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FACTOR PRICES MAY BE CONSTANT BUT FACTOR RETURNS ARE NOT

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Welfare effects of a distortion introduced in a 'small' region, country or industry are not limited to allocations within that domain. They may be large, and undiminishing, even though the area considered is very close to satisfying the idealized prerequisites for partial equilibrium analysis.

The assumption that actions of a given economic unit do not affect prices elsewhere in the system of which that unit forms a part, is one of the most useful approximations in economic analysis. This basic assumption is crucial to such areas of economic theory as the theory of household demand, the theory of production choices by competitive firms and the theory of international trade among 'small' countries. The approximation is clearly justified when the unit in question is small in a well-defined way relative to the larger system and that larger system allows a certain scope for substitutability.

It may not be widely understood that the superficially similar assumption under the same circumstances, that the *products* of prices and quantities elsewhere in the system can safely be treated as constant, may well not be justified. [1] The demonstration of this proposition is by the following example, which I believe fairly represents a typical invocation of constant prices. The example also clearly shows that the matter is not just a technical fine point, for in this case welfare conclusions are rather dramatically affected.

The analysis in question concerns the effect of imposing a tax on the employment of capital within the boundaries of a small local jurisdiction. In actuality such a tax might apply only to certain uses of capital (e.g., in producing housing services); the output involved might be only imperfectly substitutable for that of other jurisdictions (e.g., again, housing); and all sorts of other taxes might be in effect in the

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[1] This idea is, however, not new. As has been pointed out to me by Wallace Oates, an analysis similar in spirit to the present one was given by Herbert A. Simon in 1943, who attributed the idea to Harry Gunnison Brown (1924), who attributed it in turn to yet earlier contributors.

home and other districts, which might differ in many ways. But to keep things simple we shall assume: the tax applies to all uses of capital; there is a single homogeneous output; there are no other taxes in either the home or other districts; there are n districts, identical in all respects except for the tax in the home district. At the outset we avoid issues of valuing benefits by assuming the local tax proceeds are wastefully spent.

There are two classes of agents in our model world, 'landlords' and 'capitalists'. [2] The landlords own the productive opportunities of the system, described by a production function $F(K)$ giving the output obtained at a given location by the application of K units of capital services at that location. In the absence of taxes the landlords hire capital services at a price r (measured in output units) in order to maximize the surplus, $F(K) - rK$, obtained at each location. We shall assume F is at least twice continuously differentiable and subject to diminishing marginal productivity. Letting primes stand for derivatives, $F' > 0, F'' < 0$.

Under these assumptions, in a no-tax equilibrium the same level of capital services will be employed in all jurisdictions; let K stand for this common amount. The capitalists of the system own the capital stock, assumed fixed in total amount at nK, and sell capital services to landlords under competitive conditions.

Imposition of a tax in the home jurisdiction causes the effective price of capital services to the landlords there to increase from r to $\hat{r} + t$, where $\hat{r}$ is the new equilibrium capital rental rate. This will cause a reduction in capital employed from K to $K - \Delta K$, and lead to a loss in the landlords' surplus in the location. It may have effects outside the local area as well. Analysis of these effects is greatly simplified if r may be taken as fixed, i.e., $\hat{r} = r$. Then introducing the tax alters the equilibrium as is depicted in fig. 1, in which the curved line traces out the marginal product of capital in the home district. (The other lines should be self-explanatory.) Assuming that the basic argument is well known, we point out that the imposition of the tax causes a loss of surplus to the home landlords of $ABCD$, while raising revenue of $ABCE$. The 'triangle' ECD represents dead weight loss. As a result of the tax, capital employed locally is reduced by ΔK, equal to segment ED in fig. 1.

When r is literally constant this rather completely describes the effect of the tax. The local landlords (or the landlords as a group, if ownership is shared) bear the full burden of the tax, including the dead weight output loss, while the position of capitalists is unaffected. When r is not constant one has to contend with effects on the welfare of both groups and with changes in production and surplus at other locations.

[2] Generally the analysis concerns the different effects of a parameter change on fixed and mobile factors. In an international trade application we might, for example, assume fixed supplies of labor at all locations. The statements in the text about effects on 'landlords' would then apply instead to 'laborers'. In this case, however, separation of location and ownership of the fixed factor would presumably be ruled out: a resident of country A can own land in country B, but not labor services in country B.

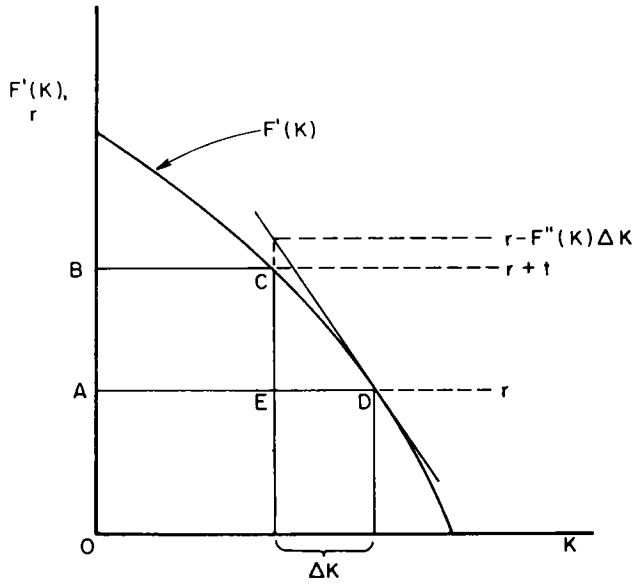

Fig. 1. Pre-tax (D) and post-tax (C) equilibrium with constant capital rental rate.

In general r will not be literally constant, but if n is large enough it will be approximately constant. Under our assumptions, the change in rental rate is given by the change in the marginal product of capital outside the home jurisdiction when ΔK is spread equally over the $n-1$ other locations. This is expressed in

$$\hat{r} - r = F'\left(K + \frac{\Delta K}{n-1}\right) - F'(K). \tag{1}$$

Since F' is continuous, for n large, $\hat{r} - r \approx 0$, and under the usual assumptions of continuously diminishing returns the approximation improves as n increases. [3] For large n, then, the assumption of constant rental rate of capital is acceptable as far as conclusions about the effects of the tax in the home district are concerned.

As we shall now see, however, no matter how large n may be, external effects will persist. Nor will these be insignificant when compared with the consequences

[3] Strictly speaking, we should systematically consider the dependence of ΔK on $\hat{r}$; the argument would be upset if ΔK were to increase sufficiently rapidly with n. A glance at fig. 1 suggests rather that ED will represent an upper bound on ΔK.

of the tax at home, as already described. In particular, when we apply the same limiting arguments to the *products* of price changes and quantities we find that, far from bearing no burden, capitalists in the aggregate lose an amount greater than the home district tax revenue plus dead weight loss. Since the home district effects previously described continue to hold (i.e., the home district surplus is reduced by *ABCD* in fig. 1) this means that owners of other districts gain from the tax. Indeed, landlords in the aggregate *gain* from the imposition of the tax.

Consider first the change in aggregate revenue of capitalists, $\Delta R(n)$, that results from a shift of ΔK out of the home district, spread equally over the other districts,

$$\Delta R(n) = nK\left[F'\left(K + \frac{\Delta K}{n-1}\right) - F'(K)\right]. \tag{2}$$

By Taylor's formula, we can always find a $\theta(n)$ between zero and one such that

$$\Delta R(n) = nKF''\left(K + \frac{\theta(n)\,\Delta K}{n-1}\right)\frac{\Delta K}{n-1}. \tag{3}$$

The limit of this revenue change is given by

$$\Delta R(\infty) = KF''(K)\,\Delta K. \tag{4}$$

The term $-F''(K)\,\Delta K$ is approximately equal to (but somewhat larger than) t, the rate of tax, as may be seen by reference to fig. 1. Hence (4) tells us that capitalists' revenues are reduced by an amount at least equal to the tax receipts that would obtain if there were *no* adjustment in home capital employment. This total exceeds the sum of tax revenues and dead weight loss, *ABCD* in fig. 1. In spite of the fact that if the system is large enough the local tax has an effect on the rental rate of capital that for the usual purposes is 'negligible', it nevertheless causes a loss to capitalists in the aggregate. Furthermore, the loss, which in our example can be calculated from purely local information, exceeds that borne by the local landlords, who would be the only losers were the rental rate truly constant.

The conclusion that landlords as a group gain from imposition of the tax follows immediately. For we have seen that the loss in surplus in the taxing district exactly equals the sum of tax revenues and dead weight output loss in the system. Thus the capitalists' loss appears in the form of extra surplus at the remaining production sites. [4] These landlords are gainers from the tax. [5]

[4] This conclusion can also be reached by direct analysis. Let $\Delta S(n)$ represent the change in aggregate surplus at the $n-1$ production sites other than the taxing jurisdiction. Then, recognizing that the equilibrium rental rate will equal the marginal product of capital,

$$\Delta S(n) = (n-1)\left[F\left(K + \frac{\Delta K}{n-1}\right) - F'\left(K + \frac{\Delta K}{n-1}\right)\left(K + \frac{\Delta K}{n-1}\right) - F(K) + F'(K)\,K\right].$$

Applying Taylor's formula twice we obtain

$$\Delta S(n) = \left[F'\left(K + \frac{\theta(n)\,\Delta K}{n-1}\right) - F'(K) - F''\left(K + \frac{\hat{\theta}(n)\,\Delta K}{n-1}\right)\left(K + \frac{\Delta K}{n-1}\right)\right]\Delta K,$$

If we add a plausible assumption about the benefits from the local tax to this story the conclusions can be rendered yet more remarkable. Assume that the landlords 'live on top of the store' and don't own productive opportunities outside their respective home districts. Assume further that the local tax finances local services equal in value at least to the surplus foregone by the local landlords. Then the *only* losers from the tax expenditure combination are the capitalists in the aggregate, and this only because they are induced to make a *transfer* to landlords in other districts.

Concluding comments. Since the argument here has been by example it cannot claim to have demonstrated that significant effects external to the small unit will always be present when the approximation of constant prices is justified. Yet the example is sufficiently representative of models used in a wide range of applications to suggest that care should be exercised when the constant price assumption is employed. Where the problem is to determine incidence of some policy change our conclusions here suggest the desirability of carrying out the full analysis of external effects. At a minimum the reader should be warned about the possibly significant consequences of local actions that are being ignored.

References

Brown, Harry Gunnison, 1924, The economics of taxation (New York).

Musgrave, Richard A. and Carl S. Shoup, 1959, Readings in the economics of taxation (George Allen and Unwin, London).

Simon, Herbert A., 1943, The incidence of a tax on urban real property, Quarterly Journal of Economics LVII. Reprinted in Musgrave and Shoup (1959).

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where $\theta(n)$ and $\hat{\theta}(n)$ are between zero and one. The limit of this expression as n goes to infinity is

$$\Delta S(\infty) = -KF''(K)\,\Delta K\,.$$

[5] It is important to keep in mind that there is no necessary connection between the location of production and the residence of landlords or capitalists. In particular, if all landlords own a share in all production sites, only the aggregate effect on landlords is of any interest.