (1994) show that the most-preferred party of half of all "tactical" voters (voters who did not vote for their most-preferred party) ranked 1st or 2nd in the constituency of these voters (see their Table 1), and that this voting behaviour is indeed more likely to arise when the margin between the top 2 finishers is large and/or their preference is weak (see their Table 4). They call "expressive tactical voting" this behaviour to distinguish it from the usual tactical vote arising when one's most preferred party ranks 3rd and has no chance of winning.

4. It has also been suggested that the sociology-of-science explanation is the Anglo-Saxon bias towards bipartism and 2-candidate elections. See Shepsle (1991, pp. 1–2).

5. For example, Table 1 of Franklin, Niemi and Whitten (1994) shows that at least 15% of the electorate voted for a different candidate than the one they preferred the most in the 1987 British general election, out of which at least 7% were "instrumental" tactical voters (their most-preferred party ranked 3rd and hence had no chance of winning). Heath and Evans (1994, p. 560) find that this 7% figure went up to 9% in the 1992 general election. Non-sincere voting is also well documented in U.S. presidential primaries (see, *e.g.* Abramson, Aldrich, Paolino and Rohde (1992)).

6. Unless the second and third candidates have exactly the same expected fraction of the vote (see Myerson and Weber (1993) and Section III.A below). This prediction is counterfactual since third candidates persistently get votes even when everybody agrees that they are far below the top two candidates (see, *e.g.* the behaviour of National Front voters described in Piketty (1995, Appendix A)).

7. Evidence for this trade-off can for example be found in the works on British general elections, as well as in Bensel and Senders (1979), who use pre-election polls to show that in 1968 the Wallace vote dropped in states where the race between Johnson and Nixon was very close. In the working paper version of this article, we also give additional evidence based upon the voting behaviour of National Front supporters in the 1995 local elections in France: voters who vote in the National Front during the first round are more likely to vote for the conventional right-wing candidate during the second round in case the election margin between the conventional right-wing candidate and the left-wing candidate was very close during the first round; conversely, they are more likely to keep voting for the National Front during the second round (even when the NF has no chance of winning the election) in case the first-round election margin was very large (see Piketty (1995, appendix A)).

designed accordingly, *i.e.* so as to facilitate efficient communication. For example, two-round systems allow for a clearer separation of the communicative and decision-making functions of the vote, while one-round systems mix up completely the two functions of the vote, which can result into major inefficiencies.

The best example of the type of inefficiency we have in mind is given by the British electoral history of the past 15 years. At least since 1983 there exists a policy package that a majority of the British electorate prefers to the Tories' policy.[8] However, the repeated desire of Labour and Alliance voters to use the single round of Britain's general elections to communicate their views prevented such a majority to take power.[9] It is highly likely that in a two-round system the Liberal/Social-Democratic Alliance would have taken power during the 1980's and would have replaced the Labour Party as the main left-wing party.[10] The fact that this did not occur is inefficient in the sense that an efficient electoral system should allow voters to transmit information and to coordinate so as to facilitate the emergence of new governing majorities when a majority of the electorate does favour new policies. This efficiency emphasis differs from the traditional political science literature on electoral systems: the latter has mostly been concerned with the trade-off between the production of a governing majority and the representation of minorities.[11] The British example illustrates that another important function of electoral systems is to facilitate the emergence of new governing majorities (when necessary). This is particularly important in party systems where old parties are very slow to adapt to a changing world.[12]

This paper is also related to the recent literature on the Condorcet Jury theorem and information aggregation through voting.[13] However, none of the existing papers in this literature is concerned with multi-period voting (a condition for communicative voting), nor with elections with more than two candidates.[14]

8. In the British general elections of 1983, 1987 and 1992, the combined vote of the Labour Party and the Liberal/Social-Democratic Alliance was higher than that of the Conservative Party (see Mair (1995)), which gives a popular majority to the Alliance since most Labour voters prefer the Alliance policies to those of the Tories. This is confirmed by data from the British Social Attitudes survey showing that Tories' supply-side and tax policies never enjoyed majority support in the 1980s, suggesting that their victories were mostly due to the unpopularity of the Labour and the electoral system (see Taylor-Gooby (1991) and especially Lipsey (1994)).

9. Both in 1983 and 1987, there were more than 150 seats won by the Tories where the combined vote of the Labour and Alliance candidates was superior to the Tory vote, enough to shift to a Labour–Alliance majority of seats. Johnson and Pattie (1991) use 1979, 1983 and 1987 constituency-level election results and aggregate polls to estimate constituency-level "flow-of-the-vote matrices"; they show that although these flows vary considerably (sometime by more than 100%) depending on the exact strategic situation of the constituency, they were just not sufficient to win many seats (in constituencies where the Alliance was 2nd and Labour 3rd in 1983, 70% of Labour voters keep voting Labour in 1987; the corresponding figure for Alliance voters is 50%; see their Table 4).

10. This probably explains the reluctance of the Labour Party to support electoral reform. However they recently adopted a proposal for a 2-ballot system very close to the French 2-round system, although nothing guarantees that it will be implemented if they win the 1997 general elections. See Section III.B below.

11. Standard references include Duverger (1963), Rae (1971) and Lijphart (1990, 1994).

12. *Cf.* the Labour Party's persistent inability to convince voters that it has moved away from old-style, union-backed policies. One could argue that this is less of a problem in countries where parties are less ideological and face less inertia in moving towards the median voter. However the price to pay for this (*i.e.* lack of party discipline) may not be worth it (*cf.* the absence of a stable majority in the last U.S. Congress).

13. Austen-Smith and Baks (1994), Feddersen and Pesendorfer (1994) and Myerson (1994*b*) generalize Condorcet's efficiency theorem in one-period, two-candidate elections (see the discussion in Section II.B below). Lohmann (1994) explores the complementarity between information aggregation through voting and actions like street protests. Piketty (1994) shows that the right to trade votes can undermine the efficiency of information aggregation through voting. Greer (1994) also argues that political institutions are designed to facilitate efficient information aggregation, taking the example of the Magna Carta. See Piketty (1999) for a short survey of this recent literature.

14. An exception is Young (1988), who is explicitly concerned with efficient information aggregation with more than 2 policies. However, Young explores the axiomatic solutions of Condorcet and Borda, not the voting behaviour that may implement them. See also Castaneiha (1998), who draws on the current paper and extends in new directions many of the results presented here.

The rest of this paper is organized as follows. Section II deals with communicative voting with 2 candidates (Section II.A gives some general results, while Section II.B investigates the efficiency of communicative voting in this context). Section III deals with the case of 3 or more candidates (Section III.A gives general results, and Section III.B applies these results to discuss the relative efficiency of various electoral systems). Section V concludes. All omitted proofs are in the Appendix.

II. COMMUNICATIVE VOTING WITH TWO CANDIDATES

II.A. Basic results

Throughout the paper we consider large electorates of finite size n and we are interested in the limits of the (Bayesian–Nash) equilibria of the voting game of size n as $n \to +\infty$.[15]

In this section we consider the following 2-period voting game:

At $t=1$, 2 policies/candidates A and C are competing, each voter $i \in (1; \dots; n)$ casts a ballot $v_i^1 = A$ or C,[16] and the winning policy P^1 is determined by the plurality rule. That is, if $n_A^1 = \#(i \text{ s.t. } v_i^1 = A)$ and $n_C^1 = \#(i \text{ s.t. } v^1 = C)$, then if $n_A^1 > n_C^1$ $P^1 = A$, if $n_c^1 > n_A^1$ $P^1 = C$, and if $n_A^1 = n_C^1$ $P^1 = A$ with probability 1/2 and $P^1 = C$ with probability 1/2. The policy P^1 is then implemented during 1 period.

At $t=2$, the winning policy P^1 competes with a 3rd policy B, each voter casts a ballot $v_i^2 = P_1$ or B, and the winning policy P^2 is determined as before. P^2 is then implemented during 1 period, and this is the end of the game. (A more general model would obviously allow for longer horizons.)

Total utility of voters is equal to $U(P^1) + \delta U(P^2)$, where δ is some discount factor, with $\delta > 0$.[17] Voters can be of two types 1 and 2. Type-1 voters prefer A to C ($U_1(A) > U_1(C)$), while type-2 voters prefer C to A ($U_2(C) > U_2(A)$). Voters do not know the exact numbers n_1 and n_2 of type-1 and type-2 voters in the electorate, but it is common knowledge that they know their type, that each voter is of type-1 with probability α and of type-2 with probability $1-\alpha$, and that types are independently distributed across all voters. We assume that $\alpha > 1/2$, so that policy A is the expected front-runner of the period-1 election.

In addition, type-1 voters prefer A to B or B to A depending on some state of the world $s = s_A$ or s_B:

$$U_1(A|s_A) > U_1(B|s_A) > U_1(C),$$
$$U_1(B|s_B) > U_1(A|s_B) > U_1(C).$$

All type-1 voters have the same uniform prior ($\mu(s_A) = 1/2$, $\mu(s_B) = 1/2$). Each type-1 voter i receives a signal $\sigma_i = \sigma_A$ or σ_B about which state is most likely to be correct,

15. Our results also apply to the case where the finite population is a Poisson random variable of mean n and we let $n \to +\infty$, as we show when we prove the various propositions in the Appendix. This uncertain-population setting has recently been proposed by Myerson (1994*a*, *b*) (from whom we borrow the techniques to approximate pivotal probabilities as $n \to +\infty$) as a more natural model to analyse large voting games.

16. Voting is assumed to be costless.

17. The results below apply to any $\delta > 0$, so in particular they also apply to the case where the period-2 election is only held with some positive probability $z > 0$ and P^1 is maintained for 2 periods with probability $1 - z$.

and these signals are distributed according to the following distributions

$$P(\sigma_i = \sigma_A | s_A) = q_A, \qquad P(\sigma_i = \sigma_B | s_A) = 1 - q_A,$$
$$P(\sigma_i = \sigma_A | s_B) = q_B, \qquad P(\sigma_i = \sigma_B | s_B) = 1 - q_B.$$

Therefore the posterior of type-A voters (*i.e.* type-1 voters who received a signal σ_A) is $(\mu(s_A|\sigma_A) = q_A/(q_A + q_B),\ \mu(s_B|\sigma_A) = q_B/(q_A + q_B))$, while that of type-$B$ voters is $(\mu(s_A|\sigma_B) = (1 - q_A)/(2 - q_A - q_B),\ \mu(s_B|\sigma_B) = (1 - q_B)/(2 - q_A - q_B))$. We assume $q_A > q_B$, so that type-A voters prefer A to B on the basis of their signal, while type-B voters prefer B to A.[18] In contrast to type-1 voters, type-2 voters do not care about the state of the world s: they always prefer B to A ($\forall s$, $U_2(B) > U_2(A)$).[19]

The typical example of such a situation is the September 1992 Maastricht Treaty referendum in France: period-1 voting is the referendum, period-2 voting is the likely future vote on European policy, policy A is the Maastricht Treaty, policy C is the status-quo, and policy B is an hypothetical third policy favouring more European integration than the *status-quo* but that some voters perceive as better than the Maastricht Treaty (say, less technocratic). Type-1 voters constitute the large majority favouring more European integration rather than the *status-quo*, type-2 voters definitely prefer the *status-quo* (so that it is natural to assume that they also prefer B to A, since B is in a certain sense "intermediate" between A and C). Signals σ_A and σ_B describe which pro-Europe voters were convinced (by newspapers, friends, . . .) that Maastricht was the best way to go and which were convinced that there did exist a third way (policy B). In September 1992, many of these type-B voters decided to vote for C in order to express their dissatisfaction with Maastricht and their faith in a third way (they knew from the polls that Maastricht was going to pass anyway). This is also the way type-B voters behave the equilibrium of our two-period voting game, as we see below (Proposition 2).

Before we describe the equilibria of the two-period voting game, note that the one-period voting game (where the world ends after P_1 is implemented) has an obvious unique equilibrium:[20] all type-1 voters (both type-A and type-B voters) vote for A and all type-2 voters vote for C. Given these strategies, the probability P_n^* of casting a pivotal vote is positive for all n, although it converges quickly to 0 as $n \to +\infty$ (see below), and therefore all voters strictly prefer to vote for their preferred policy. Since we assumed $\alpha > 1/2$, policy A wins the election with a comfortable margin with probability 1 as $n \to +\infty$.

Proposition 1. *The unique equilibrium of the* 1-*period voting game involves "sincere" voting, i.e. type*-1 *voters all vote for A and type*-2 *voters all vote for C. As* $n \to +\infty$, *policy*

18. This is true as long as the difference in preference intensity between A and B is not too strong. That is, we assume that $q_A(U_1(A|s_A) + q_B U_1(A|s_B) > q_A U_1(B|s_A) + q_B U_1(B|s_B)$ and that $(1 - q_A)U_1(B|s_A) + (1 - q_B)U_1(B|s_B) > (1 - q_A)U_1(A|s_A) + (1 - q_B)U_1(A|s_B)$, which for example always holds if $U_1(A|s_A) = U_1(B|s_B)$ and $U_1(B|s_A) = U_1(A|s_B)$, given the assumption $q_A > q_B$.

19. This assumption is made to simplify notations: all results can easily be extended to the case where type-2 voters also care about the state of the world s, in which case they would vote in the same way as type-1 voters in case $P^1 = A$. This is irrelevant since $\alpha > 1/2$ (*i.e.* type-1 voters do not need type-2 voters to form a majority in period 2 in case $P^1 = A$). However, if type-2 voters did prefer A to B regardless of the state, then they would try to manipulate comunicative voting by voting for A in period 1, which would complicate the analysis. The results can also be extended to the case where only a fraction γ of type-1 voters can change their mind by learning from others' signals, while a fraction $1 - \theta$ just prefers A to B or B to A and never change their mind. This is probably more realistic, and all what is needed for communicative voting to be an equilibrium behaviour is that a key fraction can change their mind.

20. Strictly speaking, the strategy profiles where everybody votes for the same policy are also equilibria for $n > 2$, since voters face a zero probability of being decisive; however they would disappear with any small uncertainty about the population size (and in particular in Myerson's Poisson setting; see Myerson (1994*a*, *b*)), and we ignore them in what follows (one could also invoke a criterion of weak dominance, as is usually done).

A wins the election with a margin of α *against* $1-\alpha$ *with probability* 1.

$$[\forall\varepsilon>0,\ \lim_{n\to+\infty}P_n(n_A^1/n\in[\alpha-\varepsilon;\alpha+\varepsilon[)=1].$$

Voting behaviour is however more complex in the two-period voting game. Type-B voters realize that the exact score $(n_A^1, n_C^1=n-n_A^1)$ of period-1 voting can be used as a signalling device in order to express their views and convince others to vote for B in period 2. Assume for example that in period 1, type-A voters vote for A, type-2 voters for C, while type-B voters vote for A with proba $1-\gamma$ and for C with proba γ, with $\gamma\in]0;1]$.[21] We define $\alpha_A(\gamma)$ (resp. $\alpha_B(\gamma)$) the expected fraction of the vote going to A in period 1 in case such strategies are followed and $s=s_A$ (resp. $s=s_B$)

$$\alpha_A(\gamma)=\alpha(q_A+(1-\gamma)(1-q_A)),\qquad \alpha_B(\gamma)=\alpha(q_B+(1-\gamma)(1-q_B)).$$

Note that $\alpha_A(0)=\alpha_B(0)=\alpha>1/2$ and $\alpha_B'(\gamma)<\alpha_A'(\gamma)<0$. We also define $\gamma_0,\gamma_1,\gamma_2>0$ by

$$\alpha_B(\gamma_0)=1/2,\qquad 1-\alpha_B(\gamma_1)=\alpha_A(\gamma_1),\qquad \alpha_A(\gamma_2)=1/2.$$

Note that $\gamma_0\leqq\gamma_1\leqq\gamma_2$, that $\gamma_0\leqq 1$ iff $\alpha q_B\leqq 1/2$, $\gamma_1\leqq 1$ iff $\alpha q_B\leqq 1-\alpha q_A$, and $\gamma_2\leqq 1$ iff $\alpha q_A\leqq 1/2$.

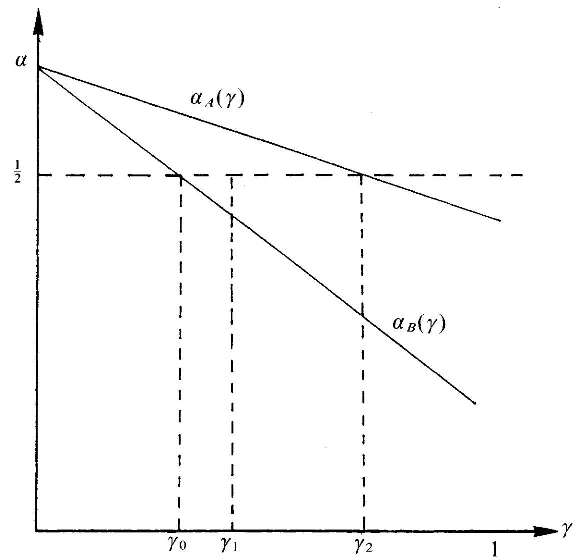

FIGURE 0
(case $\gamma_2<1$)

For any $\gamma>0$, A is expected to get more votes in state s_A than in state s_B $(\alpha_A(\gamma)>\alpha_B(\gamma))$. Therefore before period-2 election all type-1 voters learn from the election result (n_A^1, n_C^1) and update their beliefs accordingly. We prove that for n large enough there exists $n_A^*(n,\gamma)\in(1;\ldots;n)$ such that if $n_A^1>n_A^*(n,y)$ all type-1 voters' updated beliefs

21. See extension (c) in Section II.C for an interpretation of mixed strategies. These are the most natural communicative equilibria to explore, but note however that in general there also exist other equilibria where type-B voters always vote for A while type-A voters vote for C with probability γ and everybody interprets a smaller margin for A as a signal that more type-1 agents received a signal in favour of s_A. Although these "inverse communication" equilibria are very intuitive, they cannot be ruled out by standard game-theoretic arguments.

imply that they prefer A to B, while if $n_A^1 \leqq n_A^*(n,\gamma)$ they prefer B to A.[22] Thus for any sequence $(\gamma_n)_{n\geq 1}$ we consider the sequence of strategy profiles $((v_i^1, v_i^2)_{i\in(1;\ldots;n)})(y_n))_{n\geq 1}$ where the $(v_i^1)(\gamma_n)$s are defined as above with $\gamma = \gamma_n$ and the $(v_i^2)(\gamma_n)$s are defined by:

If I is a type-1 voter, $v_i^2 = B$ if $P^1 = C$

$$v_i^2 = A \quad \text{if } P^1 = A \quad \text{and} \quad n_A^1 > n_A^*(n,\gamma_n),$$

$$v_i^2 = B \quad \text{if } P^1 = A \quad \text{and} \quad n_A^1 \leqq n_A^*(n,\gamma_n);$$

If i is a type-2 voter, $v_i^2 = C$ if $P^1 = C$

$$v_i^2 = B \quad \text{if } P^1 = A.$$

Assuming everybody else follows these strategies, type-B voters decide whether to vote for A or for C by comparing the probability P_n^{d-m} of being pivotal for period-1 decision-making and the probability p_n^{com} of being pivotal for communicating (and therefore for period-2 decision making). Both probabilities go to 0 as $n \to +\infty$, but they go to 0 at different rates (in general), and this is what determines how type-B voters behave for n large. We prove in the Appendix that for any expected vote fractions (β, 1-β) for two given candidates, the proba P_n^{d-m} of being pivotal for (current, majority-rule) decision-making satisfies

$$\lim_{n\to+\infty} (\log(P_n^{d-m}))/n = \log(r^{d-m}(\beta)),$$

with

$$r^{d-m}(\beta) = 2(\beta(1-\beta))^{1/2}.$$

That is, for very large electorates, the probability P_n^{d-m} of being pivotal is well approximated by the formula $P_n^{d-m} = (r^{d-m}(\beta))^n$.[23] Since β is between 0 and 1 (by definition), $r^{d-m}(\beta) = 2(\beta(1-\beta))^{1/2}$ is also between 0 and 1, so that $r^{d-m}(\beta)$ measures the rate at which the probability of being pivotal converges to 0. The higher $r^{d-m}(\beta)$ (the closer to 1), the slower P_n^{d-m} converges to 0. Unsurprisingly, $r^{d-m}(\beta)$ is maximal for β close to 1/2 and symmetric around 1/2. That is, the probability of being pivotal converges to 0 very slowly in case the expected election margin is very close, while it converges to 0 very quickly in case one candidate is expected to get a very large majority of the vote.

Similarly, we prove that for any two possible expected vote fractions $\beta, \beta' \in [0;1]$ for a given candidate, the probability P_n^{com} of being pivotal for communicating to others whether β or β' is the true expected vote fraction satisfies

$$\lim_{n\to+\infty} (\log(P_n^{\text{com}}))/n = \log(r^{\text{com}}(\beta,\beta')),$$

with

$$r^{\text{com}}(\beta,\beta') = (\beta/\lambda)^{\lambda}((1-\beta)/(1-\lambda))^{1-\lambda}(= (\beta'/\lambda)^{\lambda}((1-\beta')/(1-\lambda))^{1-\lambda}),$$

$$\lambda = \lambda(\beta,\beta') = 1/[1 + \log(\beta'/\beta)/\log((1-\beta)/(1-\beta'))].$$

By construction, $\lambda(\beta,\beta')$ is always in between β and β' and is equal to the vote fraction that is equally likely to occur whether β or β' is the true expected vote. $r^{\text{com}}(\beta,\beta')$ is

22. In fact, $n_A^*(n,\gamma)$ is in general different for type-A and type-B voters (it is higher for type-Bs). However, $\lim_{n\to+\infty}(n_{AB}^*(n,\gamma) - n_{AA}^*(n,\gamma))/n = \lim_{n\to+\infty} P_n(n_{AA}^*(n) \leqq n_A^1 \leqq n_{AB}^*(n)) = 0$, so we ignore this in the text to simplify notations. See the Appendix for more details.

23. More precisely, the fact that $\lim_{n\to+\infty}(\log(P_n^{d-m}))/n = \log(r^{d-m}(\beta))$ implies that $P_n^{d-m} = (r^{d-m}(\beta))^{nf(n)}$, with $\lim_{n\to+\infty} f(n) = 1$.

always between 0 and 1 and measures the rate at which the probability P_n^{com} of being pivotal converges to 0: the higher $r^{\text{com}}(\beta, \beta')$ (the closer to 1), the slower P_n^{com} converges to 0. Unsurprisingly, $r^{\text{com}}(\beta, \beta')$ is maximal when β and β' are very close to another.[24] That is, the probability of being pivotal for communicating to others whether β or β' is the true expected vote converges to 0 very slowly if β and β' are very close (and conversely).

Finally, we note $r_{d-m}(\gamma) = \max[r^{d-m}(\alpha_A(\gamma)), r^{d-m}(\alpha_B(\gamma))]$ and $r_{\text{com}}(\gamma) = r^{\text{com}}(\alpha_A(\gamma), \alpha_B(\gamma))$. That is, assuming that all voters follow the strategies $((v_i^1, v_i^2)_{i\in(1;\ldots;n)}(\gamma_n))_{n\geq 1}$ described above, $r_{d-m}(\gamma)$ and $r_{\text{com}}(\gamma)$ measure the rates at which the equilibrium probabilities of being pivotal converge to 0. If $r_{d-m}(\gamma) > r_{\text{com}}(\gamma)$, then the probability P_n^{d-m} of being pivotal for current decision-making converges to 0 more slowly than the probability P_n^{com} of being pivotal for communicating.

In the Appendix, we prove the following proposition:

Proposition 2. *In the two-period voting game, all equilibria are characterized by a sequence* $(\gamma_n)_{n\geq 1}$ (*with* $\lim_{n\to+\infty} \gamma_n = \gamma^* \in [0; 1]$) *defining a strategy profile*

$$((v_i^1, v_i^2)_{i\in(1;\ldots;n)}(\gamma_n))_{n\geq 1}.$$

There always exists one (*unstable*) *"sincere" equilibrium* (*i.e.* $\gamma_n = 0 = \gamma^*, \forall n$) *and at least one* (*stable*) *equilibrium with communicative voting* $\gamma^* > 0$ *defined by*:

If $\alpha q_B > 1/2$ *and* $r_{\text{com}}(1) > r_{d-m}(1)$, then $\gamma^* = 1$ ($=\gamma_n$ for n large);

Otherwise, γ^* *is the unique* $\gamma \in]0; \gamma_0[$ s.t. $r_{\text{com}}(\gamma) = r_{d-m}(\gamma)$.

This is the unique stable equilibrium unless $\alpha q_A < 1/2$ *and* $r_{\text{com}}(1) > r_{d-m}(1)$ (*in which case there exists another stable equilibrium* $\gamma^{**} = 1$).

In this equilibrium, γ^*, *and as* $n\to+\infty$, *policy A wins with probability* 1 *in period* 1 *with a margin of* $\alpha_A(\gamma^*)\in]1/2; \alpha[$ *in state* s_A *and with a margin* $\alpha_B(\gamma^*)\in]1/2; \alpha_A[$ *in state* s_B. *That is, communicative voting biases* 2-*candidate elections towards close margins* (*as compared to the margin* α *of "sincere" voting*).

The intuition for Proposition 2 and the trade-off between communication-oriented voting and decision-making-oriented voting can be graphically illustrated by Figures 1–6, where we represent the rates $r_{d-m}(\gamma)$ and $r_{\text{com}}(\gamma)$ as a function of $\gamma \in [0; 1]$.

When $r_{\text{com}}(\gamma) > r_{d-m}(\gamma)$, it means that if all type-B voters are expected to vote for C with probability γ then as $n\to+\infty$ the probability P_n^{d-m} becomes arbitrarily small as compared to P_n^{com}, which leads type-B voters to care only about the communicative impact of their vote, *i.e.* to vote for C (and conversely if $r_{\text{com}}(\gamma) < r_{d-m}(\gamma)$). It follows that $\gamma^* = 1$ (*i.e.* full communicative voting) is an equilibrium iff $r_{\text{com}}(1) > r_{d-m}(1)$ (see Figure 1). Note that $\gamma\to r_{\text{com}}(\gamma)$ is always monotonically decreasing: as γ goes up the gap $\alpha_A(\gamma) - \alpha_B(\gamma)$ between A's expected vote fraction in states s_A and s_B rises, so that casting the decisive "communicative" vote becomes less likely. $\gamma\to r_{d-m}(\gamma)$ is increasing between 0 and γ_0 (casting a pivotal vote for decision-making becomes more likely as $\alpha_B(\gamma)$ goes towards 1/2), then decreases between. Figures 1–6 illustrate the various possible cases depending on whether $\gamma_0 > 1$ ($\alpha q_B > 1/2$) and $r_{\text{com}}(1) > r_{d-m}(1)$ (Figure 1), $\gamma_0 > 1$ and $r_{\text{com}}(1) < r_{d-m}(1)$ (Figure 2), $\gamma_0 < 1 < \gamma_1$ ($1 - \alpha q_A < \alpha q_B < 1/2$) (Figure 3),[25] $\gamma_0 < \gamma_1 < 1 < \gamma_2$

24. More precisely, assuming $\beta < \beta'$, $r^{\text{com}}(\beta, \beta')$ increases when β goes up or β' goes down (and conversely if $\beta > \beta'$).

25. Note in that case and the next one we must necessarily have $r_{\text{com}}(1) < r_{d-m}(1)$. This is because for any $\beta > 1/2 > \beta'$, $r^{\text{com}}(\beta, \beta') < \max(r^{d-m}(\beta), r^{d-m}(\beta'))$.

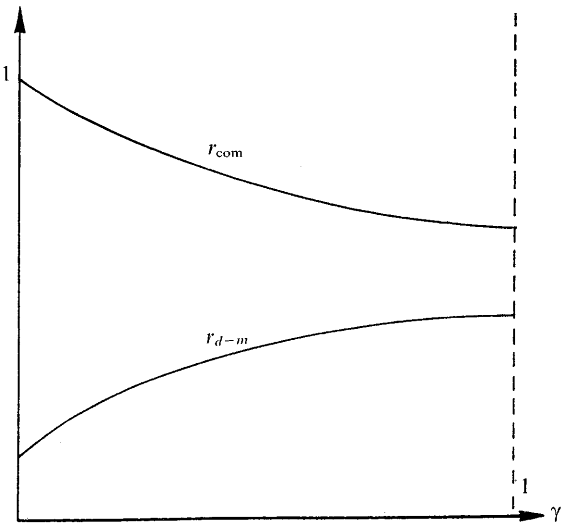

FIGURE 1

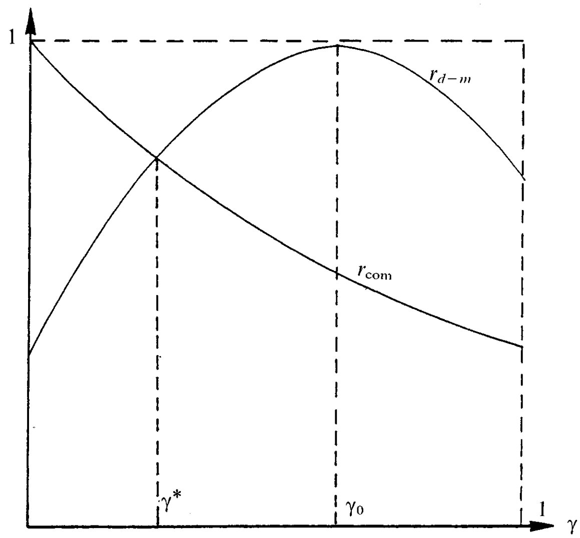

FIGURE 2

$(\alpha q_B < 1 - \alpha q_A < 1/2)$ (Figure 4), $\gamma_0 < \gamma_1 < \gamma_2 < 1$ $(\alpha q_B < 1/2 < 1 - \alpha q_A)$ and $r_{\text{com}}(1) < r_{d-m}(1)$ (Figure 5), and $\gamma_0 < \gamma_1 < \gamma_2 < 1$ and $r_{\text{com}}(1) > r_{d-m}(1)$ (Figure 6).

When the curves $r_{\text{com}}(\gamma)$ and $r_{d-m}(\gamma)$ intersect (see Figures 2–6), this means that there exists a mixed-strategy equilibrium and that there exists mixed-strategies equilibria when the 2 curves intersect: if $r_{\text{com}}(\gamma) = r_{d-m}(\gamma)$, type-$B$ voters can be made indifferent between voting for A and voting for C. One more realistic way to interpret these mixed-strategies equilibra is that there will be a "division of labour" among type-B voters: the strongest supporters of B will vote for C to express everybody's 1st preference while the others will vote for A to ensure that C does not pass. That is, one could assume some heterogeneity within type-B voters, such that there exists a continuous distribution function $F(\Delta)$ defined

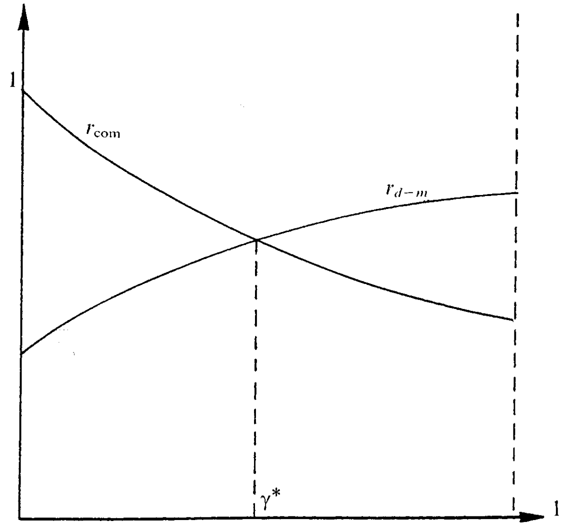

FIGURE 3

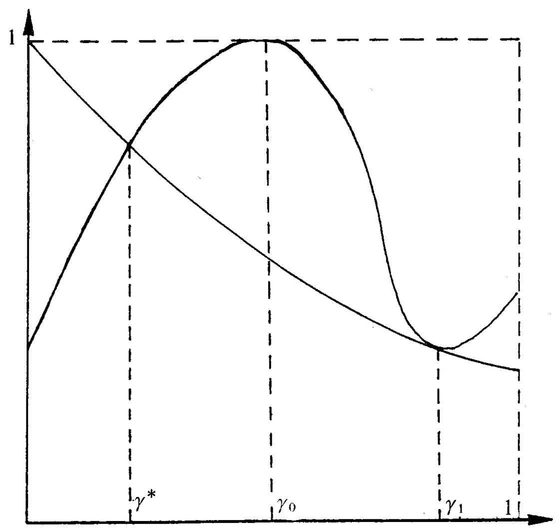

FIGURE 4

over $[\Delta_0; \Delta_1]$ of utility differentials $\Delta = U_1(B|\sigma_B) - U_1(A|\sigma_B)$.[26] All type-$B$ voters would still be 'type-B voters'', in the sense that $\Delta_0 > 0$, *i.e.* they all prefer B to A, but some prefer it more than others. Assume we are in the case where $\gamma_0 < 1$ (Figures 2–6). Then for n large enough there exists $\gamma_n \in]0; \gamma_0[$ such that if $\Delta^* = \Delta^*(\gamma_n)$ is defined by $F(\Delta^*) = 1 - \gamma_n$ then there exists an equilibrium defined by the strategy profile $((v_i^1, v_i^2)_{i\in(1;\ldots;n)}(\gamma_n))$ defined in the same way as before except that in period 1 type-B voters characterized $\Delta < \Delta^*(\gamma_n)$

26. This heterogeneity could be due to some heterogeneity of preferences, of priors, or of strength of signals, *etc*.

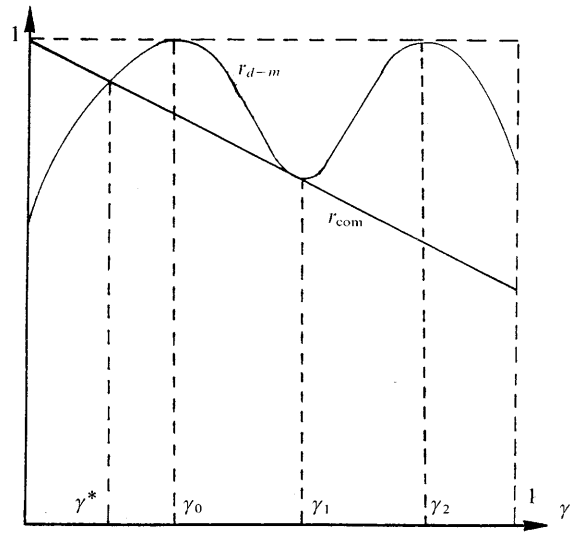

FIGURE 5

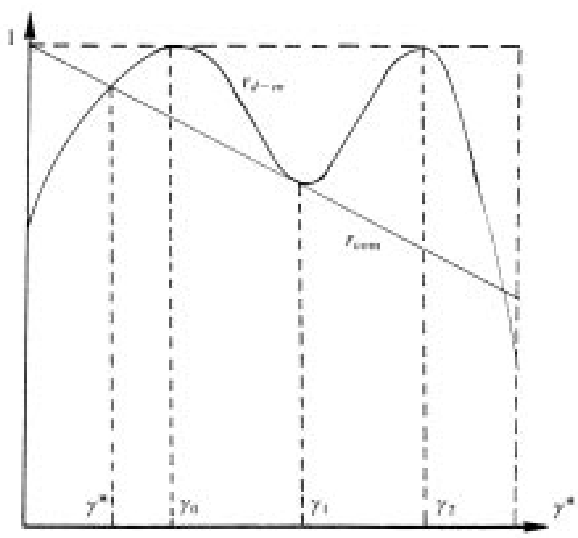

FIGURE 6

vote for A with proba 1 and type-B voters characterized by $\Delta > \Delta^*(\gamma_n)$ vote for C with proba 1.[27]

Figures 1–6 also show that "sincere" voting ($\gamma_n = \gamma^* = 0$) is always an equilibrium, but that this equilibrium is inherently "unstable", in the following sense. In this "sincere"

27. The proof is essentially the same as that of Propostion 2: for a given n consider the mapping $\gamma \to G(\gamma) = 1 - F(\Delta(\gamma))$ where $\gamma(\gamma)$ is s.t. if a fraction γ of type-B voters is expected to vote for C then type-B voters with $\Delta < \Delta(\gamma)$ strictly prefer to vote for A and type-B voters with $\Delta > \Delta(\gamma)$ strictly prefer to vote for C. Since $r_{\text{com}}(0) > r_{d-m}(0)$ and $r_{\text{com}}(\gamma_0) < r_{d-m}(\gamma_0)$, $\Delta(0) = 0$ and $\Delta(y_0) = 1$ for n large enough, *i.e.* $G(0) = 1$ and $G(\gamma_0) = 0$. By continuity, there exists y_n s.t. $G(\gamma_n)$. For the same reasons as in Proposition 2, $\lim_{n \to +\infty} \gamma = \gamma^*$ s.t. $r_{\text{com}}(\gamma^*) = r_{d-m}(\gamma^*)$.

equilibrium, type-B voters do not communicate simply because nobody expects any type-B voter to communicate (so that n_A^1 does not influence beliefs at all: nobody pays attention to the election margin). But any small deviation $\gamma_n = \varepsilon > 0$ would imply that all type-B voters strictly prefer to communicate by voting for C. This is because if a small fraction of type-B voters is expected to vote in a communicative way, then $\alpha_A(\gamma)$ and $\alpha_B(\gamma)$ are so close to one another that the probability P_n^{com} dominates the probability P_n^{d-m} (*i.e.* $r_{\text{com}}(\gamma)$ is always higher than $r_{d-m}(\gamma)$ for γ close to 0; see Figures 1–6), so that all type-B voters strictly prefer to vote in a communicative way. That is, any small deviation from sincere voting triggers a "communication cascade".[28] In contrast, the communicative equilibria $\gamma^* > 0$ described in Proposition 2 are "stable", in the sense that if a small fraction of type-B voters is expected to deviate by communicating with a probability $\gamma^* + \varepsilon$ (resp. $\gamma^* - \varepsilon$), then other type-B voters prefer to communicate less (resp. more): $r_{\text{com}}(\gamma^* + \varepsilon) < r_{d-m}(\gamma^* + \varepsilon)$ (resp. $r_{\text{com}}(\gamma^* - \varepsilon) > r_{d-m}(\gamma^* + \varepsilon)$) (see Figures 1–6).[29]

Although the exact formulas for $r_{\text{com}}(\gamma)$ and $r_{d-m}(\gamma)$ seem pretty complicated at first sight, they do correspond to very simple and plausible intuitions and can be readily applied to specific parameter values, as the following examples illustrate:

Example 1. Assume $\alpha = 70\%$, $q_A = 80\%$, $q_B = 75\%$, so that $\alpha q_A = 56\%$, $\alpha q_B = 52{\cdot}5\%$. Then $r_{d-m}(1) = 0{\cdot}9987$, $r_{\text{com}}(1) = 0{\cdot}9994$.

Therefore we are in the case of Figure 1: the unique equilibrium involves all B-types voting for C ($\gamma^* = 1$), so that A wins with a margin of 56% in state s_A and 52·5% in state s_B, as compared to the 70% margin in the sincere equilibrium.

Note that for $n = 100{,}000$, $P_n^{\text{com}}/P_n^{d-m} \simeq (0{\cdot}9994/0{\cdot}9987)^{100{,}000} = 2{\cdot}7 \times 10^{30}$: communication dominates.

Example 2. Assume $\alpha = 70\%$, $q_A = 80\%$, $q_B = 40\%$, so that $\alpha q_A = 56\%$, $\alpha q_B = 28\%$.

We are in the case of Figure 4: the unique stable equilibrium is $\gamma^* \simeq 0{\cdot}357$. A wins with a margin of 65% in state A and 55% in state B.

Example 3. Assume $\alpha = 70\%$, $q_A = 60\%$, $q_B = 40\%$, so that $\alpha q_A = 42\%$, $\alpha q_B = 28\%$. Then $r_{d-m}(1) = 0{\cdot}9871$, $r_{\text{com}}(1) = 0{\cdot}9891$.

We are in the case of Figure 6; there are 2 stable equilibria. One is $\gamma^{**} = 1$, in which C wins a margin of 58% in state s_A and 72% in state s_B. The other is $y^* \simeq 28{\cdot}6\%$, in which A wins with a margin of approximately 66% in state s_A and approximately 58% in state s_B.

So far we did not consider the possibility for type-B voters of communicating their signals by using abstention. Assume now that type-B voters abstain with probability γ (and vote for A with proba $1 - \gamma$) instead of voting for C with probability γ. The expected vote share of A in state s_A (resp. s_B) is still given by $\alpha_A(\gamma)$ (resp. $\alpha_B(\gamma)$), while that of C is now equal to $1 - \alpha$ in both states of the world. The only difference with the previous analysis is therefore that $r_{d-m}(\gamma)$ is now given by $r_{d-m}(\gamma) = \max[r(\alpha_A(\gamma), 1 - \alpha),$

28. One standard way to model this stability concept would be to introduce a small fraction ε of "communicative" voters (*i.e.* type-B voters who always vote for C with probability 1) and to let the fraction ε go to 0.

29. For the same reasons, the intersection of $r_{\text{com}}(\gamma)$ and $r_{d-m}(\gamma)$ between γ_2 and 1 on Figure 6 defines an unstable equilibrium: perturbations below or above this point would amplify themselves. The same is true for the candidate equilibrium γ_1 on Figures 4–6. Note however that the latter exists only if $\delta(U_1(B|s_B) - U_1(A|s_B)) \gg U_1(A|s_B) - U_1(C)$, as opposed to all other equilibria whose existence is guaranteed by $\delta(U_1(B|s_B) - U_1(A|s_B)) > 0$.

$r(\alpha_B(\gamma), 1-\alpha)]$ instead of $r_{d-m}(\gamma) = \max[r^{d-m}(\alpha_A(\gamma)), r^{d-m}(\alpha_B(\gamma))]$, with $r(\beta, \beta') = (\beta^{\lambda}/\lambda)((1-\beta)/(1-\lambda^{-\lambda}))$ and $\lambda = \lambda(\beta, \beta') = (\beta + \beta')/2$. One can then prove in the same way as in proposition 2 that there always exists an equilibrium with communicative voting through abstention. The following example illustrates the intuition.

Example 4. Assume $\alpha = 70\%$, $q_A = 60\%$, $q_B = 50\%$, so that $\alpha q_A = 42\%$, $\alpha q_B = 35\% > 1-\alpha = 30\%$.

There exists a unique equilibrium with communicative voting through abstention $\gamma^{**} = 1$, where A wins by 42% vs. 30% for C in state s_A and by 35% vs. 30% in state s_B. The abstention rate is 28% in state s_A and 35% in state s_B. This equilibrium with abstention is fully equivalent to the equilibrium without abstention where type-B voters vote for C with a probability $\gamma^* \in]0;1[$ of the type described in Example 3 above (from an informational viewpoint, abstention is always equivalent to some form of random voting). It is however superior to the equilibrium where type-B voters vote for C with probability 1, since communication through abstention does not threaten the victory of A (see Section III.B below).

Note that this "active" view of strategic abstention differs from the more "passive" view of strategic abstention recently proposed by Feddersen and Pesendorfer (1996), where voters with signals of lower quality choose not to vote in order to let better informed individuals take the decision.

II.B. *Efficiency of communicative voting with* 2 *candidates*

Throughout the paper we define efficiency from the viewpoint of the majority. That is, an election outcome is said to be efficient if it is a full-information majority winner. Therefore "efficiency" requires that policy A wins the election in state s_A and that policy B wins the election in state s_B.[30] In the model of the previous subsection, equilibria with communicative voting are never not more efficient than sincere voting. The reason is that even the period-1 election does not reveal any information about s_A and s_B, with probability 1 (as $n \to +\infty$) A wins the period-2 election if $s = s_A$ and B wins the period-2 election if $s = s_B$. That is, "sincere" voting is sufficient to deliver efficient information aggregation in two-candidate elections. This is just a particular case of Myerson (1994*b*)'s Theorem 2, which extends the Condorcet Jury Theorem to the case where less than 50% of the population might receive the "true" signal.[31] This general efficiency result for 2-candidate elections can also be extended to a model with $S \geqq 3$ states of the world (or, equivalently, 2 states

30. This efficiency concept is equivalent to Pareto-efficiency if one assumes that type-2 voters also have the same "true" preferences as type-A and type-B voters, but that they just received a different signal σ_C (all results can easily be extended with this alternative formulation).

31. The original Condorcet Jury Theorem simply states that if all agents have a probability $q_A = 1 - q_B > 50\%$ to receive a correct signal then sincere voting implies that the majority will make the right decision with proba 1. Myerson (1994*a*)'s Theorem 2 extends this to the case where, say, $q_A > 1/2 > 1 - q_B$ by noting that in such a case sincere voting is not an equilibrium, since type-A voters would then know that being pivotal is much more likely in s_B than in s_A and strictly prefer to vote for B; so that the equilibrium involves type-As voting for A with a positive proba to equalize the *ex ante* chances of each policy and implement efficient outcomes. [Myerson's general theorem is stated in a Poisson setting but can easily be extended to a setting with certain population size]. The results of Austen-Smith and Banks (1994) and Feddersen and Pesendorfer (1994) also exploit this informativeness of being pivotal.

and aggregate uncertainty about the distribution of signals).[32] In fact, not only communicative voting is never necessary in this setting to implement efficient outcomes, but it can actually make things worse: in Example 3 above, policy C wins the period-1 election in the equilibrium with $\gamma^{**}=1$. This arises whenever $\alpha q_A < 1/2$. Such an election outcome is inefficient: in order to make sure that policy B will win period-2 election (which would have occurred anyway), type-B voters behave in such a way that policy A loses in period 1. In contrast, communicative voting through abstention will make things worst only when $\alpha q_A < 1-\alpha$, and therefore appears to be (weakly) more efficient than voting for the opposite candidate. In practice, one obvious limitation of communicative abstention is that abstention is typically used to communicate all sorts of signals (distrust of the political process as a whole, ...), so that all available means of communication may be needed, including voting for the opposite candidate.

Proposition 3. *In repeated voting processes over* 2 *candidates with exogeneous agenda-setting, communicative voting is never more efficient than sincere voting, and it can be strictly worse.*

However, this conclusion about the inefficiency of communicative voting in two-candidate elections is not very general: communicative voting can be strictly more efficient than sincere voting once one takes into account that whether or not the period-2 election takes place at all may depend on what was revealed in period 1. For instance, assume that if A wins the period-1 election then type-A voters control agenda-setting and can decide whether or not to put A vs. B on the ballot in period 2. Assume this is costly, so that they will do so only if the period-1 election convinced them that B was better than A. In that case, it is clear that communicative voting is strictly more efficient than sincere voting: with sincere voting type-A voters could never have learned that B was better than A and an inefficient policy would have prevailed. Alternatively, one can also extend Proposition 2 to a world without uncertain preferences. That is, assume that the states of the world s_A and s_B do not represent some "truth" conditional on which all type-1 voters have the same "true" preferences, but rather represent different possible distributions of heterogeneous preferences. That is, in state s_A (resp. s_B) a fraction q_A (resp. q_B) of type-1 voters prefer A to B while a fraction $1-q_A$ (resp. $1-q_B$) prefer B to A, and these preferences cannot be altered: type-1 voters are certain about their own preferences, they are simply uncertain about what fraction of type-1 voters share their preferences. Assume also that the agenda-setter is some opportunistic government that puts on the ballot anything that ensures majority support. That is, A vs. B will be on the ballot in period 2 if and only if the period-1 elections revealed that B was likely to win (*i.e.* that $s=s_B$). Then communicative voting is an equilibrium under the same conditions as in Proposition 2, and it is efficient for the same reasons as in the previous example. These are two typical instances of what we called the first channel for communicative voting in Section I, and both imply an efficiency rationale for communicative voting.:

Proposition 4. *In repeated voting processes over* 2 *candidates with endogeneous agenda-setting, communicative voting is strictly necessary to implement first-best efficiency.*

32. In the context of our specific model, the logic of Myerson's general efficiency result can be summarized as follows. Assuming for simplicity that type-2 voters are indifferent between A and B and that $q_A > 1-q_B$ (with no generality loss), the period-2 equilibrium in case period 1 was uninformative (*i.e.* $\gamma_n=0$) involves all type-B voters voting for B and type-A voters voting for A with proba $1-\gamma'_n$ and for B with proba γ'_n, with $\lim_{n\to+\infty}\gamma'_n=\gamma^{*\prime}$ s.t. $(1-\gamma'_n)q_A = 1-q_B+y'_n q_B > 1/2$.

III. COMMUNICATIVE VOTING WITH THREE CANDIDATES

III.A. *Basic results*

We now consider the same dynamic voting game as in Section II except that voting takes place over the 3 candidates A, B, C at the same time.[33] That is, at $t = 1$ each voter i casts a ballot $v_i^1 = A$, B or C, the winner P^1 is determined by the simple plurality rule (with a coin toss in case of ties), and the same process takes place at $t = 2$.

The "canonical" coordination problem with such voting processes arises when $\alpha > 1/2$, but $\alpha q_B < 1 - \alpha$ and $\alpha(1 - q_B) < 1 - \alpha$. That is, type-1 voters form a majority preferring A and B to C, but if they all vote for their most-preferred policy (A for type-As, B for type-Bs) then C wins the election in both periods whatever the state of the world may be.

First, note that in one-period version of the voting game only one of the two candidates A and B can get a positive number of votes, except if candidates A and B have exactly the same probability of beating C. Assuming that A in s_A is expected to be stronger than B in s_B, *i.e.* $q_A > 1 - q_B$, one can prove the following result, which is a direct application of Myerson and Weber (1993, pp. 105–106):[34]

Proposition 5. (Myerson–Weber (1993)). *In the* 1-*period voting game, only* 2 *candidates obtain a positive number of votes, except if candidates A and B have exactly the same expected vote fraction in their respective state. That is, there exists* 3 *equilibria*:

(1) $v_i^1 = A$ *for all type*-1 *voters. As* $n \to +\infty$, *A wins with a margin* α *vs.* $1 - \alpha$ *for C*;
(2) $v_i^1 = B$ *for all type*-1 *voters. As* $n \to +\infty$, *B wins with a margin* α *vs.* $1 - \alpha$ *for C*;
(3) $v_i^1 = A$ *with proba* $1 - \gamma_n$ *and* $v_i^1 = B$ *with proba* γ_n *for all type-A voters, while* $v_i^1 = B$ *for all type-B voters, with* $\gamma^* = \lim_{n \to +\infty} \gamma_n$ s.t. $(1 - \gamma^*) q_A = \gamma^* q_B + 1 - q_B$. *As* $n \to +\infty$, *C wins a margin of* $1 - \alpha$ *vs.* $\alpha(1 - \lambda^*) q_A$ *for A and* $\alpha(\gamma^* q_A + 1 - q_A)$ *for B in state* s_A *and* $\alpha(\gamma^* q_B + 1 - q_B) = \alpha(1 - \gamma^*) q_A$ *for B and* $\alpha(1 - \gamma^*) q_B$ *for A in state* s_B (*in all equilibria type*-2 *voters vote for C*).

The trick behind the third equilibrium is that for each n one can always find $\gamma_n > \gamma^*$ so that the slightly higher expected vote fraction for B makes type-A voters indifferent between voting for A and voting for B. But aside from this very special case, only two candidates can obtain a positive fraction of the vote in equilibrium. This extreme form of Duverger's "psychological effect" is strongly counterfactual:[35] there are many instances where more than 2 candidates obtain a substantial fraction of the vote even though some of these candidates are unanimously known to face a lower probability of winning than all others (see Section I). We now see how this emerges naturally in the 2-period setting.

Assume that in period 1 all type-2 voters vote for C, all type-A voters vote for A, while type-B voters vote for B with probability γ for A with probability $1 - \gamma$. C's expected fraction of the vote is always $1 - \alpha < 1/2$, while A's expected fraction of the vote is

33. Similar insights apply to elections with 4 candidates and more.

34. In fact, Myerson and Weber used an equilibrium concept that is slightly different from the limit of Bayesian–Nash equilibria as $n \to +\infty$, but this result also applies if we use the latter as equilibrium concept (which seems more natural), as we do.

35. Since Duverger's seminal work on plurality-rule electoral systems and bipartism, it has become common to distinguish the "mechanical effect" (for a given distribution of votes between parties plurality-rule systems implies more disproportionality in the distribution of seats than PR systems), the "elite effect" (the elite stops forming 3rd parties if that does not yield seats, and conversely with a PR system), and the "psychological effect" (voters do not like to waste their votes), which coincides with the "strategic effect" of the calculus-of-voting literature. See the references given in Section I.

$\alpha_A(\gamma)=\alpha(q_A+(1-\gamma)(1-q_A))$ in state s_A and $\alpha_B(\gamma)=\alpha_B(q+(1-\gamma)(1-q_B))$ and B's expected fractions are $\beta_A(\gamma)=\alpha-\alpha_A(\gamma)$ and $\beta_B(\gamma)=\alpha-\alpha_B(\gamma)$. $\alpha_A(\gamma)>\alpha_B(\gamma)$, so in the same way as in Section II, we define $n^*_A(n-n^1_C,\gamma)\in(1;\ldots;n-n^1_C)$ s.t. if $n^1_A>n^*_A$ then all type-1 voters' updated beliefs are such that they prefer A to B, while if $n^1_A\leqq n^*_A$ they prefer B to A. For any sequence $(\gamma_n)_{n\geq 1}$ we consider the sequence of strategy profiles $((v^1_i, v^2_i)_{i\in(1;\ldots;n)}(\gamma_n))_{n\geq 1}$ where the $(v^1_i)(\gamma_n)$s are defined as above with $\gamma=\gamma_n$ and the $(v^2_i)(\gamma_n)$s are defined by

$$\begin{aligned}
&\text{If } i \text{ is a type-1 voter,} \quad v^2_i = A \quad \text{if } n^1_A > n^*_A(n-n^1_C,\gamma_n)\\
&\qquad\qquad\qquad\qquad\quad\; v^2_i = B \quad \text{if } n^1_A \leqq n^*_A(n-n^1_C,\gamma_n),\\
&\text{If } i \text{ is a type-2 voter,}^{36} \; v^2_i = C.
\end{aligned}$$

We show in the Appendix that approximation formulas for pivotal probabilities P^{d-m}_n and P^{com}_n that are similar in spirit to those of Section II.A can be applied, and that they lead to the following definitions for $r_{d-m}(\gamma)$ and $r_{\text{com}}(\gamma)$

$$r_{d-m}(\gamma)=\text{Max}\,(r^{d-m}(\alpha_A(\gamma),1-\alpha),\qquad r^{d-m}(\alpha_B(\gamma),1-\alpha)),$$
$$r_{\text{com}}(\gamma)=r^{\text{com}}(\alpha_A(\gamma),\alpha_B(\gamma),\alpha),$$

with

$$\forall\beta,\beta'\in[0;1],\qquad \beta+\beta'\leqq 1,$$
$$r^{d-m}(\beta,\beta')=1-\beta-\beta'+2(\beta\beta')^{1/2}\,(=r^{d-m}(\beta)\text{ if }\beta+\beta'=1)$$
$$\forall\beta,\beta',\beta''\in[0;1],\beta,\beta'\leqq\beta'',$$
$$\begin{aligned}
r^{\text{com}}(\beta,\beta',\beta'')&=1-\beta''+(\beta/\lambda)^{\lambda}((\beta''-\beta)/(1-\lambda))^{1-\lambda}\\
&=1-\beta''+(\beta'/\lambda)^{\lambda}((\beta''-\beta')/(1-\lambda))^{1-\lambda})
\end{aligned}$$

and

$$\lambda(\beta,\beta',\beta'')=1/[1+\log(\beta'/\beta)/\log((\beta''-\beta)/(\beta''-\beta'))].$$

We have the following proposition:

Proposition 6. *In the two-period voting game, the duplication of the one-period equilibrium is always an (unstable) equilibrium. There always exists a (stable) equilibrium* $((v^1_i,v^2_i)_{i\in(1;\ldots;n)}(\gamma_n))_{n\geq 1}$ such that $\gamma^*=\lim_{n\to+\infty}\gamma_n$ *is uniquely defined by*:

$$r_{\text{com}}(\gamma^*)=r_{d-m}(\gamma^*)\quad \textit{and}\quad \gamma^*\in]0;\gamma_0[,\ \textit{with}\ \gamma_0\in]0;1[\text{ s.t. }\alpha_B(\gamma_0)=1-\alpha.$$

If $r_{\text{com}}(1)\leqq r_{d-m}(1)$, *this is the unique equilibrium. Otherwise there exists another (stable) equilibrium defined by* $\gamma_n=\gamma^{**}=1$ *for n large.*

That is, Duverger's law does not hold (at least in its extreme form): equilibrium voting behaviour does lead rational voters to cast their votes for three different candidates, despite the fact that only two of them have a chance of winning. The rationale for voting for third candidates is that although such a vote is useless from the viewpoint of current decision-making, it can be decisive in a communicative way by influencing the votes in

36. To simplify the analysis, we now assume that type-2 voters are indifferent between A and B (otherwise they might want to manipulate the communication process between type-As and type-Bs by voting by A or B with positive proba in period 1).

future elections. In the same way as for Proposition 2, the trade-off between the 2 different ways of being pivotal is illustrated on Figures 5–6. $\gamma \to r_{\text{com}}(\gamma)$ is again monotonically decreasing: casting a communicative pivotal vote is more likely when $\alpha_A(\gamma)$ and $\alpha_B(\gamma)$ are very close. Regarding $\gamma \to r_{d-m}(\gamma)$, we define $\gamma_0, \gamma_1, \gamma_2$ by $\alpha_B(\gamma_0) = 1-\alpha$, $\alpha_A(\gamma_1) - (1-\alpha) = 1 - \alpha - \alpha_B(\gamma_1)$, $\alpha_A(\gamma_1) = 1-\alpha$, in the same way as in Section II.A. Unlike in Section II.A, we always have $\gamma_0 < \gamma_1 < \gamma_2 < 1$, since we assumed $1-\alpha > \alpha q_A$, $\alpha(1-q_B)$. Thus we are always in the cases of Figure 5 or 6: either $r_{\text{com}}(1) < r_{d-m}(1)$, in which case there exists a unique equilibrium $\gamma^* \in]0; y_0[$ (see Figure 5), or $r_{\text{com}}(1) > r_{d-m}(1)$, in which case there exists an additional equilibrium $\gamma^{**} = 1$ (see Figure 6). The following examples illustrate how these formulas can be applied to specific parameter values.

Example 1. Assume $\alpha = 55\%$, $q_A = 80\%$, $q_B = 60\%$, so that $1-\alpha = 45\%$, $\alpha q_A = 42{\cdot}5\%$, $\alpha q_B = 32{\cdot}5\%$.

$$r_{d-m}(1) = 0{\cdot}9996, \qquad r_{\text{com}}(1) = 0{\cdot}9936.$$

$$r_{\text{com}}(1) < r_{d-m}(1), \text{ so we are in the case of Figure 5.}$$

The unique equilibrium γ^* s.t. $r_{\text{com}}(\gamma^*) = r_{d-m}(\gamma^*)$ is $\simeq 30{\cdot}8\%$, so that as $n \to +\infty$, in state s_A A wins with a margin of 51·15% vs. 45% for C and 3·85% for B, and in state s_B A wins with a margin of 48·08% vs. 45% for C and 6·92% for B. The race between A and C is so close that the vote for B drops dramatically (with sincere voting, B gets 12·5% in state s_A and 22·5% in state s_B).

Example 2. Assume $\alpha = 55\%$, $q_A = 65\%$, $q_B = 60\%$, so that $1-\alpha = 45\%$, $\alpha q_A = 35{\cdot}75\%$, $\alpha q_B = 32{\cdot}5\%$.

$$r_{d-m}(1) = 0{\cdot}9947, \qquad r_{\text{com}}(1) = 0{\cdot}9992.$$

Now we are in the case of Figure 6: the small difference between A in state s_A and A in state s_B as compared to the gap between C and A in state s_A allows for complete communicative voting.

In the equilibrium $\gamma^{**} = 1$, and as $n \to +\infty$, C wins with a margin of 45% vs. 35·75% for A and 19·25% for B in state s_A, vs. 32·5% and 22·5% in state s_B.

Note that in both examples B gets a substantial fraction of the vote even though B is expected to get strictly less votes than A and C in both states: this could not happen in a 1-period voting game (see Proposition 5 above).

Finally, note that in the same way as Proposition 2, Proposition 6 can be readily applied to a setting without uncertain preferences. That is, assume that the states of the world s_A and s_B do not represent some "truth" conditional on which all type-1 voters have the same "true" preferences, but rather represent different possible distributions of heterogeneous preferences. That is, in state s_A (resp. s_B) a fraction q_A (resp. q_B) of type-1 voters prefer A to B while a fraction $1-q_A$ (resp. $1-q_B$) prefer B to A, and these preferences cannot be altered: type-1 voters are certain about their own preferences, they are simply uncertain about what fraction of type-1 voters share their preferences. Then Proposition 6 applies in exactly the same terms. That is, if type-1 voters expect that all other type-1 voters will coordinate in the period-2 election on voting for A if $n_A^1 > n_A^*$ and on voting for B_A^1 if $n_A \leqq n^*$, then communicative voting is indeed an equilibrium under the conditions described by Proposition 6. Such a period-2 coordination on the strongest candidate in type-1 might be due to "fairness" considerations among type-1 voters, but the point is that this is a strategic equilibrium. This is the typical example of what we called the second channel for communicative voting in Section I.

II.B. *Application*: *One-round vs. two-round electoral systems*

Communicative voting in elections with three candidates (or more) is obviously beneficial since it allows type-1 voters to coordinate on their best candidate in period 2. That is, in all equilibria with communicative voting ($\gamma^* > 0$), A wins the period-2 election in state s_A and B wins the period-2 election in state s_B with probability 1 (which would not have been the case with "sincere" voting). Therefore communicative voting can be strictly necessary in order to implement efficient information aggregation in elections with three candidates, even if agenda-setting is exogeneous.

However, communicative voting can also be very costly, as Example 2 illustrates. In case the expected margin between C and A in s_A is large as compared to the expected margin between A in s_A and A in s_B (*i.e.* $r_{\text{com}}(1) > r_{d-m}(1)$), then "sincere" voting ($\gamma^{**} = 1$) is a strategic equilibrium, and this is arguably the "focal" equilibrium when it exists. In this equilibrium, a majority of the electorate (α vs. $1-\alpha$) prefer both A and B to C and even has enough information to know whether A or B is better, but C wins with probability 1. In other words, communicative voting in a 1-round plurality system can delay the implementation of the efficient policy, possibly for very long periods. A dramatic illustration of this inefficiency is given by the British example of the past 15 years, with C = Conservative Party and A, B = Labour Party and Liberal/Social-Democratic Alliance (see Section I).

The point is that one could easily get rid of this inefficiency by designing another electoral system. Given that voters use voting not only as current decision-making scheme but also as a communication device, the inefficiency of one-round plurality systems derives from the fact they mix up both functions of the vote into a single round. In contrast, multiple-round electoral systems allow voters to separate those two functions. Assume for instance that at each period $t = 1, 2$ the election between A, B, C is ruled by a two-round majority system instead of a one-round plurality system. That is, at the first round of the period-1 election each voter i casts a ballot $v_i^{11} = A$, B, or C; at the second round each voter i casts a ballot $v_i^{12} = P^{11}$ or P^{12}, where P^{11} and P^{12} are the 2 candidates getting more votes in the 1st round, and the candidate $P^1 = P^{11}$ or P^{12} with more votes is the period-1 winner; the same process takes place at period 2. This is similar to the electoral system used in presidential, parliamentary and local elections in France, as opposed to the 1-round plurality system which is typical of Anglo-Saxon democracies.[37]

Although a proper analysis of voting equilibria in alternative multiple-round systems is well beyond the scope of this paper,[38] it is obvious that such systems can allow voters to sustain equilibria with efficient information aggregation. For instance, if all voters vote

37. There actually exists several types within the French category of 2-round systems: in presidential elections, only the top 2 candidates take part to the 2nd round; in parliamentary elections, one needs 12·5% of the registered electorate in round 1 (which in practice is very dissuasive: about 10 out of 577 constituencies have usually more than 2 candidates in round 2); in local elections, one needs only 10% of the expressed vote, so that there are usually more than 2 candidates in round 2. This can be very inefficient if voters want to communicate the strength of their signal by repeating their communicative vote in the 2nd round (see the example of National Front voters in Appendix A of Piketty (1995)).

38. A rigorous equilibrium analysis of multiple-round systems would require to take into account the strategic effects of the specific cutoff rule (in order to pass the first round) as well as the fact that there can be no second round in case one candidate gets an absolute majority of the vote at the first round.

sincerely in the first round (the "communicative round"), which with probability 1 reveals to type-1 voters the state s_A or s_B, and if in the 2nd round (the "decision-making round") type-2 voters vote for C and type-1 voters vote for their remaining candidate, then with probability 1 the efficient outcome (A in state s_A and B in state s_B) wins the election.[39] That is, such a multiple-round system allows to implement efficient outcomes every period by defining a strict separation between the communicative and the decision-making functions of the vote.

Proposition 7. *In elections over three candidates (or more), multiple-round electoral systems (such as a 2-round system with a runoff between the top 2 candidates) are strictly more efficient than a 1-round, first-past-the-post system.*

Note that the two-round system described above is almost equivalent to the new system that was advocated by the Labour Party's Electoral Committee before the 1997 general elections: in their proposed 1-round, ordinal-ballot system, voters cast two votes, second preferences are reallocated to the two candidates with most first preferences,and the winner is the one of these two candidates that gets the highest total number of votes.[40] However such a system becomes more complicated if one takes into account that there can be more than three candidates. Moreover, it does not allow for learning between the two rounds.[41] It seems simpler and more efficient to go all the way towards a two-round system. Surprisingly enough, very little attention has been devoted to two-round systems in the academic literature on electoral systems. For example, Lijphart (1990, p. 493) simply notes in his conclusion that his estimate of the "psychological effect" in plurality systems is biased downwards by his inclusion of two-round majority systems in this category, but he keeps stressing the plurality vs. PR opposition.

On the other hand, our conclusion (Proposition 7) does not take into account the arguments regarding the production of a stable majority and the representation of minorities put forward by the traditional literature, and there is some evidence that single-member-district, two-round majority systems may fail to achieve these two goals.[42] Taking these two goals into account, "the" optimum electoral system might well be a combination of a two-round majority systems with large districts (possibly with several seats going to the single winner, so as to produce strong majorities) and of a small number of additional seats allocated by a national PR formula (so as to represent minorities). We hope that the theory of communicative voting proposed in this paper can be used by others in order to develop such theories of "optimal electoral systems".

39. "Sincere" voting is the round-1 equilibrium only if $q_B > 1/2$, so that B passes the 1st round in state s_B (we already assumed $q_A > 1/2$). Otherwise type-A voters vote for B with positive probability so as to make sure that B passes the 1st round in state s_B. Regarding the unicity of this equilibrium, see Footnote 19.

40. A similar system is already used in Australia.

41. In the June 1995 local elections in the French city of Niort, the 1st round gave 34·5% to a Socialist Party national leader, 33·5% to the incumbent Socialist Party mayor and 32% to some right-wing candidate, while the 2nd round gave them 32%, 36% and 32%, with the incumbent being reelected. This is the typical example where learning between the 2 rounds can change voters' 1st preferences (some socialist voters who voted for the popular national leader in the 1st round were probably impressed by the score of the incumbent and decided that the latter should not be fired by someone with no roots in Niort). This cannot happen in a 1-round, ordinal-ballot system (randomizing over different ordinal ballots could in principle achieve the same goal with 2 states of the world).

42. See, *e.g.* the absence of a stable parliamentary majority in France between 1988 and 1993. See, *e.g.* the absence of any representative from the National Front after the parliamentary elections of 1988 and 1993 although this party accounted for 10–15% of the popular vote.

IV. CONCLUDING COMMENTS

We conclude by mentioning two other issues which, in our view, would deserve particular attention in future research:

1. *Complementarity between communicative voting and polls/debate*. On the one hand pre-election polls and political debate can be viewed as helping communicative voters to coordinate on a particular equilibrium (see the Maastricht referendum example). On the other hand, if they were perfectly reliable, then all information would be revealed before election day and voting would be useless.[43] However the cheap-talk nature of pre-election communication puts different incentive-compatibility constraints on these instruments than for voting, which can account for the strict complementarity between pre-election and election-day communication.[44]

2. *Communicative voting and theories of party positioning*. As we already mentioned (see Sections I and II), an important motive for communicative voting is the expectation that in the future mainstream parties will advocate policies in the direction of communicative voters.[45] However, this can happen only if political parties are a combination of "partisan" and "opportunistic" parties (if they are purely opportunistic, they advocate the same policy in the first place), *e.g.* if they have beliefs of their own about policies and voters. The difficulty might be to develop such "realistic" theories of party behaviour while at the same time putting some discipline on the modelling exercise.

APPENDIX

Proof of Proposition 2

First, note that all equilibria must be of the form $((v_i^1, v_i^2)_{i\in(1;\dots;n)}(\gamma_n))_{n\geq 1}$: type-$A$ voters always prefer to vote for A (whether they are pivotal for decision-making or for communicating), type-2 voters always prefer to vote for C (whether they are pivotal for decision-making or for communicating), so that only type-2 voters face a trade-off.

We first approximate pivotal probabilities as $n \to +\infty$.

Assume that a fraction β (resp. $1-\beta$) of voters is voting for A (resp. C), Then the probability $P_n^{d-m}(A)$ that if the n-th voter votes for A the winner becomes A instead of C is given by:

$$P_n^{d-m}(A) = P_n(A \text{ and } C \text{ are tied and } A \text{ loses the coin toss})$$
$$+ P_n(C \text{ is one vote ahead of } A \text{ and } A \text{ wins the coin toss in case of tie}).$$

That is:

$$\text{If } n \text{ is odd } (n-1 \text{ is even}),\ P_n^{d-m}(A) = 1/2C_{n-1}^{(n-1)/2}\beta^{(n-1)/2}(1-\beta)^{(n-1)/2};$$
$$\text{If } n \text{ is even } (n-1 \text{ is odd}),\ P_n^{d-m}(A) = 1/2C_{n-1}^{n/2}\beta^{(n-2)/2}(1-\beta)^{n/2}.$$

Using Stirling's formula ($n! = (n/e)^n(2\pi n + \pi/3)^{1/2}z(n)$, with $\lim_{n\to+\infty} z(n) = 1$), we get:

If n is odd, $P_n^{d-m}(A) = [2(\beta(1-\beta))^{1/2}]^{n-1}[(2\pi(n-1)+\pi/3)^{1/2}/2(\pi(n-1)+\pi/3)]f(n)$ with $\lim_{n\to+\infty} f(n) = 1$;

If n is even, $P_n^{d-m}(A) = 2M(1-\beta)P_{n-1}^{d-m}(A)[(n-1)/(n(n-2))^{1/2}]^n(n-2)/(n-1)$ with $M = (2\pi(n-1)+\pi/3)^{1/2}/2((\pi(n-2)+\pi/3)(\pi n+\pi/3))^{1/2}$.

That is:

$$\lim_{n\to+\infty} (\log(P_n^{d-m}(A)))/n = \log(r^{d-m}(\beta)), \qquad \text{with } r^{d-m}(\beta) = 2(\beta(1-\beta))^{1/2}.$$

The same approximation applies for $P_n^{d-m}(C)$: $\lim_{n\to+\infty} (\log(P_n^{d-m}(C)))/n = \log(r^{d-m}(\beta))$.

43. See the truthful-revelation models of McKelvey and Ordeshook (1985) and Cukierman (1990).
44. See Piketty and Spector (1995).
45. See Castanheira (1998) for a model along those lines.

More generally, the probability $P_{n,\beta,\lambda}$ that A gets a fraction λ of the vote if a fraction β of voters vote for A is given by

$$P_{n,\beta,\lambda} = C_n^{\lambda n}\beta^{\lambda n}(1-\beta)^{(1-\lambda)n}.$$

That is, by using Stirling's formula again: $\lim_{n\to+\infty} \log P_{n,\beta,\lambda} = \log[(\beta/\lambda)^{\lambda}((1-\beta)/(1-\lambda))^{1-\lambda}]$.

This implies that if a fraction β' of voters vote for A in state s_A and a fraction β vote for A in state s_B, then $n_A^*(n)$ must be such that

$$\lim_{n\to+\infty} n_A^*(n)/n = \lambda(\beta,\beta') = 1/[1+\log(\beta'/\beta)/\log((1-\beta)/(1-\beta'))].$$

To see this, assume $\beta' > \beta$ and note that the approximation formula above implies that:

$$\text{If } \lambda < \lambda(\beta,\beta'), \text{ then } (\beta/\lambda)^{\lambda}((1-\beta)/(1-\lambda))^{1-\lambda} > (\beta'/\lambda)^{\lambda}((1-\beta')/(1-\lambda))^{1-\lambda}, \quad \text{so that } \lim_{n\to+\infty} P_{n,\beta',\lambda}/P_{n,\beta,\lambda} = 0;$$

$$\text{If } \lambda > \lambda(\beta,\beta'), \text{ then } (\beta/\lambda)^{\lambda}((1-\beta)/(1-\lambda))^{1-\lambda} < (\beta'/\lambda)^{\lambda}((1-\beta')/(1-\lambda))^{1-\lambda} \quad \text{so that } \lim_{n\to+\infty} P_{n,\beta,\lambda}/P_{n,\beta',\lambda} = 0.$$

That is, if $\lim_{n\to+\infty} n_A^*(n)/n$ was different from $\lambda(\beta,\beta')$, then as $n\to+\infty$ the fact that $n_A^1 = n_A^*$ would bring overwhelming evidence that $s = s_A$ or $s = s_B$, which contradicts the definition of n_A^*.

It follows that the probability P_n^{com} of being pivotal for convincing others that $s = s_A$ or $s = s_B$ verifies:

$$\lim_{n\to+\infty} (\log(P_n^{\text{com}}))/n = \log(r^{\text{com}}(\beta,\beta')),$$

with

$$r^{\text{com}}(\beta,\beta') = (\beta/\lambda)^{\lambda}((1-\beta)/(1-\lambda))^{1-\lambda} \; (= (\beta'/\lambda)^{\lambda}((1-\beta')/(1-\lambda))^{1-\lambda}),$$

and

$$\lambda = \lambda(\beta,\beta') = 1/[1+\log(\beta'/\beta)/\log((1-\beta)/(1-\beta'))].$$

[Note that, if the population size is a Poisson random variable $n\sim$ of mean n, then the direct application of Myerson (1994*a*)'s Theorem 2 gives:

$$\lim_{n\to+\infty} (\log(P_n^{d-m}))/n = r^{d-m}(\beta) - 1;$$

$$\lim_{n\to+\infty} (\log(P_n^{\text{com}})/n = r^{\text{com}}(\beta,\beta') - 1.$$

These formulas are completely equivalent to those obtained in the case of a fixed n, given that for any real number x, $\sum_{p\geq 0} \exp(-n)n^p/p!x^p = \exp(n(x-1))$.]

Now, consider a candidate equilibrium $((v_i^1, v_i^2)_{i\in(1;\ldots;n)}(\gamma_n))_{n\geq 1}$ and assume we are in the case where $\gamma_0 > 1$ (*i.e.* $\alpha q_B > 1/2$).

Assume $r_{\text{com}}(1) > r_{d-m}(1)$. Assume $\gamma_n = 1$, and consider a particular type-B voter i. i's expected utility differential $\Delta U(A,C)$ between voting for A and voting for C is given by

$$\Delta U(A,C) = P_n^{d-m}[U_1(A|\sigma_B, d-m) - U_1(C)] + P_n^{\text{com}}\delta[U_1(A|\sigma_B,\text{com}) - U_1(B|\sigma_B,\text{com})],$$

where $U_1(A|\sigma_B, d-m)$ is the expected utility for policy A conditional on receiving signal γ_B and on being decisive for decision-making, and $U_1(A|\sigma_B,\text{com})$ (resp. $U_1(B|\sigma_B,\text{com})$) is the expected utility for policy A (resp. B) conditional on receiving signal γ_B and on being decisive for communicating. By definition, "communicative decisiveness" is uninformative so that $U_1(A|\sigma_B,\text{com}) - U_1(B|\sigma_B,\text{com}) = U_1(A|\sigma_B) - U_1(B|\sigma_B) > 0$. On the other hand, $U_1(A|\sigma_B, d-m) - U_1(C) > 0$. [$\lim_{n\to+\infty} U_1(A|\sigma_B, d-m) = U_1(A|s_A)$ if $|1/2 - \alpha_A(\gamma)| < |1/2 - \alpha_B(\gamma)|$ and $\lim_{n\to+\infty} U_1(A|\sigma_B, d-m) = U_1(A|s_B)$ otherwise, but this is irrelevant]. Since $r_{\text{com}}(1) > r_{d-m}(1)$ $\lim_{n\to+\infty}$o $P_n^{d-m}/P_n^{\text{com}} = 0$, so that for n sufficiently large $\Delta U(A,C) < 0$, *i.e.* i strictly prefers to vote for C (irrespective of the value of $\delta > 0$). It follows that for n large enough $\gamma_n = 1$ defines an equilibrium.

On the other hand, if $r_{\text{com}}(1) < r_{d-m}(1)$, then for any $\gamma\in[0;1]$ we define $\Delta U(A,C,y)$ the same utility differential in case $\gamma_n = \gamma$. Since $r_{\text{com}}(0) > r_{d-m}(0)$ and $r_{\text{com}}(1) < r_{d-m}(1)$, for n sufficiently large we have: $\Delta U(A,C,0) > 0$ and $\Delta U(A,C,1) > 0$. The continuity of $\gamma\to\Delta U(A,C,\gamma)$ then implies that for each n sufficiently large, there exists $\gamma_n\in]0;1[$ such that $\Delta U(A,C,\gamma_n) = 0$, that is, $((v_i^1, v_i^2)_{i\in(1;\ldots;n)}(\gamma_n))$ is an equilibrium. Moreover, it must be that $\lim_{n\to+\infty}\gamma_n = \gamma^*$ s.t. $r_{\text{com}}(\gamma^*) = r_{d-m}(\gamma^*)$, otherwise we would have $\lim_{n\to+\infty} \Delta U(A,C,\gamma_n) \neq 0$, which would contradict the definition of γ_n.

The other cases ($\gamma_0 \leqq 1$) can be dealt with similarly. ||

Proof of Proposition 6

The logic of the proof is the same as for Proposition 2 except that we must now compute pivotal probabilities with 3 candidates.

Assuming first that the population size is a Poisson random variable of mean n, we can directly apply Myerson (1994*a*)'s Theorem 2 to obtain:

$\forall \beta, \beta' \in [0; 1]$, $\beta + \beta' \leqq 1$, $1 - \beta - \beta' \leqq 1/3$, if a fraction β (resp. β', $1 - \beta - \beta'$) of voters vote for A (resp. C, B), then the proba P_n^{d-m} that A and C are tied for victory ($|n_A^1 - n_C^1| \leqq 1$ and $n_B^1 < n_A^1, n_C^1$) is such that:

$$\lim_{n \to +\infty} \log (P_n^{d-m})/n = -(\beta + \beta' - 2(\beta\beta')^{1/2});$$

$\forall \beta, \beta', \beta'' \in [0; 1]$, $\beta, \beta' \leqq \beta''$, if a fraction β (resp. β') of type-1 voters vote for A in state s_A (resp. s_B) and the total fraction of type-1 voters is β'', then the proba P_n^{com} that A and B are "communicatively" tied ($|n_A^1 - n_A^*| \leqq 1$) is such that:

$$\lim_{n \to +\infty} \log (P_n^{\mathrm{com}})/n = \beta'' - (\beta/\lambda)^{\lambda}((\beta'' - \beta)/(1 - \lambda))^{1-\lambda))^{1-\lambda}$$

$$= \beta'' - (\beta'/\lambda)^{\lambda}((\beta'' - \beta')/(1 - \lambda))^{1-\lambda})$$

with

$$\lambda(\beta, \beta', \beta'') = 1/[1 + \log(\beta'/\beta)/\log((\beta'' - \beta)/(\beta'' - \beta'))].$$

Coming back to the case of a fixed population size n, one can directly compute the proba P_n^{d-m} and P_n^{com} by using Stirling's formula and the same approximation techniques as for Myerson (1994*a*)'s Theorem 2 to obtain:

$$\lim_{n \to +\infty} \log (P_n^{d-m})/n = \log(1 - \beta + \beta' + 2(\beta\beta')^{1/2});$$

$$\lim_{n \to +\infty} \log (P_n^{\mathrm{com}})/n = \log(1 - \beta'' + (\beta/\lambda)^{\lambda}((\beta'' - \beta)/(1 - \lambda))^{1-\lambda})$$

$$= \log(1 - \beta'' + (\beta'/\lambda)^{\lambda}((\beta'' - \beta')/(1 - \lambda))^{1-\lambda});$$

with

$$\lambda(\beta, \beta', \beta'') = 1/[1 + \log(\beta'/\beta)/\log((\beta'' - \beta)/(\beta'' - \beta'))].$$

Again, note that the formulas for the Poisson case and the fixed-n case are completely equivalent, given that for any real number x, $\sum_{p \geq 0} \exp(-n) n^p/p! x^p = \exp(n(x-1))$.

Finally, note that these formulas (and therefore Proposition 6) can easily be extended to the case with $m > 3$ candidates. ||

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