Nonseparable Preferences and Optimal Social Security Systems*

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Abstract

In this paper, we consider economies in which agents are privately informed about their skills, which evolve stochastically over time. We require agents' preferences to be weakly separable between the lifetime paths of consumption and labor. However, we allow for intertemporal nonseparabilities in preferences like habit formation. We show that such nonseparabilities imply that optimal asset income taxes are necessarily retrospective in nature. We show that under weak conditions, it is possible to implement a socially optimal allocation using a social security system in which taxes on wealth are linear, and taxes/transfers are history-dependent only at retirement. The average asset income tax in this system is zero.

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1 Introduction

In this paper, we consider a class of economies in which agents are privately informed about their skills and those skills might evolve stochastically over time. As in Golosov, Kocherlakota, and Tsyvinski (GKT) (2003), we impose no restriction on the evolution of skills over time. GKT assume that preferences are additively separable between consumption and labor, and between consumption at different dates. We relax this assumption, and instead require only that preferences over consumption sequences be *weakly* separable (not additively separable) from agents' labor supplies. This assumption means that the marginal rate of substitution between consumption at any two dates is independent of the agent's sequence of labor supplies. However, we allow for intertemporal nonseparabilities: the marginal rate of substitution between consumption at any two dates may depend on other consumptions. We restrict attention to economies in which agents must retire at some date S (but may live thereafter).

Our goal is to study the nature of optimal asset income taxes in this setting with preference nonseparabilities. We first use an illustrative example to show that with intertemporal nonseparabilities an optimal tax that is differentiable with respect to period t asset income must depend on labor income in *future* periods. This result means that an agent must pay his period t asset income taxes at some future date, after the tax authorities learn his labor income at that future date. Hence, optimal asset income taxes are necessarily *retrospective*.

This finding leads us to consider a class of tax systems that we term *social security systems*. Agents pay a linear tax on labor income during their working lives. Then, during retirement, they receive a constant payment that is conditioned on their entire labor income history. As well, at the retirement date, agents pay taxes on their current and past asset income. These taxes are a linear function of past asset incomes; the tax *rates* are a possibly complicated function of the agents' labor income histories.

The social security systems that we study in this model are similar to the actual Social Security system in the United States. In the United States, as in the model, labor income is subject to a linear Social Security payroll tax.[1] In the United States, as in the model, the size of the benefit paid by the Social Security program in retirement is a complicated function of agents' individual labor income histories.[2] There are only two important distinctions between our social security system and the actual Social Security system in the United States. First, in our social security systems, agents are allowed to borrow against their post-retirement transfers. There is no forced-saving element in our tax system. Second, agents must pay asset income taxes in period S.

We assume that optimal incentive-feasible allocations are such that two agents with the same lifetime paths of labor income must have the same

[1] In fact, the Social Security payroll tax is linear on income not exceeding a certain limit known as the Social Security Wage Base. Income above this limit is taxed at the rate of zero. At a cost of additional notation, this feature could be introduced into our model with minor changes to our analysis.

[2] As in our model, the retirement benefit paid by the Social Security program remains constant (in real terms) throughout retirement. The size of this benefit is determined by the rules defined in the Social Security Act (U.S.C. Title 43, chapter 7). Using this source, it is not hard to verify that the size of the Social Security retirement benefit depends in complicated, nonlinear ways on the agent's full history of labor income. (The website of the Social Security Administration, www.socialsecurity.gov, provides a description of how retirement benefits are calculated.)

lifetime paths of consumption. Given an optimal allocation with this property, we can find a social security system that implements that allocation as an equilibrium. The social security system that implements an optimal allocation has the property that the average tax rate on period t asset income is zero. As well, in the optimal system, the aggregate amount of taxes collected on period t asset income is zero.

We view our analysis as making two distinct contributions. First, Golosov, Kocherlakota and Tsyvinski (GKT) (2003) initiated a literature on dynamic optimal taxation from a Mirrleesian approach.[3] However, GKT and the succeeding papers restrict attention to preferences that are additively separable between consumption and labor, and between consumption at different dates.[4] We relax these (severe) restrictions, and show that the resulting optimal tax system is necessarily retrospective in how it treats asset income.

Second, we show that optimal labor income taxes that agents face during their working years can have a simple structure. In our optimal system, agents face a period-by-period labor income tax rate that is independent of their age or their history of labor incomes. After retirement, agents receive transfers that depend in complicated ways on their histories of labor incomes. Thus, in our system, post-retirement transfers, but not pre-retirement taxes, depend on histories of labor incomes. In that, our tax system resembles

[3] See, among others, Albanesi and Sleet (2006), Golosov and Tsyvinski (2006), and Kocherlakota (2005).

[4] Golosov, Tsyvinski and Werning (2006) use a two-period parametrized example to explore numerically the structure of optimal wedges when preferences are nonseparable between consumption and leisure. Farhi and Werning (2008) derive analogs of the reciprocal Euler equation for a class of (time and state nonseparable) recursive preferences that are consistent with balanced growth. They do not discuss implementation and largely restricts attention to i.i.d. skills.

the U.S. Social Security program. Our analysis shows that social security programs can be a powerful tool for implementation of socially optimal outcomes. We believe using social security as a form of implementation may be useful in many contexts.

Our paper is not the first one to point out a role for retrospective taxes on capital income. Grochulski and Piskorski (2006) demonstrate that retrospective taxation of capital income is necessary in a Mirrleesian economy with endogenous skills, in which the technology for skill accumulation requires input of physical resources and agents can privately divert these resources to ordinary consumption. In their model, retrospective taxes on capital income are necessary because the government cannot observe agents' individual consumption, and future observations of realized labor income carry information about past marginal rates of substitution. If individual consumption were observable, retrospective capital income taxes would not be needed in their economy. In our model, we show that when preferences are time nonseparable, an optimal tax system must necessarily be retrospective, even when the government can observe individual consumption. Also, our analysis demonstrates how an optimal retrospective tax system can be implemented with a set of taxes and transfers closely resembling the structure of the U.S. Social Security system.

Huggett and Parra (2006) consider a social security system in the context of a Mirrleesian model. They, however, are interested in a quantitative evaluation of the possible inefficiency in the current U.S. Social Security system, and do not consider the question of implementation. In our paper, in contrast, we demonstrate how a (general) social security system can be used to implement an optimal social insurance scheme in a Mirrleesian economy.

Golosov and Tsyvinski (2006) show how an optimal disability insurance scheme can be implemented with a tax system that is non-differentiable in capital. They consider the case of additively separable preferences, as well as a stochastic structure tailored to the question of optimal disability insurance. In our paper, we treat the case of preferences that are time nonseparable and weakly separable between consumption and leisure. Also, we consider a more general stochastic structure for skill shocks. Our results can be viewed as demonstrating a much broader role for a social security system in the provision of social insurance than just the provision of insurance against disability.

The structure of the paper is as follows. Section 2 lays out the environment we study. Section 3 demonstrates that optimal differentiable capital income taxes must be retrospective in our environment. Section 4 provides an implementation result. Section 5 provides a characterization of an optimal social security system. Section 6 concludes.

2 Setup

In this section, we describe our basic model. The model is essentially a one-good version of GKT (2003), except that we generalize the class of preferences used by them.

The economy lasts for T periods, and there is a unit measure of agents. There is a single consumption good at each date that agents produce by expending labor. Denote period t consumption by c_t and period t labor by

l_t. All agents have a von-Neumann-Morgenstern utility function given by:

$$V(U(c_1, c_2, ..., c_T), l_1, l_2, ..., l_S),$$

where $S \leq T$, and U maps into the real line. Agents' preferences are weakly separable between consumption goods and labor. We assume that U is strictly increasing, strictly concave, and continuously differentiable in all its components. We assume that V is differentiable, increasing and concave in its first argument U, and decreasing in l_t for $t = 1, ..., S$. Note that agents can only work in periods 1 through S.

Let Θ be a finite subset of the positive real line. At time 0, Nature draws a vector θ^S from the set Θ^S for each agent. The draws are independently and identically distributed across agents, with density function π. At each date $t \leq S$, each agent privately learns his θ_t; hence, a given agent's information at time t consists of the history $\theta^t = (\theta_1, ..., \theta_t)$. An agent in period t with draw realization θ_t who works l_t units of labor can produce $\theta_t l_t$ units of consumption. We assume that both θ_t and l_t are privately known to the agent. However, the product $y_t = \theta_t l_t$ is publicly observable.

An allocation in this setting is a specification of $(c, y) = ((c_t)_{t=1}^T, (y_t)_{t=1}^S)$, where $c_t : \Theta^S \to \mathbb{R}_+$, $y_t : \Theta^S \to \mathbb{R}_+$, and

$$(c_t, y_t) \text{ is } \theta^t\text{-measurable; } c_t \text{ is } \theta^S\text{-measurable if } t > S.$$

Society can borrow and lend at a fixed gross interest rate $R \geq 1$. (We can endogenize R, but it merely serves to complicate the analysis without adding insight.) An allocation is *feasible* given that society has initial wealth W if:

$$\sum_{\theta^S} \pi(\theta^S) \sum_{t=1}^{T} c_t(\theta^S) R^{-t} \leq \sum_{\theta^S} \pi(\theta^S) \sum_{t=1}^{S} y_t(\theta^S) R^{-t} + W.$$

Because at least some information is private, only *incentive-compatible* allocations are achievable. By the Revelation Principle, we can characterize the set of incentive-compatible allocations as follows. A reporting strategy σ is a mapping from Θ^S into Θ^S such that σ_t is θ^t-measurable; let Σ be the set of reporting strategies. An allocation (c, y) is incentive-compatible if:

$$\begin{aligned} & \sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\theta^S)), (y_t(\theta^S)/\theta_t)_{t=1}^S) \\ \geq \; & \max_{\sigma \in \Sigma} \sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\sigma(\theta^S))), (y_t(\sigma(\theta^S))/\theta_t)_{t=1}^S). \end{aligned}$$

We are interested in the set of *incentive-feasible* allocations (the ones that are simultaneously incentive-compatible and feasible). The social planner's problem is to choose (c, y) so as to maximize:

$$\sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\theta^S)), (y_t(\theta^S)/\theta_t)_{t=1}^S)$$

subject to (c, y) being incentive-feasible. Let $V_{SP}(W)$ be the value of the social planner's maximized objective, given initial wealth W.

The specification of preferences in this setting is more general than in GKT (2003). In GKT, both V and U are restricted to be additively separable. In our paper, we allow U and V to be nonseparable. Our key restriction is that preferences are weakly separable between consumption and labor. Note that if U takes the form:

$$U(c_1, ..., c_T) = c_1^{1/2} + \sum_{t=2}^{T} \beta^{t-1} (c_t - 0.9c_{t-1})^{1/2},$$

then preferences exhibit habit formation with respect to consumption.

3 The Necessity of Retrospective Asset Taxation

Albanesi and Sleet (2006) and Kocherlakota (2005) consider a version of this model in which the aggregator V and sub-utility function U are both additively separable. They suppose agents can borrow and lend subject to differentiable wealth taxes. They show that, if the resulting equilibrium allocation is socially optimal, then the tax on wealth accumulated through period t must depend on individual labor income in period t. Their analysis demonstrates, however, that an optimal tax on wealth accumulated through period t can be independent of individual labor income in periods subsequent to t.

In this section, we re-examine their results while allowing for time non-separabilities. Using an example, we show that when U is not time separable, an optimal differentiable tax on period t wealth necessarily needs to depend on labor income in some of the *future* periods $t + s$, $s > 0$. We argue that this dependence implies the need for *retrospective* taxation, in which taxes on a period t activity are levied in a future period t'.

3.1 A three-period example

Let $T = S = 3$, $\Theta = \{\theta_L, \theta_H\}$, with $\theta_L < \theta_H = 1$, $R = 1$, and $\pi(1, 1, \theta_H) = \pi(1, 1, \theta_L) = 1/2$. Suppose also that preferences are:

$$\begin{aligned} V(U, l_1, l_2, l_3) &= U - v(l_1) - v(l_2) - v(l_3), \\ U(c_1, c_2, c_3) &= u(c_1) + u(c_2) + u(c_3 - \lambda c_2), \end{aligned} \tag{1}$$

where $u', -u'' > 0$, $0 \leq \lambda < 1$, and $v(0) = 0$. In this setting, let (c^*, y^*) be a socially optimal allocation in which $c^*_{3H} > c^*_{3L}$ and $y^*_{3H} > y^*_{3L}$. (In this section, we use the notation c_{3i} and y_{3i} to represent consumption and output in period 3 when $\theta = \theta_i$ for $i = H, L$.) It is straightforward to show that the solution (c^*, y^*) must satisfy the incentive constraint:

$$u(c^*_{3H} - \lambda c^*_2) - v(y^*_{3H}) = u(c^*_{3L} - \lambda c^*_2) - v(y^*_{3L}), \tag{2}$$

with equality.

Now suppose agents can trade bonds with gross interest rate $R = 1$ and are subject to labor income and wealth taxes of the form used in Albanesi and Sleet (2006) and Kocherlakota (2005). More specifically, in period 1, agents pay taxes $\mathcal{T}_1$ on labor income y_1. In period 2, they pay taxes $\mathcal{T}_2(b_2, y^2)$, if they bring bonds b_2 into period 2. The tax in period 3 is $\mathcal{T}_3(b_3, y^3)$, where b_3 represents the agent's bond-holdings at the beginning of period 3. We restrict $(\mathcal{T}_2, \mathcal{T}_3)$ to be differentiable in bond-holdings b.

Taking the gross interest rate R and taxes $\{\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3\}$ as given, the typical agent seeks to maximize his expected utility

$$u(c_1) + u(c_2) + u(c_{3H} - \lambda c_2)/2 + u(c_{3L} - \lambda c_2)/2$$
$$-v(y_1) - v(y_2) - v(y_{3H})/2 - v(y_{3L}/\theta_L)/2$$

subject to the following budget constraints

$$\begin{aligned} c_1 + b_2 &= y_1 - \mathcal{T}_1(y_1), \\ c_2 + b_3 &= y_2 + b_2 - \mathcal{T}_2(b_2, y_1, y_2), \\ c_{3H} &= y_{3H} + b_3 - \mathcal{T}_3(b_3, y_1, y_2, y_{3H}) \\ c_{3L} &= y_{3L} + b_3 - \mathcal{T}_3(b_3, y_1, y_2, y_{3L}). \end{aligned}$$

We say that the tax system $\{\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3\}$ implements (c^*, y^*) if (c^*, y^*), combined with some b_2^* and b_3^*, solves the agent's problem.

3.2 The non-implementation problem

We know from the work of Albanesi and Sleet (2006) and Kocherlakota (2005) that if $\lambda = 0$, and given a social optimum (c^*, y^*), there exists a tax system $(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ that implements that optimum. In this sub-section, we show that there is *no* tax system of the form $\{\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3\}$ that can implement a social optimum (c^*, y^*) when $\lambda > 0$.

Suppose, to the contrary, that the starred allocation (c^*, y^*, b_2^*, b_3^*) is a solution to the agents' problem under some taxes of the form $\{\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3\}$. The agent's first order condition with respect to b_2 implies that the marginal tax rate $\mathcal{T}_2$, denoted by $\mathcal{T}_{2b}$, must satisfy

$$u'(c_1^*) = (1-\mathcal{T}_{2b}(b_2^*, y_1^*, y_2^*))[u'(c_2^*) - \lambda u'(c_{3H}^* - \lambda c_2^*)/2 - \lambda u'(c_{3L}^* - \lambda c_2^*)/2], \quad (3)$$

for, otherwise, the agent could do better simply by adjusting c_1, b_2, and c_2.

Now consider an allocation $(c_1^* - \varepsilon, c_2'(\varepsilon), c_{3H}', c_{3L}^*, y_1^*, y_2^*, y_{3H}', y_{3L}^*, b_2^* + \varepsilon, b_3^*)$, where

$$\begin{aligned} c_2'(\varepsilon) &= c_2^* + \varepsilon - \mathcal{T}_2(b_2^* + \varepsilon, y_1^*, y_2^*) + \mathcal{T}_2(b_2^*, y_1^*, y_2^*), \\ c_{3H}' &= c_{3L}^*, \\ y_{3H}' &= y_{3L}^*. \end{aligned}$$

The agent's welfare from this primed allocation is given by:

$$\mathcal{W}(\varepsilon) = u(c_1^* - \varepsilon) + u(c_2'(\varepsilon)) + u(c_{3L}^* - \lambda c_2'(\varepsilon)) - v(y_1^*) - v(y_2^*) - v(y_{3L}^*)/2 - v(y_{3L}^*/\theta_L)/2.$$

Note that because of (2), this welfare, when evaluated at $\varepsilon = 0$, is the same as the agent's welfare from the starred allocation. The derivative of $\mathcal{W}$, evaluated at $\varepsilon = 0$, is:

$$\begin{aligned} \mathcal{W}'(0) &= -u'(c_1^*) + (1 - \mathcal{T}_{2b}(b_2^*, y_1^*, y_2^*))[u'(c_2^*) - \lambda u'(c_{3L}^* - \lambda c_2^*)] \\ &= -u'(c_1^*) + u'(c_1^*) \frac{u'(c_2^*) - \lambda u'(c_{3L}^* - \lambda c_2^*)}{u'(c_2^*) - \lambda u'(c_{3H}^* - \lambda c_2^*)/2 - \lambda u'(c_{3L}^* - \lambda c_2^*)/2} \\ &= u'(c_1^*) \left(-1 + \frac{u'(c_2^*) - \lambda u'(c_{3L}^* - \lambda c_2^*)/2 - \lambda u'(c_{3L}^* - \lambda c_2^*)/2}{u'(c_2^*) - \lambda u'(c_{3H}^* - \lambda c_2^*)/2 - \lambda u'(c_{3L}^* - \lambda c_2^*)/2} \right) \\ &< 0, \end{aligned}$$

where the second line follows from (3). The strict inequality is a consequence of $u'' < 0$, $c^*_{3H} > c^*_{3L}$, and $\lambda > 0$. We conclude that, by choosing the primed allocation with ε small in absolute value and less than zero, the agent can obtain higher expected utility than the welfare provided by the social optimum. It follows that no (differentiable) tax system of the kind proposed by Albanesi and Sleet (2006) and Kocherlakota (2005) can implement the social optimum when preferences are not time separable.[5]

What is happening here? In period 1, agents are supposed to hold bonds b_2, and they are supposed to work y^*_{3H} in period 3 if they are highly skilled. The tax system is designed to deter agents from holding bonds other than b_2, given that they do work y^*_{3i} when they have skills θ_i in period 3. It also deters agents from shirking when skilled in period 3, given that they hold bonds b_2. However, the tax system fails to deter *joint deviations*, in which agents simultaneously save *less* in period 1 and work less in period 3. More specifically, consider two other trading strategies besides the socially optimal allocation. Under the first alternative strategy, the agent does not alter b_2, but sets $y_{3H} = y^*_{3L}$. The social optimality condition (2) implies that the agent is indifferent between this strategy and the socially optimal one. Under the second alternative strategy, the agent chooses $y_{3H} = y^*_{3L}$ but lowers b_2. The agent's marginal utility of period 2 consumption is lower when the agent sets $y_{3H} = y^*_{3L}$. Hence, the agent likes this second strategy

[5] Golosov and Tsyvinski (2006) consider the problem of designing optimal disability insurance when disability is private information. They emphasize the role of asset tests in the optimal tax system. In their model, preferences are time separable and the optimal asset tests are non-retrospective. It is simple to use the analysis in this section to show that once preferences are not time separable, the optimal asset tests are necessarily retrospective.

better than the first. The agent is made better off by a joint deviation of saving less in period 1 and shirking in period 3.

3.3 Using retrospective taxation

In this subsection, we show how to design a differentiable tax system that deters the above joint deviation. We allow the tax on bonds b_2 to be postponed to period 3. We denote this tax by $\mathcal{T}_2^{ret}(b_2, y^3)$ (where ret stands for retrospective). Note that now the tax on bonds brought into period 2 can be conditioned on period 3 income. We show how this additional information can be used to deter the joint deviation of borrowing in period 1 and shirking in period 3 without distorting the savings decision of an agent who chooses to not shirk in period 3.

Under the modified tax system $\{\mathcal{T}_1, \mathcal{T}_2^{ret}, \mathcal{T}_3\}$, agents face the following budget constraints:

$$
\begin{aligned}
c_1 + b_2 &= y_1 - \mathcal{T}_1(y_1), \\
c_2 + b_3 &= y_2 + b_2, \\
c_{3H} &= y_{3H} + b_3 - \mathcal{T}_2^{ret}(b_2, y_1, y_2, y_{3H}) - \mathcal{T}_3(b_3, y_1, y_2, y_{3H}), \\
c_{3L} &= y_{3L} + b_3 - \mathcal{T}_2^{ret}(b_2, y_1, y_2, y_{3L}) - \mathcal{T}_3(b_3, y_1, y_2, y_{3L}).
\end{aligned}
$$

For the optimal allocation (c^*, y^*) (together with some b_2^*, b_3^*) to be a solution to the agents' utility maximization problem, it is necessary that an analog of condition (3) be satisfied. Under the modified tax system, this condition (the Euler equation with respect to b_2) takes the form of

$$\begin{aligned} u'(c_1^*) &= u'(c_2^*) - (\lambda + \mathcal{T}_{2b}^{ret}(b_2^*, y_1^*, y_2^*, y_{3H}^*))u'(c_{3H}^* - \lambda c_2^*)/2 \\ &\quad -(\lambda + \mathcal{T}_{2b}^{ret}(b_2^*, y_1^*, y_2^*, y_{3L}^*))u'(c_{3L}^* - \lambda c_2^*)/2. \qquad (4) \end{aligned}$$

Consider now the following allocation (which agents can obtain by adjusting b_2 and shirking in period 3): $(c_1^* - \varepsilon, c_2^* + \varepsilon, c'_{3H}(\varepsilon), c'_{3L}(\varepsilon), y_1^*, y_2^*, y'_{3H}, y_{3L}^*, b_2^* + \varepsilon, b_3^*)$, with

$$\begin{aligned} c'_{3H}(\varepsilon) &= c'_{3L}(\varepsilon) = c_{3L}^* - \mathcal{T}_2^{ret}(b_2^* + \varepsilon, y_1^*, y_2^*, y_{3L}^*) + \mathcal{T}_2^{ret}(b_2^*, y_1^*, y_2^*, y_{3L}^*), \\ y'_{3H} &= y_{3L}^*. \end{aligned}$$

The agent's welfare from this allocation is:

$$\begin{aligned} \mathcal{W}(\varepsilon) &= u(c_1^* - \varepsilon) + u(c_2^* + \varepsilon) + u(c'_{3L}(\varepsilon) - \lambda(c_2^* + \varepsilon)) \\ &\quad -v(y_1^*) - v(y_2^*) - v(y_{3L}^*)/2 - v(y_{3L}^*/\theta_L)/2. \end{aligned}$$

The derivative of $\mathcal{W}$, evaluated at $\varepsilon = 0$, is given by

$$\mathcal{W}'(0) = -u'(c_1^*) + u'(c_2^*) - (\lambda + \mathcal{T}_{2b}^{ret}(b_2^*, y_1^*, y_2^*, y_{3L}^*))u'(c_{3L}^* - \lambda c_2^*).$$

Consider now the Euler equations (4) and $\mathcal{W}'(0) = 0$. Straightforward algebra shows that if the tax function $\mathcal{T}_2^{ret}(b_2, y^3)$ satisfies the following marginal conditions:

$$\mathcal{T}_{2b}^{ret}(b_2^*, y_1^*, y_2^*, y_{3i}^*) = \frac{-u'(c_1^*) + u'(c_2^*)}{u'(c_{3i}^* - \lambda c_2^*)} - \lambda$$

for $i = H, L$, then (4) and $\mathcal{W}'(0) = 0$ are simultaneously satisfied. Thus, a tax system $\{\mathcal{T}_1, \mathcal{T}_2^{ret}, \mathcal{T}_3\}$, in which $\mathcal{T}_2^{ret}(b_2, y^3)$ nontrivially depends on y_3, is capable of simultaneously deterring the simple deviation in savings b_2, as well as the joint deviation of adjusting savings b_2 and shirking in period 3.

3.4 Retrospective asset income taxation in general

The lesson of the above example readily generalizes. With time separable preferences, the agent's desire to save/borrow in period $(t-1)$ is affected by whether he plans to shirk in period t. This connection implies that taxes on asset income in period t must depend on labor income in period t, even though the assets were chosen in period $(t-1)$. With time nonseparable preferences, the agent's desire to save/borrow in period $(t-1)$ generally will be affected by whether he plans to shirk in period $(t+s)$, with $s > 1$. Hence, taxes on asset income in period t must depend on labor income in period $(t+s)$, even though the assets were chosen in period $(t-1)$.

4 An Optimal Social Security System

In this section, we return to the general model and consider a socially optimal allocation (c^*, y^*). We suppose that agents trade bonds and work to produce output, subject to taxes. Our goal is to design a tax system that implements the given allocation; we refer to this tax system as a social security system because its retrospective nature means that it closely resembles the current Social Security system in the United States.

We make the following assumption about (c^*, y^*).

Condition 1 *Let* $DOM = \{y^S \in \mathbb{R}_+^S : y^S = y^*(\theta^S)$ *for some* θ^S *such that* $\pi(\theta^S) > 0\}$. *Then, there exists* $\widehat{c} : DOM \rightarrow \mathbb{R}_+^T$ *such that* $\widehat{c}((y_t^*(\theta^S))_{t=1}^S) = c^*(\theta^S)$.

This condition says that two agents with the same optimal sequence of output y^*, through the retirement period S, have the same optimal consumption sequences throughout their lifetimes. It is trivially satisfied by any incentive-compatible allocation if θ_t is i.i.d. over time. We can also prove that satisfied by an *optimal* allocation if $\pi(\theta_1, ..., \theta_S) = \sum_{\{\theta^S | \theta_1^S = \theta_1\}} \pi(\theta^S)$, so that agents know their entire lifetime sequences of skill shocks in period 1 itself. In an appendix, we provide an explicit example of an environment in which the optimal allocation (c^*, y^*) does not satisfy Condition 1.[6]

In each period, agents are able to choose output levels and are able to trade bonds. In doing so, they must pay taxes that depend on their choices. We consider a tax system with three components. The first component is a constant tax rate α on output in periods 1 through S. The second component is a function:

$$\Psi : \mathbb{R}^S \rightarrow \mathbb{R}_+$$

that maps agents' output histories (from periods 1 through S) into a constant lump-sum transfer in periods $t > S$. Finally, the third component is a function $\tau : \mathbb{R}^S \rightarrow \mathbb{R}^{T-1}$ that maps agents' output histories (from periods 1 through S) into a tax *rate* on asset income in periods 2 through period

[6] Condition 1 looks similar to Assumption 1 in Kocherlakota (2005). However, Condition 1 is weaker than that assumption; in particular, the counterexample to Assumption 1 in Appendix B of Kocherlakota (2005) is *not* a counterexample to Condition 1. Unlike Assumption 1 of Kocherlakota (2005), Condition 1 does *not* require that consumption in period t depends only on the history of outputs through period t. We gain this additional flexibility because we are going to use retrospective taxes.

T. The tax on asset income in periods 2 through S is paid in period S; the asset income taxes in period $t > S$ are paid in period t.[7]

Mathematically, given a tax system (α, Ψ, τ), agents have the following choice problem

$$\max_{(c,y,b)} \sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\theta^S)), (y_t(\theta^S)/\theta_t)_{t=1}^S)$$

subject to

$$c_t(\theta^S) + b_{t+1}(\theta^S)/R \leq (1-\alpha) y_t(\theta^S) + b_t(\theta^S)$$

for all $t < S$, all $\theta^S \in \Theta^S$;

$$\begin{aligned} & c_S(\theta^S) + b_{S+1}(\theta^S)/R + \sum_{t=2}^{S} b_t(\theta^S)(1-1/R)\tau_t(y(\theta^S))R^{S-t} \\ \leq \quad & y_S(\theta^S)(1-\alpha) + b_S(\theta^S) \end{aligned}$$

for all $\theta^S \in \Theta^S$;

$$c_t(\theta^S) + b_{t+1}(\theta^S)/R \leq b_t(\theta^S)[1 - (1-1/R)\tau_t(y(\theta^S))] + \Psi(y(\theta^S))$$

for all $t > S$, all $\theta^S \in \Theta^S$; $c_t(\theta^S), y_t(\theta^S), b_{T+1}(\theta^S) \geq 0$ for all t, all $\theta^S \in \Theta^S$; c_t, y_t, b_{t+1} θ^t-measurable if $t < S$; and $b_1 = 0$.

We refer to a tax system (α, Ψ, τ) as a *social security system*. We say that it *implements* an allocation (c, y) if there exists a bond process b such that (c, y, b) solves the agent's problem given (α, Ψ, τ).

[7]With taxes on asset income, instead on assets directly, we assume that $R > 1$. Also, since transfers Ψ start in period $S+1$, we assume that $S < T$. All our results go through, with minor changes to our analysis, if $R = 1$ or $S = T$.

Our notion of a social security system has several features in common with the current social security system in the United States. At every date before retirement, agents pay a flat tax α on their labor income y. In every period after retirement, agents receive a constant transfer payment that is conditioned on their history of labor incomes. However, there are two major differences between our social security systems and the current social security system. First, in our system, agents can credibly commit to repay debts using their future social security transfers. Second, in our system, at the time of retirement, agents pay asset income taxes that are conditioned on their full history of labor incomes. Note that, from the example in the previous section, we know that optimal asset taxes typically need this kind of dependence.

We now construct a social security system that implements the given optimal allocation (c^*, y^*). Let U_{c_t} represent the partial derivative of U with respect to c_t, and V_U represent the partial derivative of V with respect to U. Pick $\alpha^* > 0$ so that for y^S in DOM:

$$(1-\alpha^*)\sum_{t=1}^{S} U_{c_t}(\widehat{c}(y^S))y_t \leq \sum_{t=1}^{T} U_{c_t}(\widehat{c}(y^S))\widehat{c}_t(y^S).$$

(It is obvious that such an α^* exists, because we can always set α^* equal to one.) Define Ψ^* so that:

$$\Psi^*(y^S) = \left(\sum_{t=S+1}^{T} U_{c_t}(\widehat{c}(y^S))\right)^{-1}\left(\sum_{t=1}^{T} U_{c_t}(\widehat{c}(y^S))\widehat{c}_t(y^S) - (1-\alpha^*)\sum_{t=1}^{S} U_{c_t}(\widehat{c}(y^S))y_t\right),$$

if $y^S \in DOM$, and

$$\Psi^*(y^S) = -2 \sum_{t=1}^{S} R^{S+1-t} y_t,$$

if y^S is not in DOM. Here, the role of the upper bound on $(1-\alpha^*)$ is to ensure that Ψ^* is non-negative, so that the social security system delivers transfers, not taxes, after retirement.

Finally, define τ^* so that for $T > t \geq 1$:

$$\begin{aligned}
\tau^*_{t+1}(y^S) &= \frac{-U_{c_t}(\widehat{c}(y^S))/R + U_{c_{t+1}}(\widehat{c}(y^S))}{(1-1/R)U_{c_S}(\widehat{c}(y^S))R^{S-t-1}} \text{ if } t < S,\, y^S \in DOM, \qquad (5)\\
&= \frac{-U_{c_t}(\widehat{c}(y^S))/R + U_{c_{t+1}}(\widehat{c}(y^S))}{(1-1/R)U_{c_{t+1}}(\widehat{c}(y^S))} \text{ if } t \geq S,\, y^S \in DOM,\\
&= 0 \text{ if } y^S \text{ is not in } DOM,
\end{aligned}$$

for all t, y^S in DOM. These formulae guarantee that the agent's intertemporal Euler equation is satisfied, even if the agent knows the entire sequence y^S. (The marginal utilities in the denominators capture the timing of when the asset taxes are actually paid.) The example in the previous section shows that we need Euler equations to be satisfied ex-post, and not just ex-ante, because agents have the ability to *choose* their future y^S sequence.

The first theorem establishes the optimality of the social security system $(\alpha^*, \Psi^*, \tau^*)$. We use the notation $\theta^S \geq \overline{\theta}^t$ to refer to histories θ^S such that the first t components equal $\overline{\theta}^t$.

Theorem 1 *The social security system* $(\alpha^*, \Psi^*, \tau^*)$ *implements* (c^*, y^*).

Proof. The agent's choice problem can be written:

$$\max_{(c,y,b)} \sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\theta^S)), (y_t(\theta^S)/\theta_t)_{t=1}^S)$$

$$s.t.\ c_t(\theta^S) + b_{t+1}(\theta^S)/R \leq (1-\alpha^*) y_t(\theta^S) + b_t(\theta^S) \text{ for all } t < S, \text{ all } \theta^S,$$

$$c_S(\theta^S) + b_{S+1}(\theta^S)/R + \sum_{t=2}^{S} b_t(\theta^S)(1-1/R)\tau_t^*(y(\theta^S))R^{S-t}$$

$$\leq\ y_S(\theta^S)(1-\alpha^*) + b_S(\theta^S) \text{ for all } \theta^S \in \Theta^S,$$

$$c_t(\theta^S) + b_{t+1}(\theta^S)/R + (1-1/R) b_t(\theta^S)\tau_t^*(y(\theta^S))$$

$$\leq\ b_t(\theta^S) + \Psi^*(y(\theta^S)) \text{ for all } t > S, \text{ all } \theta^S \in \Theta^S,$$

$$c_t(\theta^S), y_t(\theta^S), b_{T+1}(\theta^S) \geq 0 \text{ for all } t, \theta^S,$$

$$c_t, y_t, b_{t+1}\ \theta^t\text{-measurable if } t < S.$$

Suppose that $y^S(\theta^S)$ is not in DOM for some θ^S. Then, for that sample path, the tax due at $S+1$ equals twice the accumulated value of lifetime income. Along such sample paths, consumption must be negative, which violates the non-negativity constraint. Hence, $y^S(\theta^S)$ must be in DOM for all θ^S.

Now, suppose an agent chooses an output strategy $y' : \Theta^S \to DOM$. Given this choice, our claim is that the agent's optimal consumption strategy is $\widehat{c}(y'(\theta^S))$. If this claim is true, the agent's overall choice among (c, y), given $y \in DOM$, is equivalent to choosing among reporting strategies. Since truth-telling is optimal given (c^*, y^*), it is optimal for the agent to choose $y' = y^*$, and $c' = c^*$.

So, fix an output strategy y'. The agent's consumption-bond strategy then must solve the problem:

$$\max_{(c,b)} \sum_{\theta^S \in \Theta^S} \pi(\theta^S) V(U(c(\theta^S)), (y'_t(\theta^S)/\theta_t)_{t=1}^S)$$

$$s.t.\ c_t(\theta^S) + b_{t+1}(\theta^S)/R \leq (1-\alpha^*) y'_t(\theta^S) + b_t(\theta^S) \text{ for all } t < S,\ \text{all } \theta^S,$$

$$c_S(\theta^S) + b_{S+1}(\theta^S)/R + \sum_{t=2}^{S} b_t(\theta^S)(1-1/R)\tau_t^*(y'(\theta^S))R^{S-t}$$

$$\leq\ y'_S(\theta^S)(1-\alpha^*) + b_S(\theta^S) \text{ for all } \theta^S \in \Theta^S,$$

$$c_t(\theta^S) + b_{t+1}(\theta^S)/R + (1-1/R) b_t(\theta^S)\tau_t^*(y'(\theta^S))$$

$$\leq\ b_t(\theta^S) + \Psi^*(y'(\theta^S)) \text{ for all } t > S,\ \text{all } \theta^S \in \Theta^S,$$

$$c_t(\theta^S), b_{T+1}(\theta^S) \geq 0 \text{ for all } t, \theta^S,$$

$$c_t, y_t, b_{t+1}\ \theta^t\text{-measurable if } t < S.$$

This problem has a strictly concave objective (in c) and a linear constraint set. Hence, it has a unique optimum characterized by the first-order conditions with respect to (c_t, b_{t+1}):

$$\sum_{\theta^S \geq \theta^t} \pi(\theta^S) V_U(U(c(\theta^S)), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_t}(c(\theta^S)) = \sum_{\theta^S \geq \theta^t} \nu_t(\theta^S),\ \text{if } t < S,$$

$$V_U(U(c(\theta^S)), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_t}(c(\theta^S)) = \nu_t(\theta^S),\ \text{if } t \geq S,$$

$$\sum_{\theta^S \geq \theta^t} \nu_t(\theta^S)/R = \sum_{\theta^S \geq \theta^t} \nu_{t+1}(\theta^S) - \sum_{\theta^S \geq \theta^t} \nu_S(\theta^S)(1-1/R)\tau_{t+1}^*(y'(\theta^S))R^{S-t-1}, t < S,$$

$$\nu_t(\theta^S)/R = \nu_{t+1}(\theta^S) - \nu_{t+1}(\theta^S)(1-1/R)\tau_{t+1}^*(y'(\theta^S)), t \geq S,$$

where ν_t represents the multiplier on the agent's flow constraint. We claim that it is optimal for the agent to choose the strategy $(c^{**}, b^{**}) : \Theta^S \to \mathbb{R}_+^T$

such that:

$$c^{**}(\theta^S) = \widehat{c}(y'(\theta^S))$$

and b^{**} satisfies the agent's flow constraints. To validate this claim, we need to check the agent's first order conditions and to check that $b^{**}_{T+1}(\theta^S)$ is non-negative for all θ^S. For $t < S$, the first order conditions take the form:

$$\begin{aligned}
& \sum_{\theta^S \geq \theta^t} \pi(\theta^S) V_U(U(\widehat{c}(y'(\theta^S))), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_t}(\widehat{c}(y'(\theta^S)))/R \\
= & \sum_{\theta^S \geq \theta^t} \pi(\theta^S) V_U(U(\widehat{c}(y'(\theta^S))), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_{t+1}}(\widehat{c}(y'(\theta^S))) \\
& - \sum_{\theta^S \geq \theta^t} \pi(\theta^S) V_U(U(\widehat{c}(y'(\theta^S))), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_S}(\widehat{c}(y'(\theta^S)))(1 - 1/R)\tau^*_{t+1}(y'(\theta^S)) R^{S-t-1}.
\end{aligned}$$

The definition of $\tau^*_t(y'(\theta^S))$ ensures that this equality holds for each $y'(\theta^S)$. Hence, it must hold when summed across θ^S as well. Similarly, the first order condition for $t \geq S$ is:

$$\begin{aligned}
& V_U(U(\widehat{c}(y'(\theta^S))), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_t}(\widehat{c}(y'(\theta^S))/R \\
= & V_U(U(\widehat{c}(y'(\theta^S))), (y'_t(\theta^S)/\theta_t)_{t=1}^S) U_{c_{t+1}}(\widehat{c}(y'(\theta^S)) \\
& -(1 - 1/R) V_U(U(\widehat{c}(y'(\theta^S)), (y'_t(\theta^S)/\theta_t)_{t=1}^S)) U_{c_{t+1}}(\widehat{c}(y'(\theta^S)) \tau^*_{t+1}(y'(\theta^S)).
\end{aligned}$$

Again, the definition of τ^* ensures that this first order condition is satisfied for each $y'(\theta^S)$.

Finally, we need to verify that $b^{**}_{T+1}(\theta^S)$ is zero. Multiply the period t, history θ^S flow constraint by

$$U_{c_t}(\theta^S) := U_{c_t}(\widehat{c}(y'(\theta^S)),$$

and then add the flow constraints over t, pointwise (θ^S by θ^S). Recall from (5) that:

$$\begin{aligned}
\tau^*_{t+1}(y^S) &= \frac{-U_{c_t}(\widehat{c}(y^S))/R + U_{c_{t+1}}(\widehat{c}(y^S))}{(1-1/R)U_{c_S}(\widehat{c}(y^S))R^{S-t-1}} \text{ if } t < S,\ y^S \in DOM, \\
&= \frac{-U_{c_t}(\widehat{c}(y^S))/R + U_{c_{t+1}}(\widehat{c}(y^S))}{(1-1/R)U_{c_{t+1}}(\widehat{c}(y^S))} \text{ if } t \geq S,\ y^S \in DOM, \\
&= 0 \text{ if } y^S \text{ is not in } DOM.
\end{aligned}$$

Hence, for all θ^S, if $t < S$:

$$\begin{aligned}
b_{t+1}(\theta^S)U_{c_t}(\theta^S)/R &= b_{t+1}(\theta^S)U_{c_{t+1}}(\theta^S) \\
&\quad -(1-1/R)b_{t+1}(\theta^S)U_{c_S}(\theta^S)\tau^*_{t+1}(y'(\theta^S))R^{S-t-1},
\end{aligned}$$

and if $T > t \geq S$:

$$\begin{aligned}
b_{t+1}(\theta^S)U_{c_t}(\theta^S)/R &= b_{t+1}(\theta^S)U_{c_{t+1}}(\theta^S) \\
&\quad -(1-1/R)b_{t+1}(\theta^S)U_{c_{t+1}}(\theta^S)\tau^*_{t+1}(y'(\theta^S)).
\end{aligned}$$

As well, from the definition of Ψ^*:

$$\begin{aligned}
\sum_{t=1}^{T} U_{c_t}(\theta^S)c_t(\theta^S) &= (1-\alpha^*)\sum_{t=1}^{S} U_{c_t}(\theta^S)y'_t(\theta^S) \\
&\quad + \sum_{t=S+1}^{T} U_{c_t}(\widehat{c}(y^S))\Psi^*(y'(\theta^S)).
\end{aligned}$$

As a consequence, much cancels in the pointwise sum. In particular, we are left with:

$$U_{c_T}(\widehat{c}(y'(\theta^S))b^{**}_{T+1}(\theta^S)/R = 0$$

for all θ^S.

It follows that $b^{**}_{T+1}(\theta^S) = 0$. We conclude that (c^{**}, b^{**}) solves the agent's consumption-bond problem, given the choice y'. As argued above, this finding implies that the agent's overall problem of choosing (c, b, y), given $y \in DOM$, is equivalent to the original reporting problem. Hence, (c^*, y^*) must be optimal. ■

Thus, given a socially optimal allocation that satisfies Condition 1, there is a social security system that implements it.

In the above system, taxes on period $(t+1)$ asset income, $t+1 \leq S$, are collected in period S. Suppose instead that we use a tax system in which taxes on period $(t+1)$ asset income are collected in period $(t+k)$, with $(S-t) > k \geq 1$, instead of period S. Then, the optimal tax on period $(t+1)$ asset income is defined so that:

$$\tau_{t+1}(y^S) = \frac{-U_{c_t}(\widehat{c}(y^S))/R + U_{c_{t+1}}(\widehat{c}(y^S))}{(1-1/R)U_{c_{t+k}}(\widehat{c}(y^S))R^{k-1}}. \tag{6}$$

If this tax is to be collected in period $(t+k)$, it must be true that this tax is y^{t+k}-measurable. The numerator in (6) is y^{t+k}-measurable if the nonseparability in preferences does not last too long - that is, if $U_{c_{t+1}c_{t+k}} = 0$.

However, the denominator in (6) is generally not y^{t+k}-measurable. In particular, suppose that preferences exhibit a one-period consumption habit, which implies that

$$U_{c_{t+k}c_{t+k+1}} \neq 0. \tag{7}$$

Also, assume that optimal consumption $\widehat{c}_{t+k+1}$ is not known in period $(t+k)$, so that

$$Var(\widehat{c}_{t+k+1}|y^{t+k}) > 0. \tag{8}$$

This condition requires that there is some incentive problem between periods $(t+k)$ and $(t+k+1)$ which results in optimal consumption $\widehat{c}_{t+k+1}$ not being y^{t+k}-measurable. Under conditions (7) and (8), marginal utility $U_{c_{t+k}}(\widehat{c})$ depends on information revealed in period $(t+k+1)$, and the tax rate defined in (6) is not y^{t+k}-measurable. The same argument shows that a tax collected in period $(t+k+1) < S$ would not be y^{t+k+1}-measurable.

Thus, even with limited amounts of nonseparability (one-period habit formation), asset income taxes generally depend on information through the retirement date S. What makes period S special? It is common knowledge that no further information about skills is revealed after that period. More generally, asset income taxes can be collected in any history with the property that no further information about skills will be revealed to the agent.

5 Characterizing Optimal Asset Income Taxes

In this section, we first derive a partial intertemporal characterization of solutions to the social planner's problem. We then use that characterization to prove that the average asset income tax rate is zero in the optimal social security system. We also demonstrate that, in some circumstances, optimal asset income taxes may provide an extra incentive to save by introducing a positive covariance between marginal utility of consumption and the after-tax rate of return on savings.

5.1 Zero average asset income taxes

GKT (2003) assume that:

$$V(U(c), l_1, l_2, ..., l_T) = \sum_{t=1}^{T} \beta^{t-1}[u(c_t) - v(l_t)]$$

Under this restriction on preferences, they show that if (c^*, y^*) is socially optimal, then for all $\overline{\theta}^t$ such that $\sum_{\theta^S \geq \overline{\theta}^t} \pi(\theta^S) > 0$:

$$\frac{1}{u'(c_t^*(\theta^S))} = \beta^{-1} R^{-1} \sum_{\theta^S \geq \overline{\theta}^t} \frac{\pi(\theta^S)}{\sum_{\widetilde{\theta}^S \geq \overline{\theta}^t} \pi(\widetilde{\theta}^S)} \frac{1}{u'(c_{t+1}^*(\theta^S))}.$$

We can establish a generalized version of this GKT first order condition as follows.

Theorem 2 *Suppose* $V_{SP}(W^*) > V_{SP}(W')$ *if* $W^* > W'$. *Suppose too that* (c^*, y^*) *is socially optimal given social wealth* W^*, *and* $c_t^*(\theta^S) > 0$ *for all* t, θ^S. *Then:*

$$\begin{aligned} 1 &= R^{t-S} \sum_{\theta^S \geq \overline{\theta}^t} \frac{U_{c_t}(c^*(\theta^S))}{U_{c_S}(c^*(\theta^S))} \frac{\pi(\theta^S)}{\sum_{\widetilde{\theta}^S \geq \overline{\theta}^t} \pi(\widetilde{\theta}^S)} \text{ for all } t < S, \text{ all } \overline{\theta}^t, \\ 1 &= R^{t-S} \frac{U_{c_t}(c^*(\theta^S))}{U_{c_S}(c^*(\theta^S))} \text{ for all } t \geq S, \text{ all } \theta^S. \end{aligned}$$

Proof. Because $V_{SP}(W^*) > V_{SP}(W')$, it must be true that if (c^*, y^*) is socially optimal given initial wealth W^*, then c^* must solve the following minimization problem:

$$\min_{c} \sum_{\theta^S \in \Theta^S} \pi(\theta^S) \sum_{t=1}^{T} R^{-t} c_t(\theta^S)$$
$$s.t. \quad U(c(\theta^S)) = U(c^*(\theta^S)) \text{ for all } \theta^S,$$
$$s.t.\ c_t \text{ is } \theta^t\text{-measurable.}$$

If we take first order conditions, we obtain:

$$\sum_{\theta^S \geq \overline{\theta}^t} \pi(\theta^S) = R^t \sum_{\theta^S \geq \overline{\theta}^t} \lambda(\theta^S) U_{c_t}(c^*(\theta^S)) \text{ for all } t < S, \text{ all } \overline{\theta}^t,$$
$$\pi(\theta^S) = R^t \lambda(\theta^S) U_{c_t}(c^*(\theta^S)) \text{ for all } t \geq S, \text{ all } \theta^S,$$

where $\lambda(\theta^S)$ is a multiplier on the utility constraint. By substituting the period S FOC into the period t FOC, we obtain the proposition. ■

The proposition hypothesizes that having less resources reduces social welfare; that is, it assumes that $V_{SP}(W^*) > V_{SP}(W')$ for all $W' < W^*$. This hypothesis is about an endogenous variable (the planner's maximized objective). It can be shown to be true if the utility aggregator V is additively separable between the sub-utility U and the sequence of labors $(l_1, ...l_T)$. (See the proof of Lemma 1 in GKT (2003)).

The proposition is a strict generalization of Theorem 1 of GKT. Suppose the marginal utility process $U_{c_t}(c(\theta^S))$ is θ^t-measurable for all $t < S$. This measurability restriction is satisfied if U is additively separable. Then, if

$t < S$:

$$1 = R^{t-S} U_{c_t}(\overline{\theta}^t) E\{\frac{1}{U_{c_S}} | \theta^t = \overline{\theta}^t\},$$
$$1 = R^{t+1-S} U_{c_{t+1}}(\overline{\theta}^{t+1}) E\{\frac{1}{U_{c_S}} | \theta^{t+1} = \overline{\theta}^{t+1}\}.$$

Using the Law of Iterated Expectations, this reduces to the GKT condition:

$$\frac{1}{U_{c_t}(\overline{\theta}^t)} = R^{-1} E\{\frac{1}{U_{c_{t+1}}} | \theta^t = \overline{\theta}^t\}. \tag{9}$$

We can use Theorem 2 to derive properties of the optimal social security system $(\alpha^*, \tau^*, \Psi^*)$ described in the prior section. In particular, if $t \geq S$:

$$\begin{aligned} \tau^*_{t+1}(y^*(\theta^S)) &= \frac{-U_{c_t}(c^*(\theta^S))/R + U_{c_{t+1}}(c^*(\theta^S))}{(1-1/R)U_{c_{t+1}}(c^*(\theta^S))} \\ &= 0. \end{aligned}$$

It is optimal not to tax asset income after the retirement period S. This result is intuitive. The only reason that asset income taxes exist in this setting is to deter agents from saving/borrowing and then working less. Agents don't work after period S, and so there is no reason to tax asset income in those periods.

The situation is different before retirement. If $t < S$, then:

$$\tau^*_{t+1}(y^*(\theta^S)) = \frac{-U_{c_t}(c^*(\theta^S))/R + U_{c_{t+1}}(c^*(\theta^S))}{(1-1/R)U_{c_S}(c^*(\theta^S))R^{S-t-1}}.$$

Suppose first that the marginal utility processes are such that $U_{c_{t+1}}(c^*(\theta^S))$

and $U_{c_t}(c^*(\theta^S))$ are both θ^t-measurable. Then Theorem 2 implies that:

$$\begin{aligned} U_{c_t} &= R^{S-t} E_t U_{c_S}, \\ U_{c_{t+1}} &= R^{S-1-t} E_t U_{c_S}, \end{aligned}$$

and $\tau_t^*(y^*(\theta^S)) = 0$ for all θ^S. This measurability restriction is satisfied, for example, if there is no private information problem after period s, $s < t$.

In general, though, $U_{c_{t+1}}$ and U_{c_t} will not be predictable using time t information. These marginal utilities will depend on future consumption, and future consumption will depend on individual-specific realizations of $\theta_{t+s}, s > 1$ because of the informational problem. However, we can calculate the average asset income tax rate as follows:

$$\begin{aligned} & \sum_{\theta^S \geq \bar{\theta}^t} \pi(\theta^S) \tau_{t+1}^*(y^*(\theta^S)) \\ = & \sum_{\theta^S \geq \bar{\theta}^t} \pi(\theta^S) \{ \frac{-U_{c_t}(c^*(\theta^S))/R + U_{c_{t+1}}(c^*(\theta^S))}{(1-1/R) U_{c_S}(c^*(\theta^S)) R^{S-t-1}} \} \\ = & \; 0, \end{aligned}$$

where the last equality follows from Theorem 2. If we average asset income tax rates across all agents with the common history $\overline{\theta}^t$, we get zero. Moreover, because b_{t+1} is θ^t-measurable, the total asset income tax collections in period S are also zero.

5.2 Negative intertemporal wedge and the tax-consumption covariance structure

In the additively separable case, GKT (2003) demonstrate that optimal allocations of consumption are characterized by a positive intertemporal wedge: at every date and state, the marginal return on savings exceeds the shadow interest rate of every agent in the economy. Albanesi and Sleet (2006) and Kocherlakota (2005) show how this wedge can be implemented in a linear capital income tax system in which the average tax rate is zero: marginal tax rates must be negatively correlated with consumption. This negative correlation means that capital income tax rates are high when consumption is desirable, which discourages savings and implements the positive intertemporal wedge in asset market equilibrium.

In this subsection, we use a robust example to show that the optimal intertemporal wedge can be negative when preferences are not time separable. In that example, we also show that the optimal asset income taxes τ^* implement this negative intertemporal wedge by subsidizing capital income when consumption is low and taxing it when consumption is high.

Consider again the example of Section 3. In that example, the sub-utility function U, given in (1), satisfies

$$U_{c_2}(c_1, c_2, c_3) = u'(c_2) - \lambda U_{c_3}(c_1, c_2, c_3). \qquad (10)$$

Theorem 2 implies that

$$1 = E_1\{\frac{U_{c_t}(c^*)}{U_{c_3}(c^*)}\},$$

for $t = 1, 2$. Since U_{c_1} is θ^1-measurable in this example, we have

$$\frac{1}{U_{c_1}(c_1^*)} = E_1\{\frac{1}{U_{c_3}(c^*)}\}. \tag{11}$$

We can also write

$$E_1\{\frac{U_{c_2}(c^*)}{U_{c_3}(c^*)}\} = E_1\{U_{c_2}(c^*)\}E_1\{\frac{1}{U_{c_3}(c^*)}\} + cov_1\{U_{c_2}(c^*), \frac{1}{U_{c_3}(c^*)}\}.$$

Using (10), we can evaluate the covariance term. We have

$$\begin{aligned} cov_1\{U_{c_2}(c^*), \frac{1}{U_{c_3}(c^*)}\} &= cov_1\{u'(c_2^*) - \lambda U_{c_3}(c^*), \frac{1}{U_{c_3}(c^*)}\} \\ &= -\lambda cov_1\{U_{c_3}(c^*), \frac{1}{U_{c_3}(c^*)}\} \\ &> 0, \end{aligned}$$

where the second equality follows from the fact that $u'(c_2^*)$ is a constant. The strict inequality follows from $\lambda > 0$, $c_{3H}^* > c_{3L}^*$ and the fact that the inverse function is strictly decreasing. We thus obtain that

$$\begin{aligned} 1 &= E_1\{U_{c_2}(c^*)\}E_1\{\frac{1}{U_{c_3}(c^*)}\} + cov_1\{U_{c_2}(c^*), \frac{1}{U_{c_3}(c^*)}\} \\ &> E_1\{U_{c_2}(c^*\}E_1\{\frac{1}{U_{c_3}(c^*)}\} \\ &= E_1\{U_{c_2}(c^*)\}\frac{1}{U_{c_1}(c^*)}, \end{aligned}$$

where the last line uses (11). The above strict inequality can be written as

$$U_{c_1}(c^*) > E_1\{U_{c_2}(c^*)\}. \tag{12}$$

With $R = 1$, this inequality shows that the intertemporal wedge between periods 1 and 2 is strictly negative. In the absence of taxes, agents would like to deviate from the socially optimal allocation c^* by borrowing in period 1.

This result is quite intuitive. Since marginal utility of consumption in period 3 is increasing in the level of consumption habit λc_2, providing incentives for high effort in period 3 is inexpensive (in terms of the required spread between c_{3H} and c_{3L}) when the level of habit λc_2 is high. Thus, an increase in consumption c_2 relaxes the incentive constraint (2). A similar increase in consumption c_1 has no effect on incentives. Due to this socially beneficial effect of c_2 on incentives, optimal consumption c_2^* is high, relative to c_1^*. Private agents, however, do not take this (external) effect into account. In the absence of taxes, they would like to smooth consumption by decreasing c_2^* and increasing c_1^*.

How is this negative wedge implemented? Under optimal retrospective taxes τ_2^*, the individual Euler equation

$$U_{c_1}(c^*) = E_1\{U_{c_2}(c^*)\} - E_1\{\tau_2^* U_{c_3}(c^*)\}$$

is satisfied. Using (12), we get that

$$\begin{aligned} 0 &> E_1\{\tau_2^* U_{c_3}(c^*)\} \\ &= E_1\{\tau_2^*\}E_1\{U_{c_3}(c^*)\} + cov_1\{\tau_2^*, U_{c_3}(c^*)\} \\ &= cov_1\{\tau_2^*, U_{c_3}(c^*)\}, \end{aligned}$$

where the last line follows from the zero average tax result. The optimal tax rate on b_2 co-varies negatively with the marginal utility of consumption in period 3, and hence co-varies positively with consumption in period 3. This tax makes bonds held from period 1 into period 2 a *better* precautionary hedge: taxes on savings b_2, due at $t = 3$, are low exactly when consumption c_3 is low. This tax promotes savings from period 1 into period 2, and creates the negative intertemporal wedge.

6 Conclusions

Over the past five years, there has been a great deal of work on optimal asset taxation when agents are privately informed about skills. This work has typically restricted agents' preferences to be additively separable between consumption at different dates, and between consumption and leisure. Both restrictions are severe ones. In this paper, we relax these restrictions considerably, and require only that preferences be weakly separable between consumption paths and labor paths. This class of preferences includes, for example, the possibility that preferences exhibit habit formation with respect to consumption.

We show that intertemporal nonseparabilities matter. We demonstrate that if a tax system is differentiable with respect to asset income, and implements a social optimum, then the taxes on period t asset income must depend on period t' labor income, where $t' > t$. Given this result, it is natural to look at tax systems in which period t asset income is taxed only at the time of retirement. We restrict attention to what we term *social secu-*

rity systems. In these systems, labor income before retirement is taxed at a time-independent rate. At retirement, agents' asset income is taxed linearly, but at a rate that depends on their full labor income history. After retirement, agents receive history-dependent constant transfers. We prove that, because of the weak separability of preferences, the taxes on asset income average to zero across all agents (as in Kocherlakota (2005)). Asset income taxes are purely redistributive.

In our analysis, the only form of uncertainty is idiosyncratic labor productivity risk. In the real world, there are many other forms of risk, including age of death and health shocks. We could readily extend our analysis to account for these forms of risk, as long as there is no private information associated with them. For example, with uncertain lifetimes, we could implement an optimal allocation by embedding an annuity feature into our social security system.

One criticism of the implementations used in Albanesi and Sleet (2006) and especially Kocherlakota (2005) is that they are too complex relative to capital and labor income taxes used in practice. In this paper, even though preferences are time nonseparable, all redistribution and insurance is embedded in the calculations of taxes and transfers at retirement. These calculations are admittedly complex. But there is no real sense that they are any more complex than the calculations that the Social Security administration currently does to determine post-retirement benefits. We believe that social security systems can be useful for implementation in many other dynamic settings.

Appendix

In this Appendix, we provide an example of an environment in which our Condition 1 is violated.

Let $W = 0$, $T = S = 2$. Suppose that preferences are (separable):

$$\begin{aligned} V(U, l_1, l_2) &= U - .5(l_1)^2 - .5(l_2)^2 2/3, \\ U(c_1, c_2) &= -2c_1^{-1/2} - 2c_2^{-1/2} 2/3. \end{aligned}$$

Suppose also that $R = 3/2$ and

$$\Theta = \{.5,\ 1,\ 1.051425,\ 1.1392115,\ 2\}.$$

Let π be such that

$$\begin{aligned} \pi(1.1392115,\ 2) &= 1/4, \\ \pi(1.1392115,\ 1) &= 1/4, \\ \pi(1,\ 1.051425) &= 1/4, \\ \pi(1,\ .5) &= 1/4. \end{aligned}$$

Under π, therefore, the skill level at $t = 1$, θ_1, is either 1.1392115 (high) or 1 (low). The high realization of θ_1 also means good prospects for θ_2, the skill level at $t = 2$. Conditional on $\theta_1 = 1.1392115$ the distribution of θ_2 first-order stochastically dominates the distribution of θ_2 conditional on $\theta_1 = 1$. (It does not however dominate state-by-state.)

Solving numerically for an optimum, we get the following optimal allocation:

$$
\begin{aligned}
&c_1^*(1.1392115) = 1.1622, && c_1^*(1) = 0.9515,\\
&y_1^*(1.1392115) = 1.0358, && y_1^*(1) = 1.0358,\\
&c_2^*(1.1392115, 2) = 1.4573, && c_2^*(1.1392115, 1) = 0.8231,\\
&y_2^*(1.1392115, 2) = 2.2738, && y_2^*(1.1392115, 1) = 0.8878,\\
&c_2^*(1, 1.051425) = 1.0944, && c_2^*(1, .5) = 0.7970,\\
&y_2^*(1, 1.051425) = 0.8878, && y_2^*(1, .5) = 0.2488.
\end{aligned}
$$

We thus have that the following two histories

$$(1.1392115,\ 1),$$

$$(1,\ 1.051425)$$

are assigned (i) the same output path

$$y^2 = (1.0358, 0.8878),$$

and (ii) two very different consumption paths:

$$
\begin{aligned}
c^*(1.1392115,\ 1) &= (1.1622, 0.8231),\\
c^*(1,\ 1.051425) &= (0.9515, 1.0944).
\end{aligned}
$$

The function $\hat{c}$ postulated in our Condition 1, therefore, does not exist in this example.

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