Depreciating Licenses*

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January 2018

Abstract

Perpetual licenses incent owners incentives to invest in the common value of public resources, but impede efficient reallocation of resources to higher-valued entrants. Short-term licenses improve allocative efficiency but discourage investment. We propose a *depreciating license* that improves on this tradeoff. Licensees periodically announce valuations at which they commit to sell their licenses, and pay a percent of these valuations as license fees. Depreciating licenses time-stationary investment incentives while encouraging truthful value revelation that improves allocative efficiency. The only tuning parameter, the depreciation rate, can be chosen appropriately by targeting the observed equilibrium frequency of license turnover.

Keywords: depreciating license, property rights, investment, misallocation, monopoly
JEL classifications: B51, C78, D42, D61, D82, K11

*This paper was previously circulated under the title "Ownership of the Means of Production." We appreciate helpful comments by Evan Kwerel, Eric Posner, Jeremy Bulow, Lanier Benkard, Michael Ostrovsky, Andy Skrzypacz, Mohammad Akbarpour, Matthew N. White, Ilya Segal, Gabriel Carroll, Shengwu Li, seminar participants at the London School of Economics, the Hong Kong University of Science and Technology, Microsoft Research, Stanford University, the University of Chicago Law School, the Yale School of Management, ASSA 2017, Airbnb, multiple referees, and especially Nicolaus Tideman. In addition, we especially thank Paul Milgrom for his invaluable comments, as well as for suggesting the title of the paper. Alex Moehring provided excellent research assistance. All errors are our own.

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1 Introduction

The design of licenses for public resources, such as radio spectrum and fisheries, often trades off allocative and investment efficiency. In this paper we propose a license design which navigates this tradeoff more efficiently than existing alternatives: *depreciating licenses*.

The role of property rights in promoting the efficient use of scarce resources is perhaps the oldest idea in mainstream economic thought. An intuition dating back to the Greek prehistory of the field is that property rights solve the tragedy of the commons, providing incentives for maintaining the common use value of assets.[1] The role of property rights in providing robust investment incentives is also the subject of a sizable modern literature (Grossman and Hart, 1986; Hart and Moore, 1990).

On the other hand, economists since at least George (1879), Jevons (1879) and Walras (1896) have argued that private ownership may inhibit the efficient allocation of assets.[2] While some interpreted Coase (1960) as arguing that initial property rights allocations are irrelevant, contemporary economic theory has shown that the assumptions under which transaction costs are absent are narrow. Myerson and Satterthwaite (1981) show that fully efficient trade of a privately-owned asset between a buyer and a seller is impossible if both agents have private information about their values. This contrasts with Vickrey (1961)'s demonstration that an appropriate auction allows publicly-owned assets to be efficiently allocated. In this sense, private ownership is a fundamental barrier to efficient trade.

For many publicly owned natural resources, both allocative and investment efficiency are important concerns. On the allocative side, different parties may have different values and costs for using the resource; long-term or perpetual licenses inhibit the efficient reallocation of resources to high-value users. On the investment side, costly development or maintenance activities are often required to sustain the common value of the resource for all parties; short-term licenses decrease the incentives of resource users to make such investments. Effective license systems should provide robust and long-term incentives for investment while also promoting efficient license reallocation.

Inspired by a taxation scheme proposed by Harberger (1965) and its benefits for value revelation highlighted by Tideman (1969) and Plassmann and Tideman (2008), we propose a new system for assigning resource use rights, which we call *depreciating licenses*. These licenses have indefinite length, but decay at some annual rate τ. Each year, every licensee must announce a

[1] "What is common to the greatest number gets the least amount of care. Men pay most attention to what is their own; they care less for what is common; or at any rate they care for it only to the extent to which each is individually concerned." Aristotle, *The Politics*, Book XI, Chapter 3

[2] Walras (1896) writes "Declaring individual landownership to be in the interests of agriculture means turning our backs to the...effects of free competition by preventing the land from being used as is most advantageous to society. If large properties are favoured, we will see parts of the territory becoming parks or hunting-grounds...if small properties are favoured, we will see them delivered up to the most outdated farming methods as a consequence of ignorance and traditional practice. Hence, both economic advantageousness and justice demand that the price of the service of land goes to the state..."' See Posner and Weyl (Forthcoming) for a detailed history of these ideas.

price at which she repurchases a share τ of her license from the government. The repurchase payment can also be thought of as a self-assessed license fee. Licensees' price announcements are kept in a publicly available register, and any interested buyer can purchase any license from its owner at her most recent value announcement.

In contrast to previous literature which focused only on the static allocative, revenue or private-valued investment properties of such an mechanism (Plassmann and Tideman, 2008), we show that depreciating licenses create high and time-stationary incentives for common-valued investment, improving significantly upon term-limit licenses. They also promote aftermarket trade and improve allocative efficiency, since the license fee incents licensees to announce prices close to their true values for the license.

In Section 2, we elaborate on the argument of Plassmann and Tideman to show that the price-setting behavior of licensees is governed by a simple economic intuition – licensees set prices higher or lower than their true values depending on whether the annual probability of license sale is higher or lower than the depreciation rate. Thus, the equilibrium license turnover rate is an observable approximate sufficient statistic for setting optimal license depreciation rates, though we also show unlike previous work that this allocative optimality must be balanced against incentives for common-valued investment which smoothly decay with τ.

In Section 3, we construct a simple dynamic model in which a depreciating license is repeatedly traded between many agents, who make common-valued investment and price announcement decisions in each period. In Section 4, we calibrate the model to loosely match moments of existing markets for durable assets, and find that depreciation rates set at roughly half the rate of asset turnover under perpetual ownership licenses are near optimal for a range of specifications and parameter values. This increases the net utility generated by the asset in the stationary trading equilibrium of the model by roughly a fifth of a standard deviation in willingness-to-pay of different potential asset buyers, which is approximately 4% of the price of perpetual licenses in our preferred specification.

In Section 5, we analyze a number of extensions suggesting that depreciating licenses may be robust to many other factors, such as different structures of investment technology, adverse selection and credit constraints. In Section 6 we discuss our proposal's relationship to literatures on mechanism design, taxation and intellectual property. We present longer and less instructive calculations, proofs, and calibration details in an appendix following the main text.

2 Two-stage model

Relative to perpetual licenses, depreciating licenses improve allocative efficiency but decrease investment incentives. This tradeoff is governed by the choice of depreciation rate. We illustrate these intuitions in a simple two-stage model in which a depreciating license owner makes some costly common-valued investment in an asset, and then announces her valuation for the license, paying the self-assessed license fee and potentially selling the license to an arriving buyer.

2.1 Setup

There are two agents, S and B. There is a single asset, and S initially owns a depreciating license for the asset. Values of S and B for the asset are, respectively,

$$\nu_S = \eta + \gamma_S,$$

$$\nu_B = \eta + \gamma_B.$$

The term γ_S represents S's idiosyncratic value component for the asset; it is fixed and known to S at the beginning of the game. $\gamma_B \sim F(\cdot)$ is a random variable representing heterogeneity in B's value, which is not observed by S. η is a common-value component; S chooses $\eta \geqslant 0$, incurring a convex cost $c(\eta)$ to herself. Both agents are risk neutral.[3]

For a given η, let $1_S, 1_B$ be indicators, which respectively represent whether S and B hold the license at the end of the game, and let y be any net transfer B pays to S. Final payoffs for S and B respectively are

$$U_S = (\eta + \gamma_S)\, 1_S - c(\eta) + y,$$

$$U_B = (\eta + \gamma_B)\, 1_B - y.$$

Prior to the beginning of the game, the government decides on a depreciation rate τ. Then, S and B play a two-period game. In period 1, S chooses η. In period 2, S announces a price p for the license, pays the self-assessed license fee $p\tau$ to the government, and then B can decide whether to buy the license by paying p to S.

We solve the game by backwards induction. First, fixing η and τ, we analyze the behavior of S in the period 2 price offer game.

2.2 Allocative efficiency

For any price p, B's optimal strategy is to buy the license if her value is greater than p, that is, if $\eta + \gamma_B > p$. Let $m \equiv p - \eta$ be the markup S chooses to set over the common value η. The probability of sale under markup m is then $1 - F(m)$. Fixing common value η, and depreciation rate τ, S's optimal price offer solves:

$$\max_m \left(1 - F(m)\right)(\eta + m) + F(m)(\eta + \gamma_S) - \tau(\eta + m) - c(\eta).$$

[3]See Tideman (1969) for a partial analysis of the allocative problem that allows for risk aversion.

We can change variables to work in terms of sale probabilities. Define $q \equiv (1 - F(m))$, and $M(q) \equiv F^{-1}(1-q)$. S then solves:

$$\max_q (\eta + M(q))\, q + (\eta + \gamma_S)(1-q) - \tau(\eta + M(q)) - c(\eta).$$

Note that the socially efficient outcome corresponds to setting $M(q) = \gamma_S$, or equivalently $q = 1 - F(\gamma_S)$. We can rearrange S's optimization problem to:

$$\max_q (M(q) - \gamma_S)(q - \tau) + (\eta + \gamma_S)(1-\tau) - c(\eta).$$

Only the *variable profit* term $(M(q) - \gamma_S)(q - \tau)$ depends on the sale probability q. Thus, S's optimal choice of sale probability if her value is γ_S and the depreciation rate is τ can be written as:

$$q^*(\gamma_S, \tau) \equiv \arg\max_q (M(q) - \gamma_S)(q - \tau).$$

We can think of the objective function as the net trade profits of an agent who sells share q of the asset to buyers, and buy share τ of the asset from the government, both at price $M(q)$. In the following Theorem, we show that the relationship between τ and q summarizes licensees' incentives to markup or markdown prices.

Theorem 1. *(Net trade property)*

- *If* $\tau = 1 - F(\gamma_S)$, *then* $q^*(\gamma_S, \tau) = \tau$ *and* $M(q^*(\gamma_S, \tau)) = \gamma_S$.
- *If* $\tau < 1 - F(\gamma_S)$, *then* $q^*(\gamma_S, \tau) \geqslant \tau$ *and* $M(q^*(\gamma_S, \tau)) \geqslant \gamma_S$.
- *If* $\tau > 1 - F(\gamma_S)$, *then* $q^*(\gamma_S, \tau) \leqslant \tau$ *and* $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$.

Proof. See Appendix A.1. □

Theorem 1 shows that the net effect of depreciating licenses on sellers' price-setting incentives is linked to an observable quantity: $\tau - q$, the difference between the depreciation rate and the probability of sale that it induces. If τ are lower than equilibrium sale probabilities, licensees can be thought of as selling a larger share of the asset than they are buying from the government, hence have net incentives to set prices higher than their values. Likewise, if τ is higher than sale probabilities, licensees are buying more from the government than they are selling, thus set prices below their values. When τ is equal to the probability of sale, asset licensees are neither net buyers nor net sellers of their assets; thus they set prices equal to their values, leading to full allocative efficiency.

Theorem 2 of Section 3 shows that the net trade property generalizes to our dynamic model. In a setting with many licensees with heterogeneous values, no single depreciation rate can give all licensees incentives to truthfully reveal their values; however, we show in our calibration of Section 4 that depreciation rates equal to average sale probabilities across sellers are close to allocatively optimal. Thus, the license turnover rate serves as an observable approximate sufficient statistic for setting depreciation rates.

We proceed to quantify the comparative statics of allocative welfare with respect to the depreciation rate. Assuming $F(\cdot)$ is twice continuously differentiable, S's first-order condition is

$$M'(q)(q-\tau)+(M(q)-\gamma_S)=0,$$

so that by the Implicit Function Theorem,

$$\frac{\partial q^\star}{\partial \tau}=\frac{M'(q^\star(\gamma_S,\tau))}{2M'(q^\star(\gamma_S,\tau))+M''(q^\star(\gamma_S,\tau))(q^\star(\gamma_S,\tau)-\tau)}=\frac{1}{2+\frac{M''(q^\star-\tau)}{M'}}=\frac{1}{2-\frac{M''(M-\gamma_S)}{(M')^2}},$$

where the last equality invokes the first-order condition and drops arguments. Cournot (1838) showed that this quantity equals the *pass-through* rate $\rho(q^\star(\gamma_S,\tau))$ of a specific commodity tax into price; see Weyl and Fabinger (2013) for a detailed discussion and intuition. ρ is closely related to the curvature of the value distribution; it is large for convex demand and small for concave demand. It is strictly positive for any smooth value distribution and is finite as long as S is at a strict interior optimum.[4]

The marginal gain to social welfare from a unit increase in the probability of sale is equal to the gap between γ_B and γ_S, which is by construction $(M(q^\star(\gamma_S,\tau))-\gamma_S)$. Thus, the marginal allocative gain from raising τ is $(M(q^\star(\gamma_S,\tau)-\gamma_S))\rho(q^\star(\gamma_S,\tau))$, or $(M-\gamma_S)\rho$ for short. Since $(M-\gamma_S)\rho$ is 0 at $q=1-F(\gamma_S)$, the first-order social welfare gain from raising τ approaches 0 as we approach the allocatively optimal turnover rate of $1-F(\gamma_S)$. On the other hand, when $\tau=0$, we have $(M-\gamma_S)\rho>0$, hence increasing τ from 0 creates a first-order welfare gain.

2.3 Investment efficiency

The variable profit term $(M(q)-\gamma_S)(q-\tau)$ in S's objective function is independent of η. Hence, S chooses investment to maximize the *sunk profit* term $(1-\tau)\eta-c(\eta)$. The first-order condition for this optimization problem is:

$$c'(\eta)=1-\tau.$$

We can define the investment supply function $\Gamma(\cdot)$ as:

$$\Gamma(s)\equiv c'^{-1}(s).$$

[4] Myerson (1981)'s regularity condition is sufficient but not necessary for this second-order condition, as we show in Appendix A.2.

The value of a unit of investment η is always 1, so the socially optimal level of investment is $\Gamma(1)$, whereas investment is only $\Gamma(1-\tau)$ when the depreciation rate is τ. The social value of investment is always 1, whereas S only invests up to the point where $c' = 1-\tau$. Thus, the marginal distortion from under-investment is τ. The marginal increase in investment from a rise in τ is $\Gamma' = \frac{1}{c''}$ by the inverse function theorem. Thus, the marginal social welfare loss from raising τ is $\Gamma'\tau$, or $\frac{\tau}{1-\tau}\Gamma\epsilon_\Gamma$, where ϵ_Γ is the elasticity of investment supply. Note that as $\tau \to 0$, this investment distortion goes to 0, so that there is no first-order loss in investment welfare when $\tau = 0$. Since there is a first-order gain in allocative welfare from raising the depreciation rate when $\tau = 0$, the optimal choice of τ is strictly greater than 0.

2.4 Allocation-investment tradeoff

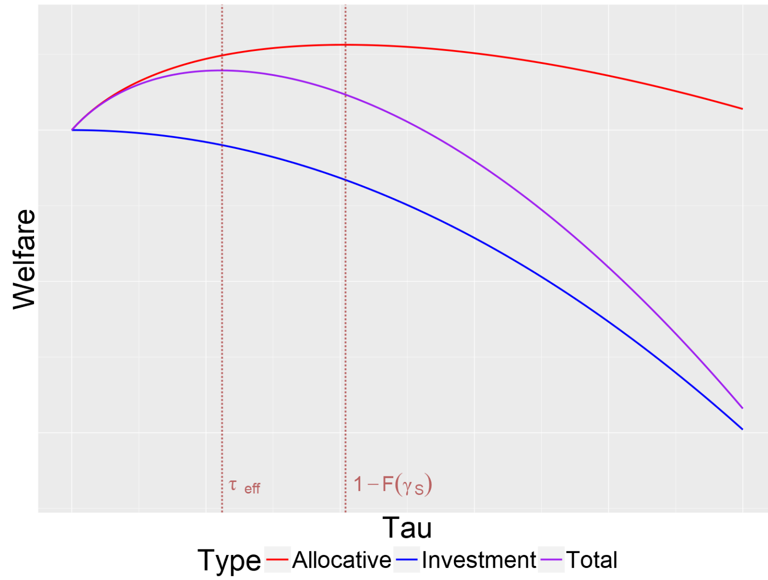
Figure 2.4 graphically illustrates the tradeoff between allocative and investment welfare. Allocative welfare increases monotonically in τ on the interval $\tau \in [0, 1-F(\gamma_S)]$. The marginal gain in allocative welfare from raising the depreciation rate is $(M(q^\star) - \gamma_S)\,\rho(q^\star)$; thus, the marginal allocative gain is 0 when $\tau = 1-F(\gamma_S)$ and $M(q^\star) = \gamma_S$. Similarly, the marginal investment loss is $\Gamma'\tau$, which is 0 at $\tau = 0$. These properties hold independently of the cost function and demand distribution; intuitively, this reflects the fact that both the marginal trades when $\tau = 1-F(\gamma_S)$, and the marginal units of investment when $\tau = 0$, have no social value. Thus, regardless of the underlying cost and demand functions, the efficient depreciation rate τ_{eff} lies strictly in the interior of the interval $[0, (1-F(\gamma_S))]$.

In Figure 2.4, allocative welfare is concave and investment losses are convex in τ, so total social welfare is a concave function of τ. While this is not true for all cost functions and demand distributions, it tends to hold for most well-behaved choices of the primitives. Since the markup $M(q^\star)$ is decreasing in τ, allocative marginal gains $(M(q^\star) - \gamma_S)\,\rho(q^\star)$ tend to be decreasing in τ, and since the marginal investment loss $\Gamma'\tau$ contains a τ term, marginal investment losses tend to be increasing in τ. Intuitively, as we raise τ from 0, the first trades that go through are the highest value trades, and the first investment losses are those which are both privately and socially marginal. As we raise τ, the allocative wedge $M(q^\star) - \gamma_S$ decreases, so new trades are less valuable, and the investment wedge τ increases, so the new investment losses are more costly to society. Thus, for relatively smooth demand forms and cost functions, the social optimization problem of maximizing the allocative gain less the investment loss will be concave.

In Appendix A.3, we formally derive sufficient conditions on the cost function $c(\cdot)$ and the inverse demand function $M(\cdot)$ for concavity of the social optimization problem. Under concavity, the following first-order condition, which resembles an optimal tax formula, uniquely characterizes the welfare-maximizing depreciation rate:

$$\frac{\tau_{\text{eff}}}{1-\tau_{\text{eff}}} = \frac{(M(q^\star(\gamma_S, \tau_{\text{eff}})) - \gamma_S)\,\rho(q^\star(\gamma_S, \tau_{\text{eff}}))}{\Gamma(1-\tau_{\text{eff}})\,\epsilon_\Gamma(1-\tau_{\text{eff}})}. \tag{1}$$

Figure 1: Allocative, Investment, and Total Welfare vs τ



Notes. Qualitative behavior of allocative (red), investment (blue) and total (purple) welfare vs. depreciation rate.

The left-hand side is a monotone-increasing transformation of τ that appears frequently in elasticity formulas in the optimal tax literature; see, for example, Werning (2007). The right-hand side is the ratio of two terms: the allocative benefit of higher depreciation rates and the investment distortion of higher depreciation rates. The allocative benefit equals the product of the mark-up and the pass-through rate, whereas the investment distortion equals the product of the equilibrium investment size and its elasticity with respect to $1 - \tau$.

3 Dynamic model

In this section, we construct a simple dynamic extension of the two-stage model. The economic intuitions are similar to those of the static model, but the dynamic model has well-defined quantities such as turnover rates, asset prices, and stationary value distributions, which we use to calibrate the model to data in Section 4.

3.1 Agents and utilities

Time is discrete, $t = 0, 1, 2 \ldots \infty$. All agents discount utility at rate δ. There is a single asset, which an agent S_0 owns at time $t = 0$. In each period, a single buyer B_t arrives to the market and bargains with the period-t licensee S_t to purchase the license, through a procedure we detail in Subsection 3.2 below. Hence, the set of agents is $\mathscr{A} = \{S_0, B_0, B_1, B_2 \ldots\}$. We will use S_t as an alias for the period-t licensee, who may be a buyer $B_{t'}$ from some period $t' < t$. We will use A to denote a generic agent in $\mathscr{A}$.

In period t, agent A has period-t use value γ_t^A for the asset. The values of entering buyers $\gamma_t^{B_t}$ are drawn i.i.d. from some distribution F. Values evolve according to a Markov process: for any agent A with period-t use value γ_t^A, her use value in the next period γ_{t+1}^A is drawn from the transition probability distribution $G(\gamma_{t+1} \mid \gamma_t)$.

Assumption 1. $F(M) = 1$ *for some finite* M.

Assumption 2. $\gamma_t > \gamma_t'$ *implies* $G(\gamma_{t+1} \mid \gamma_t) >_{FOSD} G(\gamma_{t+1} \mid \gamma_t')$.

Assumption 3. $G(\gamma' \mid \gamma)$ *is continuous and differentiable in* γ *for any* γ'.

In any period, there is a single user of the asset. Let 1_t^A denote agent A being the user of the asset in period A. Given 1_t^A and γ_t^A, agent A's utility is:

$$\sum_{t=0}^{\infty} \delta^t \left[1_t^A \gamma_t^A + y_t^A\right].$$

Where, y_t^A is any net monetary payment made to agent A in period t.

3.2 Game

The license has some fixed depreciation rate τ. We define the following *dynamic depreciating license game*. At $t = 0$, agent S_0 owns the license, and observes her own use value $\gamma_t^{S_0}$ for the asset. In each period t:

1. **Buyer arrival:** Buyer B_t arrives to the market; his use value $\gamma_t^{B_t}$ is drawn from $F(\cdot)$, and is observed by himself but not the period-t seller S_t.

2. **Seller price offer:** The licensee S_t makes a take-it-or-leave-it price offer p_t to buyer B_t, and immediately pays license fee τp_t to the government.

3. **Buyer purchase decision:**

   - If B_t chooses to buy the license, she pays p_t to S_t. B_t becomes the period-t asset user, $1_t^{B_t} = 1$, and enjoys period-t use value $\gamma_t^{B_t}$ from the asset. B_t becomes the licensee in period $t+1$, that is, $S_{t+1} \equiv B_t$. Seller S_t receives payment p_t from B_t, and seller S_t leaves the market forever, with continuation utility normalized to 0.

   - If B_t chooses not to purchase the license, S_t becomes the period-t asset user, $1_t^{S_t} = 1$, and she enjoys period-t use value $\gamma_t^{S_t}$ from the asset. S_t becomes the licensee in period $t+1$, that is, $S_{t+1} \equiv S_t$. Buyer B_t leaves the market forever, with continuation utility normalized to 0.

4. **Value updating**: $\gamma_{t+1}^{S_{t+1}}$, the period $t+1$ value for licensee S_{t+1}, is drawn from the distribution $G\left(\gamma_{t+1} \mid \gamma_t^{S_{t+1}}\right)$ according to her period-t value $\gamma_t^{S_{t+1}}$.

3.3 Equilibrium

Equilibrium in the dynamic depreciating license game requires that, in all histories, all sellers make optimal price offers, and all buyers make optimal purchase decisions. Since τ, F, G are constant over time, the problem is Markovian: the optimal strategies of buyers and sellers may depend on their types $\gamma_t^{S_t}, \gamma_t^{B_t}$ respectively, but not on the period t. Hence we can characterize equilibria of the game by a stationary value function $V(\gamma)$ which describes the value of being a type γ seller in any given period.

In any period t, we can think of S_t as choosing a probability of sale q_t, where buyers in period t make purchase decisions according to the inverse demand function $p(q_t)$. If the continuation value in period $t+1$ for seller type γ_{t+1} is $V(\gamma_{t+1})$, the optimization problem that S_t faces is:

$$\max_{q_t} q_t p(q_t) + (1-q_t)\left[\gamma_t + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma_{t+1}) \mid \gamma_t\right]\right] - \tau p(q).$$

Simplifying and omitting t subscripts, optimality for sellers requires $V(\gamma)$ to satisfy the following Bellman equation:

$$V(\gamma) = \max_q (q-\tau) p(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right]\right]. \tag{2}$$

Buyer optimality pins down the relationship between $p(\cdot)$ and $V(\cdot)$. If buyer B_t with value γ_t purchases the license, he receives value γ_t in period t, and then becomes the seller in period $t+1$, receiving utility $\delta V\left(\gamma_{t+1}^{B_t}\right)$. Hence the period-t willingness-to-pay of buyer type γ_t is:

$$WTP(\gamma_t) = \gamma_t + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V(\gamma_{t+1}) \mid \gamma_t\right].$$

Thus, in equilibrium, optimality for the buyer implies that the inverse demand function $p(\cdot)$ satisfies:

$$p(q) = \left\{p : \ P_{\gamma \sim F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right] > p\right] = q\right\}. \tag{3}$$

Fixing τ, a value function $V(\cdot)$ which satisfies Equations 2 and 3 defines an equilibrium of the dynamic depreciating license game.

Under Assumptions 1–3, we can prove that the net trade property from the two-stage depreciating license game applies to the dynamic game: net sellers set prices above their continuation values, and net buyers set prices below their continuation values.

Theorem 2. *(Dynamic net trade property) Under Assumptions 1–3, in any τ-equilibrium of the dynamic depreciating license game,*

- *For type γ with $\tau = 1 - F(\gamma)$, we have $q^*(\gamma) = \tau$ and $p(q^*(\gamma)) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*
- *For types γ with $\tau < 1 - F(\gamma)$, we have $q^*(\gamma) \geqslant \tau$ and $p(q^*(\gamma)) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*
- *For types γ with $\tau > 1 - F(\gamma)$, we have $q^*(\gamma) \leqslant \tau$ and $p(q^*(\gamma)) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}[V(\gamma') \mid \gamma]$.*

Proof. See Appendix A.4. □

As we discuss in Appendix A.4, equilibria are guaranteed to exist. However, we were only able to prove uniqueness of equilibrium under an additional assumption:

Assumption 4. *$\gamma_{t+1} \leqslant \gamma_t$ with probability 1.*

The proof of equilibrium uniqueness given Assumption 4 is presented in the online appendix. This assumption is strong, but is satisfied in all specifications we use for our calibration. In versions of the calibration in which $\gamma_{t+1} > \gamma_t$ with some probability but decreases on average, we can numerically solve for equilibria and the equilibria appear to be unique; thus, our calibration results do not appear to be very sensitive to Assumption 4.

3.4 Investment

Suppose that at the beginning of each period t, the current licensee S_t can make common-valued investment η_t in the asset at cost $c(\eta_t)$. As before, we assume that all investments are fully observable to all agents. We will allow investments to have long-term effects on common values: suppose that, in period $t' > t$, investment η_t increases the common value of the asset for all agents by some $H_{t'-t}(\eta_t)$. In Appendix A.5, we show that common-valued investment affects the equilibrium of the trading game by shifting all offered prices by some constant.

The social value of investment is the discounted sum:

$$\sum_{t=0}^{\infty} \delta^t H_t(\eta).$$

and, the social FOC sets:

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t H_t'(\eta).$$

The following proposition states that depreciating licenses distorts longer term investments more than shorter-term investments. Intuitively, if licensees make investments that pay off t periods in the future, they have to pay license fees for $t+1$ periods on their investments, generating an investment wedge of $(1-\tau)^{t+1}$ relative to the social optimum.

Proposition 1. *In any τ-equilibrium of the dynamic depreciating license game, all agents choose a constant level of investment η such that:*

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta).$$

Proof. See Appendix A.5. □

4 Calibration

In this section, we computationally solve our dynamic model under parameters chosen to match moments of various markets for durable assets. We model trade under full private ownership as the solution to our dynamic depreciating license game when the depreciation rate is set to 0. We then evaluate the effects of increasing the depreciation rate from 0 on allocative and investment welfare.

4.1 Methodology

The dynamic depreciating license game requires us to specify a number of unknowns: the discount rate δ, the distribution of entering buyer values $F(\gamma)$, the transition probability distribution $G(\gamma' \mid \gamma)$, the investment cost function $c(\eta)$, and the investment benefit functions $H_t(\eta)$.

We use the standard choice of annual discount rate $\delta = 0.95$. We assume that the distribution of entering buyer values $F(\cdot)$ is log-normal, with log mean normalized to 0. The log standard deviation σ is a spread parameter controlling the amount of dispersion in idiosyncratic values. While this F does not satisfy boundedness, as required by Assumption 1, we will approximate F using a bounded grid distribution.

We will use a variety of stochastic decay processes for the transition distribution $G(\gamma' \mid \gamma)$. In our baseline calibration, we will assume that if an agent has value γ_t in period t, her period $t+1$ value is $\chi\gamma$, where χ has a beta distribution with mean β. Thus, values decay by a factor β in expectation in each period. We will also show results from a variety of specifications using a bimodal mixtures of beta distributions, and a specification in which values either stay constant or jump to 0. These choices of stochastic decay distributions are designed to vary the dispersion in the stationary distribution of seller values; our results show that this dispersion significantly influences the welfare gains from implementing depreciating licenses.

Given these choices for F and $G(\gamma' \mid \gamma)$, the parameters to be determined are the log standard deviation σ of F, and various shape parameters for the decay processes $G(\gamma' \mid \gamma)$. For each transition distribution, we fix all but one parameter affecting the mean of $G(\gamma' \mid \gamma)$ in advance; these choices for different specifications of $G(\gamma' \mid \gamma)$ are described in Appendix B.3. We then determine σ and the parameter moving the mean of $G(\gamma' \mid \gamma)$ by choosing the values of two moments.

First, we target the relative standard deviation of buyer valuations. A large empirical literature studies static auctions for various use rights for government resources; these papers tend to find fairly high dispersion in the willingness-to-pay of different buyers for identical assets. The ratio of the standard deviation of idiosyncratic buyer values to its mean is found to be roughly 0.5 for timber auctions (Athey, Levin and Seira, 2011), 0.18 for highway procurement contracts (Krasnokutskaya and Seim, 2011), and roughly 0.2 for oil drilling rights (Li, Perrigne and Vuong, 2000). For our baseline specification, we will require that the standard deviation of the allocative component of equilibrium willingness-to-pay of entering buyers is 0.20 times its mean. We will refer to this as the $SDmean$ moment.

Second, we require the annual turnover rate in equilibrium when $\tau = 0$ in our baseline specification to be 5%; this is likely to be fairly low for many of the assets we consider, hence this choice should give a conservative estimate the extent of allocative distortions, and thus the welfare gains from depreciating licenses. We will refer to this as the $saleprob$ moment.

Increasing the rate of value decay lowering the mean of $G(\gamma' \mid \gamma)$ should increase the efficient probability of sale, the equilibrium probability of sale, as well as the optimal license depreciation rate. Increasing the lognormal standard deviation σ should increase the dispersion of values, the dispersion of prices, and the total achievable allocative welfare gains. Thus, intuitively, the $saleprob$ moment should be matched mostly by the parameter controlling the posterior mean of $G(\gamma' \mid \gamma)$, while the $SDmean$ moment should be matched mostly by σ. We confirm these intuitions and show the effects of changing moment values on our calibration results in Figure 4, which we discuss further in Subsection 4.2 below.

We will assume that investment has geometrically depreciating value over time: $H_t(\eta) = \theta^t \eta,\ \theta < 1$. For our calibrations, we will set $\theta = 0.85$, which is similar to depreciation rates from in the literature on capital depreciation (Nadiri and Prucha, 1996). We will report results for our baseline specification assuming that investment value constitutes a fraction 0%, 10%, 40% or 70% of total average asset value; we find that optimal depreciation rates and welfare gains are relatively insensitive to the total value of investment welfare. In Appendix B.1, we derive analytical expressions for the equilibrium investment level and investment welfare for any value of τ. In Appendix B.2 we describe further details of the numerical procedure we use to analyze the game and solve for equilibria.

4.2 Results

In Figure 2, we plot the equilibrium stationary distribution of use values for different values of τ in the baseline specification. As we increase τ from 0 to 15%, probability mass moves from relatively low values towards higher values, as a result of lower markups and increased frequency of sales to high-value entering buyers. However, starting at around 5%, increasing τ also moves mass from the highest values towards somewhat lower values, though this effect does not become pronounced until τ reaches 15%. Intuitively, this is because the highest value licensees set prices below their values, causing the license to occasionally be purchased by buyers with values lower than that of the licensee.

In Figure 3, we show the behavior of various quantities in stationary equilibrium as functions of τ, assuming that investment is 40% of total asset value. The topmost panel shows allocative, investment and total welfare, in units of percentages of the average license transaction price when the depreciation rate is 0. Allocative welfare is maximized at a $\tau = 8.7\%$, which optimizes the trade-off between moving mass away from low value quantiles and away from the highest quantiles. The horizontal line labeled $\mathtt{eff\_alloc\_welfare}$ represents the max possible allocative welfare, calculated by solving for the steady-state distribution of use values assuming that the asset is always transferred to the agent with higher value in any period. The allocatively optimal depreciation rate τ_{alloc} achieves over 80% of the total possible allocative welfare gains. If we take into account investment welfare, the optimal depreciation rate is 4.0%, and this increases total welfare by 5.4% of the baseline average asset price. Investment losses are not globally convex, likely due to the complex interactions of depreciating licenses with persistent investment. However, allocative welfare is still concave, and thus total social welfare is also concave in τ for depreciation rates below and near the efficient depreciation rate.

The second panel shows the equilibrium sale frequency and the average quantile markup set by sellers, as well as a line of slope 1 representing the depreciation rate τ itself. When the depreciation rate is set equal to the efficient probability of trade, the equilibrium average trade probability is also equal to the efficient probability of trade, and the average quantile markup is near 0. However, the optimal depreciation rate is lower than the efficient probability of trade, likely because the right-skew of the lognormal distribution means that the losses from excessive turnover by high value sellers outweigh the gains from eliminating inefficiently low turnover rates by low value sellers.

In the third panel of Figure 3 we show the behavior of various stock/flow quantities as we vary τ. License prices rapidly decrease and revenues from license fees rapidly increase as we increase τ. Intuitively, if agents have to pay license fees at rate τ every period, this is roughly equivalent to discounting by rate $\delta(1-\tau)$; thus, increasing τ has a similar effect to increasing discounting, and rapidly lowers license prices.

In Table 1, we show results for different choices of the fraction of $Invfrac$, the share of investment value in asset prices. The optimal depreciation rate ranges from 2.1% to 8.7%. Total gains from depreciating licenses range from 1.8% to 12.9%. In all cases, setting the depreciation rate equal to 2.5%, or half the license turnover rate in existing markets, achieves most of the welfare gains from the optimal rate.

In Table 2, we report results from other specifications for the transition process $G(\gamma' \mid \gamma)$.

Qualitatively, depreciating licenses perform worse under transition processes which induce more dispersion in seller values. However, for all transition processes we tried, depreciating license gains can achieve at least 50% of all achievable welfare gains, amounting to at least 1% of asset prices. Moreover, depreciation rates set at 2.5% achieve most of the gains from setting the optimal depreciation rate.

We propose a simple and conservative rule-of-thumb for depreciating license design: depreciation rates should be set to roughly half the rates of trade in markets for similar assets owned under long-term or perpetual licenses. From Table 1, across most specifications, the optimal depreciation rate is close to the private market trade rate of 5%, but a rate half this achieves most welfare gains. The probability of trade in equilibrium when $\tau = 0$ is lower than the socially optimal probability of trade; by further halving this amount, such a rule will tend to choose depreciation rates significantly below the allocative optimum. However, total welfare is increasing and concave in τ for depreciation rates between 0 and the optimal rate, so any depreciation rate smaller than the optimal value is welfare-improving relative to pure private ownership, and in fact fairly low rates can capture a large fraction of all possible welfare gains from depreciating licenses.

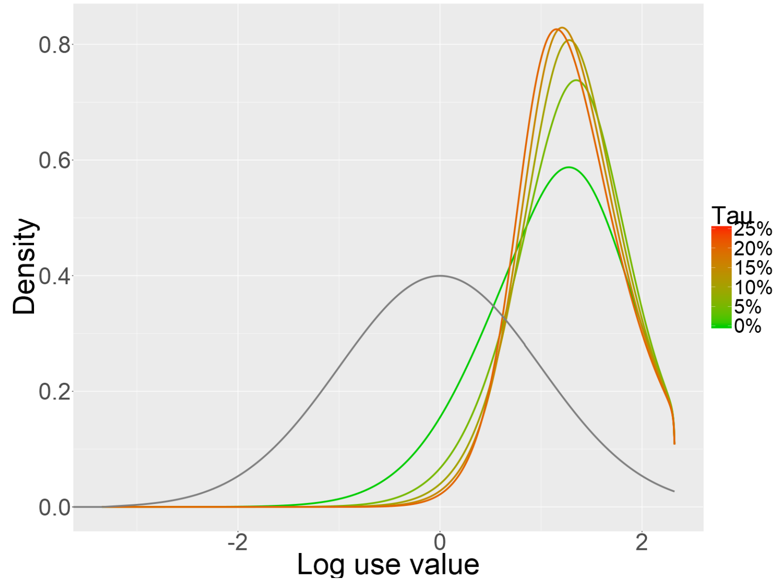
In Figure 4, we show the behavior of calibration outcomes as we change the two input moments: $SDmean$, the relative standard deviation of prices, and $saleprob$, the probability of asset trade when $\tau = 0$. As we change the $SDmean$ moment, the optimal depreciation rate increases slightly, and welfare gains increase approximately proportionately with $SDmean$. As we increase $saleprob$, welfare gains increase slightly, and optimal depreciation rates increase significantly. The allocatively optimal depreciation rate increases approximately proportionately with $saleprob$, whereas the total welfare-optimizing depreciation rate increases slightly less than proportionately. In all cases, our rule-of-thumb setting depreciation rates equal to half of observed sale probabilities achieves a large fraction of all possible welfare gains.

The particular quantities we find for welfare gains and optimal depreciation rates in our calibration will likely vary for different classes of assets; however, we emphasize two qualitative takeaways from the calibration. First, depreciating licenses can achieve welfare gains comprising a significant fraction of allocative distortions in private markets. Second, our rule-of-thumb, setting depreciation rates equal to half the observed rates of asset trade, chooses conservative depreciation rates that achieve a large fraction of all possible welfare gains.

5 Robustness

In this section, we discuss the robustness of our results to relaxing various assumptions of our model. In Subsection 5.1, we discuss depreciating licenses when there may be multiple agents and goods. In Subsection 5.2, we discuss our assumptions regarding observability of investment. In Subsection 5.3, we discuss our assumptions regarding additive separability of allocative and

Figure 2: Stationary Use Value Distribution



Notes. Stationary distributions of log use values in trading equilibrium, for different values of the depreciation rate τ. The gray line shows the distribution of entering buyers, which is lognormal with log mean 0.

Table 1: Calibration results

Invfrac	Optimal τ	Total gain	Alloc gain	Inv loss	$\tau = 2.5\%$ gain
0%	8.60%	12.85%	12.85%	0.00%	9.30%
10%	7.00%	10.60%	11.38%	-0.79%	8.19%
40%	4.00%	5.36%	6.60%	-1.24%	4.93%
70%	2.10%	1.79%	2.47%	-0.68%	1.76%

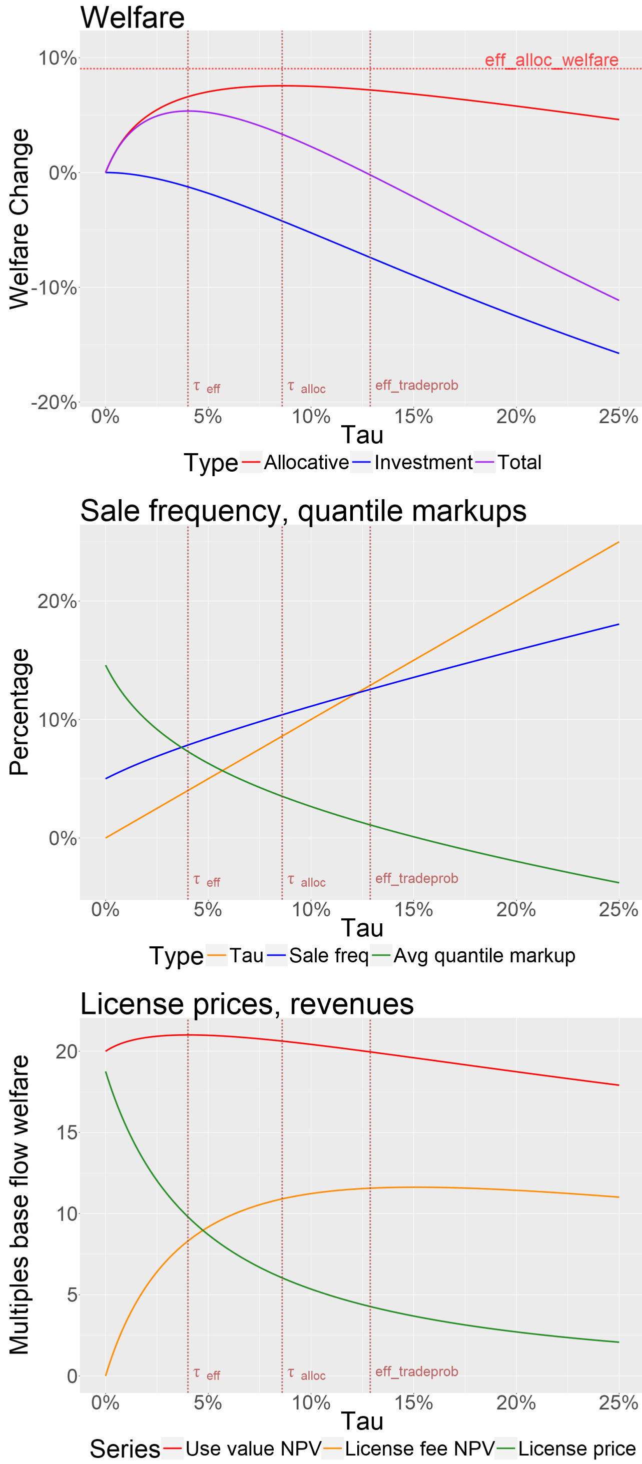
Notes. All gains are in units of percentages of the average asset transaction price at $\tau = 0$. All columns show welfare changes from the optimal τ, except for the last column, which shows the total welfare gain from imposing a 2.5% rate. $\mathtt{Invfrac}$ is the ratio of total investment value to the average asset price at $\tau = 0$.

Table 2: Alternative specifications

Transition process	Optimal τ	Total gain	Alloc gain	Inv loss	$\tau = 2.5\%$ gain	% max gain
Baseline	4.00%	5.36%	6.60%	-1.24%	4.93%	83.42%
Mixbeta	3.10%	3.37%	4.17%	-0.80%	3.31%	73.95%
Jump	1.90%	1.86%	2.18%	-0.33%	1.76%	52.92%

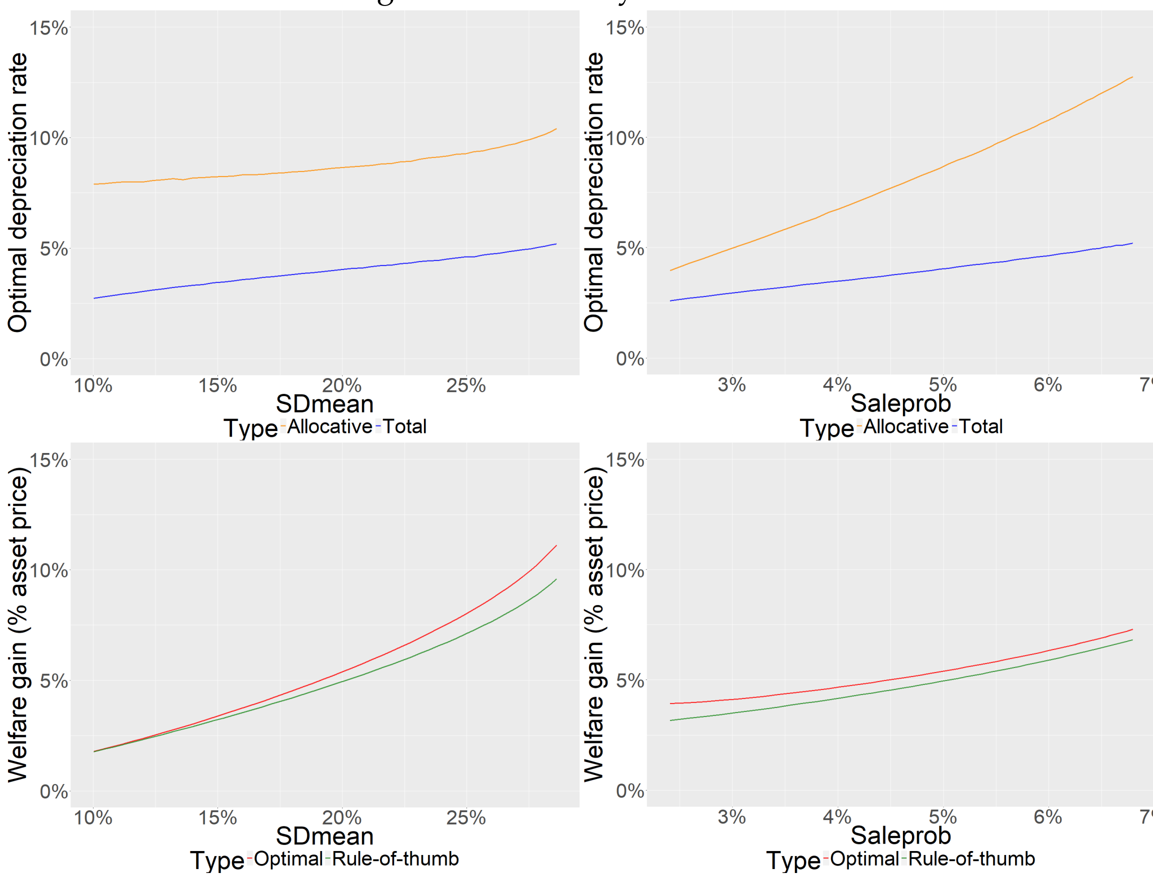
Notes. All gains are in units of percentages of average asset prices at $\tau = 0$. For all specifications, we assume $\mathtt{Invfrac} = 0.4$, so investment is 40% of average asset price. The "% max gain" column shows what fraction of total possible allocative welfare gains, under the social planner's optimum assuming all welfare-improving trades are made, are captured by the allocatively optimal depreciation rate.

Figure 3: Comparative statics vs τ



Notes. In the first panel, all welfare changes are in units of percentages of the average asset transaction price at $\tau = 0$. In the third panel, license prices are average prices of licenses conditional on sale occurring.

Figure 4: Sensitivity to moments



Notes. In the left column, the $SDmean$ moment is varied holding fixed $Saleprob = 0.05$. In the right column the $Saleprob$ moment is varied holding fixed $SDmean = 0.2$. The top row shows the allocative and total optimal depreciation rates, and the bottom row shows the welfare gain from the optimal and rule-of-thumb depreciation rates, as a fraction of the initial asset price.

investment welfare. In Subsection 5.4, we show that depreciating licenses lower, but do not distort, private-valued investments.

5.1 Multiple agents and goods

Our analysis above assumes that there is a single potential buyer of the license. In many settings, several bidders may be competing to buy the license. In such settings, we might implement depreciating licenses by allowing potential buyers to participate in an auction for the asset, with reserve price equal to the value announcement of the current licensee. We briefly analyze optimal depreciation rates in such an auction model.

Suppose the license belongs to S, and assume for simplicity that $\gamma_S = 0$. There are multiple potential buyers $B_1 \ldots B_n$, with values drawn i.i.d. from distribution F. The license is sold in a second-price auction, where S can set a reserve price p. S pays a license fee based on her reserve price p. Let y_1 represent the highest bid, and let y_2 represent the second-highest bid. S's objective function is

$$\pi_S = y_2 1_{y_1, y_2 > p} + p 1_{y_1 > p > y_2} + \eta 1_{y_1, y_2 < p} - p\tau.$$

Taking expectations over y_1, y_2 and then taking derivatives with respect to p yields

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = P\left(y_1 > p > y_2\right) - \tau - m\frac{dP\left(y_1, y_2 < p\right)}{dp},$$

where, as in Section 2, we define the markup $m \equiv p - \eta$.

Setting this to 0 and rearranging, we have that

$$m\frac{dP\left(y_1, y_2 < p\right)}{dp} = P\left(y_1 > p > y_2\right) - \tau.$$

This implies that the markup m depends on the difference between τ, the depreciation rate, and $P\left(y_1 > p > y_2\right)$, the probability that the asset is sold at the reserve price, rather than the total probability of sale. Intuitively, this is because S's reserve price announcement only affects her profits if the reserve price is binding. Thus, if there are multiple buyers, allocatively optimal depreciation rates can be lower than the probability of sale. In the limit as buyer competition eliminates the welfare loss from seller markups, optimal depreciation rates decrease to 0. However, many of the assets we consider trade infrequently in secondary markets, hence it is unlikely that competition between high-value buyers will be sufficient to eliminate all bargaining inefficiency.

Throughout the paper, we study trade of a single asset. This can be thought of as a reduced-form representation of a richer model in which agents with unit demand consider buying and selling multiple differentiated products; for example, agents may consider purchasing spectrum use rights at different frequencies in different regions, or oilfields with different capacities and drilling costs. In this richer model, when bargaining over a given asset, both the buyer and the seller have the outside option of purchasing any other asset in the market, though the seller may face transaction or adjustment costs for doing so. The buyer's willingness-to-pay and the seller's minimum acceptable sale price in bargaining for the asset in question take into account these outside options; thus, in our reduced-form model, we do not explicitly model transaction or adjustment costs or the availability of outside options.

An important simplification of our model is that agents have unit demand, and we do not consider preferences for bundles of goods. Posner and Weyl (2017) propose an extension where licensees could determine their own packaging of licenses they hold. Analyzing such a proposal is interesting, but beyond the scope of our analysis here.

5.2 Observability

Throughout the paper, we have assumed that investment is commonly observed by agents; in such a setting the government could conceivably achieve first-best investment incentives by directly rewarding observed investment. We follow the literature on property rights and incomplete contracts (Grossman and Hart, 1986; Hart and Moore, 1990) in assuming away this possibility; see Maskin and Tirole (1999) and Hart and Moore (1999) for further discussion. We believe that this assumption is particularly justified in the context of license design. The government structures licenses for resources such as radio spectrum to dictate the rules of asset use and trade over long time horizons; it may be difficult to predict optimal uses and investment for these assets as technology shifts over time, and granting the discretion necessary to adapt to changing circumstances to agencies regulating licenses may create opportunities for capture and corruption. Moreover, license design often simultaneously affects large classes of heterogeneous assets; the nature of optimal investment may be very different, for example, for oilfields or radio spectrum at different geographical locations. While the local costs and benefits of common-valued investments, such as oil wells or radio antenna, are likely relatively well-observed to market participants, asset heterogeneity makes it difficult for the government to observe and enforce optimal investment separately for each individual asset. Our mechanism is a simple system that gives users of heterogeneous assets robust, though suboptimal, incentives for investment.[5]

Government policy towards resource rights has attempted to directly enforce common-valued investment to the extent possible; examples include the FCC's complex buildout and coverage requirements for radio spectrum licensees (2017*a*), and various systems of quantity limits and minimum size requirements for recreational fishing (2017*b*). If common-valued investment is observable and homogeneous enough to be directly enforced, property rights are not necessary

[5] This argument loosely relates to the literature on complexity-based foundations of incomplete contracts (Segal, 1999), as well as recent work on robustness-based justifications of simple contracts (Carroll, 2015).

for providing investment incentives, and total welfare is maximized by selling use rights in frequent auctions; this is effectively a rental system, as is commonly used to allocate durable goods such as cars and hotel rooms. When governments can partially enforce investment, the investment distortion is lessened and optimal depreciation rates are higher.

A related issue is the interaction of depreciating licenses with asymmetric information between buyers and sellers, that is, the lemons problem (Akerlof, 1970). While this is an important and potentially complex issue, we will not analyze it in this paper; see Posner and Weyl (2017) for a more extensive discussion. We can briefly discuss a simple extreme case – in settings where asset quality is imperfectly observed but persistent over time, depreciating licenses decrease the potential losses to the buyer from purchasing a lemon, since the depreciating license has a significantly lower price than a perpetual license. Hence, the total monetary risk to buyers from unobserved asset quality is lower for assets owned under depreciating licenses, which may decrease the extent of market breakdown due to the lemons problem.[6] Depreciation also penalizes attempts by a licensee to signal quality by raising price, potentially lessening the tendency of adverse selection to impede asset turnover.

5.3 Additive separability

For simplicity, we have analyzed a model in which investment and allocative efficiency are additively separable, but our results are mostly robust to relaxing this assumption. Since the second-stage allocative game is played conditional on arbitrary first-stage investment, the net trade property holds irrespective of the structure of investment. We modelled common-valued investment as additive to both buyer and seller values, and we analyze private-valued investments in Subsection 5.4 below; these two can be combined to approximate any kind of investment whose values to agents do not depend on their types.

Considering investments with differential benefits or costs for different agent types is more complex, and we will not attempt to formally analyze such settings. We can briefly consider some settings in which allocative efficiency and investment are partially complementary. In some settings, agents who have higher idiosyncratic values may optimally make larger investments into the assets; for example, more popular wireless carriers may optimally invest more in building efficient spectrum infrastructure. In other settings, agents' investment costs may differ; for example, different firms may have different costs for drilling an oilfield. In settings where allocative value and investment are complementary, improving allocation through depreciating licenses will tend to increase investment, counteracting the negative direct effect of license fees on investment. Hence, assigning assets using depreciating licenses could actually increase the value of total investments made by license holders.

[6] This echoes George (1879), who argues that high taxes on land would discourage land speculation.

5.4 Selfish investments

Suppose the licensee S can invest in increasing her private value for the asset: she can increase her own use value for the asset by λ at cost $c(\lambda)$. As before, B has value $\eta + \gamma_B$ for the asset, and all other features of the game are identical to those in Section 2. Fixing γ_S and λ, S's second stage profits are once again:

$$\pi_S(\lambda, \gamma_S, \tau) \equiv \max_q p(q)\, q + (\eta + \gamma_S + \lambda)(1 - q) - p(q)\, \tau.$$

Let $q^*(\tau, \gamma_S)$ represent S's choice of q for any given τ, γ_S. In the investment stage, S chooses λ to maximize $\pi_S(\lambda, \gamma_S, \tau) - c(\lambda)$. But, using the envelope theorem, we have that

$$\begin{aligned} \frac{d\pi_S(\lambda, \gamma_S, \tau)}{d\gamma} &= \frac{\partial}{\partial \lambda}\left[p(q^*(\tau, \gamma_S))\, q^*(\tau, \gamma_S) + (\eta + \gamma_S + \lambda)(1 - q^*(\tau, \gamma_S)) - p(q^*(\tau, \gamma_S))\, \tau\right] \\ &= 1 - q^*(\tau, \gamma_S). \end{aligned}$$

Hence, the first-order condition for S's choice of private-valued investment λ is

$$c'(\lambda) = 1 - q^*(\tau, \gamma_S).$$

This equation defines the constrained efficient level of investment, conditional on S keeping the license with probability $1 - q^*(\tau, \gamma_S)$. In words, depreciating licenses leads licensees to set lower prices and sell their licenses more often; licensees correspondingly reduce private-valued investments, in a manner that is both privately and socially optimal. Hence depreciating licenses do not distort license holders' choices of private-valued investments.

In addition to private-valued investments, we might also consider investments by licensees that affect the value of the asset to potential buyers, but not to the licensees themselves. A natural objection to depreciating licenses is that licensees who are unwilling to sell their assets may set low prices to minimize license fee payments, and then purposefully damage their assets, making them less attractive to buyers in order to deter purchase. However, at optimal or rule-of-thumb depreciation rates, τ is smaller than the probability of sale $q^*(\tau, \gamma_S)$ for most licensees. Thus, most licensees are net sellers of their assets in any given period; if marginal buyers' values increase, licensees gain more from increased sale prices than they lose from increased license fee payments. As a result, most licensees have net positive incentives to make investments which increase the value of the asset only for potential buyers. Another implication is that, under reasonable depreciation rates, most licensees set prices above their values, and thus receive higher total utility from selling their assets than keeping them; hence most trades induced by depreciating licenses will make both buyers and sellers better off.

Certain assets, such as trademarks, may have relatively homogeneous low values to all potential licensees ex ante. Most of the value from these items comes from private-valued investments that their licensees make; for example, websites build brand capital as their users become accustomed to visiting a given domain name over time. In such contexts, both the socially efficient probability of asset trade and the turnover rates of assets in market equilibrium will tend to be low. Rule-of-thumb choices of depreciation rates will be correspondingly low. While depreciating licenses cannot greatly improve allocative efficiency in such contexts, they will not lead to serious adverse effects on the efficiency of asset trade. Depreciating licenses with rule-of-thumb depreciation rates are thus adaptive to the primitives of asset markets, playing a large role only when high equilibrium turnover rates suggest that efficient dynamic reallocation is an important concern.

Conversely, in a world in which depreciating licenses were ubiquitous, the overall level of private-valued investments that organizations and individuals make in assets that they use may decrease significantly. For example, if spectrum use rights at different frequencies were traded regularly in liquid markets for depreciating licenses, device makers would have increased incentives to develop devices that are interoperable at many frequencies, rather than tuned to any specific frequency band that they have acquired long-term use rights for. Our arguments in this subsection suggest that these behaviors are efficient responses to lowered probabilities of long-term asset use.

6 Discussion

In this section, we relate our proposal to previous economic analysis and practices related to property rights in mechanism design, asset taxation, and intellectual property. We also compare depreciating licenses to the more common term-limit licenses, and discuss its application to resource use rights. Posner and Weyl (2017) discuss this and other potential applications of depreciating licenses, as well as its relationship to existing legal institutions. Milgrom, Weyl and Zhang (2017) discuss in detail a proposal to use depreciating licenses to allocate priority access rights for the 3.5GHz radio spectrum band.

6.1 Mechanism design

This paper is most closely related to Plassmann and Tideman (2008, 2011)'s work on self-assessment, which was inspired by Harberger (1965)'s original proposal. The primary distinction between our proposal and previous work is that the existing literature proposes self-assessment for infrequent cases of government takings, while we propose self-assessment as part of a license design to promote decentralized resource reallocation between private users. The static environment studied by Plassmann and Tideman (2008) is similar to our static model and they prove a theorem similar to our net trade property. However, because of there interest in one-off purchases, they do not consider the dynamic issues, such as capital-investments, asset prices and license turnover rates, on which we focus. Interestingly, this more complicated dynamic setting may actually make self-assessment easier to implement as the fee rate can be set based on observed license turnover rates, as would be impossible in the context of one-time government takings and the flexibility for private agents to take property may be a more effective enforcement device than government takings. In fact, Chang (2012) found that when a self-assessment system with only government takings was implemented in Taiwan, it was subject to significant government lenience and corruption.

This paper is also related to a body of work that analyzes the role of property rights in asymmetric information bargaining problems (Cramton, Gibbons and Klemperer, 1987; Segal and Whinston, 2011, 2016). This literature focuses on optimal mechanisms, while we study a specific extensive form, in which sellers commit to the prices they announce. While this extensive form could be relaxed in the implementation of depreciating licenses (e.g. sellers might be allowed to post a price, make self-assessed fee payments, and then bargain with potential buyers for a sale below this price), our focus is not on solving for an optimal mechanism. Instead our interest is in proposing an analyzing a simple mechanism that achieves most of the gains possible from the larger class of mechanisms and we hope is robust; see Posner and Weyl (2017) for a discussion of robustness. More sophisticated schemes that achieve higher welfare levels may be possible in certain settings, but we leave the analysis of such schemes and their robustness to future work.

6.2 Asset taxation

The depreciating license system resembles, and is inspired by, a class of proposals for self-assessed taxation of land and other assets. Discussion of such self-assessment systems dates back to at least ancient Greece (Posner and Weyl, Forthcoming). More broadly, a number of schemes have aimed to use market outcomes to assess the values of assets for tax purposes; an example is the common practice of setting property taxes based on the most recent sale price of a house.[7] A problem common to most market-based valuation schemes is that nontrivially large taxes based on market outcomes influence the behavior of market participants.

The self-assessment scheme underlying the depreciating license design was first suggested by Harberger (1965) as a way to enforce property taxes. He informally noted, echoing George (1879)'s arguments for common ownership of land, that this might improve allocative efficiency. To our knowledge, this observation was never formalized. In fact, following up on Harberger's work, Levmore (1982) wrote that "It is perhaps unfortunate that these... effects to self-assessment [on turnover rates] exist," and other critiques of self-assessed taxation (Epstein, 1993; Chang, 2012) also suggest that the effects of self-assessment on market outcomes are generally undesirable.

Rather than using self-assessment to value assets for taxation, we propose to use self-assessed

[7] California's Proposition 13 is partially based on this principle; see for example Wasi and White (2005).

license fees to increase the efficiency of license trade. We show that the "side effects" of self-assessment on asset turnover are governed by a simple economic intuition, the net trade property: asset owners announce prices above or below their values depending on whether the self-assessed tax rate is lower or higher than the probability of license turnover. Self-assessment will therefore not typically perfectly reveal private values for tax purposes, since no fixed tax rate gives all license owners incentives for truthful value announcement. However, self-assessment in the context of depreciating licenses can robustly improve allocative efficiency, since any license fee rate lower than the license turnover rate will induce most asset owners to announce prices closer to their values.

6.3 Intellectual property

Since intellectual property is non-rivalrous in consumption, the investment-allocation tradeoff from property rights is particularly clear: the socially optimal allocation is to allow all parties to use all innovations at no cost, but such a system gives innovators no incentives to develop innovations. A fairly large literature has addressed the question of optimal ownership rights over intellectual property, largely finding that partial ownership systems, such as limited-term patents, are optimal. In a sense, our argument in this paper is that a similar allocative-investment tradeoff is relevant for many assets which are rivalrous in consumption, and a similar partial ownership system, depreciating licenses, improves on the extremes of full private or common ownership.

Depreciating licenses qualitatively resemble patent buyout schemes (Kremer, 1998; Hopenhayn, Llobet and Mitchell, 2006), under which innovators' exclusivity rights can be purchased by entrants or the government under certain circumstances. Depreciating licenses could be useful to help ensure turnover of licenses at fair prices to future investors in sequential innovations, in the spirit of the mechanisms studied by Hopenhayn, Llobet and Mitchell, or to facilitate patent pools/avoid thickets. Depreciating licenses, however, do not address market failures from the non-rival nature of intellectual property; see Weyl and Tirole (2012) for a more elaborate partial property mechanism with this aim.

6.4 Term-limit licenses

Use rights for most natural resources are typically assigned using term-limit licenses. The tradeoff in the choice of term limit is similar to the choice of depreciation rate in our scheme: shorter term limits improve allocative efficiency at the cost of investment incentives. However, depreciating licenses are likely to dominate term limit licenses, in the sense that they can achieve better allocative efficiency for a given level of investment incentives.

Term limit licenses grant full property rights to licensees for a set period of time and no property rights thereafter. This may be undesirable in many settings – for example, a fishery licensee at the end of her term has no stake in the future of the fishery, hence may dramatically overfish to the point of destroying the resource. In contrast, depreciating licenses can achieve significant allocative efficiency gains while maintaining fairly large ownership stakes over long time horizons. Depreciating licenses also have the benefit of conferring stationary property rights over time, which is desirable both for investment and allocative efficiency. Under term-limit licenses, trade is fully efficient when license terms end, but license holders can charge high markups, blocking or delaying many valuable trades, in all other periods. Depreciating licenses induce time-constant lower markups; in each period, some welfare-improving trades are blocked, but buyers of sufficiently high value and sellers of sufficiently low value are able to trade quickly regardless of when they meet. Similar arguments suggest that small time-stationary distortions to investment incentives are generally preferable to occasional large distortions.[8]

7 Conclusion

We propose depreciating licenses primarily as a system for assigning use rights for natural resources; we could alternatively characterize these as assets that correspond to *land* within the three classical factors of production. The defining feature of land relative to capital is that it is not created as the product of labor, hence it has no natural owner ex ante from either an efficiency or equity perspective. The allocation of these resources between competing uses is thus a ubiquitous problem for government policy. Modern economic thought has favored solving this allocation problem using auctions (Coase, 1959), and a large body of work has studied the optimal design of static auctions for allocating resource use licenses. Comparatively little work has analyzed the optimal design of the licenses themselves.

As we discuss above, depreciating licenses significantly improve upon term-limit or perpetual licenses in navigating the tradeoff between allocative and investment efficiency. The depreciating license system is also simple to implement – after licenses are initially allocated, the system is essentially self-regulating. Government intervention is unnecessary unless large-scale license redesign, such as land or spectrum repackaging, is required. In these cases, the government can simply purchase depreciating licenses at the prices announced by their licensees, repackaging and then auctioning new licenses to replace them. Thus, depreciating licenses have the potential to replace the centralized auction procedures used at present for resource allocation with a decentralized market-like system, in which resource allocation continuously adapts to changes in the technological and competitive environment over time.

In this paper, we have proposed depreciating licenses as a simple and robust system for assigning resource use rights which improves on the more common system of term-limit licenses.

[8] This argument resembles that of Gilbert and Shapiro (1990), who show in the context of intellectual property rights that, since the social welfare loss from increased monopoly power in any period is likely to be convex, policies which grant innovators time-invariant decreased monopoly power over intellectual property are often preferable to policies granting full monopoly power for a limited period of time.

Depreciating licenses only have a single tuning parameter, the depreciation rate; a simple rule-of-thumb is that depreciation rates for a given class of assets should be set equal to approximately half the turnover rate of similar assets in private markets. We show that depreciating licenses can achieve significant welfare gains over perpetual licenses – approximately 4% of asset prices in our baseline specification.

We have abstracted away from a number of important issues in this paper. We assumed that the common value of assets is fully observed by all participants, ignoring "lemons" problems from asymmetric information about common values. We abstract away from the repeated strategic interactions between agents that would arise in markets where the pool of potential licensees is small. We assumed that only the current user of the asset can make investments that affect the common value of the asset; in settings such as land allocation, agents besides the asset owner may be able to take actions, such as emitting pollution which affect the value of an asset. We model repeated trade of a single asset; certain assets such as radio spectrum display high degrees of complementarity, which may cause much greater losses from market power than occur with single assets because of hold-out problems (Kominers and Weyl, 2012). We hope that future research will analyze the performance of depreciating licenses in settings where these assumptions are not satisfied. Finally, this paper is only a first step in examining the possibilities for augmenting markets with alternative systems of property ownership, and we hope that future work continues to explore this promising area.

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Appendix

A Proofs and derivations

A.1 Net trade property

In this subsection, we prove Theorem 1, the net trade property. First, suppose $\tau = 1 - F(\gamma_S)$.

- If S chooses sale probability $q = \tau$, she makes no net trades, and receives 0 variable profits. Moreover, the markup is $M(q) = F^{-1}(1-\tau) = \gamma_S$, so that also $M(q) - \gamma_S = 0$.

- If S chooses a higher sale probability, so that $q - \tau > 0$, we have $M(q) \leqslant \gamma_S$, so variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$. In words, S becomes a net seller at a price lower than her value.

- Symmetrically, if S chooses a lower sale probability $q - \tau < 0$, she becomes a net buyer at a price higher than her value, and once again variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$.

Now suppose that $\tau < 1 - F(\gamma_S)$.

- By the first part of the Theorem, license owners with higher values $\gamma'_S = F^{-1}(1-\tau)$ have $\tau = 1 - F(\gamma'_S)$, hence choose $q^*(\gamma'_S, \tau) = 1 - F(\gamma^*_S)$. By construction, $\gamma_S \leqslant \gamma'_S$; since the variable profit function is supermodular in q and $-\gamma_S$, $q^*(\gamma_S, \tau) \geqslant q^*(\gamma'_S, \tau) = \tau$.
- By the first part of the Theorem, if we set a lower rate $\tau' = 1 - F(\gamma_S)$, we have $q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$ and $M(q^*(\gamma_S, \tau')) = \gamma_S$. By construction, $\tau \leqslant \tau'$. Since $M(q)$ is a decreasing function, the variable profit function is supermodular in q and τ, hence $q^*(\gamma_S, \tau) \leqslant q^*(\gamma_S, \tau') = 1 - F(\gamma_S)$. This implies that $M(q^*(\gamma_S, \tau)) \geqslant M(q^*(\gamma_S, \tau')) = \gamma_S$.

An analogous argument shows that $\tau > 1 - F(\gamma_S)$ implies that $q^*(\gamma_S, \tau) \leqslant \tau$ and $M(q^*(\gamma_S, \tau)) \leqslant \gamma_S$.

A.2 Myerson regularity and existence of $\frac{\partial q^\star}{\partial \tau}$

In this subsection, we prove our statement in Subsection 2.2 that Myerson (1981)'s regularity condition is sufficient for $\frac{\partial q^\star}{\partial \tau}$ to be finite for all depreciation rates below the efficient probability of sale $\tau = 1 - F(\gamma_S)$. Myerson's regularity condition states that marginal revenue is monotone. A monopolist seller with value γ_S for the asset has revenue $(M(q) - \gamma_S)\, q$. Taking a derivative yields $M'(q)\, q + (M(q) - \gamma_S)$. Taking the second derivative, we have

$$2M'(q) + M''(q)\, q < 0.$$

Now consider the monopolist's problem under a depreciation rate $\tau < 1 - F(\gamma_S)$. By Theorem 1, $q(\tau) > \tau$; hence, $0 < q(\tau) - \tau < q(\tau)$. We want to show the following quantity exists:

$$\frac{\partial q^*}{\partial \tau} = \frac{M'(q)}{2M'(q) + M''(q)(q - \tau)}.$$

So we have to show that the denominator is bounded away from 0. From our full-support assumptions on ϵ, $M'(q)$ exists and is negative for all q. If $M''(q) \leqslant 0$, we know $q - \tau > 0$, so $M''(q)(q - \tau) \leqslant 0$, and the numerator and denominator are both strictly negative; hence, their ratio is positive and nonzero and $\frac{\partial q^*}{\partial \tau}$ exists. So suppose $M''(q) > 0$. Then

$$2M'(q) + M''(q)(q - \tau) < 2M'(q) + M''(q)\, q < 0$$

Where we first use that $0 < q(\tau) - \tau < q(\tau)$, and then apply Myerson regularity. Hence, the denominator $2M'(q) + M''(q)(q - \tau)$ is strictly negative, and the ratio $\frac{\partial q^*}{\partial \tau}$ exists and is positive.

A.3 Concavity of social welfare in τ

In this subsection, we discuss conditions on the cost and demand functions such that social welfare is a concave function of the depreciation rate. From the text, the marginal benefit of increasing the depreciation rate is $M\left(q^{\star}\left(\tau\right)\right)\rho\left(q^{\star}\left(\tau\right)\right)$ and the marginal cost is $\Gamma'\left(1-\tau\right)\tau$. Recalling that $\rho=\frac{\partial q^{\star}}{\partial\tau}$, the second-order condition is

$$M'\rho^{2}+\rho' M\rho+\Gamma''\tau-\Gamma'.$$

The first term is always negative ($\rho>0>M'$) and represents the quadratic nature of the allocative distortion discussed in the text. The final term is always negative as $\Gamma'>0$ and represents the quadratic nature of the investment distortion. The two central terms are more ambiguous. However, Fabinger and Weyl (2016) argue ρ' is typically negative for most plausible demand forms (those with a bell-shaped distribution of willingness to pay, as we assume in most calibrations) and thus, given that $M,\rho>0$, the second term is likely to be negative as well. The third term is ambiguous. By the inverse function theorem, given that $\Gamma=\left(c'\right)^{-1}$,

$$\Gamma''=-\frac{c'''}{\left(c''\right)^{3}}.$$

Assuming a convex cost function, this quantity is negative if and only if $c'''>0$. Thus, a grossly sufficient condition (assuming ρ') for the first-order conditions to uniquely determine the optimal depreciation rate is that $c'''>0$. However, note this term is multiplied by τ, which is typically below 10% in our calibrations. Thus c''' would have to be quite negative indeed to cause the problem to be nonconvex.

A.4 Dynamic net trade property

In this subsection, using Assumptions 1–3, we prove Theorem 2, the net trade property for the dynamic depreciating license game.

A.4.1 $V(\cdot)$ is strictly increasing

To begin with, we show, that any equilibrium $V\left(\cdot\right)$ must be increasing. This will allow us to consider only increasing candidate $\hat{V}\left(\cdot\right)$ functions for the remainder of the proof.

Claim 1. In any stationary equilibrium, $V\left(\gamma\right)$ is strictly increasing.

Proof. Consider a stationary equilibrium described by value function $V\left(\cdot\right)$. This defines an inverse demand function $p_{V(\cdot),F(\cdot)}\left(q\right)$. We will define the following Bellman operator for candidate value functions $\hat{V}(\cdot)$ for the seller's optimization problem:

$$\mathcal{R}\left[\hat{V}(\cdot)\right] \equiv \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right]. \tag{4}$$

Note that $\mathcal{R}$ fixes the demand distribution $p_{V(\cdot),F(\cdot)}$ at the true equilibrium value function $V(\cdot)$, and only depends on $\hat{V}$ through the seller's continuation value $\delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$. As a result, $\mathcal{R}$ is a standard Bellman equation satisfying Blackwell's sufficient conditions for a contraction mapping.

Consider a candidate value function $\hat{V}(\cdot)$ which is nondecreasing in γ. Supposing $\tilde{\gamma} > \gamma$, the single-period value $(q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\gamma$ is strictly higher under $\tilde{\gamma}$ relative to γ for all q, and the continuation value $\delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$ is weakly higher under $\tilde{\gamma}$, since $G(\gamma' \mid \tilde{\gamma}) >_{FOSD} G(\gamma' \mid \gamma)$. Hence, $\mathcal{R}\left[\hat{V}(\cdot)\right](\tilde{\gamma}) > \mathcal{R}\left[\hat{V}(\cdot)\right](\gamma)$, hence $\mathcal{R}\left[\hat{V}(\cdot)\right]$ is strictly increasing in γ. Hence $\mathcal{R}$ takes nondecreasing $\hat{V}$ functions to strictly increasing $\hat{V}$ functions; hence the true value function V, which is the unique fixed point of $\mathcal{R}$, must be strictly increasing in γ. □

A.4.2 The pseudo-Bellman operator $\mathcal{T}$

As we discuss in Subsection 3.3, stationary equilibria of the dynamic depreciating license game must satisfy two conditions. First, the sellers' value function must be satisfied for any γ:

$$V(\gamma) = \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right]\right].$$

Second, the WTP distribution $p_{V(\cdot),F(\cdot)}$ must be consistent with the value function $V(\gamma)$, that is,

$$p_{V(\cdot),F(\cdot)}(q) = \left\{p : \; P_{V(\cdot),F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[V\left(\gamma'\right) \mid \gamma\right] > p\right] = q\right\}.$$

We will define the following *pseudo-Bellman operator* $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}(\cdot)\right](\gamma) \equiv \max_q (q-\tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right]. \tag{5}$$

The operator $\mathcal{T}$ is similar to the seller's Bellman operator $\mathcal{R}$ in Equation 4. The difference is that $\mathcal{R}$ fixes the inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$ at its true equilibrium value, whereas $\mathcal{T}$ calculates the inverse demand distribution $p_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ assuming that buyers also act according to continuation value $\hat{V}(\cdot)$. We will likewise define the "candidate optimal sale probability function" $q^*_{\mathcal{T}}\left(\gamma;\, \hat{V}(\cdot)\right)$ assuming continuation value $\hat{V}(\cdot)$, as:

$$q^*_{\mathcal{T}}\left(\gamma;\, \hat{V}(\cdot)\right) \equiv \arg\max_q (q-\tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right].$$

In words, $\mathcal{T}\left[\hat{V}(\cdot)\right]$, $q^*_{\mathcal{T}}\left(\gamma;\, \hat{V}(\gamma_t)\right)$ and $p_{\hat{V}(\cdot),F(\cdot)}(q)$ describe the values and optimal behavior of buyers and sellers, assuming that the continuation value of a license owner of type γ in the next period is $\hat{V}(\gamma)$. Equilibria of the depreciating license game are fixed points of the $\mathcal{T}$ operator.

Since $\mathcal{T}$ characterizes the equilibrium of a game rather than a single-agent optimization problem, it is not necessarily a contraction mapping, and the standard contraction-based proofs of uniqueness in bounded discounted dynamic programs do not apply. However, equilibrium existence is not a problem – Assumptions 1, 2 and3, imply that $\mathcal{T}$ is a smooth function of $\hat{V}$, hence Brouwer's fixed point theorem implies that $\mathcal{T}$ must have a fixed point in the convex compact set of bounded $\hat{V}$ functions.

A.4.3 $\mathcal{T}$ Net trade property

In Claim 2, we show that, for any increasing candidate $\hat{V}$ function, the corresponding candidate optimal sale probability $q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right)$ respects the net trade property of Theorem 1. Since Claim 2 also applies to the true value function $V(\cdot)$ and policy function $q^*(\cdot)$, this proves Theorem 2, the dynamic net trade property.

Claim 2. ($\mathcal{T}$ net trade property) Suppose that $\hat{V}(\cdot)$ is strictly increasing. Then $q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right)$ satisfies:

- If $\tau = 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) = \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.
- If $\tau < 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \geqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.
- If $\tau > 1 - F(\gamma)$, then $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \leqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.

Proof. We prove this by constructing an analogy to a two-stage depreciating license game. Fixing any increasing candidate value function $\hat{V}$, the optimization problem for a license owner with value γ is:

$$q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right) = \arg\max_q (q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right].$$

Note that, by definition, we also have $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma)) = \gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$. In words, $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$ is the WTP of buyer quantile $F(\gamma)$, which is just the use value γ plus the continuation value $\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$. Hence, we can write $q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right)$ as:

$$q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right) = \arg\max_q (q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\,p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma)).$$

Subtracting the term $(1-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$, which does not depend on q, we get:

$$q^*_{\mathcal{T}}\left(\gamma;\hat{V}(\cdot)\right) = \arg\max_q (q-\tau)\left(p_{\hat{V}(\cdot),F(\cdot)}(q) - p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right).$$

This can be interpreted as the optimization of a variable profit function from a two-stage depreciating license game, for a seller with use value $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$ for keeping the asset, faced with buyer values distributed as $p_{\hat{V}(\cdot),F(\cdot)}(q)$, $q \sim U[0,1]$. Let $H_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ represent the distribution of $p_{\hat{V}(\cdot),F(\cdot)}(q)$; Theorem 1 implies that:

- If $\tau = 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right) = \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right)\right) = \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.
- If $\tau < 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right) \geqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right)\right) \geqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.
- If $\tau > 1 - H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right)$, we have $q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right) \leqslant \tau$ and $p_{\hat{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{J}}\left(\gamma;\, \hat{V}(\cdot)\right)\right) \leqslant \gamma + \mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$.

To complete the proof, we must show that $H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right) = F(\gamma)$. Since $p_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ is an increasing function, for a seller of value γ,

$$p_{\hat{V}(\cdot),F(\cdot)}(q) \leqslant p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma)) \iff q \leqslant F(\gamma),$$

hence,

$$H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right) = P\left[p_{\hat{V}(\cdot),F(\cdot)}(q) \leqslant p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right] = P\left[q \leqslant F(\gamma)\right] = F(\gamma).$$

While the notation is somewhat cumbersome, the intuition behind this sequence of equalities is straightforwards. The WTP function $p(\cdot)$ is an increasing function of the F-quantile q. Thus, for a license owner of type γ, the arriving buyer's willingness to pay $p_{\hat{V}(\cdot),F(\cdot)}(q)$ is lower than the license owner's own continuation value $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$ if and only if the arriving buyer has F-quantile lower than the license owner's quantile $F(\gamma)$. Thus, the probability $H\left(p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))\right)$ that the arriving buyer's WTP is lower than the license holder's WTP is exactly $F(\gamma)$.

□

A.5 Persistent investment

In order to accomodate investment, we need a nonstationary definition of equilibria in the depreciating license game. Let $\boldsymbol{\zeta} = (\zeta_0, \zeta_1, \zeta_2 \ldots)$ represent the path of common use values over time, and suppose that this is common knowledge. The use value for any agent A_t in any period is thus $\zeta_t + \gamma_t^{A_t}$. We will define the nonstationary value function $V_t(\gamma_t, \boldsymbol{\zeta})$ as the value of being a seller with type γ_t in period t, if the path of common use values is $\boldsymbol{\zeta}$. Analogously to above, we will define the inverse demand function in period t as:

$$p_{t,V_t(\cdot,\zeta),F(\cdot)}(q_t) = \left\{p_t :\; P_{V_{t+1}(\cdot,\zeta),F(\cdot)}\left[\gamma_t^{B_t} + \zeta_t + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[V_{t+1}\left(\gamma_{t+1}^{B_t}, \boldsymbol{\zeta}\right) \mid \gamma_t\right] > p_t\right] = q_t\right\}.$$

Equilibrium then requires that, in each history,

$$V_t(\gamma_t, \zeta) = \max_{q_t} (q_t - \tau) p_{V_{t+1}(\cdot,\zeta),F(\cdot)}(q_t) + (1 - q_t) \left[\gamma_t + \zeta_t + \delta \mathbb{E}_{G(\cdot|\cdot)} [V_{t+1}(\gamma_{t+1}, \zeta) \mid \gamma_t]\right]. \quad (6)$$

We conjecture an equilibrium of this game of the following form:

$$V_t(\gamma_t, \zeta) = V(\gamma) + \sum_{t'=0}^{\infty} \delta^{t'} (1-\tau)^{t'+1} \zeta_{t+t'}.$$

One can verify that if $V(\cdot)$ satisfies the "allocative" equilibrium Equation 5, then $V_t(\gamma_t, \zeta_t)$ satisfies Equation 6. Intuitively, as in the two-stage case, if the depreciation rate is τ, agent A_t only owns $(1-\tau)$ of the asset in period t. However, if the asset has some common value in period $t + t'$, agent A_t has to pay license fees t' times on the asset before enjoying its use value; hence she effectively only owns $(1-\tau)^{t'+1}$ of any common value of the asset in period t'.

For simplicity, we analyze the investment decision of the $t = 0$ agent; the problem is additive and identical for all agents in all periods, hence all agents make the same choice of investment in each period. Suppose investment level η_0 produces common value $\zeta_t = H_t(\eta)$ in the future. Agents' FOC for investment is:

$$c'(\eta_0) = \frac{\partial V_0(\gamma_t, \zeta(\eta_0))}{\partial \eta_0}.$$

This implies that

$$c'(\eta_0) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta_0), \quad (7)$$

proving Proposition 1.

B Calibration details

B.1 Persistent investment algebra

In our calibrations, we assume that investment decays geometrically at rate $\theta < 1$; that is, persistent investment η_0 generates period t value:

$$H_t(\eta_0) = \theta^t \eta_0.$$

Hence, following Equation 7 in Appendix A.5, the present value of a unit of investment is:

$$\sum_{t=0}^{\infty} \eta_0 \delta^t \theta^t (1-\tau)^{t+1} = \frac{\eta_0 (1-\tau)}{1 - \delta\theta(1-\tau)},$$

and agents' investment FOCs are thus:

$$c'(\eta_0) = \frac{1-\tau}{1-\delta\theta(1-\tau)}.$$

We will suppose that the cost function is:

$$c(\eta) = \frac{\eta^2}{2(1-\delta\theta)\, g},$$

for some value of parameter g. This is a convenient functional form which leads to a simple analytical solution. Total social investment welfare for investment level η_0 is

$$\mathit{Investment\ Welfare} = \left(\eta_0 - \frac{\eta_0^2}{2g}\right)\left(\frac{1}{1-\delta\theta}\right). \tag{8}$$

The socially optimal level of investment is $\eta_0 = g$. The maximum possible investment NPV is thus:

$$\frac{g}{2(1-\delta\theta)}.$$

As we discuss in Subsection 4.1, we choose g such that the maximum possible net present value of investment is some target fraction $invfrac$ of the average transaction price.

Given some depreciation rate τ, constant for all time, the seller's FOC for investment is:

$$\frac{\eta}{(1-\delta\theta)\, g} = \frac{1-\tau}{1-\delta\theta(1-\tau)},$$

$$\implies \eta = g\frac{(1-\tau)(1-\delta\theta)}{1-\delta\theta(1-\tau)}.$$

We can plug this into Equation 8 to calculate total investment welfare for any given value of τ.

B.2 Numerical procedures

As we discuss in Appendix A.4, the equilibria of the dynamic depreciating license game are the fixed points of the pseudo-Bellman operator $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}\right] = \max_q (q-\tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]\right].$$

We numerically solve our calibrations by iterating $\mathcal{T}$ on grid-supported F distributions. We use gradient descent with our numerical equilibrium solver to find moments σ, β to match the $SDmean$ and $saleprob$ moments, as we describe in Subsection 4.1 of the text. Given a candidate value function $\hat{V}$ and decay rate β, we can evaluate the continuation value $\mathbb{E}_{G(\cdot|\cdot)}\left[\hat{V}(\gamma') \mid \gamma\right]$

for any type γ, and thus also the inverse demand function $p_{\hat{V}(\cdot),F(\cdot)}(q)$. Thus, we can find the optimal sale probability $q^*_{\mathcal{T}}(\gamma; \hat{V})$ for any $\hat{V}$, and thus calculate $\mathcal{T}(\hat{V})$. Starting from a linearly increasing $V(\cdot)$ function, we iteratively apply $\mathcal{T}$ until convergence.

Once we have solved for $V(\gamma)$, this gives us equilibrium sale probability functions $q^*(\gamma, V)$ for every type γ. Together, the equilibrium $q^*(\gamma, V)$, the transition probability distribution $G(\gamma' \mid \gamma)$ and the distribution of entering buyer values $F(\gamma)$ define a ergodic Markov chain over values γ of the period-t owner of the asset S_t. We construct this transition probability matrix of this Markov chain, and solve for its unique stationary distribution, which we call $H_\tau(\gamma)$. We plot these stationary distributions for various values of τ in Figure 2. Total achievable allocative welfare is calculated as the average welfare from the stationary distribution of the Markov chain generated by assuming that all welfare-improving trades happen; that is, trade occurs whenever a buyer arrives with value higher than the seller.

Once we have solved for the equilibrium $V(\cdot)$, we can recover the equilibrium sale probability function $q^*(\gamma, V)$ and inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$, and we can use these, together with $H_\tau(\gamma)$, to recover the stationary averages of various quantities that we plot in Figure 3. Specifically, these quantities are averages of the following variables with respect to $H_\tau(\gamma)$:

- **Use value**: $\gamma + \eta$.
- **Sale probability**: $q^*(\gamma,\ V)$.
- **Quantile markup**: $(1 - q^*(\gamma,\ V)) - (1 - F(\gamma))$.
- **License fee NPV**: $\tau p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$.

For stationary average use values, if sale occurs in period t, we should use the buyer's use value, not the seller's, in calculating the stationary average. This is accommodated by multiplying the stationary distribution $H_\tau(\gamma)$ by a "buyer transition" adjustment matrix, which reflects the probability that seller type γ "transitions" through sale of the asset to any buyer type γ' with $p_{V(\cdot),F(\cdot)}(1 - F(\gamma')) > p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$.

For **license prices**, we observe in the real world prices only for successful transactions; correspondingly, we would like to take an average of asset prices weighted by the probability of sale for each seller value γ. Thus, average license prices in Figure 3, panel 2 are calculated as:

$$\frac{\int p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))\, q^*(\gamma, V)\, dH_\tau(\gamma)}{\int q^*(\gamma, V)\, dH_\tau(\gamma)}.$$

We include investment value in the asset price by multiplying investment flow value by a factor $\frac{1}{1-\delta(1-\tau)}$, and then adding the flow cost of investment. Note that, since license fees are collected regardless of sale, we do not weight offered prices by sale probabilities $q^*(\gamma, V)$ when we calculate average license fee revenues. Values labelled "NPV" are calculated by taking average flow values and multiplying by $\frac{1}{1-\delta}$.

For the sensitivity graphs in Figure 4, in order to vary the SDmean moment, for a grid of values of σ, we search for a value of β which keeps $\mathit{saleprob}$ at its initial calibration value while varying SDmean. Likewise, for the $\mathit{saleprob}$ graphs, we use a grid of β values, searching for σ values to vary $\mathit{saleprob}$ while holding SDmean constant.

B.3 Transition distribution details

Here, we describe the choices for $G(\gamma' \mid \gamma)$ in different specifications of our calibration. In all cases, the transition process is multiplicatively separable with respect to current-period value γ_t; that is, if an agent has value γ_t in period t, her value in period $t+1$ is $\chi\gamma_t$ for some random variable χ. We can thus describe transition distributions by describing the distribution of χ.

- In specification $\mathit{baseline}$, χ has a Beta distribution with shape parameters $20\omega, 20(1-\omega)$.
- In specification $\mathit{mixbeta}$, χ is a ω-weighted mixture of two beta distributions, with shape parameters $30 \times 0.98, 30 \times 0.02$ and $10 \times 0.25, 10 \times 0.75$.
- In specification jump, χ is a Bernoulli random variable with mean ω.

Intuitively, specification $\mathit{baseline}$ is a relatively smooth unimodal decay process, specification $\mathit{betamix}$ is smooth but bimodal, and specification jump is as disperse as possible. The more disperse transition distributions induce more disperse stationary distributions of asset owner values. Depreciating licenses function more poorly when seller values are more disperse. We believe this is because the optimal depreciation rate depends on sellers' valued; when seller values are more disperse, no single depreciation rate is close to correct for all seller types, thus sellers on average have worse incentives for truthful value revelation.