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ScienceDirect
Review of Economic Dynamics 11 (2008) 566–583
Review of Economic Dynamics
www.elsevier.com/locate/red

Consumption over the life cycle: The role of annuities

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Received 2 August 2006; revised 21 December 2007

Available online 10 January 2008

Abstract

We explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a general equilibrium overlapping generations model in which markets are otherwise complete. Empirical studies find that consumption displays a hump shape over the life cycle. Our model exhibits life cycle consumption that is consistent with this pattern. Our calibrated model, which includes an unfunded social security system, displays a hump shape but the peak occurs later in the life cycle than in the data. Adding a bequest motive causes this decline to begin at a younger age.

JEL classification: E2; D91

Keywords: Life cycle consumption; General equilibrium; Annuity markets; Social security; Bequests

1. Introduction

Among the many sources of uncertainty that an individual faces when planning for consumption in old age, one of the more significant is uncertainty about how long the individual will live. This source of uncertainty could be easily insured against if the individual were to purchase an annuity that provides a constant flow of income until death. But, annuity markets in the US are quite thin. A standard explanation for the lack of annuity markets is adverse selection—those with long expected lifetimes will be attracted to annuities, which might cause them to be unattractively priced for most people.[1]

In this paper we explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a calibrated general equilibrium overlapping generations model where markets are otherwise complete. A large literature has documented that individual household consumption increases early in life, with a peak sometime around age 50 and a decline after that.[2] This is generally regarded as posing a puzzle for

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[1] See, for example, Friedman and Warshawsky (1990) and Mitchell et al. (1999).

[2] Thurow (1969) is an early example from this empirical literature. More recent contributions include Attanasio and Browning (1995), Attanasio et al. (1999), Gourinchas and Parker (2002), and Fernández-Villaverde and Krueger (2002). In addition, a recent paper by Aguiar and Hurst (2005) argues that, while consumption expenditures may be hump shaped, home production is used to smooth actual consumption relative to expenditures.

1094-2025/$ – see front matter
doi:10.1016/j.red.2007.12.004

a standard life cycle model of consumption because, with complete markets, the model implies that consumption should be smooth over a lifetime. Depending on the relative magnitudes of the household's time discount rate and the market interest rate, consumption can be constant, or monotonically decreasing or increasing as an individual ages.

If an annuity market (or its equivalent) is unavailable, this intuition no longer applies. If survival probabilities decrease as an individual ages, individuals will more heavily discount the future as they grow older. This allows for the possibility, depending on the value of the interest rate, that consumption might increase early in life when survival probabilities are high and the effective rate of discount is low. As survival probabilities fall, the slope of the consumption profile may become negative.

Because social security provides some insurance against uncertain lifetimes and may provide an adequate substitute for missing annuity markets, we study the shape of the consumption profile in a model with a pay-as-you-go social security system as in the US. In this model, consumption peaks later in the life cycle in comparison to estimated consumption profiles and a similar model that abstracts from social security. Introducing a joy of giving bequest motive decreases the age of the hump in consumption.

We are not the first to note the impact of annuity markets on consumption over the life cycle. Yaari (1965) is perhaps the first to study the impact of uncertain lifetime on the shape of the life cycle consumption profile in an overlapping generations model. Levhari and Mirman (1977) extend Yaari's work by providing results on how risk averse consumers respond to a change in the distribution of lifetime uncertainty. They obtain results showing how uncertain lifetimes affect the level of consumption at a particular age, as opposed to how consumption changes over the course of the life cycle. Barro and Friedman (1977) demonstrate in the context of a simple life cycle model that, when perfect insurance markets are allowed, life cycle consumption under uncertain lifetime is the same as under certainty.

Davies (1981) may be the first to use a life cycle model with uncertain lifetimes to interpret actual consumption and savings behavior, in particular the savings behavior of retired individuals. More recently, İmrohoroğlu et al. (1995) develop an applied general equilibrium model with long but randomly-lived households to study the welfare effects of social security reform. They were able to generate age-consumption profiles with a hump by closing annuity markets, though they also had individual income uncertainty and borrowing constraints. Bütler (2001) provides a continuous-time overlapping generations model and gives an example of how a lack of annuity markets can yield a hump-shaped consumption profile. In this paper, our goal is to assess, using a calibrated general equilibrium model with social security, the extent to which a lack of annuity markets by itself can account for the observed hump shaped consumption profile.[3]

Most of the consumption literature, however, has explored other factors that potentially play an important role in determining how consumption changes over the life cycle. One possibility is that the hump shape may be due to demographic factors–Attanasio and Browning (1995) and Attanasio et al. (1999) argue that the change in the size of a household over time is a significant determinant of the hump in consumption. However, more recent research has generally found that demographic factors alone cannot account for the pattern of lifetime consumption.[4]

Thurow (1969), for example, suggested that the age-consumption profile may be hump shaped due to borrowing constraints. That is, individuals are prevented from shifting as much wealth as they would like from later in life to finance consumption earlier. Another possibility is that individuals face income uncertainty and must die with non-negative assets. This creates a motive for precautionary savings that could lead to consumption rising with income early in life. Both Attanasio et al. (1999) and Gourinchas and Parker (2002) emphasize this point. Fernández-Villaverde and Krueger (2001) argue that households accumulate durables early in life as a way of insuring against income uncertainty. In their model, the stock of durables provides insurance by acting as collateral for consumption loans.

Heckman (1974) and Bullard and Feigenbaum (2003) explore the possibility that substitutability between consumption and leisure, rather than market incompleteness, may play an important role. That is, if preferences are such that consumption and leisure are substitutes, individuals may choose to consume more during the periods of their life when they spend the largest fraction of their time engaged in market work.

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[3] A recent paper, Feigenbaum (2007), expands on the analysis presented in this paper by studying the role of mortality risk in a continuous time life cycle model where labor supply is inelastic and households are borrowing constrained.

[4] For example, compare the findings of Attanasio and Browning (1995) with those of Attanasio et al. (1999). The first paper concludes that the hump can be entirely explained by demographic factors while the second finds an important role for income uncertainty.

The main finding of our paper is that consumption over the life cycle in our calibrated model displays a hump shape, but the timing of the hump is different from what has been estimated from US data. In particular, in the model economy, consumption peaks later in life than in the data and the ratio of peak consumption to age 25 consumption is much larger. In the same calibration, if annuity markets were present, consumption would monotonically increase throughout life.

To explore the robustness of this finding, we also consider cases with no social security and with a bequest motive sufficiently large to account for the saving behavior of the elderly. Both of these changes cause consumption to peak at an earlier age. However, even with the bequest motive, social security sufficiently substitutes for the lack of annuity markets that consumption continues to increase until quite late in life.

The remainder of the paper is organized as follows. The next section surveys some of the empirical literature estimating life cycle consumption profiles. From this we obtain some basic summary statistics that we can also compute for our model economies. In the third section, we present a simple partial equilibrium model to provide intuition on how a lack of annuity markets can deliver a hump-shaped consumption profile. A general equilibrium model, one that incorporates social security, is described in Section 4. In Section 5 we set up our quantitative exercise and present results in Section 6. A bequest motive is introduced and its quantitative implications are studied in Section 7. Section 8 provides some concluding remarks.

2. Empirical consumption profile

Gourinchas and Parker (2002) and Fernández-Villaverde and Krueger (2002), among others, have estimated life cycle consumption profiles using data from the Consumer Expenditure Survey, shown in Fig. 1.

Fernández-Villaverde and Krueger (2002) obtain a profile for nondurable consumption expenditures of an adult equivalent for ages 22 through 87. According to their estimates, consumption peaks at age 52 and the ratio of consumption at this maximum to consumption at age 25 is equal to 1.29. We use these numbers as data benchmarks with which to compare our model results because we abstract from durable consumption in our model. Also, their estimation procedure controls for both family composition and cohort (growth) effects. We abstract from the first in our theoretical analysis, and, although we incorporate technological progress in our model, we also correct for growth

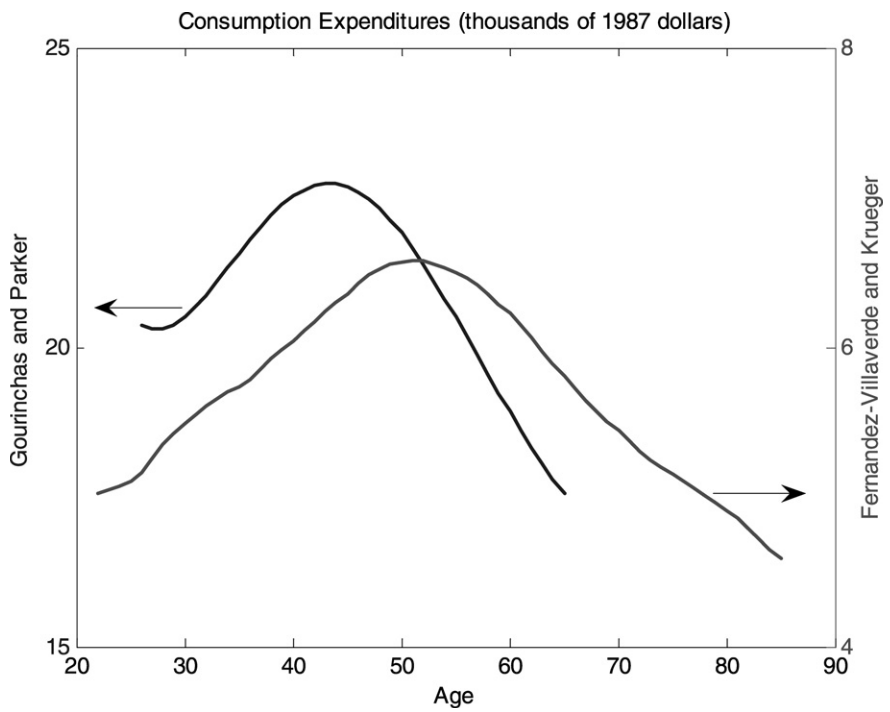

Fig. 1. Consumption over the life cycle.

in computing our theoretical consumption profiles. Gourinchas and Parker (2002), using a broader definition of consumption, compute a household consumption profile for ages 26 through 65. They find that consumption peaks near age 45 and the ratio of peak consumption to age 25 consumption is close to 1.12.[5] This is consistent with Fernández-Villaverde and Krueger (2002) who find that durable consumption peaks earlier than consumption of nondurables.

3. A simple partial equilibrium model

To clarify why a lack of annuity markets can lead to a hump shaped lifetime consumption profile we first study a very simple endowment economy. Each period one agent is born that lives a maximum of I periods. Assume that the lifetime endowment pattern is given by

$$y_i = \begin{cases} 1 & \text{for } i < I_M, \\ 0 & \text{for } i \geqslant I_M \end{cases}$$

where I_M is the mandatory retirement age. That is, individuals receive one unit of income in each period until they retire, at which point they must finance consumption with accumulated savings. A new born in this economy solves the following problem:

$$\max \sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=0}^{i-1} s_j \right) \ln c_i$$

subject to

$$c_i + \Lambda_i a_{i+1} = R(a_i + b) + y_i, \quad i = 1, 2, \ldots, I,$$
$$a_1 = 0.$$

Here c_i is consumption of an age-i individual, a_i is beginning of period asset holdings, y_i is endowment income, and s_j is the conditional probability of surviving from age j to age $j+1$. Non-annuitized assets of individuals who die in a given period are distributed to all living individuals as a lump sum transfer b. The interest factor, R, is taken parametrically in this partial equilibrium model. We allow for zero, partial or complete annuitization of wealth by assuming a value for $\lambda \in [0, 1]$, which is the fraction of assets that are annuitized. For a given value of λ, the savings required of an individual who would like a_{i+1} assets available at the beginning of the following period is $\Lambda_i a_{i+1}$, where $\Lambda_i = 1 - \lambda(1 - s_i)$. This implies that

$$\Lambda_i a_{i+1} = \lambda s_i a_{i+1} + (1 - \lambda) a_{i+1}.$$

The first term on the right hand side of this expression is savings in the form of annuitized assets and the second term is savings in the form of assets that are not annuitized. Note that s_i is the actuarially fair price for a one-period annuity sold to an individual of age i. If $\lambda = 1$, then there are complete annuity markets. As long as $\lambda < 1$, there will be unintended bequests b. These are computed as

$$b = \frac{\sum_{i=1}^{I-1} [\prod_{j=1}^{i} s_{j-1}](1 - \lambda)(1 - s_i) a_{i+1}}{\sum_{i=1}^{I} \prod_{j=1}^{i} s_{j-1}}.$$

Given values for the model parameters and the interest factor R, it is straight forward to solve for the lifetime consumption path that would be chosen by individuals in this economy. We assume that individuals start their economic life at age 21 (corresponds to $i = 1$) and live to a maximum age of 100 ($I = 80$). They retire at age 65 ($I_M = 45$). We use survival probabilities published by the Social Security Administration for the cohort born in 1950.

In this case, the life cycle consumption-saving decision is determined by a sequence of Euler equations that can be written as follows:

[5] Fernández-Villaverde and Krueger (2002) measure consumption as quarterly expenditures on food, alcoholic beverages, tobacco, utilities, personal care, household operations, public transportation, gas and motor oil, and entertainment. Gourinchas and Parker (2002) use a broader measure of consumption and consider all annual expenditures by a household except those for health care, mortgage interest and education. In Fig. 1, we adjust for the differences in frequency and base year calculations.

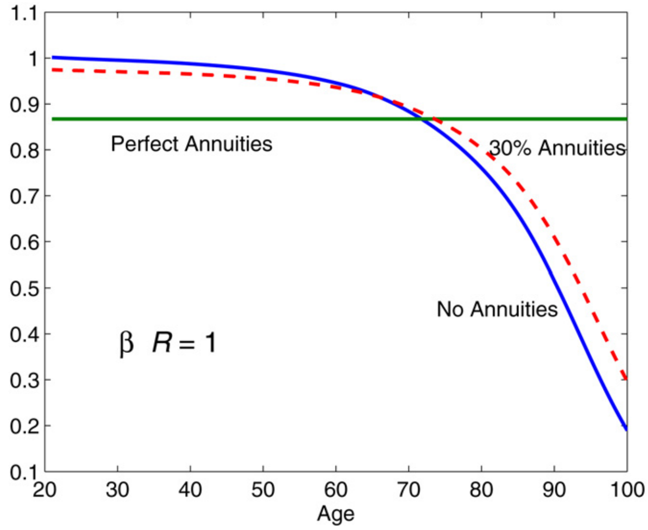

Fig. 2. Consumption profiles.

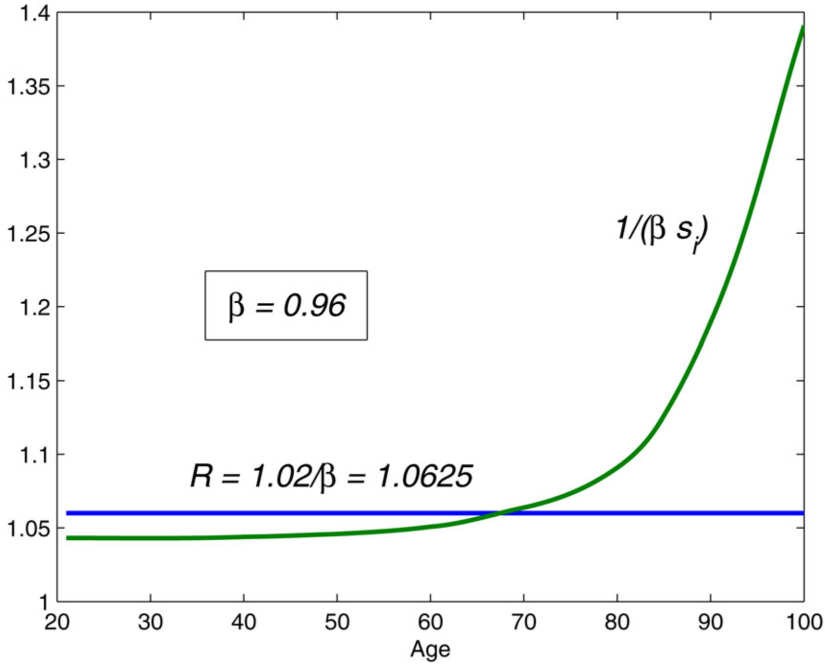

Fig. 3. Subjective and market discount factors.

$$\frac{c_{i+1}}{c_i} = \frac{\beta s_i R}{\Lambda_i},$$

where the right-hand side reduces to $\beta s_i R$ in the absence of annuity markets and to βR when all assets are annuitized.

We are interested in determining how the consumption profile depends on the value of λ and the value of the interest factor R relative to the subjective discount factor β. Assuming $\beta = 0.96$, we first consider a value of R such that $\beta R = 1$. Fig. 2 shows the consumption profile in this case for three values of λ: 0, 0.3, and 1.

One can see that, if there are perfect annuity markets ($\lambda = 1$), consumption will be constant over the individual's lifetime, which is also clear from the Euler equation, as the right hand side equals one in this case. If there are no annuities available ($\lambda = 0$), individual consumption is declining over time. The intuition for this is that individuals, because they face a probability of not surviving to enjoy the fruits of their savings, discount the future more heavily

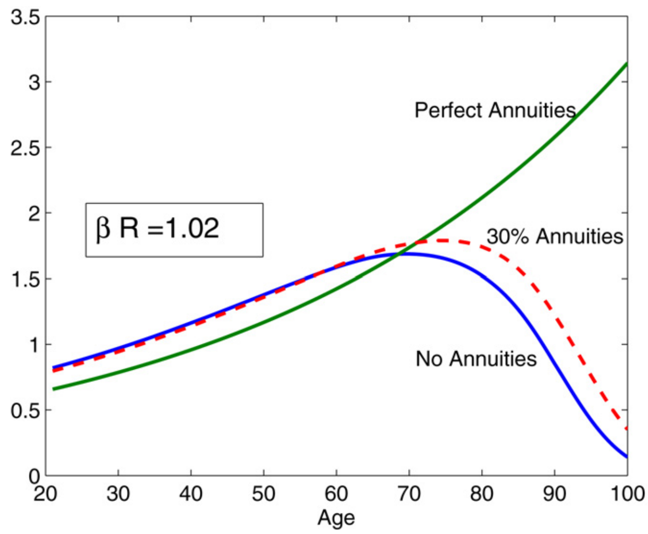

Fig. 4. Consumption profiles.

than if actuarially fair annuities are available. This finding is robust to allowing individuals to hold a substantial amount of their saving in the form of annuities. For example if individuals annuitize thirty percent ($\lambda = 0.3$) of their wealth at all ages, their consumption profile is very similar to that in the case of no annuities.

If the consumption profile is to be hump shaped, consumption must increase early in life. To illustrate this, we choose R so that $\beta R = 1.02$. In this case, consumption would rise throughout life if individuals had access to perfect annuity markets. This is because the price of the annuity, which is falling as an individual ages, compensates for the increasing effective rate of discount due to survival probabilities falling as an individual ages.

A lack of annuity markets, however, means that individuals are not compensated for their increasing effective rate of discount and consumption may decline in the later stages of life. This happens when an individual's effective rate of discount is larger than the interest rate. Fig. 3 shows the period-by-period trade-off that the individual faces in his consumption-saving choice for $\beta = 0.96$ and $R = 1.02/\beta$. As long as the market discount factor given by the gross real interest rate exceeds the subjective discount factor adjusted for the conditional survival probability of that age, consumption grows. Once the reverse is true, consumption has reached its peak and starts to decline.

Fig. 4 illustrates consumption profiles when there are perfect annuity markets, when there are no annuities, and when 30% of assets is annuitized.

4. A general equilibrium model

The preceding section provides the basic intuition about how missing annuity markets leads to a hump in consumption. In this section, we will describe a fully calibrated general equilibrium life cycle model of the sort studied by Auerbach and Kotlikoff (1987), İmrohoroğlu et al. (1995), Ríos-Rull (1996), and Fernández-Villaverde and Krueger (2001), among others. Studying this issue in a general equilibrium setting is crucial because model features or institutions (such as social security) will influence the consumption profile through its effect on the interest rate.

4.1. The environment and demographics

We use a stationary overlapping generations setup. At each date t, a new generation of individuals is born and the population growth rate is η. Individuals face long but random lives with a maximum possible age I. Lifespan uncertainty is described by s_i, the conditional probability of surviving from age i to $i+1$. We assume the survival probabilities and the population growth rate are time-invariant.[6] Total population is given by

[6] For studies that examine the quantitative impact of time-variation in either demographic variable on social security reform, see Kotlikoff et al. (1999) and De Nardi et al. (1999), among others.

$$N_t \sum_{i=1}^{I} \frac{\prod_{j=1}^{i} s_{j-1}}{(1+\eta)^{i-1}},$$

where N_t denotes the number of individuals born in period t. Given that we assume stationary demographics, the fraction of the total population that is of age i is constant over time. These cohort shares, $\{\mu_i\}_{i=1}^{I}$, are given by

$$\mu_i = \frac{s_{i-1}}{(1+\eta)} \mu_{i-1}, \quad \text{for } i = 2, \ldots, I, \tag{1}$$

and

$$\sum_{i=1}^{I} \mu_i = 1.$$

4.2. Technology

There is a representative firm with access to a constant returns to scale Cobb–Douglas production function:

$$Y_t = K_t^{\alpha} (A_t H_t)^{1-\alpha}, \tag{2}$$

where K_t and H_t are aggregate capital and labor inputs, respectively, and α is capital's output share. There is exogenous labor-augmenting technological growth at the rate $g > 0$:

$$A_{t+1} = (1+g) A_t. \tag{3}$$

The capital stock depreciates at the rate δ and follows the law of motion

$$K_{t+1} = (1-\delta) K_t + X_t, \tag{4}$$

where X_t is aggregate investment in period t.

4.3. Households

Individuals differ by their date of retirement. There are M possible retirement dates (I_m for $m = 1, \ldots, M$) and individuals know the date of their retirement at birth. The fraction of individuals with retirement date I_m is denoted by π_m.

An individual of type m born at time t solves the following problem:

$$\max \sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=1}^{i-1} s_j \right) \frac{[c_{i,m,t+i-1}^{\phi} (1 - h_{i,m,t+i-1})^{1-\phi}]^{1-\gamma}}{1-\gamma}, \tag{5}$$

subject to

$$\begin{aligned} c_{i,m,t+i-1} + \Lambda_i a_{i+1,m,t+i} = {} & R_{t+i-1}(a_{i,m,t+i-1} + b_{t+i-1}) \\ & + (1-\tau_s) w_{t+i-1} \varepsilon_i h_{i,m,t+i-1} + S_{i,m,t+i-1}, \end{aligned} \tag{6}$$

where β is the subjective discount factor, R_{t+i-1} is the interest factor, τ_s is the social security payroll tax, $S_{i,m,t+i-1}$ is the social security benefit paid to an individual of age i and type m, $a_{i+1,m,t+i}$ is the amount of assets to be available at age $i+1$, ε_i is the efficiency weight of an individual at age i, and $h_{i,m,t+i-1}$ is hours supplied by an age-i individual of type m at time $t+i-1$. As in Section 3, we use $\lambda \in [0, 1]$ to indicate the degree of completeness of private annuity markets, and $\Lambda_i = 1 - \lambda(1 - s_i)$. We assume that accidental bequests, if they exist, are returned to all surviving individuals, regardless of age, in a lump sum denoted by b_{t+i-1}.

Finally, we assume that all individuals are born with zero wealth and will exhaust all accumulated wealth at the maximum achievable age I, so that $a_{1,m,t} = a_{I+1,m,t+I} = 0$ for all m and t.

4.4. Social security

There is an unfunded social security system in our economy. Benefits are linked to average lifetime earnings in a manner consistent with the Social Security Administration's (SSA) computation. An individual born at date t receives total labor income over the life cycle equal to

$$\sum_{j=1}^{I_m-1} w_{t+j-1}\varepsilon_j h_{j,m,t+j-1},$$

where I_m is the retirement age for this individual and $t+I_m-1$ is the date of retirement. To obtain the indexed annual income (similar to the notion of Average Indexed Monthly Earnings calculated by the SSA), we need to multiply past earnings up to the time of retirement by a 'productivity factor', with earnings that are in the more distant past getting a higher factor. For an individual who retires at age I_m at date $t+I_m-1$, past earnings are scaled up so that the most recent income before retirement (at date $t+I_m-2$) is multiplied by $(1+g)^0$, income from the period preceding that one is multiplied by $(1+g)^1$, and so on, until the first working age income for this individual, $w_t\varepsilon_1 h_{1,m,t}$, is multiplied by $(1+g)^{I_m-2}$. Therefore, for an individual who retires at time $t+I_m$, total indexed labor income over the life cycle is given by

$$\sum_{j=1}^{I_m-1} w_{t+j-1}(1+g)^{I_m-1-j}\varepsilon_j h_{j,m,t+j-1}.$$

Retirement benefits for an age i individual who retires at age I_m in date $t+i-1$ is a fraction θ_m of average lifetime indexed income (the replacement rate depends on the age of retirement).

$$S_{i,m,t+i-1} = \begin{cases} \frac{\theta_m}{I_m-1}\sum_{j=1}^{I_m-1} w_{t+j-1}(1+g)^{I_m-1-j}\varepsilon_j h_{j,m,t+j-1} & \text{for } i \geqslant I_m, \\ 0 & \text{for } i < I_m. \end{cases}$$

Given the replacement rates, we impose the pay-as-you-go requirement

$$\sum_{m=1}^{M}\pi_m \sum_{i=I_m}^{I}\mu_i S_{i,m,t} = \tau_s \sum_{m=1}^{M}\pi_m \sum_{i=1}^{I_m-1}\mu_i w_t \varepsilon_i h_{i,m,t}$$

to endogenously calculate the social security tax rate. In this formula, $\{\pi_m\}_{m=1}^M$ is the fraction of individuals who retire at age I_m.

Given that current labor market decisions affect future social security benefits, an individual facing problem (5)–(6) solves an intertemporal first order condition for hours worked rather than the usual static first order condition.

4.5. Competitive equilibrium

A competitive equilibrium with stationary demographics consists of a social security tax rate τ_s, and sequences indexed by t for unintended bequests b_t, household allocations $\{\{c_{i,m,t}, a_{i+1,m,t+1}, h_{i,m,t}\}_{i=1}^I\}_{m=1}^M$, factor demands K_t and H_t, and factor prices w_t and R_t such that

1. The household allocation solves its maximization problem.
2. Factor demands solve the stand-in firm's profit maximization problem, which implies that

$$w_t = (1-\alpha)\left(\frac{K_t}{H_t}\right)^{\alpha} A_t^{1-\alpha},$$

$$R_t = \alpha\left(\frac{A_t H_t}{K_t}\right)^{1-\alpha} + 1 - \delta.$$

3. Markets clear:

Capital Rental Market $$K_t = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I} (a_{i,m,t} + b_t)\mu_i,$$

Labor Market $$H_t = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} \mu_i \varepsilon_i h_{i,m,t},$$

Goods Market $$C_t + X_t = Y_t,$$

where

$$C_t = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} c_{i,m,t},$$
$$X_t = K_{t+1} - (1-\delta)K_t,$$
$$b_t = \sum_{m=1}^{M} \pi_m \sum_{i=1}^{I-1} \frac{\mu_i (1-s_i)(1-\lambda) a_{i+1,m,t}}{1+\eta}.$$

4. The social security system is unfunded:

$$\tau_s = \frac{\sum_{m=1}^{M} \pi_m \sum_{i=I_m}^{I} \mu_i S_{i,m,t}}{\sum_{m=1}^{M} \pi_m \sum_{i=1}^{I_m - 1} \mu_i w_t \varepsilon_i h_{i,m,t}}.$$

5. Quantitative exercise

In this section, we describe how we solve for the balanced growth path and the calibration of the model.

5.1. Solving for the steady-state equilibrium

The competitive equilibrium defined above has the property that $c_{i,m,t}$, $a_{i,m,t}$, $S_{i,m,t}$, K_t, and w_t, for all i and m, grow at the constant rate of technological progress, g. For each variable Z_t define

$$\widehat{Z}_t \equiv \frac{Z_t}{A_t}.$$

Introducing this change of variables to the model enables us to solve for steady state values of $\{\{\hat{c}_{i,m}, h_{i,m}, \hat{a}_{i+1,m}, \widehat{S}_{i,m}\}_{i=1}^{I}\}_{m=1}^{M}$, $\widehat{K}$, H, $\hat{b}$, and prices $\widehat{w}$ and R. In particular, the life cycle profiles of consumption and hours described in section 6 are computed by taking $\sum_{m=1}^{M} \pi_m \hat{c}_{i,m}$ and $\sum_{m=1}^{M} \pi_m h_{i,m}$.

5.2. Calibration

Individuals are assumed to begin their economic life at age 21 (that is, $i = 1$ corresponds to age 21) and live until a maximum age of 100 ($i = 80$). The conditional survival probabilities from age i to age $i+1$, $\{s_i\}_{i=1}^{I}$, are taken from estimates provided by the Social Security Administration (SSA) for a cohort born in 1950 (see Bell and Miller, 2002).[7] The population growth rate, η, is assumed to be 1.2 percent per year. The age specific efficiency weights for labor hours, $\{\varepsilon_i\}_{i=1}^{I}$, are based on estimates from Hansen (1993).

[7] The survival probabilities used in the model are simple averages of the male and female survival rates in the life table. We assume that households do not have more information about their mortality risk than what is summarized in the life tables and therefore abstract from asymmetric information as well as individual effort affecting survival probabilities. Both of these might be important features of a model in which a lack of annuity markets emerges endogenously.

Table 1
Benchmark calibration

Demographics		
first age	$i=1$	21
maximum age	$I=80$	100
population growth rate	η	0.012
conditional survival probabilities	$\{s_i\}_{i=1}^I$	SSA, cohort born in 1950
efficiency weights	$\{\varepsilon_i\}_{i=1}^I$	Hansen (1993)
Technology		
capital share parameter	α	0.36
depreciation rate	δ	0.047
productivity growth rate	g	0.0165
Preferences		
subjective discount factor	β	0.9726
coefficient of relative risk aversion	γ	1.0
share of consumption	ϕ	0.368

Social Security Parameters ($\theta = \Sigma_{m=1}^M \theta_m / M = 0.45$)

I_m	62	63	64	65	66	67	68	69	70
π_m	0.46	0.14	0.06	0.19	0.06	0.02	0.02	0.01	0.05
θ_m	0.39	0.43	0.46	0.49	0.52	0.56	0.59	0.62	0.65

Retirement can occur at $M = 9$ possible ages, which correspond to ages 62–70 ($I_m \in \{42, 43, \ldots, 50\}$).[8] The fraction of individuals that retire and begin collecting social security at age I_m, π_m, was obtained from the SSA.[9] In addition, we calculated the age adjusted social security replacement rate, θ_m, from SSA data on benefits as a percentage of the Primary Insurance Amount (PIA) by the age at which benefits begin.[10] In particular, we chose $\{\theta_m\}_{m=1}^M$ to be consistent with these benefit percentages, the fraction of individuals that retire at these ages, and the US average social security replacement rate of 0.45.

Since the primary focus of this paper is on the role of annuities, as opposed to the role of nonseparable utility studied in Bullard and Feigenbaum (2003), we set the risk aversion parameter, γ, equal to one in our benchmark calibration. That is, the period utility function is separable, $U(c_i, h_i) = \phi \log c_i + (1-\phi)\log(1-h_i)$.

The remaining parameters of preferences and technology are chosen so that our model is consistent with various facts characterizing the US macroeconomy. The growth rate of labor augmenting technological progress, g, is chosen so that the model is consistent with the measured growth rate of real per capita income. Given this and the average capital output ($K/Y = 3.32$) and investment output ($X/Y = 0.25$) ratios measured from US data, we obtain the depreciation rate as follows:

$$\delta = \frac{X/Y}{K/Y} - g - \eta - g\eta.$$

The capital share parameter, α, is set equal to 0.36, which is consistent with measures of capital's share from NIPA data. Finally, the preference parameters β and ϕ are chosen to target the capital–output ratio and the fraction of time spent on market activities (taken to be 0.31).

Table 1 summarizes our calibration.

In alternative calibrations, we explore the relative impact of nonseparable utility in addition to lack of annuity markets on the shape of the consumption profile. Hence, we also consider cases where $\gamma = 4$ and $\gamma = 7$. This requires

[8] In our model, the retirement age is the exogenously determined age at which individuals begin collecting social security and also stop working. In the US economy, these two events do not need to coincide. French (2004) studies a life cycle model where labor participation and the date at which social security payments begin are separate decisions made by utility maximizing individuals.

[9] Age 62 is the first age at which individuals are eligible to collect social security. By delaying, an individual will receive higher payments (θ_m increases with m), but there is no incentive to delay beyond age 70. Hence, the fraction of individuals that begin collecting social security after age 70 is trivial. The values we used for $\{\pi_m\}_{m=1}^M$ were obtained directly from Social Security Administration and are similar to numbers found in Table 6B5 of the 2004 Annual Statistical Supplement of the Social Security Administration.

[10] These data can be found at the following web site: http://www.ssa.gov/OACT/ProgData/ar_drc.html.

Table 2
Alternative calibrations

	β	ϕ
$\gamma = 1$	0.9726	0.368
$\gamma = 4$	1.0127	0.380
$\gamma = 7$	1.0545	0.386

re-calibrating the parameters β and ϕ in order to hit our targets. Table 2 summarizes the values used in these alternative calibrations. In particular, as γ is increased, β must be increased to achieve the target capital–output ratio, and ϕ increased to match the fraction of time engaged in market activities.

6. Results

Here we describe consumption profiles for three model economies. The first is our benchmark economy with social security and no annuity markets. The second one is with complete annuity markets and no social security. The third case has no annuity markets and no social security.

In each case, we consider three values for the risk aversion parameter, $\gamma = 1$, 4, and 7. The utility function is separable between consumption and leisure when $\gamma = 1$, so leisure (retirement, in particular) has no effect on the marginal utility of consumption. The other two cases involve non-separable utility, so changes over the life cycle in the amount of time individuals spend working will also affect the shape of the consumption profile.[11] A summary of our quantitative findings is contained in Table 3.

Table 3 reports various statistics including the three that were used to calibrate the model in Section 6.1: the investment–output ratio, the capital–output ratio, and the average fraction of time spent working. Since we calibrated the model under the assumptions of Case 1 (social security and no annuities), the results hit our targets exactly in this case.[12] In addition, we report the social security tax rate, the steady state interest rate, the age at which consumption reaches its maximum, and the ratio of maximum consumption to age 25 consumption.[13]

Our main quantitative result is displayed in Fig. 5 (and the first panel of Table 3) which also shows the consumption profile estimated from the Consumer Expenditure Survey by Fernandez-Villaverde and Krueger (2002). Consumption from the model economies has been normalized so that the life cycle profile has the same mean as the one estimated

Table 3
Quantitative findings

Data	θ	τ_s	X/Y	K/Y	R	H	S_1	S_2
	0.45	0.10	0.25	3.32		0.310	52	1.29
Case 1: Social Security and No Annuities								
$\gamma = 1$	0.45	0.107	0.25	3.32	1.062	0.310	67	1.61
$\gamma = 4$	0.45	0.107	0.25	3.32	1.062	0.310	61	1.60
$\gamma = 7$	0.45	0.107	0.25	3.32	1.062	0.310	61	1.59
Case 2: No Social Security and Perfect Annuities								
$\gamma = 1$	0.0	0.0	0.280	3.718	1.050	0.310	NA	1.34
$\gamma = 4$	0.0	0.0	0.300	3.979	1.044	0.318	59	1.34
$\gamma = 7$	0.0	0.0	0.309	4.103	1.041	0.323	59	1.39
Case 3: No Social Security and No Annuities								
$\gamma = 1$	0.0	0.0	0.269	3.573	1.054	0.297	60	1.21
$\gamma = 4$	0.0	0.0	0.311	4.131	1.041	0.311	57	1.28
$\gamma = 7$	0.0	0.0	0.355	4.719	1.030	0.327	57	1.30

Notes. NA: the ratio is still rising at maximum attainable age. S_1 is the age at which life cycle consumption attains its maximum. S_2 is the ratio of maximum consumption to consumption at age 25.

11 The effect of this non-separability on consumption profiles is studied in Heckman (1974) and Bullard and Feigenbaum (2003).

12 Note that, as shown in Table 2, we have different calibrated parameters depending on the value of γ.

13 Note that the social security tax rate that maintains the pay as you go system in Case 1 is close to the actual social security tax rate of about 10%.

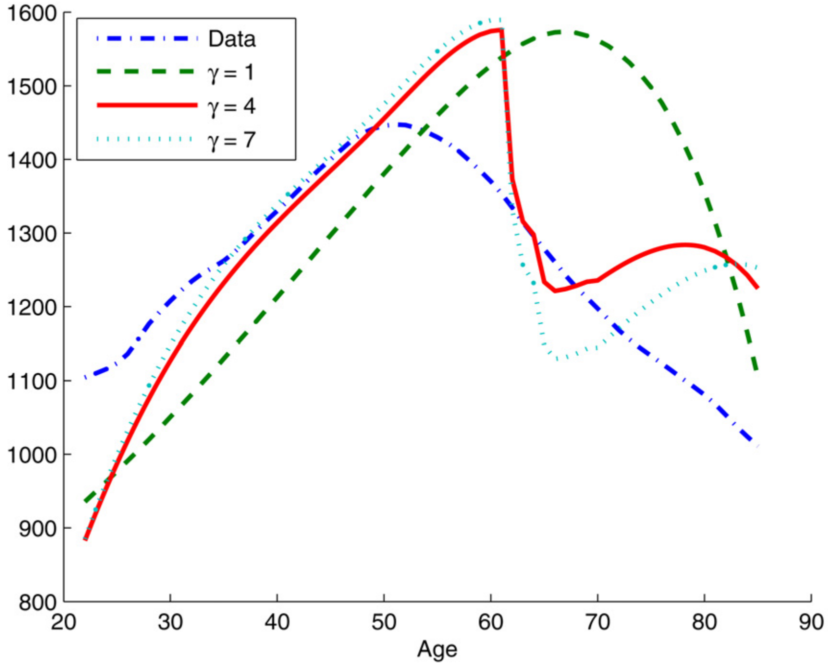

Fig. 5. Social security and no annuity markets.

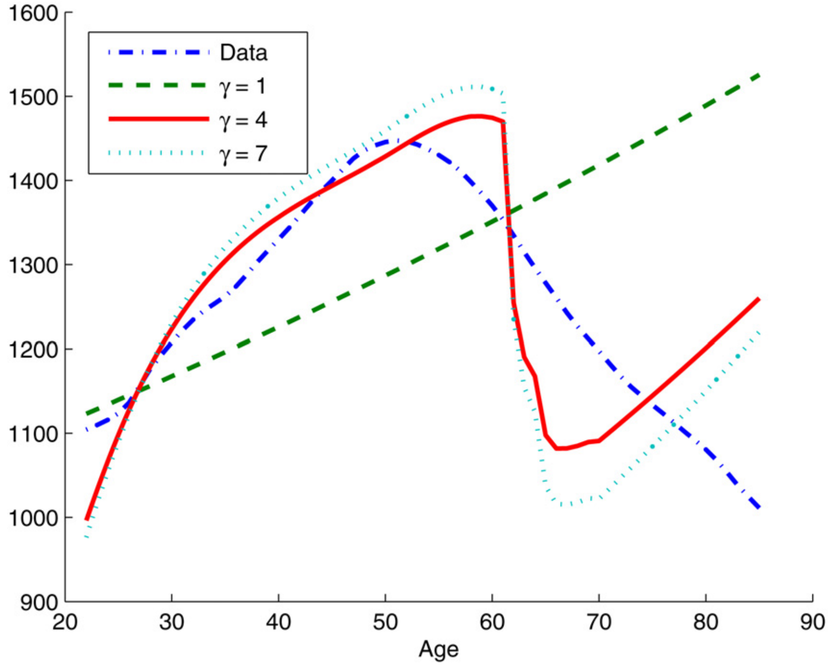

Fig. 6. No social security and perfect annuities.

from actual data. In an economy with social security but no annuity markets, consumption peaks later than that in the data and the ratio of maximum consumption to age 25 consumption is too large. In the figure, we also see an upward sloping profile after retirement (except for the $\gamma = 1$ case) that we do not see in actual data.

To better understand how the model works, we consider Case 2, which has perfect annuity markets and no social security. In this case, consumption profiles are hump shaped only when utility is non-separable. Fig. 6 shows that

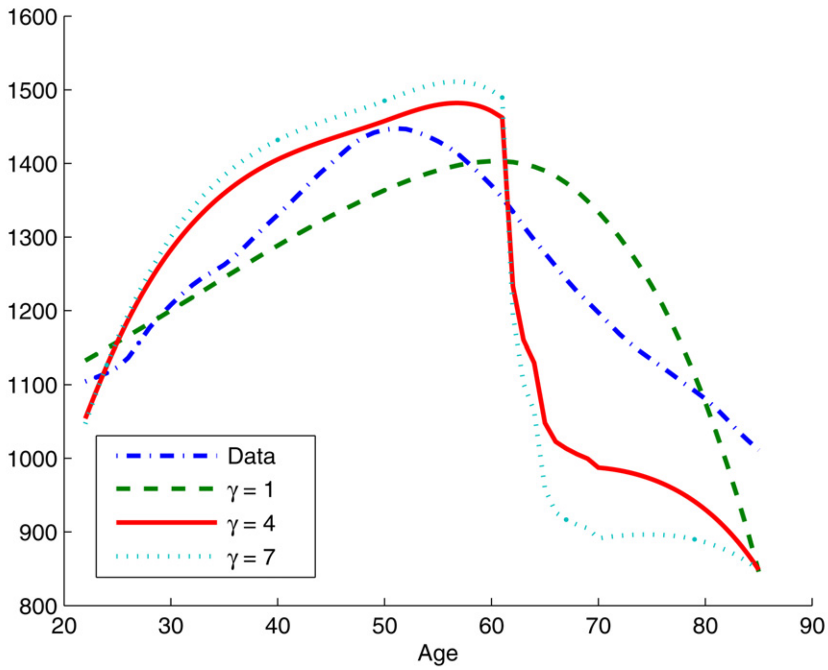

Fig. 7. No social security and no annuity markets.

consumption monotonically increases throughout life in the $\gamma = 1$ case. This reflects the fact that for our calibrated capital–output ratio the equilibrium interest rate is higher than the subjective discount rate. In the other two profiles shown, consumption peaks at about age 59 and then rises again after retirement (consumption peaks at age 52 in the data). This is very different from what is observed in actual data.

When public and private annuities are shut down simultaneously, all consumption profiles have a hump shape (see Fig. 7). The intuition for why a hump is observed in the $\gamma = 1$ case is the same as was discussed in Section 3; the effective rate of discount that combines the subjective rate of discount and the unconditional probability of survival eventually exceeds the market rate of discount measured by the interest rate. We interpret this counterfactual case as indicating that *if* there were no access to annuities whatsoever, life cycle uncertainty might account substantially for the hump in consumption estimated in the data. This conclusion is reinforced by our robustness experiments that we describe in Section 6.1.

Our model also has implications for the life cycle profile of hours worked. It turns out that the hours profiles are very similar in all the cases considered, so we only show the profiles for the benchmark economy with no annuity markets and social security in Fig. 8. These profiles are almost completely determined by the efficiency weights and by our retirement assumptions. Annuity markets, or lack thereof, have essentially no impact on the shape of the hours profile.

6.1. Robustness

In order to assess the robustness of our results on the lack of annuity markets, we relax the discipline imposed by our calibration procedure. In particular, we solve the model for a wide range of plausible parameter values without targeting a particular capital–output ratio, investment–output ratio, or fraction of time spent on market activities. We create a discrete grid for each parameter as shown in Table 4 and solve our model for all parameter combinations on these grids.

In Table 5 we summarize our findings regarding the shape of the steady state life cycle consumption profile. We report the minimum and maximum values for the age at which life cycle consumption attains its maximum (S_1) over all parameter combinations considered. In addition, we report the corresponding ranges of values for the ratio of maximum consumption to consumption at age 25 (S_2).

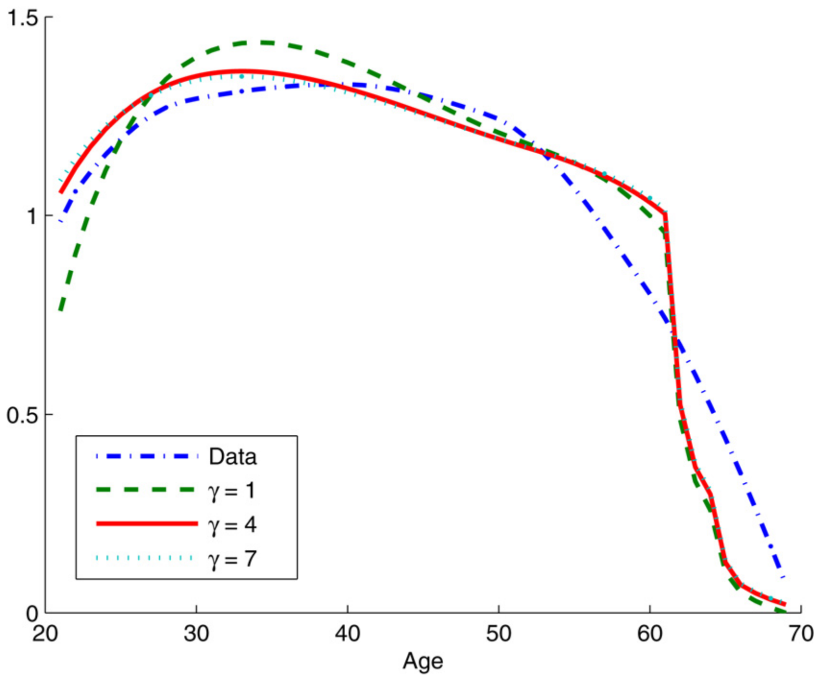

Fig. 8. Hours profile, social security and no annuity markets.

Table 4
Parameter space for robustness check

Parameter	Min value	Max value	Increment
γ	1	4	3
β	0.98	1.02	0.01
δ	0.03	0.09	0.02
α	0.30	0.40	0.02
ϕ	0.30	0.40	0.02

Table 5
Results of robustness check

Case	min S_1	S_2	max S_1	S_2
Social security, $\gamma = 1$	60	1.21	74	2.54–2.74
Social security, $\gamma = 4$	57	1.34	61	1.51–1.78
No social security, $\gamma = 1$	22	1.09–1.10	70	1.92–1.99
No social security, $\gamma = 4$	37	1.1	60	1.49

Notes. S_1 is the age at which life cycle consumption attains its maximum ($S_1 = 52$ in data). S_2 is the ratio of maximum consumption to consumption at age 25 ($S_2 = 1.29$ in data).

This exercise reinforces the findings from our calibrated model. If one ignores the existence of social security, a life cycle model with no annuity markets appears to account for the key quantitative properties of consumption over the life cycle. However, when social security is present, consumption is predicted to peak too late in life and the size of the hump in consumption is too large.

7. Bequests

The results of the previous section indicate that, while a lack of annuity markets causes consumption to be hump shaped over the life cycle, consumption appears to peak quite a bit later than empirical studies have found.

Another problem with these results is that older households are not saving as much (holding as many assets) as they do in the data. For example, for the benchmark case in Table 3 with social security and no annuities ($\gamma = 1$), the ratio of assets held by individuals age 75 and older to average wealth is 0.69. But according to Table 702 of the Statistical Abstract of the United States (2006), this ratio is around 1.75.[14] Clearly, our model is inconsistent with the saving behavior of older households.[15]

There are a variety of reasons why older households might save more than they do in our model economy. While we have taken into account uncertainty about longevity, we have not taken into account uncertainty about medical expenditures or that individuals may care about their descendants. In this section, we will introduce a "joy of giving" bequest motive in order to give older households an additional motive to save.[16]

To see how this might affect consumption over the life cycle, suppose that a household's preferences are represented by

$$\sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=1}^{i-2} s_j \right) \left\{ s_{i-1} U(c_i) + (1 - s_{i-1}) V(a_i) \right\},$$

where U and V are increasing, concave and differentiable functions. The function V captures the bequest motive. The intertemporal first order condition can be written as

$$U'(c_i) = \beta \left\{ s_i R U'(c_{i+1}) + (1 - s_i) V'(a_{i+1}) \right\}. \tag{7}$$

If $V'(a) = 0$, the slope of the consumption profile depends on the value of s_i relative to βR (see discussion in Section 3). If $V'(a) > 0$, then there is an additional value of saving that will cause households to delay consumption (the consumption profile will have a positive slope) even for a range of values of s_i that are less than $1/(R\beta)$. Hence, for a given interest factor and sequence of survival probabilities, it would appear that the introduction of a bequest motive will delay the peak of the life cycle consumption profile. Since the puzzle remaining from Section 6 is that consumption in our model peaks too late in life, it would appear that a bequest motive is not going to help us resolve this anomaly.

However, ours is a general equilibrium model. The interest rate is not exogenous and will fall if we hold all other parameters constant and introduce a bequest motive that increases savings and, hence, the capital stock. The general equilibrium effect comes from a reduction in the interest factor as the capital stock increases with a more intense bequest motive.

In our numerical exercise, we calibrate the model by choosing β to deliver a particular capital–output ratio. With a bequest motive, we will require a smaller value of β to achieve our target. Hence, the condition $s_i < 1/(\beta R)$ will be satisfied at an earlier age since survival probabilities fall as an individual ages. Whether or not this general equilibrium effect dominates the effect described above can only be determined through a numerical exercise. We describe this exercise and the results obtained in the next subsection.

7.1. A simplified model with a bequest motive

We will consider a simplified version of our model in order to understand the role of bequests in affecting the consumption profile. In particular, we restrict our attention to log utility, one retirement type, and exogenous labor. We assume there are no annuity markets. We use a 'joy of giving' formulation to represent a bequest motive.[17] The household's problem is

$$\max \sum_{i=1}^{I} \beta^{i-1} \left(\prod_{j=1}^{i-2} s_j \right) \left\{ s_{i-1} \log(c_{i,t+i-1}) + (1 - s_{i-1}) \psi \log(a_{i+1,t+i}) \right\}$$

[14] These wealth ratios are computed using data from the Survey of Consumer Finance.

[15] This ratio is somewhat sensitive to the timing of bequests. In the experiments reported in Table 3, bequests, which are all accidental, are paid in lump sum fashion to all living households. If we assume that bequests are paid uniformly only to individuals of age 52–58, which are the ages most individuals receive bequests in the US, the ratio of assets held by individuals age 75 and older to average wealth is 0.87 rather than 0.69.

[16] We calibrate the bequest motive to account for all saving by the elderly, so we are exaggerating the importance of this particular motive.

[17] The inclusion of a bequest motive presents computational difficulties in characterizing steady-state equilibria especially when the bequest motive is very strong. To deal with this problem, we simplified the model in all the dimensions that we could but retained the crucial features.

subject to

$$c_{i,t+i-1} + a_{i+1,t+i} = R_{t+i-1}(a_{i,t+i-1} + b_{i,t+i-1}) + (1-\tau_s)w_{t+i-1}\varepsilon_i + S_{i,t+i-1},$$

where $\psi > 0$ is a parameter that represents the intensity of the 'bequest' motive. Conditional on survival, individuals begin collecting social security benefits at age I_R:

$$S_{i,t+i-1} = \begin{cases} 0 & \text{for } i = 1, 2, \ldots, I_R - 1, \\ \frac{\theta}{I_R - 1}\sum_{j=1}^{I_R-1}(1+g)^{I_R-j-1} w_{t+j-1}\varepsilon_j & \text{for } i = I_R, I_{R+1}, \ldots, I, \end{cases}$$

and the social security tax rate is computed as

$$\tau_s = \frac{\sum_{i=I_R}^{I} \mu_i S_{i,t+i-1}}{\sum_{i=1}^{I_R-1} \mu_i w_{t+i-1}\varepsilon_i}.$$

Factor prices are the same as in Section 4 with $H = \sum_{i=1}^{I_R-1} \mu_i \varepsilon_i$. We will present our findings for two cases, with and without social security. In both cases, we set $\eta = 0.012$, $g = 0.0165$, and $\alpha = 0.36$ as before, and target $K/Y = 3.32$ and $I/Y = 0.25$ (which together imply $\delta = 0.047$ as before). We calibrate the bequest motive (ψ) so that the ratio of assets held by households aged 75 and older to the total capital stock is equal to 1.78. This choice of ψ, however, is likely to be an upper bound because there are many additional reasons that will induce households to keep a large level of assets late in their life cycle such as the illiquidity of the housing markets, the risk of out-of-pocket medical expenses, etc. Also, we assume that both accidental and intentional bequests are received as lump sum transfers by all households between ages 52 and 58 (see footnote 15).

7.2. Results with social security

Setting the replacement rate $\theta = 0.45$, a subjective discount rate of $\beta = 0.9672$ and an intensity of bequest $\psi = 3.80$ allow us to obtain $K/Y = 3.32$ and $S_3 = 1.78$. Table 6 reports the quantitative general equilibrium findings.

This case is analogous to the case shown in Table 3 for Case 1, $\gamma = 1$ and differs from this case only because of the simplifications we have made to the model and the different timing of accidental bequests. As in Table 3, the results shown in the first row of Table 6 indicate that consumption peaks at a late age (70) when there is no bequest motive. The introduction of a bequest motive calibrated to $S_3 = 1.78$, however, does not have much of an effect on the age at which consumption peaks (68 as opposed to 70). Introducing a bequest motive does not appear to overturn our main findings from Section 6, that the partial annuity provided by social security causes the peak in consumption to appear

Table 6
Timing of the hump: with social security

ψ	R	K/Y	S_1	S_2	S_3
		(calibrated with $\psi = 0$)			
0	1.0619	3.32	70	1.90	1.02
		(calibrated to $S_3 = 1.78$)			
0	1.0715	3.05	70	1.90	1.11
1	1.0682	3.14	69	1.81	1.37
2	1.0658	3.20	69	1.76	1.54
3	1.0636	3.27	69	1.71	1.69
3.8	**1.0619**	**3.32**	**68**	**1.68**	**1.78**
5	1.0592	3.40	67	1.64	1.90
8	1.0532	3.61	60	1.57	2.12
10	1.0496	3.74	57	1.55	2.20
15	1.0418	4.07	55	1.52	2.32

Notes. S_1 is the age at which life cycle consumption attains its maximum. S_2 is the ratio of maximum consumption to consumption at age 25. S_3 is the share of wealth held by households aged 75 and older.

Table 7
Timing of the hump: no social security

ψ	R	K/Y	S_1	S_2	S_3
		(calibrated with $\psi = 0$)			
0	1.0619	3.32	64	1.34	1.42
		(calibrated to S_3)			
0	1.0762	2.93	65	1.41	1.59
1	1.0711	3.06	61	1.43	1.55
2	1.0686	3.12	59	1.42	1.61
3	1.0663	3.19	56	1.41	1.68
4	1.0640	3.25	55	1.41	1.73
5	**1.0619**	**3.32**	**54**	**1.40**	**1.78**
10	1.0525	3.63	53	1.40	1.93
15	1.0451	3.93	53	1.39	2.00

Notes. S_1 is the age at which life cycle consumption attains its maximum. S_2 is the ratio of maximum consumption to consumption at age 25. S_3 is the share of wealth held by households aged 75 and older.

too late in the life cycle relative to empirical findings. This conclusion could, however, be reversed with a stronger bequest motive and a much larger share of wealth being held by households aged 75 or older.[18]

The remaining lines of Table 6 show that, if all other parameters are held constant, increasing ψ increases the capital-output ratio and the fraction of the capital stock held by the elderly, and decreases the interest factor. Increasing ψ also reduces the age at which consumption peaks and slightly decreases the size of the hump (S_2).

7.3. *Results without Social Security*

In the previous subsection we found that adding a bequest motive had relatively little impact on the timing of the hump in consumption over the life cycle. Table 7 reports our findings when we consider an economy without social security. In this case, we set $\beta = 0.9562$ and $\psi = 5$ to hit our calibration targets. Now, a much more dramatic effect is found. In particular, the age at which consumption attains its maximum falls from 64 to 54.

8. Concluding remarks

The empirical life cycle consumption profile in the US has a hump that peaks around age 50. This is typically considered a puzzle since the complete markets life cycle model would produce a consumption profile that is monotonic over the life cycle. In this paper we explore the quantitative implications of uncertainty about the length of life and a lack of annuity markets for life cycle consumption in a calibrated general equilibrium life cycle model where markets are otherwise complete. In addition, since social security may substitute for missing annuity markets, we introduce an unfunded social security system in our model.

If an annuity market (or a partial substitute) is unavailable, then the decline in the survival probabilities over the life cycle as an individual ages leads to a heavier discounting of the future as they grow older. This allows for the possibility, depending on the value of the interest rate, that consumption might increase early in life when survival probabilities are high and the effective rate of discount is low. As survival probabilities fall, the slope of the consumption profile may become negative.

In our calibrated model, if complete annuity markets exist, consumption would increase over the entire life cycle. When the annuity market is shut down, consumption displays a hump shape where consumption peaks well before retirement. Social security, since it substitutes for the missing annuity market, causes consumption to continue increasing until after retirement, although consumption still displays a hump shape. In particular, the consumption profile displayed by our model peaks significantly later than what has been estimated from US consumption data. We

[18] One could argue that an $S_3 = 1.78$ is too low on the grounds that this ratio does not take into account the smaller household size in older households. Using a typical equivalence scale to adjust the family size in households that are 75 and above, S_3 becomes 2.50. Keeping $\alpha = 0.36$ and $\theta = 0.45$, the (new) joint targets of $K/Y = 3.32$ and $S_3 = 2.50$ are obtained with $\beta = 0.9487$ and $\psi = 12$. Now, the hump occurs at age 56 and S_2 is 1.67. So we need a strong preference for bequests and high asset holdings by the elderly to get the hump in the presence of social security.

find that these conclusions are robust to the introduction of a bequest motive calibrated to account for the fraction of wealth held by the elderly, although consumption now peaks at a somewhat lower age.

We find that a lack of annuity markets is quantitatively important for life cycle consumption, but this feature by itself cannot account for the estimated life cycle profiles. Other factors, such as borrowing constraints and income uncertainty, must also be important.

Acknowledgments

We would like to thank Paul Gomme, an Associate Editor of the journal, and an anonymous referee for their comments on a previous version. We also thank Jesus Fernández-Villaverde, Pierre-Olivier Gourinchas, and Jonathan Parker for providing us with life cycle consumption data.

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