Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth[1]

One of the most exciting results of the macro-economic theories which have recently been elaborated in Cambridge is a very simple relation connecting the rate of profit and the distribution of income to the rate of economic growth, through the inter-action of the different propensities to save. The interesting aspect of this relation is that—by utilizing the Keynesian concepts of income determination by effective demand and of investment as a variable independent of consumption and savings—it gives a neat and modern content to the deep-rooted old Classical idea of a certain connection between distribution of income and capital accumulation. In this sense, it represents a break with the hundred-year-old tradition of marginal theory, and it is no wonder that it has immediately become the target of attacks and eulogies of such strongly emotional character. Approval and rejection have almost invariably coincided with the commentators' marginalistic or non-marginalistic view.

The purpose of this paper is to present a more logical reconsideration of the whole theoretical framework, regarded as a system of necessary relations to achieve full employment. A proof will be given that the model, as originally formulated, cannot be maintained. However, once the necessary modifications are introduced, the conclusions which emerge appear much more general and—it seems to me—much more interesting than the authors themselves thought them to be.

1. A Post-Keynesian Theory of Income Distribution and of the Rate of Profit

The profit and distribution theory which is common to a number of macro-dynamic models recently elaborated in Cambridge[2] has emerged as a development of the Harrod-

[1] I have received helpful comments and criticism on a first draft of this paper from almost all my colleagues in Cambridge. I should like to thank them all.

[2] The theory of distribution is due to Nicholas Kaldor, who put it forward in "Alternative Theories of Distribution," *The Review of Economic Studies*, 1955-56. The relation of the rate of profit to the rate of growth has a longer story. In the thirties, J. von Neumann and also N. Kaldor, while still accepting a marginal productivity theory of the rate of interest, analysed the case of a slave economy, showing that the rate of growth is maximum when it is equal to the rate of interest. (J. von Neumann, "A Model of General Economic Equilibrium," *The Review of Economic Studies* 1945-46, a paper first presented at a seminar of Princeton University in 1932; N. Kaldor, "The Controversy on the Theory of Capital," *Econometrica* 1937, p. 228 and ff.). It was, however, only with the recent macro-dynamic models that the causal link has been reversed. A relation of *dependence* of the rate of profit on the rate of growth appeared first, in the form of verbal statements supplemented with an arithmetical example, in Joan Robinson, *The Accumulation of Capital*, London 1956, p. 255, and then, in the shape of a formal equation, in Nicholas Kaldor, "A Model of Economic Growth," *The Economic Journal*, 1957. The same formal relations have been adopted by D. G. Champernowne, "Capital Accumulation and the Maintenance of Full Employment," *The Economic Journal*, 1958, and by Richard F. Kahn, "Exercises in the Analysis of Growth," *Oxford Economic Papers*, 1959.

Domar model of economic growth.[1] As is well known, all these models are theories of *long-run equilibrium.* They consider full employment systems where the possibilities of economic growth are externally given by population increase and technical progress. Therefore, the amount of investment—in physical terms — necessary in order to keep full employment through time, is also externally given. The interesting device which has made the analytical formulation of these models so simple and manageable consists in assuming that the externally given possibilities of growth increase at a *steady proportional rate* through time, i.e., according to an exponential function. When this happens, and the corresponding investments are actually carried out, all economic quantities grow in time at the same proportional rate of growth, so that all the ratios among them (investment to income, savings to income, rate of profits, etc.) remain constant. The system expands though keeping all proportions constant.

Now, for any given rate of population growth and of (neutral) technical progress[2]—i.e. for any given *natural rate of growth,* in Harrod's terminology—there is only one saving ratio which keeps the system in equilibrium growth. This sounds an awkwardly rigid conclusion. But the Cambridge economists have gone on to show that an externally given *aggregate* saving ratio is not incompatible with independently given individual propensities to save, because the aggregate ratio is simply a weighted average of individual ratios, where the weights represent individual shares in national income. Therefore, within certain limits, there is always a distribution of income at which the system produces the required amount of savings.

On this problem it is useful to follow the neat and simple formulation given by Mr. Kaldor.[3] Consider total net income (Y) as divided into two broad categories, wages and profits (W and P); and total net savings as also divided into two categories, workers' savings (S_w) and capitalists' savings (S_c), so that

$$Y \equiv W + P, \tag{1}$$

$$S \equiv S_w + S_c. \tag{2}$$

Suppose now simple proportional savings functions $S_w = s_w W$ and $S_c = s_c P$ (where s_w and s_c, both being no less than zero and no more than unity, and $s_w \neq s_c$, represent the propensities to save of the workers and of the capitalists respectively). Suppose, moreover, that the amount of investment necessary to cope with population growth and technical progress—which we may call I —is actually carried out. (This assumption is taken to mean that in the long run—provided that the rate of profit stays above a certain minimum level, below which capitalists would refuse to invest—decisions concerning

[1] Roy F. Harrod, *Towards a Dynamic Economics*, London 1948; E. D. Domar, " Capital Expansion, Rate of Growth and Employment," *in Econometrica* 1946.

[2] The concept of " neutral " technical progress—defined as such that, at the same rate of interest, the capital-output ratio remains unchanged—is another concept introduced by Harrod (*op. cit.* pp. 22-23). Jointly with the hypothesis of an exponential growth of productivity and of population, it makes the formulation of all these model extremely simple.

[3] N. Kaldor, " Alternative Theories of Distribution "*op. cit.*

investment are governed by the possibilities of expansion of the markets.) The condition under which the system will remain in a dynamic equilibrium, namely

$$I = S, \tag{3}$$

has straightforward implications. Substituting from the saving functions:

$$I = s_w W + s_c P = s_w Y + (s_c - s_w)P,$$

whence

$$\frac{P}{Y} = \frac{1}{s_c - s_w}\,\frac{I}{Y} - \frac{s_w}{s_c - s_w}, \tag{4}$$

and

$$\frac{P}{K} = \frac{1}{s_c - s_w}\,\frac{I}{K} - \frac{s_w}{s_c - s_w}\,\frac{Y}{K}, \tag{5}$$

which means that there is a distribution of income between wages and profits—equation (4) —and a corresponding rate of profit—equation (5)—at which the equilibrium condition (3) remains satisfied through time. Two particular cases of special interest arise when $s_w = 0$, so that (5) becomes

$$\frac{P}{K} = \frac{1}{s_c}\,\frac{I}{K},$$

and when $s_c = 1$ (besides $s_w = 0$) in which case, (5) simply reduces to

$$\frac{P}{K} = \frac{I}{K}.$$

It is the two more general equations (4) and (5) which have been considered so far as expressing what we may call the post-Keynesian theory of income distribution and of the rate of profit. To them we must add two restrictions, in order to limit the validity of the mathematical formulations to the range in which they have an economic meaning:

$$s_w < \frac{I}{Y}, \tag{6}$$

and

$$s_c > \frac{I}{Y}. \tag{7}$$

Restriction (6) excludes the case of a dynamic equilibrium with a null or negative share of profit, and restriction (7) excludes the case of a dynamic equilibrium with a null or negative share of wages. In practice, if (6) were not satisfied, the system would enter a situation of chronic Keynesian underemployment. Similarly, if (7) were not satisfied the system would enter a situation of chronic inflation. (As a matter of fact the latter limit becomes operative much before s_c even approaches $\frac{I}{Y}$, because there is a minimum level below which the wage-rate cannot be compressed). It is within these limits that the above model is meant to apply, and that equations (4) and (5) show the existence of a distribution of income and a rate of profit which, through time, will keep the system in equilibrium.

But Mr. Kaldor has gone further. He has pointed out that, if there is in the system a price mechanism by which the level of prices with respect to the money wages (i.e. profit margins) is determined by demand; and if $s_c > s_w$—which after all is implied by (6)-(7)— then income distribution (4) and rate of profit (5) will not only exist but also will be the ones that the system actually tends to produce.

2. A Correction

There is a logical slip, in the theory reported above, which has so far passed unnoticed. The authors have neglected the important fact that, in any type of society, when any individual saves a part of his income, he must also be allowed to own it, otherwise he would not save at all. This means that the stock of capital which exists in the system is owned by those people (capitalists or workers) who in the past made the corresponding savings. And since ownership of capital entitles the owner to a rate of interest, if workers have saved—and thus own a part of the stock of capital (directly or through loans to the capitalists)—then they will also receive a share of the total profits. Therefore total profits themselves must be divided into two categories: profits which accrue to the capitalists and profits which accrue to the workers.

It is this distinction that is missing in the theory just considered. By attributing all profits to the capitalists it has inadvertently but necessarily implied that workers' savings are always totally transferred as a gift to the capitalists. Clearly this is an absurdity. To eliminate it, we must reformulate the model from the beginning and clear up the confusion which has been made of two different concepts of distribution of income : distribution of income between profits and wages, and distribution of income between capitalists and workers. The two concepts only coincide in the particular case in which there is no saving out of wages.

3. Reformulating the Model

For a correct reformulation of the model, we must resume equations (1), (2), (3) and add a further identity:

$$P \equiv P_c + P_w,$$

where P_c and P_w stand for profits which accrue to the capitalists and profits which accrue to the workers. The saving functions now become $S_w = s_w(W + P_w)$ and $S_c = s_c P_c$; and the equilibrium condition becomes

$$I = s_w(W + P_w) + s_c P_c = s_w Y + (s_c - s_w)P_c,$$

from which—by following exactly the same steps which led to equations (4) and (5)—we obtain

$$\frac{P_c}{Y} = \frac{1}{s_c - s_w}\,\frac{I}{Y} - \frac{s_w}{s_c - s_w}, \tag{8}$$

and

$$\frac{P_c}{K} = \frac{1}{s_c - s_w}\,\frac{I}{K} - \frac{s_w}{s_c - s_w}\,\frac{Y}{K}. \tag{9}$$

As the reader can see, the right hand sides of (8) and (9) exactly coincide with the right hand sides of (4) and (5), but the left hand sides do not. This means that the expressions used so far *do not refer to total profits*. They only refer to that part of profits which accrue to the capitalists. Let us examine the implications.

As far as the distribution theory is concerned, equation (8) still retains a definite, but restricted, meaning. It now only expresses the distribution of income between capitalists and workers. The distribution of income between profits and wages is something different, and to obtain it, one must add the share of workers' profit into income $\left(\frac{P_w}{Y}\right)$ to both sides of equation (8). As to the theory of the rate of profit, the consequences of our reformulation are even more serious. Expression (9) simply represents the ratio of a *part* of profits (P_c) to *total* capital, but this concept has no useful or interesting meaning. The expression which is really needed is one for the ratio of total profits to total capital (rate of profit), and to obtain it, we must again add a ratio $\left(\frac{P_w}{K}\right)$ to both sides of equation (9). In other words, we have to find suitable expressions for

$$\frac{P}{Y} = \frac{P_c}{Y} + \frac{P_w}{Y}, \tag{10}$$

and

$$\frac{P}{K} = \frac{P_c}{K} + \frac{P_w}{K}.$$

Let us start with the latter equation. We know $\frac{P_c}{K}$ already from (9). Thus, writing K_w for the amount of capital that the workers own indirectly—through loans to the capitalists—and r for the rate of interest on these loans, we obtain

$$\frac{P}{K} = \frac{1}{s_c - s_w}\frac{I}{K} - \frac{s_w}{s_c - s_w}\frac{Y}{K} + \frac{rK_w}{K}.$$

An expression for $\frac{K_w}{K}$ can easily be found. In dynamic equilibrium:

$$\frac{K_w}{K} = \frac{S_w}{S} = \frac{s_w(Y - P_c)}{I} = \frac{s_w s_c}{s_c - s_w}\frac{Y}{I} - \frac{s_w}{s_c - s_w},$$

which, after substitution into the previous expression, finally gives us:

$$\frac{P}{K} = \frac{1}{s_c - s_w}\frac{I}{K} - \frac{s_w}{s_c - s_w}\frac{Y}{K} + r\left(\frac{s_w s_c}{s_c - s_w}\frac{Y}{I} - \frac{s_w}{s_c - s_w}\right). \tag{11}$$

By exactly following the same procedure, the expression for equation (10) comes out as

$$\frac{P}{Y} = \frac{1}{s_c - s_w}\frac{I}{Y} - \frac{s_w}{s_c - s_w} + r\left(\frac{s_w s_c}{s_c - s_w}\frac{K}{I} - \frac{s_w}{s_c - s_w}\frac{K}{Y}\right). \tag{12}$$

These are the two general equations we were looking for. By now, we have all the elements which are necessary to correct the post-Keynesian theory of income distribution and of the rate of profit. Equation (5) of section 1, expressing the rate of profit, must be replaced by equation (11); and equation (4) must be replaced by two distinct equations: equation (8) for the distribution of income between workers and capitalists, and equation (12) for the distribution of income between wages and profits.

4. Rate and Share of Profits in Relation to the Rate of Growth

The most immediate consequence of the reformulation which has just been carried out is that, in order to say anything about share and rate of profits, one needs first *a theory of the rate of interest*. In a long run equilibrium model, the obvious hypothesis to make is that of a rate of interest equal to the rate of profit. If we do make such a hypothesis, equations (11) and (12) become very simple indeed. By substituting $\frac{P}{K}$ for r, in equation (11), we get

$$\frac{P}{K}\left(1 - \frac{s_w s_c}{s_c - s_w}\,\frac{Y}{I} + \frac{s_w}{s_c - s_w}\right) = \frac{1}{s_c - s_w}\,\frac{I}{K} - \frac{s_w}{s_c - s_w}\,\frac{Y}{K},$$

$$\frac{P}{K}\,\frac{s_c(I - s_w Y)}{I} = \frac{I - s_w Y}{K}.$$

Whence, provided that

$$I - s_w Y \neq 0, \tag{13}$$

(otherwise the ratio $\frac{P}{K}$ would be indeterminate) the whole expression simply becomes

$$\frac{P}{K} = \frac{1}{s_c}\,\frac{I}{K}. \tag{14}$$

And by an analogous process, equation (12) reduces to

$$\frac{P}{Y} = \frac{1}{s_c}\,\frac{I}{Y}. \tag{15}$$

The reader will notice that these results are formally similar to those which have been shown in section 1 as particular cases. But now they have been reached without making any assumption whatsoever on the propensities to save of the workers. This is the most striking result of our analysis. It means that, in the long run, workers' propensity to save, though influencing the distribution of income between capitalists and workers—equation (8)—does not influence the distribution of income between profits and wages—equation (15). Nor does it have any influence whatsoever on the rate of profit—equation (14) !

5. A Fundamental Relation Between Profits and Savings

The novelty of the conclusion reached in the previous section makes it perhaps worth our while trying to investigate a little more closely the logic behind it.

Let me point out immediately that the model has been built on the institutional principle, inherent in any production system, that wages are distributed among the members of society in proportion to the amount of labour they contribute and profits are distributed in proportion to the amount of capital they own. The latter proposition implies something which has passed unnoticed so far, namely that, in the long run, profits will turn out to be distributed in proportion to the amount of savings which are contributed. In other words, no matter how many categories of individuals we may consider, in a long run exponential growth, the ratio of the profits that each category receives to the savings that it provides will always be the same for all categories. In our model:

$$\frac{P_w}{S_w} = \frac{P_c}{S_c}. \tag{16}$$

This indeed sets a proportionality relation between profits and savings which is fundamental to the whole problem of profits and distribution. It means that, for each category, *profits are in the long run proportional to savings.* Let me stress that this relation does not depend on any behavioural assumption whatsoever; it simply and logically follows from the institutional principle that profits are distributed in proportion to ownership of capital.

This principle, however, still leaves the actual value of ratio (16) indeterminate, as it only requires that this ratio be the same for all categories. It is at this point that the particular types of income out of which savings are made become relevant. If there is in the system a category of individuals who—owing to the position they occupy in the production process—derive all their incomes, and therefore savings, exclusively from profits, the saving behaviour of just this group of individuals will set up, independently of (16), another and more definite relation between savings and profits. The only way in which this new behavioural relation can be compatible with (16) is for it to determine the actual value of the ratio of profits to savings for the whole system. We can see this immediately by substituting our saving functions into (16). We obtain

$$\frac{P_w}{s_w(W + P_w)} = \frac{P_c}{s_c P_c},$$

which may also be written in either of the two following ways:

$$s_w (W + P_w) = s_c P_w, \tag{17}$$

or

$$s_w W = [(1 - s_w) - (1 - s_c)]P_w. \tag{18}$$

These expressions now allow us some insight into the reason why workers' propensity to save does not, and capitalists' propensity to save does, play a role in determining *total* profits. Expression (17) says that, in the long run, when workers save, they receive an amount of profits (P_w) such as to make their total savings exactly equal to the amount that the capitalists would have saved out of workers' profits (P_w) if these profits remained to them. Expression (18) is even more explicit. Savings out of wages always turn out to be equal to workers' extra consumption out of profits (extra consumption meaning consumption in excess of what the capitalists would have consumed if those profits remained to them). Another way of interpreting these results is to say that whatever workers' propensity to save (s_w) may be, there is always a distribution of income and a distribution of profits which makes the ratio $\frac{P_w}{s_w(W + P_w)}$ equal to any pre-determined ratio $\frac{P_w}{S_w}$. Or, to look at the problem the other way round, for any given s_w, there are infinite proportions between profits and savings which can be used in (16) and which at the same time can make $\frac{P_w}{s_w(W + P_w)}$ equal to $\frac{P_w}{S_w}$. All this is, after all, a complicated way of saying that, on the part of the workers, the rate of profit is indeterminate. They will always receive, in the long run, an amount of profits proportional to their savings, whatever the rate of profit may be.

The situation is entirely different when we consider the capitalists. The fact that all capitalists' savings come out of profits sets a straight relation between savings and profits. No other variable enters into it, in contrast with the previous case where the wage share was also in the picture. It follows that, for any given s_c, there is only one proportionality relation between profits and savings—this relation being required by (16)—which can also make the ratio $\frac{P_c}{s_c P_c}$ equal to $\frac{P_c}{S_c}$. This proportionality relation can be nothing but s_c, which will therefore determine the ratio of profits to savings for all the saving groups, and consequently also the income distribution between profits and wages and the rate of interest for the whole system. The reader may complete his view of the problem by thinking for a moment of the practically irrelevant but interesting case, in which capitalists' profits are nil (i.e. $P_c = 0$). In this case, the behavioural relation ($s_c P_c$) determining the rate of profit drops out of the picture altogether and the rate of profit becomes indeterminate. (The parameter s_w, which remains, cannot determine the rate of profit!) We have met this case already in the process of finding $\frac{P}{K}$, in section 4, where non-fulfilment of (13) would exactly imply $P_c = 0$ and an indeterminate rate of profit.

6. Implications

We may now synthesize the implications of the foregoing analysis in two conclusions. First of all, the irrelevance of workers' propensity to save gives the model a much wider generality than was hitherto believed. Since the rate of profit and the income distribution between profits and wages are determined independently of s_w, there is no need for any hypothesis whatever on the *aggregate* savings behaviour of the workers. The non-capitalists might well be divided into any number of sub-categories one likes; the subdivision might even be carried as far as to consider single individuals; yet equations (14)-(15) would not change. Of course the particular behaviour of the sub-categories or single individuals would influence the distribution of income among the various workers, and between the workers and the capitalists, as equation (8) shows. But the distribution of income between total wages and total profits, and the rate of profit would remain exactly the same.

Secondly, the relevance of the capitalists' propensity to save, which is the only one to appear in the final formulae (14) and (15), uncovers the absolutely strategic importance for the whole system of the decisions to save of just one group of individuals: the capitalists. The particular saving function of this group transforms the open proportionality relation (16) into a definite function in which the proportion that profits must bear to savings *in the whole system*, is given by the saving propensity of one single category of individuals.[1] The similar decisions to save of all the other individuals, the workers, do not count in this respect. Whatever the workers may do, they can only share in an amount of total profits which for them is predetermined; they have no power to influence it at all.

These conclusions, as the reader may clearly realise, now suddenly shed new light on the old Classical idea, hinted at already at the beginning, of a relation between the savings of that group of individuals who are in the position to carry on the process of production and the process of capital accumulation. This idea has always persisted in economic literature but in a vague and muddled form. Economists have never been able to bring it out clearly. In particular they have always thought—and the post-Keynesian theories

[1] It may be useful to remind the reader that the whole analysis refers to states of long-run equilibrium. The relevant behavioural process must not necessarily be looked for in association with particular physical persons, but rather in association with specific kinds of decisions: those concerning savings out of profits as such. It must, moreover, be noticed that in a modern economic system, where large corporations have a certain autonomous power to retain profits, the profit retention ratios of the corporations and the individuals' propensities to save out of (distributed) profits add up.

examined in section 1 seemed to confirm—that the relation between capitalists' savings and capital accumulation depended on particularly simplifying and drastic assumptions about negligible savings by the workers. The novelty of the present analysis has been to show that the relation is valid independently of any of those assumptions. It is valid whatever the saving behaviour of the workers may be.

7. The Conditions of Stability

Our analysis would be incomplete if, after showing that there exists a distribution of income between profits and wages which keeps the system in long run equilibrium, we did not also specify the limits within which such distribution has economic meaning and the conditions under which it is stable.

On this problem we may recall the discussion that Mr. Kaldor has carried out already. The limits (6) and (7) of section 1 must here be confirmed. Moreover, we must confirm that if there is in the system a price mechanism by which the level of prices with respect to the level of wages (profit margins) rises or falls according as to whether demand exceeds or falls short of supply, and if equilibrium investments are actually carried out, then the system is stable. For it will tend to get back to its dynamic equilibrium path whenever displaced from it.

But we are now in a position to examine these problems in a much better way. The propositions stated above may be expressed as follows:

$$\frac{d}{dt}\left(\frac{P}{Y}\right) = f\left(\frac{I}{Y} - \frac{S}{Y}\right), \tag{19}$$

with the properties

$$f(0) = 0,$$
$$f' > 0,$$

which simply means that, as time (t) goes on, the profit margins, and therefore the share of total profits, remain constant, increase or decrease according as to whether total savings produced by the system turn out to be equal, smaller, or greater than total investments.

Equation (19) is a simple differential equation. By solving it with respect to deviations from the equilibrium share of profits,[1] the only requirement for stability emerges as

[1] Call $\left(\frac{P}{Y}\right)^*$ the equilibrium value of $\frac{P}{Y}$ at which $\frac{I}{Y} - \frac{S}{Y} = 0$. By expanding (19) in Taylor series around this equilibrium value, and neglecting the terms of higher order than the first, we obtain

$$\frac{d}{dt}\left[\frac{P}{Y} - \left(\frac{P}{Y}\right)^*\right] = f(0) +$$
$$+ f'(0)\left[\frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{I}{Y}\right) - \frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{S}{Y}\right)\right]_{\left(\frac{P}{Y}\right)^*}\left[\frac{P}{Y} - \left(\frac{P}{Y}\right)^*\right],$$

where the last but one square brackets contain derivatives taken at the particular point $\left(\frac{P}{Y}\right)^*$, so that the whole term is constant. Calling now

$$\left[\frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{I}{Y}\right) - \frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{S}{Y}\right)\right]_{\left(\frac{P}{Y}\right)^*} = m,$$

and integrating, we obtain

$$\left[\frac{P}{Y} - \left(\frac{P}{Y}\right)^*\right]_t = \left[\frac{P}{Y} - \left(\frac{P}{Y}\right)^*\right]_0 e^{f'mt}.$$

Since $f' > 0$, the only condition for this expression to tend to zero as time goes on (i.e. for the system to be stable) is $m < 0$.

$$\frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{I}{Y}\right) < \frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{S}{Y}\right), \tag{20}$$

which means that the response of $\frac{I}{Y}$ to deviations of $\frac{P}{Y}$ from its equilibrium value must be smaller than the response of $\frac{S}{Y}$. But, in our model, there can be no response of $\frac{I}{Y}$ to $\frac{P}{Y}$ because I has been defined as that amount of investments which has to be undertaken in order to keep full employment over time. This amount of investments, as a proportion of total income, is uniquely determined from outside the economic system, by technology and population growth[1]; and the share of profits can in no way alter it. Therefore $\frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{I}{Y}\right) = 0$. We are thus left with the right hand side of (20) required to be greater than zero. After substituting from the savings functions,

$$\frac{d}{d\left(\frac{P}{Y}\right)}\left(\frac{S}{Y}\right) = \frac{d}{d\left(\frac{P}{Y}\right)}\left(s_w \frac{W}{Y} + s_w \frac{P_w}{Y} + s_c \frac{P - P_w}{Y}\right) > 0. \tag{21}$$

[1] Those readers who have been brought up in the neo-classical tradition might think that I am implicitly assuming the existence of only one technique of production. Since I am not, it may be useful to clarify the issue by making explicit the implications of the foregoing analysis for what has been called the *neo-classical* theory of economic growth (as expounded, for example, by Professor Solow in a " Contribution to the Theory of Economic Growth, " *The Quarterly Journal of Economics* 1956, or by Professor Meade in *A Neo-Classical Theory of Economic Growth*, London 1961). Suppose there exists an infinite number of possible techniques expressed by a traditional production function

$$Y = F(K, L), \tag{1'}$$

assumed to be homogeneous of the first degree and invariant to time; and suppose that labour (L) is increasing at an externally given rate of growth n, so that $L(t) = L(0)e^{nt}$. The whole previous analysis remains unaltered. We may now read $F(K, L)$ whenever we have written Y, and we may write, if we like, the final expression (14) as

$$s_c[F(K, L) - W] = I. \tag{2'}$$

However, as we have assumed more information about technology, we can now inquire further into the composition of I. By defining $k = \frac{K}{L}$, so that $K = kL$, we may write

$$I = \frac{dK}{dt} = k\frac{dL}{dt} + L\frac{dk}{dt} = knL + L\frac{dk}{dt}.$$

Substituting into (2'), we obtain

$$s_c[F(K, L) - W] = knL + L\frac{dk}{dt}.$$

But on the steady growth path $\frac{dk}{dt} = 0$, so that the equilibrium relation is

$$s_c[F(K, L) - W] = \frac{K}{L}nL,$$

whence

$$\frac{P}{K} = \frac{n}{s_c}. \tag{3'}$$

"The equilibrium rate of profit is determined by the natural rate of growth divided by the capitalists' propensity to save; independently of anything else in the model. This basic relation is confirmed. Therefore, the link between (14) and (15) that the extra technical equation (1') has introduced, can only go one way. Since it cannot influence the rate of profit, it can only contribute to determining the investment-income ratio. In this way the equilibrium amount of investment is uniquely and exogenously determined. All this means that the foregoing analysis has singled out one of those asymmetrical chains of relations to which in science the concept of causality is associated. This causality chain may here be expressed as follows. The externally given rate of population growth and the capitalists' propensity to save determine first of all the rate of profit. At this rate of profit, the optimum technique is chosen (in such a way as to satisfy the marginal productivity conditions). Then, the optimum technique, together with the rate of population growth, uniquely determine the equilibrium investment-income ratio. In this system, therefore, technical relation (1') simply comes to determine one more variable—the equilibrium quantity of capital."

We may first consider an intermediate step. In the short run, the share of profits which accrue to the workers is fixed, as it takes time for the rate of interest to adapt itself to the rate of profit (even if the two coincide in the long run). And since $\dfrac{d}{d\left(\dfrac{P}{Y}\right)}\left(\dfrac{W}{Y}\right) = -1$, condition (21) becomes

$$s_c - s_w > 0.$$

This is exactly the stability condition given by Mr. Kaldor. The above analysis proves that it is only a short-run condition.

But let us consider the long run, when the share of workers' profits is no longer fixed and P_w adapts itself to a proportion of K_w equal to the proportion that P_c bears to K_c. By substitution from (18), condition (21) simply becomes

$$s_c > 0.$$

This is all that is required. We may conclude that, in a system where full employment investments are actually carried out, and prices are flexible with respect to wages, the only condition for stability is $s_c > 0$, a condition which is certainly and abundantly satisfied even outside the limits in which the mathematical model has an economic meaning.

8. The Case of a Socialist System

Going back now to our basic model, the reader will notice how few, after all, are the assumptions which have been used. These assumptions become even fewer if we consider the case of a socialist system.

In a socialist society, all the members of the community belong to the category of workers. There is no place for capitalists; the responsibility for carrying on the production process and the direct ownership of all means of production are taken over by the State. However, the State, as such, cannot consume: consumption can be carried out only by individuals. Therefore, if any amount of the national product is not distributed to the members of the community, either as wages or as interest on their loans to the State, that amount is *ipso facto* saved. This means that the parameter s_c becomes unity ($s_c = 1$), as an inherent property of the system; so that even the one behavioural parameter that still remained in our final formulae, disappears.

Equations (14) and (15) become:

$$\frac{P}{Y} = \frac{I}{Y}, \tag{22}$$

and

$$\frac{P}{K} = \frac{I}{K}, \tag{23}$$

with the evident meaning that, in equilibrium, total profits are equal to total investments, and the rate of profit (and of interest) is equal to the ratio of investment to capital, i.e. equal to the *natural* rate of growth. It follows that total wages always turn out to be equal to total consumption and total profits always turn out to be equal to total savings. However, this does not mean that all wages are consumed and all profits are saved! The (22)-(23), have been reached without any assumption whatsoever on individual decisions to save. Each individual may be left completely free to decide the proportion of his income (wages plus interest) that he likes to save, without in the least affecting the (22)-(23). This result is simply the counterpart, for a socialist system, of what has been pointed out in section 4 as the most striking outcome of our analysis.

An explanation can be given by following the same procedure used in section 5, which here becomes even simpler. By putting $s_c = 1$ in (18), the interesting property immediately emerges that, in a dynamic equilibrium, individual savings out of wages are exactly equal to individual consumption out of interest; so that total consumption (out of wages and out of interests) turns out to be equal to total wages.

The important corollary that follows is that there is no need for a socialist State to exert any interference whatsoever in individual decisions to consume and to save. Only one limit must be respected, a limit which is the same encountered in the case of a capitalist system and expressed by inequality (6). The community, as a whole, cannot remain in equilibrium if it insists in saving more than what is required by the *natural* rate of growth; if it did, the system would fall into a situation of chronic under-employment due to lack of effective demand.[1] Provided that this limit is not overcome, no restriction need be put on individual savings. The only effect of these savings is to require from the State the issuing of a national debt for a part of the stock of capital, and the consequent distribution of a part of profits, which will however come back under the form of lent savings.

To conclude, we may put these results in the following way. In a full employment economic system in which all net revenues that accrue to the organizers of the process of production are saved, there exists one particular rate of interest, which we may indeed call the *natural rate of interest*—since it turns out to be equal to the natural rate of growth—which has the following property. If it is applied both in the process of pricing and in the payment of interest on loans, it causes the system, *whatever the individual decisions to save may be*, to produce a total amount of savings which is exactly equal to the amount of investment needed to cope with technical progress and population growth.

9. Models and Reality

At this point, the reader may have become a little impatient and may begin to wonder: But what is after all the practical relevance of the whole macro-economic exercise?

There are two different problems raised by this question. The first one concerns aggregation. It must be noticed that the foregoing investigation is not " macro-economic " in the sense of representing a first simplified rough step towards a more detailed and disaggregated analysis. It is macro-economic because it could not be otherwise. Only problems have been discussed which are of a macro-economic nature; an accurate investigation of them has nothing to do with disaggregation. They would remain the same—i.e. they would still arise at a macro-economic level—even if we were to break down the model into a disaggregated analysis, and therefore introduce the necessary additional information (or assumptions) about consumers' choice of goods and producers' choice of techniques. I might add that in fact, the present paper has originated from a *multi-sector* growth model, on which I have been working for some time, and whose results have turned out to be incompatible with the post-Keynesian theories examined in section 1.

A second and separate problem concerns the interpretative value of the model. When

[1] We are considering, of course, a closed system. In an open system, in which the State might lend abroad, full employment might be kept even if total savings go beyond required total investments.

Mr. Kadlor presented his theory of income distribution, he pointed out that the interpretative value of the theory depends on the Keynesian hypotheses on which it is built. In particular it depends on the crucial hypothesis (post-Keynesian rather than Keynesian) that investment can be treated as an independent variable governed by technical progress and population growth.

But this is not the approach that I should like to take here. Whether we are or whether we are not prepared to accept the model in this behavioural sense, there are important practical implications which are valid in any case. I should look, therefore, at the previous analysis simply and more generally as a logical framework to answer interesting questions about what *ought* to happen if full employment is to be kept over time, more than as a behavioural theory expressing what actually happens.

The case of a socialist system, which came last in our analysis, is the most straightforward on this respect. The amount of investments that must be undertaken in order to maintain full employment—once this has been reached—is indeed that which is required by technical progress and population growth. And if these investments are carried out, the rate of profit (when uniformly applied) must be equal to the natural rate of growth, if total demand is to be such as to allow the full utilization of the productive capacity and of the labour force. These results do not depend on any behavioural assumption whatsoever. They are true whatever individual behaviour may be; as a simple matter of logical necessity.

In the case of a capitalist system, the additional problem arises of whether the capitalists will or will not spontaneously undertake the amount of investments necessary to cope with the natural possibilities of growth. We may of course discuss at length in this case the circumstances under which equilibrium will or will not be automatically reached. But again we should not let these discussions obscure the conclusions, which are valid in any case, about the relations that must be satisfied if full employment is to be kept. If full employment is to be maintained, *that* amount of investment *must* be undertaken. And if it is undertaken; there is—for any given proportion of capitalists' income which tends to be saved[1]—only one rate of profit, i.e. one distribution of income between profits and wages, that keeps the system on the dynamic path of full employment.

This, it seems to me, is the relevant way to look at the model which has been elaborated above. The whole analysis has been carried out with constant reference to a situation of full employment because full employment is the situation that matters, and that indeed, now-a-days, forms one of the agreed goals of any economic system. The conclusions, therefore, acquire an important practical relevance whether the system is automatically able to reach full employment or whether it is not. In the latter case, I should say that they become even more important, because it is then that practical measures have to be taken and it becomes essential to have clear ideas about the direction in which to move.

Cambridge. LUIGI L. PASINETTI

[1] Let me point out that, in this context, the objection so commonly advanced against the theory formulated in section 1—namely, that to classify the members of a modern society in only two groups is an arbitrary and crude simplification—entirely loses its ground. The central outcome of the previous analysis has just been to show that, as far as the determination of the rate of profit is concerned, the distinction between individuals who save exclusively out of profits and individuals who save out of wages is the only one that matters.