Ownership of the Means of Production*

E. Glen Weyl† Anthony Lee Zhang‡

July 2016

Abstract

Private property creates monopoly power, harming allocative efficiency. Common ownership can restore allocative efficiency, but it destroys incentives for investments in the capital value common to all potential owners of an asset. Property rights should thus balance ex-ante capital investment and ex-post allocative efficiency through partial common ownership. We propose self-assessed property taxes, with a universal right to force a sale at the self-assessed value, as a simple mechanism for implementing partial property rights in dynamic asset markets with many potential buyers. We loosely calibrate a model of self-assessed taxation to the US housing markets and find that a 5-10% annual tax rate is robustly near optimal, implying collectivization of a majority of the capital stock and increasing welfare by approximately 6%. However, the simplest application of self-assessed taxation may be to assets with limited capital investment opportunities and administratively assigned property rights, such as radio spectrum or internet addresses, where taxes should be set equal to the rate of annual asset turnover.

Keywords: property rights, market power, investment, asymmetric information bargaining

JEL classifications: B51, C78, D42, D61, D82, K11

*We appreciate helpful comments by Evan Kwerel, Eric Posner, seminar participants at the London School of Economics, Paul Milgrom's workshop, Microsoft Research, the University of Chicago Law School and especially Nicolaus Tideman. All errors are our own.

†Microsoft Research New York City, 641 Avenue of the Americas, New York, NY 10011 and Department of Economics, Yale University; glenweyl@microsoft.com, http://www.glenweyl.com.

‡Stanford Graduate School of Business, 655 Knight Way, Stanford, CA 94305; anthonyzhang0@gmail.com.

What is common to the greatest number gets the least amount of care. Men pay most attention to what is their own; they care less for what is common; or at any rate they care for it only to the extent to which each is individually concerned.

– Aristotle, *The Politics*, Book XI, Chapter 3

Property is only another name for monopoly.

– William Stanley Jevons, preface to the second edition of *The Theory of Political Economy*

1 Introduction

Private ownership of the means of production is perhaps the oldest continually maintained doctrine in mainstream economic thought, dating back to the Greek prehistory of the field and pervading contemporary thought. For example, Jacobs (1961) and de Soto (2003) argue the undermining or lack of property rights discourages investment in rich and poor countries equally. This has led Acemoglu and Robinson (2012) to consistently list property rights as the leading example of the "inclusive institutions" they argue foster economic development. On the other hand, the founders of contemporary economic analysis (Jevons, 1879; Walras, 1896) believed that private property was inherently in conflict with the market principle at the heart of their systems, because property is monopoly. This argument was formalized by Myerson and Satterthwaite (1981) who show that private property inhibits free competition, thus decreasing the efficiency of allocation, and conversely, Vickrey (1961) showed that under common ownership, full allocative efficiency can be achieved through an auction.

In this paper, we propose a simple system of self-assessed taxation that continuously interpolates between private and common ownership, trading off the investment incentives from the former with the allocative benefits from the latter. We loosely calibrate our model to moments of the US housing markets and argue that a 5-10% annual tax is robustly near optimal. This implies collectivization of approximately two thirds of capital value, and increases average asset use values by approximately 5.5%. For assets where investment is not important (e.g., radio spectrum) or where investment can be directly rewarded through objective assessment, the tax rate should be higher, approximately equal to the probability of per-period asset turnover.

The role of property rights in incenting agents to invest in assets is an old idea in economics, famously explored in the theoretical literature by Grossman and Hart (1986) and Hart and Moore (1990). Although individuals have incentives for purely "selfish" investment (those that raise only that user's value for using the good) under competitive common ownership (viz. Vickrey's commons) (Milgrom, 1987; Rogerson, 1992), Che and Hausch (1999) show such schemes typically do not give individuals incentives to make investments that might benefit other potential users.[1]

[1] Che and Hausch distinguish between "selfish" and "cooperative" investments. Our capital investment is the

In particular, common ownership offers *zero* incentive for an individual to make a "capital" investment that increases the value of the of the asset to *all* individuals equally, because such an investment raises the value of the good to the investing individual's competitor as much as it raises the value to the investing individual. If the individual does not have an ownership stake in the good, such an investment is a pure waste from her perspective because any competitive mechanism with common ownership must be allocatively neutral in the face of such symmetric increases in all individuals' values. Thus, some form of private ownership is crucial to encourage capital investment.

On the other hand, a separate strand of literature has argued that private property is harmful to allocative efficiency. Myerson and Satterthwaite (1981) show that no mechanism can achieve fully efficient bargaining between a seller and buyer when the seller has property rights. Cramton, Gibbons and Klemperer (1987) show that this failure can be attributed to the structure of property rights – if both agents have partial ownership of the asset, efficient bargaining is possible.[2] This suggests that decreasing the level of private ownership can improve welfare in market for assets, such as radio spectrum or consumer housing, for which allocative efficiency is an important concern. However, it is not clear whether such ideas can be applied in a dynamic setting when many potential buyers of an asset arrive over time, because there is a large and evolving potential set of purchasers with whom property rights would need to be shared. Furthermore, as Segal and Whinston (2013) discuss, no existing research combines combines the allocative efficiency costs of private property with the investment efficiency benefits.

In this paper, we suggest *self-assessed taxation* as a simple tool for interpolating between common and private ownership of property, trading off the benefits of common ownership for allocative efficiency and the benefits of private ownership for investment efficiency. In our baseline static model, sellers make capital investments in assets that they own, and then set prices at which a buyer can purchase the assets. While investment is efficient under full private ownership, the final allocations of assets are distorted by the market power of the seller. A *self-assessed tax* on sellers requires them to pay some fraction of any sale price they announce as tax revenue to the community, independently of whether the good is sold. Because Harberger (1965) first proposed this form of self-assessed taxation, we will refer to it as a *Harberger tax*. In the presence of such a tax, sellers effectively become partially "buyers" as well as sellers; they will tend to announce lower prices, alleviating market power distortions and improving allocative efficiency. However, sellers pay taxes on the capital value of the assets they own; hence, they do not appropriate the full marginal benefits of their costly capital investments, implying that investment is distorted by taxation.

Harberger taxation is fairly simple to implement in practice. Similar systems have been used

sum of a selfish and cooperative investment, because it augments the good's value to both the investing and other users. Any of these types of investment may be synthesized with linear combinations of the other two, but in our context, the classification makes expositing the analysis simpler.

[2]This idea is further explored by Kaplow and Shavell (1996) and generalized by Segal and Whinston (2011).

in a variety of settings dating as far back as ancient Rome,[3] although to our knowledge we are the first to propose using self-assessed taxation to improve allocative efficiency in asset markets.[4] Importantly, implementing Harberger taxes only requires identifying the owner of any given asset, not the large collection of potential buyers of the asset. Additionally, we show in our Theorem 1 that the tax which maximizes allocative efficiency is particularly simple to determine, as at the optimum it simply equals the probability of asset turnover that it induces.

We then construct a dynamic model of Harberger taxation in which new buyers interested in acquiring the asset from its current owner repeatedly arrive to the market. We prove the uniqueness of equilibrium for any tax level and that the intuitions from the static model extend directly to the dynamic model. However, beyond the basic structure of equilibrium, optimal taxes are challenging to characterize analytically and we thus proceed to numerically calibrate the dynamic model to loosely match moments of the US housing markets. We find that Harberger taxes in the range of 5-10% are robustly near optimal. This increases the net utility generated by the asset by approximately 5.5% (or approximately 6.3% of market values), and decreases the transaction prices of assets in equilibrium by approximately two thirds.

In our calibration, the detrimental effect of Harberger taxes on investment value implies that the optimal Harberger tax rate is lower than the probability of asset turnover that it induces. However, for a family of assets for which capital investments are relatively unimportant, such as radio spectrum and internet domain names, optimal tax rates should be substantially higher, approximately equal to the rate of asset turnover that they induce. In addition, if the community can observe and directly incent capital investments using objective assessments and investment tax credits, the negative effects of Harberger taxation on investment can be mitigated, also implying higher optimal tax rates.

In Section 2, we construct our baseline static model and use this to illustrate the fundamental trade-offs of our analysis. In Section 3, we introduce our dynamic model and calibration to housing markets. In Section 4, we discuss various extensions, such as the effect of community observability of investment, the effect of taxation on private-value investments, and relaxing our assumptions on the nature of buyers and sellers. In Section 5, we discuss our proposal's relationship to other work on mechanism design, capital taxation and intellectual property. In Section 6 we discuss applications of our results to different forms of capital. We conclude in Section 7. We present longer and less instructive calculations, proofs and calibration details in an appendix following the main text.

[3] See Epstein (1997).

[4] The closest prior idea is Tideman (1969)'s demonstration that Harberger taxes tend to increase the probability of sale, but Tideman does not explicitly model the consequent welfare effects.

2 Baseline model

In the baseline model, a seller S holds an asset and makes a single take-it-or-leave-it offer to a buyer B. In the absence of a Harberger tax, S is a monopolist, and announces a price higher than her value for the asset. If S pays a Harberger tax of τ on p, she has a lower incentive to overstate her valuation. Thus, Harberger taxes alleviate the monopoly distortion and improve *allocative efficiency*: the asset more often ends up with the individual with a higher value. However, imposing Harberger taxes on S decreases her incentives to make capital investments in the asset, thus harming *investment efficiency*.

2.1 Setup

There is a seller S who owns a single asset, and a buyer B. Values of S and B for the asset are, respectively,

$$v_S = \eta + \gamma_S$$

$$v_B = \eta + \gamma_B.$$

$\gamma_B \sim F(\cdot)$ is a random variable representing heterogeneity in B's value, which is not observed by S. η is a common-value component; S chooses $\eta \geqslant 0$, incurring a convex cost $c(\eta)$ to herself. Both agents are risk neutral.[5]

For a given η, let $1_S, 1_B$ be indicators, which respectively represent whether S and B hold the asset at the end of the game, and let y be any net transfer B pays to S. Final payoffs for S and B respectively are

$$U_S = (\eta + \gamma_S) 1_S - c(\eta) + y$$

$$U_B = (\eta + \gamma_B) 1_B - y.$$

Prior to the beginning of the game, the community decides on a *Harberger tax* level τ. Then, S and B play a two-period game. In period 1, S chooses η. In period 2, S announces a price p for the asset, pays taxes $p\tau$ to the cadaster, and then B can decide whether to buy the asset by paying p to S. The revenue the cadaster raises is distributed to the broader community in a manner we do not specify here. Since S is a member of a large community, we will ignore any impact the revenue raised has on her pricing incentive.[6]

We solve the game by backwards induction. First, fixing η and τ, we will analyze behavior in the period 2 price offer game.

[5] See Tideman (1969) for a partial analysis of the allocative problem that allows for risk aversion.

[6] In fact, even this small issue can be addressed by stipulating that no individual is paid out of the pool of taxes she paid herself.

2.2 Allocative efficiency

For any price p, B's optimal strategy is to buy the asset if her value is greater than p, that is, if $\eta + \gamma_B > p$. Let $m \equiv p - \eta$ be the markup S chooses to set over the common value η. The probability of sale under markup m is then $1 - F(m)$. Fixing common value η, and Harberger tax level τ, S's optimal price offer solves:

$$\max_m (1 - F(m))(\eta + m) + F(m)(\eta + \gamma_S) - \tau(\eta + m) - c(\eta)$$

We can change variables to work in terms of sale probabilities. Define $q \equiv (1 - F(m))$, and $M(q) \equiv F^{-1}(1 - q)$. S then solves:

$$\max_q (\eta + M(q))\, q + (\eta + \gamma_S)(1 - q) - \tau(\eta + M(q)) - c(\eta)$$

Note that the socially efficient outcome corresponds to setting $M(q) = \gamma_S$, or equivalently $q = 1 - F(\gamma_S)$. We can rearrange S's optimization problem further:

$$\max_q (M(q) - \gamma_S)(q - \tau) + (\eta + \gamma_S)(1 - \tau) - c(\eta)$$

Only the "variable profit" term $(M(q) - \gamma_S)(q - \tau)$ depends on the sale probability q. We can think of this term as the net trade profits of an agent who owns a fraction $1 - \tau$ of the asset, facing market demand described by $M(q)$. S can always choose $q = \tau$, in which case she makes no net trades and earns zero variable profits. If S chooses $q > \tau$, she is a net seller of a share $q - \tau$ at price $M(q)$ relative to her value γ_S; likewise, if she chooses $q < \tau$, she is a net buyer of a share $\tau - q$ of the asset. Intuitively, if S has value γ_S higher than the market price $M(\tau)$ at her initial share $1 - \tau$, or equivalently if S's quantile among buyer values $1 - F(\gamma_S)$ is lower than τ, we expect S to be a net buyer; likewise, we expect S to be a net seller if $1 - F(\gamma_S) > \tau$. In Theorem 1, we formalize this intuition, showing in particular that setting $\tau = 1 - F(\gamma_S)$ will induce S to choose $q^*(\gamma_S, 1 - F(\gamma_S)) = 1 - F(\gamma_S)$, which is the socially efficient outcome.

Theorem 1. *(Quantile markup property)* $q^*(\gamma_S, \tau)$ *always lies between* τ *and* $1 - F(\gamma_S)$. *Specifically,*

1. *If* $\tau = 1 - F(\gamma_S)$, *then* $q^*(\gamma_S, 1 - F(\gamma_S)) = \tau = 1 - F(\gamma_S)$
2. *If* $\tau < 1 - F(\gamma_S)$, *then* $\tau \leqslant q^*(\gamma_S, \tau) \leqslant 1 - F(\gamma_S)$
3. *If* $\tau > 1 - F(\gamma_S)$, *then* $1 - F(\gamma_S) \leqslant q^*(\gamma_S, \tau) \leqslant \tau$

Proof. For a given γ_S, τ we consider S's optimization of the variable profit function:

$$\max_q (M(q) - \gamma_S)(q - \tau)$$

First, suppose that $1 - \tau = F(\gamma_S)$.

- If S chooses sale probability $q = \tau$, she makes no net trades, and receives 0 variable profit. Moreover, the markup is $M(q) = F^{-1}(1-\tau) = \gamma_S$, so that $M(q) = \gamma_S$.
- If S chooses a higher sale probability, so that $q - \tau > 0$, we have $M(q) \leqslant \gamma_S$, so variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$. In words, S becomes a net seller at a price lower than her value.
- If S chooses a lower sale probability, so that $q - \tau < 0$, we have $M(q) \geqslant \gamma_S$, so variable profits $(M(q) - \gamma_S)(q - \tau) \leqslant 0$. In words, S becomes a net buyer at a price higher than her value.

Hence, the optimal strategy for S is to choose $q = \tau$. Suppose now that $\tau < 1 - F(\gamma_S)$.

- Suppose we raise the tax rate to $1 - F(\gamma_S)$. By the first part of the claim, $q^*(\gamma_S, 1 - F(\gamma_S)) = 1 - F(\gamma_S)$. The variable profit function is supermodular in τ and q, so if we lower the tax rate from $1 - F(\gamma_S)$ to τ, q^* must decrease, so $q^*(\gamma_S, \tau) \leqslant 1 - F(\gamma_S)$.
- If we increased γ_S to $F^{-1}(1-\tau)$, again by the first part of the claim, we would have $q^*\left(F^{-1}(1-\tau), 1-\tau\right) = 1 - \tau$. The variable profit function is supermodular in $-\gamma_S$ and q, hence if we lower S's value from $F^{-1}(1-\tau)$ to γ_S, q^* must increase, so $q^*(\gamma_S, \tau) \geqslant 1 - \tau$.

The third part of the claim, for $\tau > 1 - F(\gamma_S)$, follows symmetrically to the second. □

An important consequence of Theorem 1 is that, for any given seller value γ_S, the allocatively optimal tax can be found without knowledge of the distribution $F(\cdot)$: if the community iteratively sets the tax equal to the current probability of sale, $\tau_{k+1} = q^\star(\tau_k)$, the tax τ_k will converge to the efficient probability of sale $1 - F(\gamma_S)$.

To quantify these comparative statics we now assume $F(\cdot)$ is twice continuously differentiable. S's first-order condition is

$$M'(q)(q - \tau) + (M(q) - \gamma_S) = 0,$$

so that by the Implicit Function Theorem,

$$\frac{\partial q^\star}{\partial \tau} = \frac{M'(q^\star)}{2M'(q^\star) + M''(q^\star)(q^\star - \tau)} = \frac{1}{2 + \frac{M''(q^\star)(q^\star - \tau)}{M'(q^\star)}} = \frac{1}{2 - \frac{M''(M - \gamma_S)}{(M')^2}},$$

where the last equality invokes the first-order condition and drops arguments. Cournot (1838) showed that this quantity equals the *pass-through* rate $\rho(q^\star)$ of a specific commodity tax into price; see Weyl and Fabinger (2013) for a detailed discussion and intuition. ρ is closely related to the curvature of the value distribution; it is large for convex demand and small for concave demand. It is strictly positive for any smooth value distribution and is finite as long as S is at a strict interior optimum.[7]

[7] Myerson (1981)'s regularity condition is sufficient but not necessary for this second-order condition, as we show in Appendix A.1.

The marginal gain to social welfare from a unit increase in the probability of sale is equal to the gap between γ_B and γ_S, because the tax raised is simply a transfer. This gap is, by construction, $(M(q^\star) - \gamma_S)$. Thus, the marginal allocative gain from raising τ is $(M(q^\star) - \gamma_S)\,\rho(q^\star)$ or $(M - \gamma_S)\,\rho$ for short.

Note that $(M - \gamma_S)\,\rho$ is 0 at $q = 1 - F(\gamma_S)$, so the first-order social welfare gain from taxation approaches 0 as we approach the allocatively optimal tax of $1 - F(\gamma_S)$. On the other hand, when $\tau = 0$, we have $(M - \gamma_S)\,\rho > 0$ and increasing τ creates a first-order welfare gain.

2.3 Investment efficiency

Note the variable profits defined in the previous subsubsection were independent of η. On the other hand, the sunk profits, $(1-\tau)\,\eta - c(\eta)$, depend on η. Because this component is entirely separable from the other component, regardless of what happens in the second stage of the game, S finds it optimal to choose η such that:

$$c'(\eta) = 1 - \tau.$$

We can define the investment supply function $\Gamma(\cdot)$ as:

$$\Gamma(s) \equiv c'^{-1}(s).$$

The value of a unit of investment η is always 1, so the socially optimal level of investment is $\Gamma(1)$, whereas investment is only $\Gamma(1-\tau)$ when the tax rate is τ. By strict convexity of c, Γ is strictly increasing, so the higher the tax, the more downward distorted the investment.

Again turning to the quantitative side, the marginal increase in investment from a rise in τ is $\Gamma' = \frac{1}{c''}$ by the inverse function theorem. The social value of investment is always 1, whereas S only invests up to the point where $c' = 1 - \tau$. Thus, the marginal distortion from under-investment is τ. Thus, the marginal social welfare loss from raising τ is $\Gamma'\tau = \frac{\tau}{1-\tau}\Gamma\epsilon_\Gamma$, where ϵ_Γ is the elasticity of investment supply. Note that as $\tau \to 0$, this investment distortion goes to 0, so that there is no marginal investment distortion near the investment optimum of zero tax, whereas at any other tax (such as the allocatively efficient tax level $1 - F(\gamma_S)$), there is a strictly positive marginal investment distortion, as long as c does not have a convex kink in the relevant neighborhood.

2.4 Tradeoff between allocative and investment welfare

Fixing S's value γ_S, the socially optimal level of property rights balances the cost of taxation for investment efficiency with its benefit (below $1 - F(\gamma_S)$) to allocative efficiency. It thus solves a

Figure 1: Allocative, Investment, and Total Welfare vs Tax

classic optimal tax formula:

$$\frac{\tau_{\text{eff}}}{1-\tau_{\text{eff}}} = \frac{\left(M\left(q^{\star}\left(\gamma_S, \tau_{\text{eff}}\right)\right) - \gamma_S\right) \rho\left(q^{\star}\left(\gamma_S, \tau_{\text{eff}}\right)\right)}{\Gamma\left(1-\tau_{\text{eff}}\right) \epsilon_{\Gamma}\left(1-\tau_{\text{eff}}\right)}. \tag{1}$$

Under weak additional regularity conditions we discuss in Appendix A.1, this equation has a unique solution. The left-hand side is a monotone-increasing transformation of τ that appears frequently in elasticity formulas in the optimal tax literature; see, for example, Werning (2007). The right-hand side is the ratio of two terms: the allocative benefit of higher taxes and the investment distortion of higher taxes. The allocative benefit equals the product of the mark-up and the pass-through rate, whereas the investment distortion equals the product of the equilibrium investment size and its elasticity with respect to $1-\tau$. By the logic of the previous subsections, $\tau_{\text{eff}} \in \left(0, 1-F\left(\gamma_S\right)\right)$.

Proposition 1. *The optimal tax rate τ_{eff} that maximizes social welfare is strictly positive and strictly below $1-F\left(\gamma_S\right)$, and satisfies:*

$$\frac{\tau_{\text{eff}}}{1-\tau_{\text{eff}}} = \frac{\left(M-\gamma_S\right)\rho}{\Gamma \epsilon_{\Gamma}}.$$

Figure 1 graphically illustrates the tradeoff between allocative and investment welfare. Allocative welfare increases monotonically in τ on the interval $\tau \in \left[0, 1-F\left(\gamma_S\right)\right]$, with slope 0 at $\tau = 1-F\left(\gamma_S\right)$. Investment welfare decreases monotonically in τ, with slope 0 at $\tau = 0$. Thus, τ_{eff} always lies strictly in the interior of the interval $\left(0, 1-F\left(\gamma_S\right)\right)$.

3 Dynamic model

In this section, we construct a dynamic model of Harberger taxation, and show that the core intuitions of the static model extend to this more general setting. This allows us to study the effect of Harberger taxation on turnover rates and the stationary distribution of values, as well as the influence of Harberger taxation on asset prices. We calibrate the model loosely to features of US housing markets, and show that Harberger taxes in the range of 5-10% robustly increase total value, and that larger Harberger taxes can achieve a large fraction of total possible allocative welfare.

3.1 Model

3.1.1 Agents and Utilities

Time is discrete, $t = 0, 1, 2 \ldots \infty$. All agents discount utility at rate δ. There is a single asset, which an agent S_0 owns at time $t = 0$. In each period, a single buyer B_t arrives to the market and bargains with the period-t seller S_t to purchase the asset, through a procedure we detail in Subsubsection 3.1.2 below. Hence, the set of agents is $\mathscr{A} = \{S_0, B_0, B_1, B_2 \ldots\}$. We will use S_t as an alias for the period-t seller, who may be a buyer $B_{t'}$ from some period $t' < t$. We will often use A to denote a generic agent in $\mathscr{A}$.

In period t, agent A has period-t use value γ_t^A for the asset. The values of entering buyers $\gamma_t^{B_t}$ are drawn i.i.d. from some distribution F. Values evolve according to a Markov process: for any agent A with period-t usage utility γ_t^A, her use value in the next period γ_{t+1}^A is drawn according to the transition probability distribution $G(\gamma_{t+1} \mid \gamma_t)$.

Assumption 1. $G(\gamma \mid \gamma) = 1 \ \forall \gamma$, *that is,* $\gamma_{t+1} \leqslant \gamma_t$ *with probability 1.*

Assumption 2. $\gamma_t > \gamma_t'$ *implies* $G(\gamma_{t+1} \mid \gamma_t) >_{FOSD} G(\gamma_{t+1} \mid \gamma_t')$.

Assumption 3. $G(\gamma' \mid \gamma)$ *is continuous and differentiable in* γ *for any* γ'.

Assumptions 1 and 2 imply that, for any given agent, use values decay uniformly over time, in the sense that future values are always lower than current values, and higher current values imply uniformly higher future values in the sense of stochastic dominance. Assumption 3 guarantees that the value decay process is smooth.

In any period, there is a single owner of the asset. Let 1_t^A denote agent A being the owner of the asset in period A. Agent A's utility for ownership path 1_t^A and utility path γ_t^A, is:

$$\sum_{t=0}^{\infty} \delta^t \left[1_t^A \gamma_t^A + y_t^A \right]$$

Where, y_t^A is any net monetary payment made to agent A in period t.

To avoid dealing with repeated strategic interactions, after the period in which agent A arrives to the market as a buyer, we will allow agent A to remain in the market only so long as $1_t^A = 1$; once $1_t^A = 0$, agent A leaves the market forever. Thus, in each period t, only two agents exist in the market: the period t seller S_t, and the arriving buyer B_t. Any pair of agents interacts at most once.

A (possibly random) *allocation rule* $\Phi(h_t)$ determines in each time t, history h_t whether to allocate the good to S_t or B_t. Intuitively, since Assumption 2 states that higher present values imply uniformly higher future values, a social planner aiming to maximize average use values should assign the asset to whichever of S_t, B_t has higher current-period value in any given period. This is formalized in the following proposition.

Proposition 2. *The socially optimal allocation rule* $\Phi(\cdot)$ *allocates the good to whichever of* $\{S_t, B_t\}$ *has higher use value* γ_t *in every period* t.

Proof. See Appendix Section A.2. □

3.1.2 Game

The community chooses some Harberger tax level τ, constant for all time. For any tax level τ, we will define the following *dynamic Harberger tax game*. At $t = 0$, agent S_0 owns the asset, and observes her own use value $\gamma_t^{S_0}$ for the asset. In each period t:

1. **Buyer arrival:** Buyer B_t arrives to the market; his use value $\gamma_t^{B_t}$ is drawn from $F(\cdot)$, and is observed by himself but not the period-t seller S_t.

2. **Seller price offer:** Seller S_t makes a take-it-or-leave-it price offer p_t to buyer B_t, and immediately pays tax τp_t to the community.

3. **Buyer purchase decision:**

   - If B_t chooses to buy the asset, she pays p_t to S_t. B_t becomes the period-t asset owner, $1_t^{B_t} = 1$, and enjoys period-t use value $\gamma_t^{B_t}$ for the asset. B_t becomes the seller in period $t+1$, that is, $S_{t+1} \equiv B_t$. Seller S_t receives payment p_t from B_t, and seller S_t leaves the market forever, with continuation utility 0.

   - If B_t chooses not to purchase the asset, S_t becomes the period-t asset owner, $1_t^{S_t} = 1$, and she enjoys period-t use value $\gamma_t^{S_t}$. S_t becomes the seller in period $t+1$, that is, $S_{t+1} \equiv S_t$. Buyer B_t leaves the market forever, with continuation utility 0.

4. **Value updating**: $\gamma_{t+1}^{S_{t+1}}$, the period $t+1$ value for seller S_{t+1}, is drawn from $G\left(\gamma_{t+1} \mid \gamma_t^{S_{t+1}}\right)$ according to her period-t value $\gamma_t^{S_{t+1}}$.

3.1.3 Equilibrium

Equilibrium in the dynamic Harberger tax game requires that, in all histories, all sellers make optimal price offers, and all buyers make optimal purchase decisions. Since τ, F, G are constant over time, the problem has a Markovian structure: the optimal strategies of buyers and sellers may depend on their types $\gamma_t^{S_t}, \gamma_t^{B_t}$ respectively, but not on the period t. Hence we can apply dynamic programming techniques, characterizing equilibria of the game by a stationary value function $V(\gamma)$ which describes the value of being a type γ seller in any given period.

In any period t, we can think of seller S_t as choosing a probability of sale q_t, where buyers in period t make purchase decisions according to the inverse demand function $p(q_t)$. If the continuation value in period $t+1$ for seller type γ_{t+1} is $V(\gamma_{t+1})$, the seller's optimization problem in period t is:

$$\max_{q_t} q_t p(q_t) + (1-q_t)\left[\gamma_t + \delta \mathbb{E}_{G(\gamma_{t+1}|\gamma_t)}\left[V(\gamma_{t+1}) \mid \gamma_t\right]\right] - \tau p(q)$$

Simplifying and omitting t subscripts, optimality for the seller requires $V(\gamma)$ to satisfy the following Bellman equation:

$$V(\gamma) = \max_q (q-\tau) p(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]\right] \tag{2}$$

Buyer optimality pins down the relationship between $p(\cdot)$ and $V(\cdot)$. If buyer B_t with value γ_t purchases the asset, he receives value γ_t in period t, and then becomes the seller in period $t+1$, receiving utility $\delta V\left(\gamma_{t+1}^{B_t}\right)$. Hence the period-$t$ willingness-to-pay of buyer type γ_t is:

$$WTP(\gamma_t) = \gamma_t + \delta \mathbb{E}_{G(\gamma_{t+1}|\gamma_t)}\left[V(\gamma_{t+1}) \mid \gamma_t\right]$$

Thus, in equilibrium, optimality for the buyer implies that the inverse demand function $p(\cdot)$ satisfies:

$$p(q) = \left\{p : \ P_{\gamma \sim F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right] > p\right] = q\right\} \tag{3}$$

Fixing τ, a value function $V(\cdot)$ which satisfies equations (2) and (3) defines an equilibrium of the dynamic Harberger tax game.

Theorem 2. *For any τ, F, G satisfying our assumptions, there exists a unique equilibrium of the dynamic Harberger tax game.*

Proof. We prove this theorem in Appendix Section A.3, where we also describe a numerical procedure that solves for the unique equilibrium for any τ. □

The following theorem states that the "net trade" intuition from the static Harberger tax game applies exactly to the dynamic case.

Theorem 3. *(Dynamic quantile markup property) In any τ-equilibrium of the dynamic Harberger taxation game, Theorem 1 holds, that is, the optimal sale probability function $q^*(\gamma)$ satisfies:*

1. *For type γ such that $\tau = 1 - F(\gamma_S)$, we have $q^*(\gamma) = \tau = 1 - F(\gamma)$*
2. *Types γ with $\tau < 1 - F(\gamma_S)$ are net sellers, that is, $\tau \leqslant q^*(\gamma) \leqslant 1 - F(\gamma)$*
3. *Types γ with $\tau > 1 - F(\gamma)$ are net buyers, that is, $1 - F(\gamma) \leqslant q^*(\gamma) \leqslant \tau$*

Proof. This follows from the more general Claim 2 in Appendix Subsection A.3. □

Together, these theorems demonstrate that the basic intuitions of the static model carry over to the dynamic Harberger tax game. However, the dynamic game is generally not analytically solvable, and comparative statics of outcomes with respect to τ are difficult to calculate analytically. Instead, we proceed in Subsection 3.2 by calibrating the dynamic Harberger tax game to realistic parameters, numerically calculating the equilibria for different levels of τ, and studying the effect of changing τ on steady-state asset use values and other equilibrium outcomes.

3.1.4 Investment

Throughout the description of the game thus far, we have ignored investment. Common-valued investment enters into the utility functions of all agents identically, so any common value component only affects equilibria of the game by raising prices by some constant. Thus, we can disregard the structure of investment in solving for equilibria of the dynamic Harberger tax game.

In the dynamic Harberger game, we will allow investments to have long-term effects on common values. Specifically, suppose that agent A_t can make common-valued investment η_t in the asset at cost $c(\eta_t)$. However, now suppose that, some number of periods $t' \geqslant 0$ in the future, investment η_t increases the common value of the asset by some $H_{t+t'}(\eta_t)$. The social value of investment is then the discounted sum:

$$\sum_{t=0}^{\infty} \delta^t H_t(\eta)$$

and, the social FOC sets:

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t H_t'(\eta)$$

The following proposition shows that the Harberger tax distorts longer term investments more than shorter-term investments.

Proposition 3. *In any τ-equilibrium of the dynamic Harberger taxation game, all agents choose a constant level of investment η such that:*

$$c'(\eta) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta)$$

Proof. See Appendix A.4. □

3.2 Calibration

3.2.1 Specification and moment matching

In this section, we study a computational simulation of dynamic Harberger taxation loosely calibrated to US housing markets.

Our dynamic Harberger taxation game has two main unknowns: the distribution of entering buyer values $F(\gamma)$, and the transition probability distribution $G(\gamma' \mid \gamma)$. In addition, we have to choose the discount rate δ, and the investment cost function $c(\eta)$ and benefit functions $H_t(\eta)$.

We will assume that the distribution of entering buyer values $F(\cdot)$ is lognormal, with log mean normalized to 0. The log standard deviation parameter σ then serves the role of a spread parameter, controlling the amount of idiosyncratic dispersion in values, which thereby determines the social loss from allocative inefficiency. We model the transition probability distribution $G(\gamma' \mid \gamma)$ as follows: given an agent in period t with value γ_t at the $F^{-1}(\gamma)$th quantile of $F(\cdot)$, her period $t+1$ value γ_{t+1} is calculated by drawing a value ψ from the beta distribution with parameters $[\kappa\beta,\ \kappa(1-\beta)]$, then setting γ_{t+1} equal the $\psi F^{-1}(\gamma)$ quantile of $F(\cdot)$. The result is that values decay in quantile terms, with expected quantile decay rate β. The shape parameter κ, which represents the prior sample size in the "Bernoulli prior" interpretation of the beta distribution, is somewhat arbitrary, and we use $\kappa = 10$ in our simulations.

We are left with two parameters to determine: the log standard deviation σ of F, and the quantile decay rate β of G. To match these parameters, we target two moments of housing markets. Willekens et al. (2015) finds that, in the Danish housing market, the idiosyncratic variance in house prices is at least 10%. We will refer to this moment as $sdmean$, and we will match it by requiring the standard deviation of transaction prices in equilibrium for $\tau = 0$ to equal $sdmean$. Using American Housing Survey data, Emrath (2013) estimate that the average house is sold approximately once every 13 years. We will refer to this moment as $saleprob$, and we will match it by requiring the average sale probability in each period in the $\tau = 0$ equilibrium to equal $saleprob$.

Intuitively, under our parametrization, increasing the quantile decay rate β increases the probability of efficient sale, and hence should increase the efficient tax level. Increasing the lognormal standard deviation σ increases the dispersion of values about its mean; this should

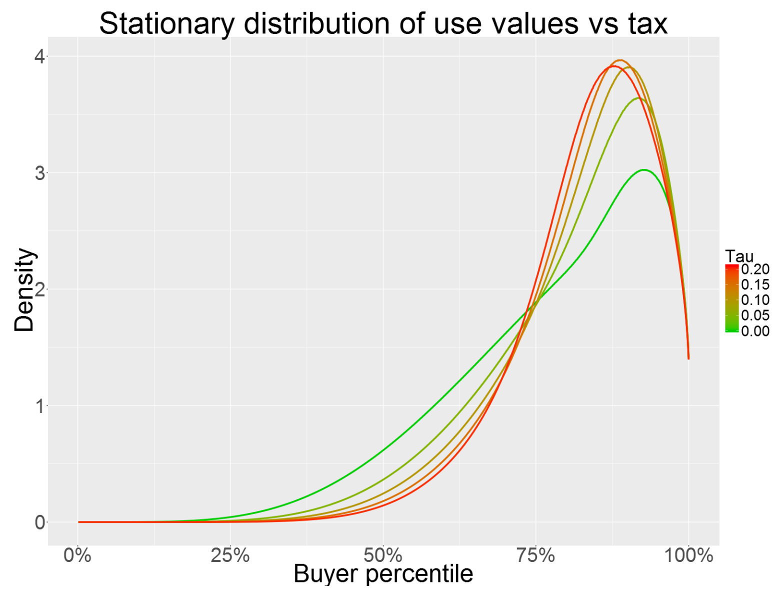

Figure 2: Stationary Use Value Distributions vs τ

increase the dispersion of prices, with a smaller effect on the probability of sale. Hence, we should expect that the $saleprob$ moment is matched mostly by the decay rate β, whereas the $sdmean$ moment is matched more by σ. We confirm these intuitions in Figure 4, which we discuss in Section 3.2.2. However, as both σ and β simultaneously influence the probability of sale and the dispersion of prices, we jointly choose σ and β in order to match the $saleprob$ and $sdmean$ moments.

On the investment side, we will assume that investment has geometrically depreciating value over time: $H_t(\eta) = \theta^t \eta, \ \theta < 1$. For our calibrations, we will set $\theta = 0.85$, which is similar to numbers used in the literature on capital depreciation (Nadiri and Prucha, 1996). We will assume the investment cost function is quadratic, $c(\eta) = \frac{\eta^2}{2g}$, and we will choose g such that the total value from investment is some fraction $invfrac$ of the mean allocative welfare in stationary distribution at $\tau = 0$. Glaeser and Shapiro (2003) show that ownership in a neighborhood roughly influences housing values by 25%, hence we use $invfrac = 0.25$ in our calibration. In Appendix B.2, we analytically solve for the equilibrium investment level and investment welfare for any value of τ.

Finally, we will use the standard choice of $\delta = 0.95$. In Appendix B.1, we describe further details of the numerical procedure we use to analyze the game and solve for equilibria.

3.2.2 Results

In Figure 2, we show the stationary distribution of use values vs. tax rate τ. As τ increases from 0 to .15, the stationary distribution puts more mass on relatively higher values of γ. However note that a further increase to .2 has more mixed effects, decreasing the mass on the very highest values and on the lowest values, as the higher rate of turnover it induces eliminates inefficiently low values but also excessively encourages sales by the highest value sellers. In fact this phenomenon even emerges at lower tax rates, but is so modest as to make little difference. This is why we will find that allocative efficiency is maximized at a 15% tax rate.

In Figure 3, we show the behavior of various parameters as we change τ. In the topmost panel, we show the behavior of allocative, investment and total welfare with respect to τ. Allocative welfare is maximized at approximately $\tau = 0.15$, and this value of τ achieves over 80% of welfare gains relative to the social planner's optimum. Note that this is despite the fact that theoretically this could be a very small fraction of the greatest possible gains as seller values, and thus efficient probabilities of sale, are very heterogeneous. If we consider investment welfare as well, the optimal tax is $\tau = 0.072$, and this increases total welfare by 5.5%, which is 6.3% of the average asset transaction price with pure private ownership. However even this reduced tax achieves more than 70% of the maximum allocative welfare gains. Thus the limits on allocative gains placed by heterogeneous seller values (viz. the imperefctions of the coarse Harberger tax relative to a more optimal allocative mechanism) and investment distortion are roughly comparable. Asset prices under the optimal tax are approximately 37% of asset prices under pure private ownership.

In the second panel of Figure 3 we show the behavior of quantile markups and sale frequency with respect to τ. Average quantile markups decrease and sale frequency increases as we raise τ. The average quantile markup is 0, and the sale frequency is equal to the tax rate, when τ is set equal to the efficient probability that the good turns over, similar to the intuition in the static model from Theorem 1. However, the allocatively optimal tax is slightly lower than the efficient probability of sale, likely because the right-skew of the lognormal distribution means that the losses from excessive turnover shown in Figure 2 by high value sellers outweigh the gains from eliminating inefficiently low turnover rates by low value sellers.

In the third panel of Figure 3 we show the behavior of various stock/flow quantities as we vary τ. Importantly, asset prices rapidly decrease and tax revenues rapidly increase as we increase τ. Intuitively, if agents have to pay tax τ every period, this is roughly equivalent to discounting by rate $\delta(1-\tau)$; thus, increasing τ has a similar effect to increasing discounting, and rapidly lowers asset prices. The net present value of tax revenues greatly exceeds the price of the asset even for relatively small values of τ, supporting the interpretation that Harberger taxes effectively act by continuously decreasing the degree of private ownership of assets, moving towards a "partial rental" system from the community.

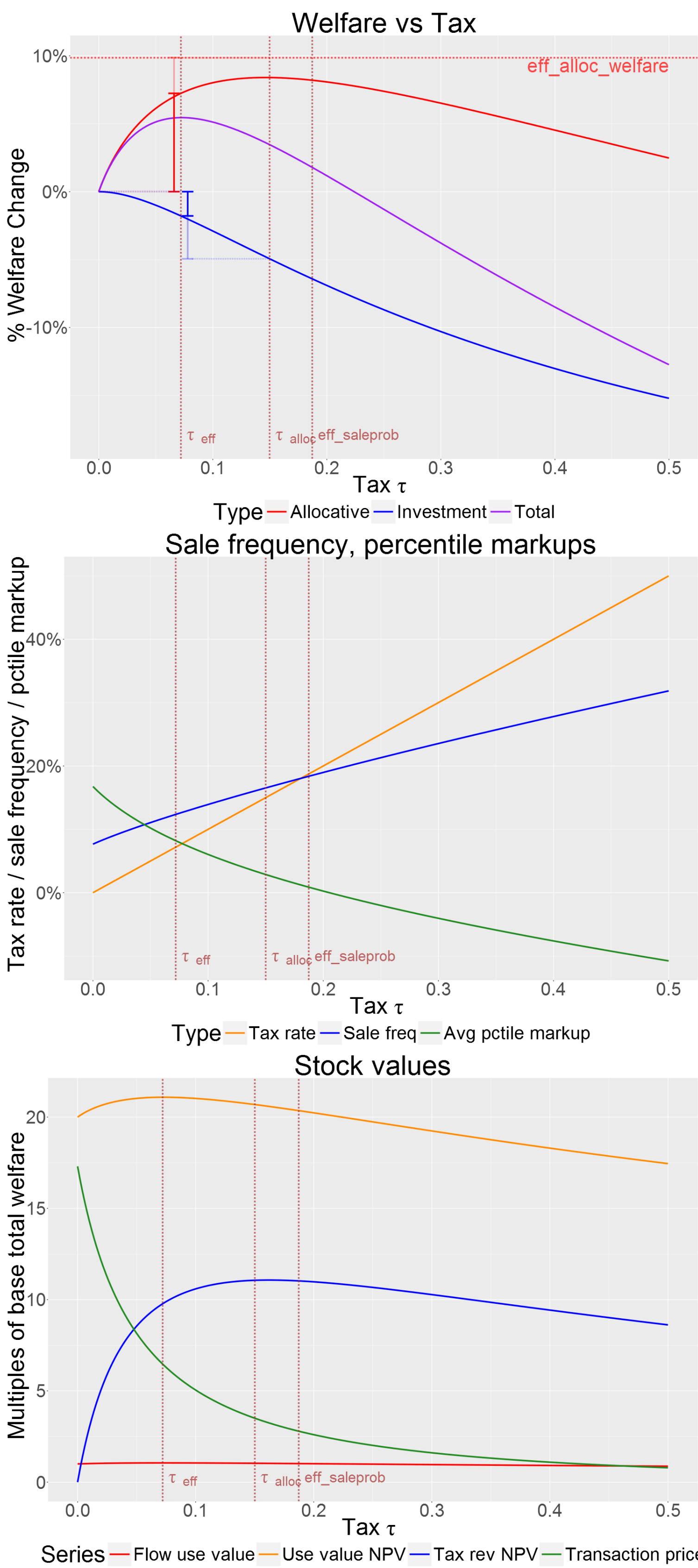

Figure 3: Comparative statics vs τ

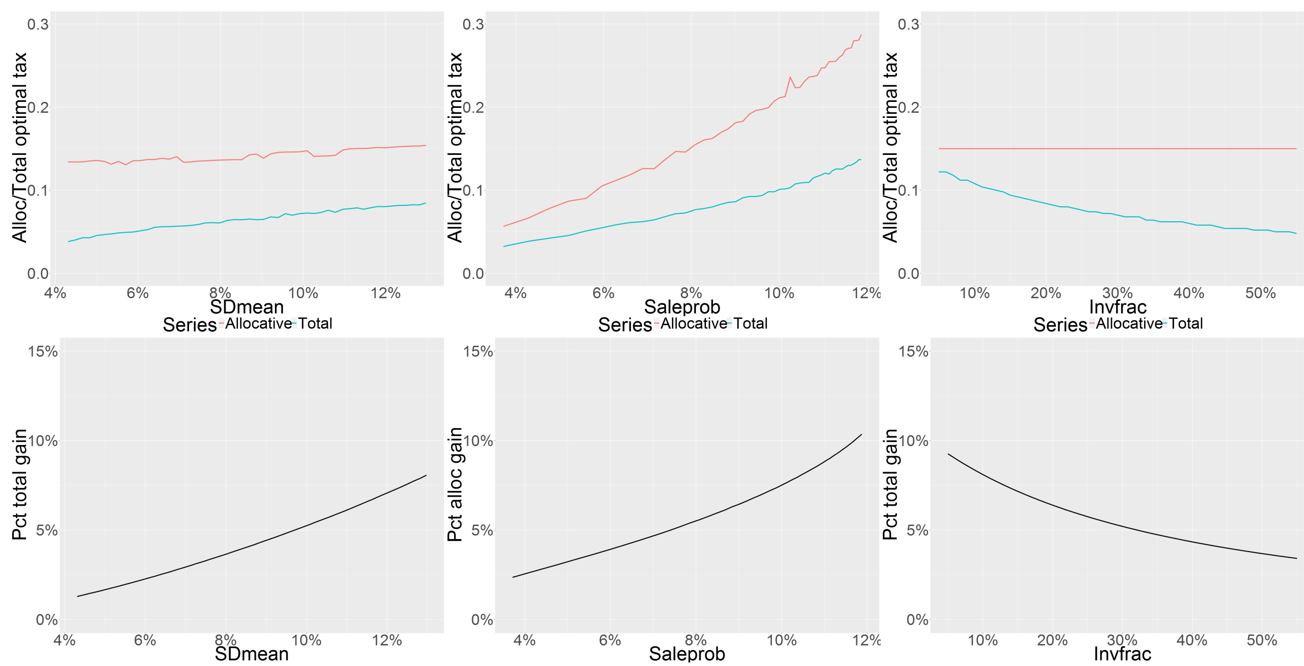

Figure 4: Sensitivity to moments

In Figure 4, we vary the input moments used in the calibrations, and show that allocative/total optimal taxes and total welfare gains depend on the input moments in intuitive ways. Changing the $sdmean$ moment moves the percent total welfare gain, with a relatively small effect on the optimal tax rate. Changing the $saleprob$ moment moves both optimal taxes and welfare gains. Changing the $invfrac$ moment lowers the total optimal tax relative to the allocatively optimal tax. While optimal taxes and their associated welfare gains vary across all of these calibrations, note that the optimal tax almost never falls below 5% or rises above 10% and when it does exit these bounds it does so by a very small amount. As this suggests, we find that over the full range of parameters we try any tax in the range of 5-10% is superior to full private ownership and is quite close to optimal.

4 Extensions

In this section, we consider several extensions that investigate the robustness of our analysis in the baseline case as well as enrich it in various directions. For simplicity, all these extensions build off of the static model of Section 2, rather than the dynamic model of the previous section.

In Subsection 4.1, we show that if the community is able to observe and directly incent the seller to make common-valued investments, it can alleviate the investment efficiency losses from Harberger taxation, leading to higher optimal tax levels. Subsection 4.2 shows private-valued investments by the seller are efficient in our mechanism, regardless of the level of the Harberger tax. In Subsection 4.3, we show that increasing competition on the buyer side lowers the optimal level of the Harberger tax.

4.1 Partial observability

In our analysis above, the public cannot directly reward investments made by individuals, and thus distorts investment when taxing self-assessed property. In some cases, though, the public may be able to directly reward capital investments. After all, the leading property of capital investments is that they affect the *objective* value to *all individuals* and not just to the idiosyncratic value of the seller. Furthermore, for the purposes of imposing more traditional property taxes for real estate agents or other paid experts, making an objective appraisal of the market value of a property is common practice.

Beyond these common practices, a number of mechanisms could be used to elicit valuations from private agents, such as keeping cadastral values secret but having individuals offer bids on them that would be accepted upon exceeding cadastral value but that would also be used to estimate capital appreciation. Although such schemes could be gamed through collusion between potential buyers and the seller, law enforcement may be able to discourage such manipulations. Additionally, Levmore suggests other schemes for eliciting competing assessments, and the literature on mechanism design is continually developing more elaborate methods for elicitation in circumstances like these; see, for example, Crémer and McLean (1988).

If some mix of objective appraisal and such elicitation mechanisms could provide at least a noisy signal of capital value, the public may be able to directly reward investments through a tax deduction for investment value and thus avoid the investment distortion from the Harberger tax, which in turn raises its optimal rate closer to the allocatively efficient level. This argument captures some of Hayek (1945)'s intuition that local knowledge is what limits the prospects of common ownership. In this subsection, we formalize this intuition by following the analysis of Baker (1992) to construct optimal direct property subsidies to mitigate the moral hazard problem created by Harberger taxation.

As before, suppose S chooses η at cost $c(\eta)$. However, now suppose common value υ is determined by

$$\upsilon = \zeta\eta,$$

where ζ is a random variable representing the value of investment in different states of the world. There is *local information* about ζ; that is, S and B both observe ζ prior to investment, but the public does not.[8]

The community can observe η and a signal ξ. Prior to period 1, the community can choose an *incentive scheme* $\eta\psi(\xi)$, meaning that if the community observes ξ, it will pay S some amount $\psi(\xi)$ for each unit η of investment S makes. This policy is simply a negative property (property subsidy) based on an objective appraisal of υ.

Fix a realization of the signal value ξ. If the community chooses reward function $\psi(\xi)\eta$, and the Harberger tax rate is τ, investment level $\Gamma(\psi(\xi) + (1-\tau)\zeta)$ will be induced. For expositional simplicity, we now focus on the case when costs of investment are quadratic and thus Γ is linear: $\eta(\zeta) = g\zeta$ for some $g > 0$, or, equivalently, that cost is $c(\gamma) = \frac{\gamma^2}{2g}$. Baker (1992) uses Taylor approximations to show that similar conclusions hold for more general investment cost functions.

Given any choice of $\psi(\xi)$, S chooses investment:

$$\Gamma(\psi(\xi) + (1-\tau)\zeta) = g(\psi(\xi) + (1-\tau)\zeta).$$

Hence, for a fixed τ, the optimal ψ solves pointwise over realizations of ξ the maximization problem:

$$\max_{\psi} \mathbb{E}\left[g(\psi(\xi) + (1-\tau)\zeta)\zeta - \frac{(g(\psi(\xi) + (1-\tau)\zeta))^2}{2g} \mid \xi\right].$$

[8] An alternative formulation of this model is that the cost of investment is uncertain and is known to the seller and buyer, but the community only observes a noisy signal of the cost. Although this interpretation is more natural in many settings, we focus on the "investment value" interpretation to stay closer to Baker's analysis and because it is simpler to present.

This program has the simple linear solution $\psi(\xi) = \tau\mathbb{E}(\zeta \mid \xi)$, which induces investment

$$g\left(\tau\mathbb{E}(\zeta \mid \xi) + (1-\tau)\zeta\right) = g\left(\mathbb{E}(\zeta \mid \xi) + (1-\tau)(\zeta - \mathbb{E}(\zeta \mid \xi))\right).$$

Investment is thus equal to the conditional expectation of investment value conditional on the signal ξ, plus a multiple $1-\tau$ of the deviation $\zeta - \mathbb{E}(\zeta \mid \xi)$ from the conditional mean. For higher values of τ, investment is closer to the conditional mean.

The social welfare loss from this noisy estimation is

$$IVL = \frac{\tau^2}{2} g\mathbb{E}\left[\zeta^2 - (E(\zeta|\xi))^2\right] = \frac{\tau^2}{2} g\left(1-r^2\right) Var(\zeta),$$

where $r^2 = \mathbb{E}\left[(\mathbb{E}(\zeta \mid \xi))^2\right]$ is the fraction of the variance in ζ that is predictable by ξ. If we take the derivative with respect to τ, we get

$$\frac{dIVL}{d\tau} = -\tau g\left(1-r^2\right) Var(\zeta). \tag{4}$$

As r^2 increases, $\frac{dIVL}{d\tau}$ decreases in magnitude, and the socially optimal choice of the Harberger tax given by equation

$$\frac{\tau^\star}{1-\tau^\star} = \frac{(M-\gamma_S)\rho}{\tau g(1-r^2) Var(\zeta)}$$

moves closer to the allocatively efficient level $(1 - F(\gamma_S))$. Thus, to the degree that the public can observe and reward capital investment, the detrimental effect of Harberger taxes on investment efficiency diminishes, and the optimal Harberger tax level is higher. This finding is consistent with Lange (1967)'s argument that the improvement of observation and computation through the improvement of information technology would increasingly make common ownership of the means of production feasible.[9] In fact, ongoing work by a team of researchers led by Nikhil Naik, of which Naik et al. (2015) is a preliminary output, is attempting to automate high-quality property value assessment using Google's Streetview images combined with computer vision and machine learning.

[9] In particular, in this limit, George (1879)'s argument for the taxation of land value while allowing people to keep investment value (see Subsection 5.2 below) becomes possible and thus all land, and no investment should be taxed. However, in contrast to this argument for full common ownership, our analysis suggests that for any $r^2 < 1$, for any seller type γ_S, the optimal Harberger tax approaches only $1 - F(\gamma_S)$ and not 1. Thus even as information technology improves, fully common ownership may not be desirable. However, note that our calculations in Subsection 3.2 above suggest that a tax at the allocatively efficient turnover rate would likely expropriate the overwhelming majority of the value of private capital. For example, at a 5% discount rate, an allocatively efficient turnover rate of 30% would imply the expropriation of 85% of private capital value. Furthermore, for $r^2 = 1$, a full common ownership is efficient as well, so in a more general environment, and at the true limits of information technology or elicitation mechanisms described above, Lange's argument may hold up in its simpler form.

4.2 Selfish investments

Thus far, we have assumed the seller's investment only affects the common value of the good. Here, we show that if the seller can make pure private-value investments, that is, investments that affect only her value for the good, these investments are efficient, conditional on the final probability that the seller holds the asset. This efficiency guarantee is true regardless of the level of the tax, which is why we largely ignore such selfish investments in our analysis above.

Suppose S can invest in increasing her private value for the good: she can increase her own use value for the good by λ at cost $c(\lambda)$. As before, B has value $\eta + \gamma_B$ for the good, and all other features of the game are identical to those in Section 2. Fixing γ_S and λ, S's second stage profits are once again:

$$\pi_S(\lambda, \gamma_S, \tau) \equiv \max_q p(q)\, q + (\eta + \gamma_S + \lambda)(1 - q) - p(q)\, \tau.$$

Let $q^*(\tau, \gamma_S)$ represent S's choice of q for any given τ, γ_S. In the investment stage, S chooses λ to maximize $\pi_S(\lambda, \gamma_S, \tau) - c(\lambda)$. But, using the envelope theorem, we have that

$$\begin{aligned}\frac{d\pi_S(\lambda, \gamma_S, \tau)}{d\gamma} &= \frac{\partial}{\partial \lambda}\left[p\left(q^*(\tau, \gamma_S)\right) q^*(\tau, \gamma_S) + (\eta + \gamma_S + \lambda)\left(1 - q^*(\tau, \gamma_S)\right) - p\left(q^*(\tau, \gamma_S)\right)\tau\right] \\ &= 1 - q^*(\tau, \gamma_S).\end{aligned}$$

Hence, the first-order condition for S's choice of private-valued investment λ is

$$c'(\lambda) = 1 - q^*(\tau, \gamma_S).$$

This equation defines the constrained efficient level of investment, conditional on S keeping the asset with probability $1 - q^*(\tau, \gamma_S)$. Thus, Harberger taxation does not directly affect selfish investment efficiency. The only impact of Harberger taxes on such selfish investments is indirectly, through the probability of sale. For example, with absolute private property, selfish investments will tend to be too high relative to the social optimum, because $q^\star$ is depressed below $1 - F(\gamma_S)$. Thus, with pure private ownership, individuals will tend to invest in becoming excessively "attached" to their possessions relative to an optimal world where possessions turn over more frequently, as Blume, Rubinfeld and Shapiro (1984) first observed. However, this distortion occurs only because of the change in the turnover rate and not directly because of the tax.

A few more informal observations are in order:

1. Our analysis shows that no issue of "hold-up" in the spirit of Grossman and Hart (1986); Hart and Moore (1990, 1988) with purely selfish investments exists in this model. The reason is that the investor (the seller) makes a take-it-or-leave-it offer that the buyer thus

cannot appropriate the seller's investment. This property would not hold if the buyer made a take-it-or-leave-it offer. However, the very structure of the Harberger tax makes a seller offer the natural bargaining protocol, so we don't consider this a serious concern.

2. Milgrom (1987) and Rogerson (1992) show that a Vickrey auction with common ownership or efficient bargaining through the similar Expected Externality mechanism (Arrow, 1979; d'Aspremont and Gérard-Varet, 1979) implies ex-ante selfish and privately observed investments are efficient. Our result indicates this result is driven entirely by the fact that these protocols lead to efficient allocations and not at all by the impact of these protocols on investment decisions conditional on investments, given that privacy ensures one's bargaining partner cannot change his offer, even if the investor does not make a take-it-or-leave-it offer.

3. A natural concern with Harberger taxation is that individuals may try to "sabotage" the value of their goods to others. However, our analysis shows that they never have a local incentive to do this as long as the tax rate is weakly below the turnover rate. To see this, note that sabotage is equivalent to a negative unit of capital investment plus a positive unit of selfish investment. The value derived from such an investment is, by our above analysis, $1 - q^\star - (1 - \tau) = -(q^\star - \tau)$. Intuitively as long as $\tau < q^\star$ the price the seller sets is weakly above the monopoly price and thus the seller benefits when her property is taken, though not as much as were $\tau = 0$. Thus she would not choose to sabotage her property in equilibrium.[10]

4. One objection to Harberger taxation is that individuals with high idiosyncratic utilities for assets must pay a high tax to avoid takings. To the extent that such idiosyncratic attachments are the result of random shocks an individual may experience, taxing away the benefits of such shocks could be either an advantage or disadvantage of Harberger taxation, depending on how this random gain in idiosyncratic utility interacts with the marginal value of consumption and thus whether individuals would like to insure against such shocks (as the Harberger tax effectively does). Regardless, our risk-neutral framework does not adequately capture these issues.

   However, to the extent that such attachment is partially under the control of the owner, this may actually be an additional benefit of Harberger taxation. In particular, common experience suggests individuals tend to form much stronger attachments to objects that do not decay than to those that are durable, and to objects they own versus those they rent. Because of monopoly distortions, property tends to turn over much less frequently than is socially optimal. Thus, alleviating monopoly distortions may optimally lead individuals to

[10]Note this argument is local and it could be possible that a joint global deviation to a very low price and extreme sabotage could be optimal in some cases. Levmore (1982) provides a detailed plan for how such extreme sabotage could be avoided.

become less attached to their possessions. This excessive attachment to material possessions is one of the principal flaws many religious and social thinkers (especially from the Buddhist tradition) perceive in capitalist societies. Therefore, unsurprisingly though heartening, moving optimally toward common ownership would tend to alleviate such attachment.

4.3 Many buyers

Our analysis above assumes that there is a single potential buyer of the asset. In many realistic cases, several bidders may be competing to buy the asset. In this subsection, we analyze the effect of buyer-side competition on the optimal Harberger tax rate using a simple auction model.

Suppose the asset belongs to the seller S, and assume for simplicity that $\gamma_S = 0$. There are multiple buyers $B_1 \ldots B_n$, with values drawn i.i.d. from distribution F. The asset is sold in a second-price auction, where S can set a reserve price p. S pays a tax on the reserve price p. Let y_1 represent the highest bid, and let y_2 represent the second-highest bid. S's objective function is

$$\pi_S = y_2 1_{y_1, y_2 > p} + p 1_{y_1 > p > y_2} + \eta 1_{y_1, y_2 < p} - p\tau.$$

Taking expectations over y_1, y_2 and then taking derivatives with respect to p yields

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = P\left(y_1 > p > y_2\right) - \tau - m\frac{dP\left(y_1, y_2 < p\right)}{dp},$$

where, as in Section 2, we define the markup $m \equiv p - \eta$. Substituting for the probability expressions, the derivative becomes

$$\frac{d\mathbb{E}\left[\pi_S\right]}{dp} = nF^{n-1}\left(p\right)\left(1 - F\left(p\right)\right) - \tau - mnF^{n-1}\left(p\right) f\left(p\right).$$

If we set this to 0, we get

$$\tau = nF^{n-1}\left(p\right)\left(1 - F\left(p\right)\right) - mnF^{n-1}\left(p\right) f\left(p\right). \tag{5}$$

Allocative efficiency is achieved when $p = \eta$ and thus $m = 0$, which requires

$$\tau = nF^{n-1}\left(\eta\right)\left(1 - F\left(\eta\right)\right).$$

As $n \to \infty$, this expression goes exponentially to 0. Thus, the allocatively optimal Harberger tax goes to 0 as competition grows. This conclusion is intuitive, given the follow-up to Jevons (1879)'s quote in our epigraph:

But when different persons own property of exactly the same kind, they become subject to the

> *important Law of Indifference...that in the same open market...there cannot be two prices for the same kind of article. Thus monopoly is limited by competition, and no owner, whether of labour, land, or capital, can, theoretically speaking, obtain a larger share of produce for it than what other owners of exactly the same kind of property are willing to accept.*

Larsen (2015) confirms this intuition empirically, and shows that sufficient competition in the market for used automobiles (if they are put up for an auction with dozens of bidders) limits the market-power distortion from property to 2%-4% of first-best allocative efficiency. Thus, in very competitive environments, or any environment where buyers have little idiosyncratic value for a particular piece of property, optimal Harberger taxes will be smaller than in an environment with greater market power.

Conversely, however, we assume throughout that the buyer values the asset under consideration on its own rather than in conjunction with other properties. If, on the other hand, the asset is complementary with many others, as is common in property development and the reassembly of spectrum (Kominers and Weyl, 2012*b*), then monopoly distortions substantially increase because no individual seller is pivotal in such a sale. This creates a "hold-out" problem (Mailath and Postelwaite, 1990). Introducing such complementarity would increase the optimal Harberger tax, and in the one case in which we are aware of Harberger taxation being proposed as a means of improving allocative efficiency, it was intended precisely to address such "eminent domain" issues (Tideman, 1969; Plassmann and Tideman, 2011). Thus, whether optimal Harberger taxes are above or below the levels we describe above depends largely on how large issues of complementarity or competition are relative to each other. We focus on a simple monopoly case as a compromise between these issues.

5 Connections

In this section, we relate our proposal to previous economic analysis and practices related to property rights in mechanism design, capital taxation, and intellectual property.

5.1 Other mechanisms

As we discuss in our introduction, this paper is inspired in part by a body of work that analyzes the role of property rights in asymmetric information bargaining problems (Cramton, Gibbons and Klemperer, 1987; Segal and Whinston, 2011). In particular, our quantile markup property of Theorem 1 builds on Segal and Whinston (2011)'s conclusion that property rights equal to the expected efficient decision can support efficient trade. We connect this observation to the empirical turnover rate of the asset at the optimal tax in our static model and extend this insight to a dynamic setting in which a diffuse set of buyers arrives over time. The central benefit of Harberger taxation in this setting is that it only requires the mechanism administrator to maintain a relationship with the current asset owner rather than the full set of potential asset purchasers.

This setting differs from other dynamic extensions of bilateral trade models (Athey and Miller, 2007; Skrzypacz and Toikka, 2015) which focus on repeated strategic interactions between a fixed pair of individuals. Our emphasis is on the possibility, which seems important in asset markets like housing and spectrum, that an asset may change hands between a potentially large number of individuals over an extended period of time. To allow for this tractably, we abstract from the detailed strategic interactions between any given pair of individuals.

While Harberger taxation has the benefit of extending naturally to a dynamic setting, it does not achieve the full static efficiency that is possible in a richer partial property rights mechanism. We view self-assessed taxation as a tractable first step towards designing practical dynamic partial property rights mechanisms; we leave the question of whether more sophisticated schemes can be made practical to future research.[11]

5.2 Capital taxation

While the taxation of wealth has been proposed for a variety of reasons, mostly related to redistribution or benefits-based taxation, proposals to use it to increase the efficiency of utilization of assets have a narrower history. While some early socialist thinkers motivated their arguments for common ownership of property partly by concerns about monopoly, the first clear expressions of the idea common ownership could improve allocative efficiency appear, to our knowledge, in Jevons (1879) and Walras (1896).

These ideas were popularized and most forcefully advanced by George (1879) who argued that effective common ownership of land could be implemented by taxing away land rents. Beyond the revenue it raised, he suggested this would also improve the efficiency of allocation by breaking up wasteful "feudal" land holdings. However, to avoid discouraging investment, George argued this tax should apply only to pure land rents. While George's ideas were highly influential for a period, they ran into two practical obstacles: that there was no way to separate

[11] Another arrangement that could avoid market power distortions and thus obviate the need for common ownership is an approximation to the "counter-speculation" subsidies Vickrey (1961) proposed. In particular, a subsidy in the amount $\frac{M'(1-F(\gamma_S))(1-F(\gamma_S))}{2}$ that is paid to the seller if and only if a sale takes place has the same effect as as an allocatively efficient Harberger tax. Such a subsidy could conceivably be implemented without distorting investment incentives and thus could potentially be superior to an optimal Harberger tax.

We are concerned, however, that such a scheme would be impracticable for a variety of informational reasons. First, it requires knowing the value distribution, but without a simple means to iteratively calculate the value, because it also depends on the value of M'. Second, M' is particularly difficult to measure and requires a lot of cadastral authorities, especially given its value could be significantly context dependent in a way known to the seller but not to the cadastral authorities; see Weyl and Tirole (2012) for a detailed related discussion. Third, and perhaps most important, the scheme would be open to tremendous manipulation. Two-way sales could take place in succession and generate net subsidies to the participants. Finally, and perhaps most importantly, although this scheme would avoid common ownership in some sense, it would involve much more discretionary official intervention than would a Harberger tax. We therefore do not consider it a credible or less radical alternative.

pure land rents from the fruits of various investments and that there was no way to assess the value of land used by its owners.

This problem has led to various schemes for using market prices to assess property taxes, such as using previous sale value. This assessment method creates a variety of perverse incentives (e.g., to hold on to property when it rises in value and dispose of it when it falls) and does a poor job estimating the present property value. This assessment problem is particularly acute in developing countries, where corruption in the assessment process routinely undermines property tax collection. As a solution, Harberger (1965) proposed self-assessment and compulsory sale as a self-enforcing means of collecting taxes on real property. Tideman (1969), a self-described Georgian, highlighted how such a tax might increase turnover of property. However he did not consider the impact on investment, a model with dynamics or explicitly model allocative efficiency. This meant that much of the following literature continued to focus on the revenue-raising rather than efficiency-enhancing effects of Harberger taxation.

In fact, Levmore writes of the impact of self-assessment on turnover rates: "It is perhaps unfortunate that these side effects to self-assessment exist." The author goes on to discuss methods of minimizing these "side effects." This use of Harberger taxes has been criticized theoretically (Epstein, 1993) and empirically (Chang, 2012), primarily because it is unclear that self-assessed taxation schemes create incentives for truthful value revelation.[12]

We essentially propose to invert the core argument of this line of work: rather than using information from market transactions to more effectively tax capital, we propose applying a tax on capital purely to increase the efficiency with which market transactions take place.[13] Viewed in this light, the primary flaw of self-assessed taxation schemes discussed in Epstein (1993) and Chang (2012) – that proposed prices tend not to be equal to true values – is a core feature of our mechanism. At the welfare-maximizing level of the Harberger tax in our model, sellers are left

[12]Chang (2008) provides, to our knowledge, the only formal illustration of this argument, but the logic is clear from Theorem 1: the assessment is truthful if and only if the tax rate equals the efficient probability of sale, and any individual is entitled to take the property. In fact, in Harberger's original proposal, any individual may take the property, but the tax rate was set to be equal to a usual property tax rate, on the order of a few percent, which is almost certainly far below the efficient probability of sale (it is below the probability of sale even in the present, which is distorted downward by market power). Thus, we would expect assessments at rates far above truthful values under Harberger's system. On the other hand, Chang (2012) finds that in nearly all historical cases in which self-assessment has been used, only extraordinary actions (e.g., litigation or eminent domain takings) have triggered takings at the self-assessed value. In these cases, the probability of sale is far below the property tax rate, and thus we would expect, and Chang finds, that assessments are clearly below true values. To our knowledge, there is no example of Harberger's system of *universally invokable and universally applicable* being implemented practically.

[13]Various other rationales for capital taxation in the literature include local property taxes as a source of funds for local public goods provision (Lindahl, 1919; Bergstrom, 1979; Arnott and Stiglitz, 1979), a mechanism for dynamic redistribution (Judd, 1985; Chamley, 1986; Golosov and Tsyvinski, 2015), and a mechanism for governments without commitment power to avoid the temptation of appropriative redistribution (Farhi et al., 2012; Piketty, 2014; Scheuer and Wolitzky, Forthcoming). These arguments are largely orthogonal to our analysis, though in some cases, capital taxation using our mechanism can simultaneously accomplish some of the goals outlined in these works; in other cases, our tax will tend to create an excessive wedge along these dimensions that should be compensated by a subsidy (e.g., on savings) to ensure wedges are of optimal size. See, for example, our discussion of objectively assessed property subsidies in Subsection 4.1 above.

with residual monopoly power, and announce prices higher than their values. However, this residual market power distortion is necessary in order to maintain optimal incentives for capital investment. In the context of our model, the purpose of self-assessed taxes is not necessarily to elicit truthful value assessments – for allocative efficiency to increase, it is enough that the value assessments are lower than the prevailing prices in a market absent the tax. Similarly our proposal does not achieve George's full ambition of socializing the ownership of "land", as it is restrained by the inability to fully distinguish investment from land, but it is still able to improve on private ownership.

5.3 Intellectual property

Unlike physical property, *intellectual* property has always been limited in scope and duration precisely to limit monopoly power. In a sense, our argument is simply that intellectual and physical property should be treated more symmetrically.

However, unlike physical property, intellectual property is non-rivalrous in consumption, which implies the relevant activity distorted downward by property is not a turnover of the good from one rivalrous owner to another, but rather wide availability of the good.[14] This difference in the nature of the distortion in turn means allocative efficiency can be achieved by simply setting the price of the property to zero. On the other hand, it makes rewarding investment much more complicated.

In particular, we only considered the impact of Harberger taxation on a simple type of investment, namely, one in a uniform increase in values. By contrast, Weyl and Tirole (2012) argue that inventions differ along multiple dimensions in terms of the market for the products they produce. Although physical property usually has a tangible value and easily observable investments, the value of intellectual property is usually not apparent until a long process of marketing, adoption, and market testing has sorted out its value-added.

To make matters worse, charging a Harberger tax as a fraction of the total value of intellectual property requires knowing the total size of the market for the product if it were offered for free, so that the tax can be applied as a fraction of this total size. Unlike the probability of turnover, which is bounded between 0 and 1 and is plausibly in a knowable range for most goods, the value of market sizes will vary by orders of magnitude for observably similar products. For example, many apps on Apple's App Store receive only a few downloads, whereas others "go viral" and are downloaded a billion times. Without knowledge of this market size (which almost solves the problem itself, because a prize could be given directly), a Harberger tax would likely be laughably small for some markets while leaching all profits out of others.[15] Thus while

[14] However, as Hopenhayn, Llobet and Mitchell (2006) point out, intellectual property *rights* may be rivalrous given that at most a single monopoly rent exists that must be divided among sequential innovators, and the rate of turnover of intellectual property rights themselves may be distorted downwards as we discuss in the next section.

[15] If a public authority knew the efficient market size (call it σ) for a good, our scheme would be equivalent to

Harberger taxation shares motivational similarities with temporal and breadth limitations on intellectual property rights, it is not easily applicable to intellectual property as such. More appropriate are more centralized schemes that rely more heavily on central administrators, such as those proposed by Kremer (1998) and Weyl and Tirole (2012).

6 Applications

In this section, we discuss three categories of applications of our approach. The first is in the private sector and could be implemented by profit-making firms. The second concerns goods for which capital investment plays a negligible role, and thus the primary focus is on allocative efficiency. For these goods, Harberger taxes seem particularly attractive, easy to implement, and perhaps even unlikely to raise substantial controversy. However, these goods are relatively limited, and thus we conclude by discussing a broader implementation that could be applied economy-wide and would trade off investment incentives and allocative efficiency as in our calibrations above. Our discussion here is relatively brief; see Posner and Weyl (2016) for a detailed discussion of these applications and the relationship to existing legal institutions.

6.1 Private sector

While we have framed most of this paper in terms of the general market for housing, a natural and likely more near-term feasible class of applications of efficiency-enhancing Harberger taxation lies within the private sector. Two categories of such applications seem natural.

The first is a consumer, peer-to-peer platform of shared property. One such arrangement would be a broader or more radical version of various "sharing economy" platforms that have recently received significant commercial attention. A firm producing a particular durable commodity could, rather than offer it for traditional sale, sell goods only to members of the platform. Similarly, instead of selling goods entirely, the firm could sell only a right to use contingent on paying a Harberger tax, and offer the good for sale to any platform member at the self-assessed value. The platform would make revenue off the sales price, the Harberger tax, and the membership fee. As Armstrong (1999) shows, if this platform covered sufficiently many goods, setting the sale price and Harberger tax to maximize member surplus, which could then

setting a tax equal to $p\sigma\tau$, where τ is the Harberger tax as previously and p is the price chosen by the monopolist. The required level of Harberger taxation to achieve taxation would be $\tau = 1$ (but would all eliminate monopoly profits and thus innovation incentive). A lower tax would still incent lower prices than pure intellectual property, but the innovator would be left with some rents, implementing the trade-off we analyzed above.

Even if such a scheme could be implemented, it would involve a substantially worse trade-off than with physical property, because the allocatively efficient tax is so high. However, it seems almost certainly impractical given the difficulty of estimating σ. For example, a σ estimated at 1,000 would have no appreciable impact on the bottom line, and therefore prices of a product that ended up having a mass market. On the other hand, it would drive out all profits even at a modest 10% tax rate for a product with niche appeal to only a hundred clients.

be extracted through the membership fee, would typically be in the interests of the platform.[16]

A potentially broader application involves applying Harberger taxation within corporations. Coase (1937) famously argued that an important reason for the existence of firms was the "transaction costs of the market." While a large literature since his work has analyzed what these transaction costs comprise, Coase highlighted costs of bargaining avoided within firms. Such costs may naturally be interpreted as the monopoly distortion the Harberger tax addresses, as in the double marginalization problem of Cournot (1838) and Spengler (1950) or as wasteful investments in reducing asymmetric information to avoid these distortions. Thus, firms may be seen as a form of private common property aimed at increasing allocative efficiency within the firm. Groves and Loeb (1979) take this interpretation to its logical extreme by arguing that a Vickrey auction should be used to allocate resources within firms.

However, as emphasized by Grossman and Hart (1986) and subsequent work in the property rights literature surveyed by Segal and Whinston (2013), such common ownership can reduce investment incentives of various stakeholders within the firm, just as a Harberger tax does. This suggest that a formalized institution of Harberger taxation may be an effective way to formalize and optimize internal markets for resources within firms that are often managed through informal reputational contracts. Because the relevant taxes would be collected by the firm and all individuals participating in the market employed by it, the necessity of gate-keeping to capture the associated efficiency benefits which limits the applicability of the platform model described above would be unlikely to be a significant concern.

6.2 Assets with limited investment component

Although many categories of property require substantial investments to maintain or improve, the value of some types of property is largely independent of investments. The leading example is radio spectrum. At present, no known means exist to make permanent improvements to the spectrum itself, though individuals may make selfish investments in adapting themselves to broadcasting their programming on that band and may make some investment in marketing devices that tune well into that band.[17] Other examples include internet names and addresses, some types of undeveloped land in rural areas, certain natural resource extraction rights, especially for automatically self-renewing resources with little permanent temporal linkage and many intellectual property rights. For the remainder of this subsection, we focus on the case of spectrum, but most of our arguments apply more broadly to these other cases.

In the early 1990s, the Federal Communications Commission (FCC) auctioned off most of the

[16] Although getting such a platform off the ground could present challenges, because it would have substantial network effects, dynamic pricing of membership might be able to overcome these challenges (Weyl, 2010).

[17] Given that some of these investments are potentially transferable and may have limited or no value outside of that band it may be appropriate to tie such physical investments to the license itself. Posner and Weyl (2016) discuss potential property frameworks for such a regime.

radio spectrum in the US; this strategy has been followed by governments around the world. Prior to this auction, standard practice had been to allocate spectrum by lottery and allow private bargaining in the spirit of Coase (1960) to reallocate spectrum to its most valuable use. As Milgrom (2004) emphasizes, a central motivation behind the design work was that monopoly distortions emphasized by Myerson and Satterthwaite (1981) stood in the way of this reallocation. The FCC and the economists it hired thought an auction could ensure an efficient allocation.

However, at best such an allocation could ensure *static* efficiency of the allocation to its first owner. Repeated auctions for very temporary usage rights would be necessary to ensure efficient allocations in the future. For a variety of administrative and complexity reasons, as well as because of some residual concerns over investments, such auctions are widely viewed as impractical.[18] In fact, the single major auction (Milgrom and Segal, 2015) since that time held by the United States, to reassemble and re-purpose spectrum currently in private hands, took an act of Congress and the better part of a decade to organize and execute, despite broad consensus about the need for changes in ownership blocked by the market power of current owners.[19] This strongly suggests that a regime, like Harberger taxation, that encourages more efficient reallocation dynamically could significantly improve the efficiency with which spectrum is allocated over time. Empirical determination of allocatively efficient Harberger taxes is extremely simple, as we highlighted in Subsubsection 2.2: it simply requires iteratively setting the tax rate to equilibrium turnover rate.[20] Furthermore, given that spectrum property rights are administrative rather than absolute, this regime should be relatively straightforward to implement and would save on the substantial costs expended on auction design and participation. Spectrum thus seems perhaps the most natural place to begin experimenting with Harberger taxation.

6.3 Economy-wide implementation

In the previous subsection, we considered the simplest case for applying Harberger taxation publicly. Now we consider how broad its scope should be in the long term. For the most liquid forms of currency and government bonds, Harberger taxation is neither harmful nor beneficial: these assets carry no market power, but also require no investment. One should

[18]Beyond political issues, designing a practical auction dealing with the full set of complexities in an environment like this (collusion, the combinatorial nature of bids, etc.) has proved daunting (Ausubel and Milgrom, 2005; Levin and Skrzypacz, Forthcoming). As Posner and Weyl (2016) discuss, a more decentralized system like Harberger taxation might be able to overcome some of these issues by allowing reallocation to occur in a more flexible way, where individual agents could determine patterns of complementarity and substitution rather than this being dictated by a centralized computation.

[19]In fact, a major lobbying organization, the Dynamic Spectrum Alliance, is dedicated solely to ensuring more flexible reallocation of spectrum rights.

[20]As we discussed in Section 3, the allocatively efficient tax may actually be below the turnover rate in a dynamic model. However, these seem to be extremely close in most examples and the welfare loss from using the turnover rate appears even smaller. More investigation of this in a model calibrated to the market for spectrum would be instructive.

therefore be indifferent to Harberger taxation of them except for the issues raised our discussion in Subsection 5.2 above about avoiding savings distortions. On the other hand, our argument suggests the application of Harberger taxation to essentially all other forms of capital, even very liquid ones, such as shares of companies. Although most individuals do not exercise market power over these commodities, some do, and these individuals also make decisions about investing their time in augmenting the value of these securities.

Thus all property would be registered in a cadaster with a regularly updated value. The cadaster would be made available, perhaps through a smartphone app, and the standard right of property would be replaced with a right to property that has not been purchased at the cadastral value, combined with a right to appropriate any property of another at its cadastral value. Cadastral proceeds would fund the enforcement of this system as taxes fund present-state enforcement of property law. Excess revenue could be returned to the community in any desired fashion, but it would likely be most desirable to use the revenues to reduce or eliminate existing distortionary means of public funds, as this would further increase the social value created by the Harberger tax.[21] Furthermore such a system need not begin with a rate high enough to imply large changes to the value of capital; it could start at a very low rate, of say 1% per annum, and only rise gradually once it was widely believed its effects had been beneficial.

However, our calibrations suggest that the rate would eventually optimally make a large impact on aggregate welfare and its distribution. To see this we briefly consider the magnitude of revenue in a world in which all capital is subject to Harberger taxation. Suppose, following our dynamic calibration of Section 3, the entire capital stock of the world is subject to a 7% annual Harberger tax. This would generate annual revenue of approximately 60% of capital income. According to Piketty and Zucman (2014), capital's share is a bit greater than 25% of national income in most countries, implying revenue of approximately 15% of national income annually. This would account for more than half of the revenue that governments in the United States collect and would also eliminate most of the value of private capital, reducing its levels to the lows of the middle twentieth century.

The total welfare gains from our proposed policy are roughly 6% of the value of assets at their average market prices. Again assuming capital accounts for a quarter of national income, this would imply annual gains of roughly 2% of national income, or about $300 billion annually if the gains applied to all capital. Realistically some capital is significantly more liquid than is housing, though housing alone accounts for roughly a third of capital income (Rognlie, 2015), though some is less liquid as well. We thus would guess that 1% of national income or $150

[21] As we discuss above, raising such large revenues need not lead to a large redistribution of those resources, because the revenue collected could be distributed in a manner specified by rules or given back as savings or property subsidies to offset other distortions. However, once such a large revenue stream is collected, a community would not necessarily wish to disperse it with the same inequality with which capital ownership itself is presently distributed. Because the Harberger tax offers a non-distortive (actually efficiency enhancing) means of generating a large pool of revenue, some communities might use it for redistributive purposes even if its justification is not redistributive.

billion in the United States, or roughly $1 trillion globally at purchasing power parity, is a good lower bound for the benefit of optimal Harberger taxation.

7 Conclusion

In this paper, we argue the means of production should generally be owned neither in common nor privately, but rather through a mixed system that trades off the allocative benefits of common ownership against the investment incentives created by private ownership. We then show that a simple proposal for self-assessed capital taxes put forward by Harberger (1965) (for a very different purpose) can implement this system. Finally, we calibrate the optimal level of this tax in a variety of settings, and we find a figure around 5-10% per annum performs quite well in a range of settings.

Our analysis above considers only inanimate and not human capital. However, human capital receives a larger fraction of national income than inanimate capital and is likely as important a source of market power, given the unique talents many individual workers possess and the distortions to these talents, caused by labor income taxes. Indeed, most societies that have practiced common ownership of inanimate capital (e.g., the Israeli *kibbutzim* and the Soviet Union) have also socialized earning capacity to a significant extent. In these societies, human capital was largely directed according to social needs, rather than the choice of the human capitalist. Of course, these arrangements famously undermined human capital accumulation (Abramitzky and Lavy, 2014). Nonetheless, many methods exist for objectively assessing human capital that could be used to offer human capital subsidies to overcome this problem. In any case, partial common ownership would be a far smaller deterrent to investment than full common ownership. A fascinating question for future research is thus whether a workable system of more partial common ownership of human capital could be devised along the lines above. Such a system would have to deal with, among other challenges, the differing amenities of different workplaces (Sorkin, 2015) that make human capitalists far from indifferent across competing purchasers of their labor. However, on the upside, it could be used not only to address distortions to labor but also to various environmental externalities impacting human life by giving a market basis for the valuation of such externalities.

Our proposal does not, of course, exhaust the possibilities for mitigating market power by changing the nature of property entitlements. As we discussed in Subsection 5.1 above, Segal and Whinston (2011) show that with an appropriate bargaining procedure and ownership explicitly shared with potential future buyers rather than the public at large, greater efficiency gains than from Harberger taxation are possible. Additionally, a targeted Harberger tax, with rates that differ across different categories of sellers, might also dramatically outperform a uniform tax.

Whether such schemes are practical or could be simplified is an interesting question for future research. As Heller (2008) argues, the market power property creates is particularly socially costly in settings where complementary goods must be assembled. Although reasonable Harberger taxation would largely obviate this problem by forcing value revelation prior to the announcement of public projects and thereby prevent most holdout problems, Kominers and Weyl (2012*a*) show other means of relaxing property rights may also significantly or completely eliminate these concerns while maintaining full investment incentives.

Our analysis assumed only the present owner of an asset may enjoy it and invest in it. In many circumstances, enjoyment and investment may be temporally shared in a variety of ways (Ostrom, 1990). Extending our analysis to such settings and devising appropriate forms of taxation in those contexts would be valuable. Another important practical issue absent from our analysis is transactions costs involved in transferring the possession of goods. Although we suggested in Subsection 6.3 that these costs may be diminishing with technology, they still pose significant challenges in many contexts, and any practical proposal would have to confront how they should be born and how they should influence the frequency with which compulsory purchases should be allowed to occur.

Our analysis also assumed risk neutrality. This assumption may be reasonable when applied to relatively small capital goods or for wealthy individuals, but for many poorer individuals, this approximation is quite problematic regarding their attitude toward a house, which may represent a large fraction of their lifetime income. Tideman (1969) analyzes the Harberger-taxed-monopoly problem using a reduced form for risk aversion, but this approach does not allow him to study optimal taxation. In fact the impact of risk aversion and liquidity constraints on our conclusions seems quite ambiguous, as they may either imply that individuals with exceptionally high idiosyncratic values but low liquidity are forced to pay large taxes inefficiently, or that capital goods are more affordable as their capitalized asset values are lower. Also the availability of insurance against non-sale might largely mitigate these concerns. A formal analysis with risk aversion would therefore be an important topic for future research.

Finally, although our calibrational analysis suggests a 5-10% tax is nearly optimal for many goods, and is a simple procedure to determine the allocatively efficient tax level for goods with small investment components (see Subsection 6.2 above), serious empirical analysis of the size of market-power distortions and investment elasticities for goods is crucial to pinning down this number with any precision. We hope future research will clarify these crucial elasticities as, for example, the literature on the elasticity of taxable income (Saez, Slemrod and Giertz, 2012) has clarified crucial elasticities for optimal redistributive taxation.

References

Abramitzky, Ran, and Victor Lavy. 2014. "How Responsive is Investment in Schooling ot Changes in Redistributive Policies and in Returns?" *Econometrica*, 82(4): 1241–1272.

Acemoglu, Daron, and James A. Robinson. 2012. *Why Nations Fail: The Origins of Power, Prosperity, and Poverty.* New York, NY: Crown Business.

Armstrong, Mark. 1999. "Price Discrimination by a Many-Product Firm." *Review of Economic Studies*, 66(1): 151–168.

Arnott, Richard J., and Joseph E. Stiglitz. 1979. "Aggregate Land Rents, Expenditure on Public Goods, and Optimal City Size." *Quarterly Journal of Economics*, 93(4): 471–500.

Arrow, Kenneth J. 1979. "The Property Rights Doctrine and Demand Revelation under Incomplete Information." In *Economics and Human Welfare*. , ed. Michael Boskin, 23–39. New York: Academic Press.

Athey, Susan, and David A Miller. 2007. "Efficiency in repeated trade with hidden valuations." *Theoretical Economics*, 2(3): 299–354.

Ausubel, Lawrence M., and Paul Milgrom. 2005. "The Lovely but Lonely Vickrey Auction." In *Combinatorial Auctions*. , ed. Peter Cramton, Richard Steinberg and Yoav Shoham, 17–40. Cambridge, MA: MIT Press.

Baker, George P. 1992. "Incentive Contracts and Performance Measurement." *Journal of Political Economy*, 100(3): 598–614.

Bergstrom, Ted. 1979. "When Does Majority Rule Supply Public Goods Efficiently?" *Scaninavian Journal of Ecnomics*, 81(2): 216–226.

Blume, Lawrence, Daniel L. Rubinfeld, and Perry Shapiro. 1984. "The Taking of Land: When Should Compensation be Paid?" *Quarterly Journal of Economics*, 99(1): 71–92.

Chamley, Christophe. 1986. "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives." *Econometrica*, 54(3): 607–622.

Chang, Yun-chien. 2008. "A Framework of Takings Compensation." http://lsr.nellco.org/nyu_lewp/141/.

Chang, Yun-chien. 2012. "Self-Assessment of Takings Compensation: An Empirical Study." *Journal of Law, Economics and Organization*, 28(2): 265–285.

Che, Yeon-Koo, and Donald B. Hausch. 1999. "Cooperative Investments and the Value of Contracting." *American Economic Review*, 89(1): 125–147.

Coase, R. H. 1937. "The Nature of the Firm." *Economica*, 4(16): 386–405.

Coase, Ronald H. 1960. "The Problem of Social Cost." *Journal of Law and Economics*, 3: 1–44.

Cournot, A. Augustin. 1838. *Recherches sur les Principes Mathematiques de la Theorie des Richess.* Paris.

Cramton, Peter, Robert Gibbons, and Paul Klemperer. 1987. "Dissolving a Partnership Efficiently." *Econometrica*, 55(3): 612–632.

Crémer, Jacques, and Richard P. McLean. 1988. "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions." *Econometrica*, 56(6): 1247–1257.

d'Aspremont, Claude, and Louis-André Gérard-Varet. 1979. "Incentives and Incomplete Information." *Journal of Public Economics*, 11(1): 25–45.

de Soto, Hernando. 2003. *The Mystery of Capital: Why Capitalism Triumphs in the West and Fails Everywhere Else.* New York: Basic Books.

Emrath, Paul. 2013. "Latest Calculations Show Average Buyer Expected to Stay in a Home 13 Years." *HousingEconomics.com*.

Epstein, Richard A. 1993. "Holdouts, Externalities, and the Single Owner: One More Salute to Ronald Coase." *Journal of Law and Economics*, 36(1): 553–586.

Epstein, Richard A. 1997. "A Clear View of The Cathedral: The Dominance of Property Rules." *Yale Law Journal*, 106(7): 2091–2120.

Fabinger, Michal, and E. Glen Weyl. 2016. "The Average-Marginal Relationship and Tractable Equilibrium forms." http://ssrn.com/abstract=2194855.

Farhi, Emmanuel, Chris Sleet, Iván Werning, and Sevin Yeltekin. 2012. "Nonlinear Capital Taxation without Commitment." *Review of Economic Studies*, 79(4): 1469–1493.

George, Henry. 1879. *Progress and Poverty: An Inquiry into the Cause of Industrial Depressions and of Increase of Want with Increase of Wealth: The Remedy.* New York: Robert Schalkenbach Foundation.

Glaeser, Edward L., and Jesse M. Shapiro. 2003. "The Benefits of the Home Mortgage Interest Deduction." *Tax Policy and the Economy*, 17: 37–82.

Golosov, Mikhail, and Aleh Tsyvinski. 2015. "Policy Implications of Dynamic Public Finance." *Annual Review of Economics*, 7: 147–171.

Grossman, Sanford J., and Oliver D. Hart. 1986. "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration." *Journal of Political Economy*, 94(4): 691–719.

Groves, Theodore, and Martin Loeb. 1979. "Incentives in a Divisionalized Firm." *Management Science*, 25(3): 221–230.

Harberger, Arnold C. 1965. "Issues of Tax Reform in Latin America." In *Fiscal Policy for Economic Growth in Latin America: Papers and Proceedings of a Conference Held in Santiago, Chile, December, 1962*. Baltimore, MD: Johns Hopkins University Press.

Hart, Oliver, and John Moore. 1988. "Incomplete Contracts and Renegotiation." *Econometrica*, 56(4): 755–785.

Hart, Oliver, and John Moore. 1990. "Property Rights and the Nature of the Firm." *Journal of Political Economy*, 98(6): 1119–1158.

Hayek, F. A. 1945. "The Use of Knowledge in Society." *American Economic Review*, 35(4): 519–530.

Heller, Michael. 2008. *The Gridlock Economy: How Too Much Ownership Wrecks Markets, Stops Innovation and Cost Lives.* Philadelphia, PA: Basic Books.

Hopenhayn, Hugo, Gerard Llobet, and Matthew Mitchell. 2006. "Rewarding Sequential Innovators: Prizes, Patents, and Buyouts." *Journal of Political Economy*, 114(6): 1041–1068.

Jacobs, Jane. 1961. *The Death and Life of Great American Cities.* New York: Random House.

Jevons, William Stanley. 1879. *The Theory of Political Economy.* . 2nd ed., London: Macmillan and Co.

Judd, Kenneth L. 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics*, 38(1): 59–83.

Kaplow, Louis, and Steven Shavell. 1996. "Property Rules versus Liability Rules: An Economic Analysis." *Harvard Law Review*, 109(4): 713–790.

Kominers, Scott Duke, and E. Glen Weyl. 2012*a*. "Concordance among Holdouts." http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1591466.

Kominers, Scott Duke, and E. Glen Weyl. 2012*b*. "Holdout in the Assembly of Complements: A Problem for Market Design." *American Economic Review Papers & Proceedings*, 102(3): 360–365.

Kremer, Michael. 1998. "Patent Buyouts: A Mechanism for Encouraging Innovation." *Quarterly Journal of Economics*, 113(4): 1137–1167.

Lange, Oskar. 1967. "The Computer and the Market." In *Socialism, Capitalism and Economic Growth: Essays Presented to Maurice Dobb.* , ed. C. H. Feinstein, 158–161. Cambridge, UK: Cabmridge University Press.

Larsen, Bradley. 2015. "The Efficiency of Real-World Bargaining: Evidence from Wholesale Used-Auto Auctions." http://goo.gl/Ukshrl.

Levin, Jonathan, and Andrzej Skrzypacz. Forthcoming. "Properties of the Combinatorial Clock Auction." http://web.stanford.edu/~jdlevin/Papers/CCA.pdf.

Levmore, Saul. 1982. "Self-Assessed Valuation Systems for Tort and Other Law." *Virginia Law Review*, 68(4): 771–861.

Lindahl, Erik. 1919. "Positive Lösung." In *Die gerechtigkeit der Besteuerung*. Chapter 4, 85–98. Lund.

Mailath, George J., and Andrew Postelwaite. 1990. "Asymmetric Information Bargaining Problems with Many Agents." *Review of Economic Studies*, 57(3): 351–367.

Milgrom, Paul. 1987. "Adverse Selection without Hidden Information." http://goo.gl/2gsnX9.

Milgrom, Paul. 2004. *Putting Auction Theory to Work.* Cambridge, UK: Cambridge University Press.

Milgrom, Paul, and Ilya Segal. 2002. "Envelope theorems for arbitrary choice sets." *Econometrica*, 70(2): 583–601.

Milgrom, Paul, and Ilya Segal. 2015. "Deferred-Acceptance Auctions and Radio Spectrum Reallocation." http://goo.gl/eZvDYU.

Myerson, Roger B. 1981. "Optimal Auction Design." *Mathematics of Operations Research*, 6(1): 58–73.

Myerson, Roger B., and Mark A. Satterthwaite. 1981. "Efficient Mechanisms for Bilateral Trading." *Journal of Economic Theory*, 29(2): 265–281.

Nadiri, M Ishaq, and Ingmar R Prucha. 1996. "Estimation of the depreciation rate of physical and R&D capital in the US total manufacturing sector." *Economic Inquiry*, 34(1): 43–56.

Naik, Nikhil, Scott Duke Kominers, Ramesh Raskar, Edward L. Glaeser, and César A. Hidalgo. 2015. "Do People Shape Cities, or Do Cities Shape People? The Co-Evolution of Physical, Social, and Economic Change in Five Major U.S. Cities." http://web.mit.edu/naik/www/assets/pdf/naik_urban_change_nber_15.pdf.

Ostrom, Elinor. 1990. *Governing the Common: The Evolution of Institutions for Collective Action.* Cambridge, UK: Cabmridge University Press.

Piketty, Thomas. 2014. *Capital in the Twenty-First Century.* Cambridge, MA: Harvard University Press.

Piketty, Thomas, and Gabriel Zucman. 2014. "Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010." *Quarterly Journal of Economics*, 129(3): 1255–1310.

Plassmann, Florenz, and T. Nicolaus Tideman. 2011. "Marginal Cost Pricing and Eminent Domain." *Foundations and Trends in Microeconomics*, 7(1): 1–110.

Posner, Eric A., and E. Glen Weyl. 2016. "Property is Another Name for Monopoly: Facilitating Efficient Bargaining with Partial Common Ownership of Corporations, Land, and Spectrum." Please contact Glen Weyl at glenweyl@microsoft.com for a preliminary draft.

Rogerson, William P. 1992. "Contractual Solutions to the Hold-Up Problem." *Review of Economic Studies*, 59(4): 777–793.

Rognlie, Matthew. 2015. "Deciphering the Fall and Rise in the Net Capital Share." *Brookings Papers on Economic Activity*, 50(1 (Spring)): 1–69.

Saez, Emmanuel, Joel Slemrod, and Seth H. Giertz. 2012. "The Elasticity of Taxable Income With Respect to Marginal Tax Rates: A Critical Review." *Journal of Economic Literature*, 50(1): 3–50.

Scheuer, Florian, and Alexander Wolitzky. Forthcoming. "Capital Taxation under Political Constraints." *American Economic Review*.

Segal, Ilya, and Michael D. Whinston. 2013. "Property Rights." In *The Handbook of Organizational Economics*. , ed. Robert Gibbons and John Roberts. Princeton, NJ: Princeton Univeresity Press.

Segal, Ilya, and Michael Whinston. 2011. "An Expected-Efficient Status Quo Allows Efficient Bargaining." *Theoretical Economics*, 6(1): 109–125.

Skrzypacz, Andrzej, and Juuso Toikka. 2015. "Mechanisms for repeated trade." *American Economic Journal: Microeconomics*, 7(4): 252–293.

Smith, James E., and Kevin F. McCardle. 2002. "Structural Properties of Stochastic Dynamic Programs." *Operations Research*, 50(5): 796–809.

Sorkin, Isaac. 2015. "Ranking Firm Using Revealed Preference." https://goo.gl/lOIKkM.

Spengler, Joseph J. 1950. "Vertical Integration and Antitrust Policy." *Journal of Political Economy*, 50(4): 347–352.

Stokey, Nancy L., and Robert E. with Edward C. Presscott Lucas, Jr. 1989. *Recursive Methods in Economic Dynamics.* Cambridge, MA: Harvard University Press.

Tideman, T. Nicolaus. 1969. "Three Approaches to Improving Urban Land Use." PhD diss. University of Chicago.

Vickrey, William. 1961. "Counterspeculation, Auctions and Competitive Sealed Tenders." *Journal of Finance*, 16(1): 8–37.

Walras, Léon. 1896. *Etudes d'Économie Sociale. Théories de la Répartition de la Richesse Sociale.* Lausanne, Switzerland: Corbaz.

Werning, Iván. 2007. "Pareto Efficient Income Taxation." http://economics.mit.edu/files/1281.

Weyl, E. Glen. 2010. "A Price Theory of Multi-Sided Platforms." *American Economic Review*, 100(4): 1642–1672.

Weyl, E. Glen, and Jean Tirole. 2012. "Market Power Screens Willingness-to-Pay." *Quarterly Journal of Economics*, 127(4): 1971–2003.

Weyl, E. Glen, and Michal Fabinger. 2013. "Pass-Through as an Economic Tool: Principles of Incidence under Imperfect Competition." *Journal of Political Economy*, 121(3): 528–583.

Willekens, Bert, Roel Helgers, Maarten Goos, and Erik Buyst. 2015. "Intermediation in Markets with Buyer and Seller Selection: Theory and an Application to Real Estate Brokerage." https://goo.gl/YhpFV1.

Appendix

A Proofs and derivations

A.1 Baseline model

Here, we prove our statement in Subsection 2.2 that Myerson (1981)'s regularity condition is sufficient for $\frac{\partial q^\star}{\partial \tau}$ to be finite for all tax values below the efficient probability of sale $\tau = 1 - F(\gamma_S)$. Myerson's regularity condition states that marginal revenue is monotone. A monopolist seller with value γ_S for the good has revenue $(M(q) - \gamma_S)\, q$. Taking a derivative yields $M'(q)\, q + (M(q) - \gamma_S)$. Taking the second derivative, we have

$$2M'(q) + M''(q)\, q < 0.$$

Now consider the monopolist's problem under a Harberger tax $\tau < 1 - F(\gamma_S)$. By Theorem 1, $q(\tau) > \tau$; hence, $0 < q(\tau) - \tau < q(\tau)$. We want to show the following quantity exists:

$$\frac{\partial q^*}{\partial \tau} = \frac{M'(q)}{2M'(q) + M''(q)\,(q - \tau)}.$$

So we have to show that the denominator is bounded away from 0. From our full-support assumptions on ϵ, $M'(q)$ exists and is negative for all q. If $M''(q) \leqslant 0$, we know $q - \tau > 0$, so $M''(q)\,(q-\tau) \leqslant 0$, and the numerator and denominator are both strictly negative; hence, their ratio is positive and nonzero and $\frac{\partial q^*}{\partial \tau}$ exists. So suppose $M''(q) > 0$. Then

$$2M'(q) + M''(q)\,(q - \tau) < 2M'(q) + M''(q)\, q < 0$$

Where we first use that $0 < q(\tau) - \tau < q(\tau)$, and then apply Myerson regularity. Hence, the denominator $2M'(q) + M''(q)\,(q - \tau)$ is strictly negative, and the ratio $\frac{\partial q^*}{\partial \tau}$ exists and is positive.

Now we turn to regularity conditions for the social-maximization problem. From the text, the marginal benefit of increased Harberger taxation is $M(q^\star(\tau))\, \rho(q^\star(\tau))$ and the marginal cost is $\Gamma'(1-\tau)\,\tau$. Recalling that $\rho = \frac{\partial q^\star}{\partial \tau}$, the second-order condition for maximization is

$$M'\rho^2 + \rho' M \rho + \Gamma''\tau - \Gamma'.$$

The first term is always negative ($\rho > 0 > M'$) and represents the "quadratic" nature of the allocative distortion discussed in the text. The final term is always negative as $\Gamma' > 0$ and represents the "quadratic" nature of the investment distortion. The two central terms are more ambiguous. However, Fabinger and Weyl (2016) argue ρ' is typically negative for most plausible demand forms (those with a bell-shaped distribution of willingness to pay, as we assume in most calibrations) and thus, given that $M, \rho > 0$, the second term is likely to be negative as well. The third terms is genuinely more ambiguous. By the inverse function theorem, given that $\Gamma = (c')^{-1}$,

$$\Gamma'' = -\frac{c'''}{(c'')^3}.$$

Assuming a convex cost function, this quantity is negative if and only if $c''' > 0$. Thus, a grossly sufficient condition (assuming ρ') for the first-order conditions to uniquely determine the optimal tax is that $c''' > 0$. However, note this term is multiplied by τ, which is typically on the order of 10% in our calibrations. Thus c''' would have to be quite negative indeed to cause the problem to be nonconvex.

A.2 Proof of Proposition 2

Proof. The single-period value γ_t is trivially increasing in value γ_t. This fact, together with our Assumption 2 that higher γ_t's imply uniformly higher γ_{t+1}, implies that the social planner's V is component-wise increasing. For completeness, we sketch the fairly standard proof of this result. Similar arguments can be found in, for example, Stokey and Lucas (1989) and Smith and McCardle (2002).

Define the social planner's Bellman operator $\mathcal{S}$:

$$\mathcal{S}\left(W\left(\gamma_t^{S_t}, \gamma_t^{B_t}\right)\right) = \max\left[\gamma_t^{S_t} + \delta\mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right),\ \gamma_t^{B_t} + \delta\mathbb{E}\left(W\left(\gamma_{t+1}^{B_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{B_t}\right)\right].$$

Because, by assumption, γ_t is uniformly bounded above, this expression is a bounded discounted problem, and by standard arguments, $\mathcal{S}$ is a contraction mapping with a unique fixed point.

Suppose W is componentwise increasing in each component. Then, supposing $\hat{\gamma}_t^{S_t} > \gamma_t$, by the FOSD property of G, we have

$$\mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \hat{\gamma}_t^{S_t}\right) > \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right).$$

Hence,

$$\hat{\gamma}_t^{S_t} + \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \hat{\gamma}_t^{S_t}\right) > \gamma_t^{S_t} + \mathbb{E}\left(W\left(\gamma_{t+1}^{S_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right),$$

and likewise for B_t and $\gamma_t^{B_t}$. Hence $\mathcal{S}\left(W\left(\gamma_t^{S_t}, \gamma_t^{B_t}\right)\right)$ is componentwise increasing. Hence V, the unique fixed point of $\mathcal{S}$, must be componentwise increasing.

Because V is componentwise increasing and G is FOSD-increasing in γ_t, we have that $\gamma_t^{S_t} > \gamma_t^{B_t}$ implies

$$\gamma_t^{S_t} + \delta\mathbb{E}\left(V\left(\gamma_{t+1}^{S_{t+1}}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{S_t}\right) > \gamma_t^{B_t} + \delta\mathbb{E}\left(V\left(\gamma_{t+1}^{B_t}, \gamma_{t+1}^{B_{t+1}}\right) \mid \gamma_t^{B_t}\right).$$

Hence, the social planner's optimal strategy in each period is to assign the asset to the agent with higher γ_t. $\square$

A.3 Uniqueness of dynamic Harberger tax equilibrium

In this Section, we prove Theorem 2, that there exists a unique equilibrium for the dynamic Harberger tax game for any $F(\cdot), \tau$.

A.3.1 $V(\cdot)$ is strictly increasing

To begin with, we show, that any equilibrium $V(\cdot)$ must be increasing. This will allow us to consider only increasing candidate $\hat{V}(\cdot)$ functions for the remainder of the proof.

Claim 1. In any stationary equilibrium, $V(\gamma)$ is strictly increasing.

Proof. The proof is essentially the same as that of Section A.2. Consider a stationary equilibrium described by value function $V(\cdot)$. This defines an inverse demand function $p_{V(\cdot),F(\cdot)}(q)$. We will define the following Bellman operator for candidate value functions $\hat{V}(\cdot)$ for the seller's optimization problem:

$$\mathcal{R}\left[\hat{V}(\cdot)\right] \equiv \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]\right] \tag{6}$$

Note that $\mathcal{R}$ fixes the demand distribution $p_{V(\cdot),F(\cdot)}$ at the true equilibrium value function $V(\cdot)$, and only depends on $\hat{V}$ through the seller's continuation value $\delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]$. As a result, $\mathcal{R}$ is a standard Bellman equation satisfying Blackwell's sufficient conditions for a contraction mapping.

Consider a candidate value function $\hat{V}(\cdot)$ which is nondecreasing in γ. Supposing $\tilde{\gamma} > \gamma$, the single-period value $(q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\gamma$ is strictly higher under $\tilde{\gamma}$ relative to γ for all q, and the continuation value $\delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]$ is weakly higher under $\tilde{\gamma}$, since $G(\gamma' \mid \tilde{\gamma}) >_{FOSD} G(\gamma' \mid \gamma)$. Hence, $\mathcal{R}\left[\hat{V}(\cdot)\right](\tilde{\gamma}) > \mathcal{R}\left[\hat{V}(\cdot)\right](\gamma)$, hence $\mathcal{R}\left[\hat{V}(\cdot)\right]$ is strictly increasing in γ. Hence $\mathcal{R}$ takes nondecreasing $\hat{V}$ functions to strictly increasing $\hat{V}$ functions; hence the true value function V, which is the unique fixed point of $\mathcal{R}$, must be strictly increasing in γ. $\square$

A.3.2 Overview of proof

As we discuss in Section 3.1.3, stationary equilibria of the dynamic Harberger tax game must satisfy two conditions. First, the sellers' value function must be satisfied for any γ:

$$V(\gamma) = \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right]\right]$$

Second, the WTP distribution $p_{V(\cdot),F(\cdot)}$ must be consistent with the value function $V(\gamma)$, that is,

$$p_{V(\cdot),F(\cdot)}(q) = \left\{p:\ P_{V(\cdot),F(\cdot)}\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right] > p\right] = q\right\}$$

We will define the following "pseudo-Bellman" operator $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}(\cdot)\right](\gamma) \equiv \max_q (q - \tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right] \tag{7}$$

The operator $\mathcal{T}$ is similar to the seller's Bellman operator $\mathcal{R}$ in Equation 6. The difference is that $\mathcal{R}$ fixes the inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$ at its true equilibrium value, whereas $\mathcal{T}$ calculates the inverse demand distribution $p_{\hat{V}(\cdot),F(\cdot)}(\cdot)$ assuming that buyers also act according to continuation value $\hat{V}(\cdot)$. We will likewise define the "candidate optimal quantile markup function" $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right)$ assuming continuation value $\hat{V}(\cdot)$, as:

$$q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\cdot)\right) \equiv \arg\max_q (q - \tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right]$$

In words, $\mathcal{T}\left[\hat{V}(\cdot)\right]$, $q^*_{\mathcal{T}}\left(\gamma;\ \hat{V}(\gamma_t)\right)$ and $p_{\hat{V}(\cdot),F(\cdot)}(q)$ describe the values and optimal behavior of buyers and sellers, assuming that the continuation value of being type γ in the next period is $\hat{V}(\gamma)$. Equilibria of the Harberger tax game are fixed points of the $\mathcal{T}$ operator.

Since $\mathcal{T}$ characterizes the equilibrium of a game rather than a single-agent optimization problem, it is not necessarily a contraction mapping, and the standard contraction-based proofs of uniqueness in bounded discounted dynamic programs do not apply. However, in Claim 2 we show that $\mathcal{T}$ is well-behaved in an important way: for any increasing candidate $\hat{V}$ function, $\mathcal{T}\left[\hat{V}\right]$ respects the quantile markup property of Theorem 1 in the main text.

A number of properties of $\mathcal{T}$ follow from Claim 2. In Claim 4 we will show that $\mathcal{T}$ is a contraction mapping for any seller types γ for which $\gamma < F^{-1}(1-\tau)$; that is, for all sellers with values below the $1-\tau$'th buyer quantile. In Claim 3 we show that the value function for these seller types can be solved for without reference to the value function above $F^{-1}(1-\tau)$. Thus, Claims 3 and 4 show that $\mathcal{T}$ uniquely pins down $V(\cdot)$ on $\gamma \in \left[0, F^{-1}(1-\tau)\right]$. Then, in Claim 5, we show that for any type $\tilde{\gamma} \geqslant F^{-1}(1-\tau)$, the derivative $V'(\tilde{\gamma})$ can be calculated using only knowledge of $V(\cdot)$ on values $\gamma \in [0, \tilde{\gamma}]$ lower than $\tilde{\gamma}$. Thus, once we know $V(\cdot)$ on the interval $\gamma \in \left[0, F^{-1}(1-\tau)\right]$, we can integrate $V'(\tilde{\gamma})$ upwards from $F^{-1}(1-\tau)$ to recover the entire unique equilibrium $V(\cdot)$ function.

A.3.3 Proof of uniqueness

The following claim shows that the intuition from the static case carries over to the dynamic problem: sellers with value at the $1-\tau$ quantile of buyer values don't mark up; sellers above mark down to sell some amount between their quantile and $1-\tau$; and sellers below mark up.

Claim 2. ($\mathcal{T}$ quantile markup property) Suppose that $\hat{V}(\cdot)$ is strictly increasing. Then $q^*_{\mathcal{T}}\left(\gamma;\, \hat{V}(\gamma)\right)$ satisfies:

$$1 - q^*_{\mathcal{T}}\left(F^{-1}(1-\tau)\,;\, \hat{V}(\gamma)\right) = 1-\tau$$

$$1-\tau \leqslant 1 - q^*\left(F^{-1}(\pi)\,,\, \hat{V}(\gamma)\right) \leqslant F^{-1}(\pi)\ \ \forall \pi > 1-\tau$$

$$F^{-1}(\pi) \leqslant 1 - q^*\left(F^{-1}(\pi)\,,\, V(\gamma)\right) \leqslant 1-\tau\ \forall \pi < 1-\tau$$

Proof. This is essentially an application of Theorem 1. Fixing any increasing candidate value function $\hat{V}$, the optimization problem for seller value γ is:

$$\max_{q}\,(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right]\right]$$

By definition, we also have $p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma)) = \gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]$. In words, the price function is the WTP of buyer quantile q, which is the use value $F^{-1}(q)$ plus continuation value $\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]$. So, we can write:

$$\max_{q}\,(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\,p_{\hat{V}(\cdot),F(\cdot)}(F(\gamma))$$

This is a static Harberger tax problem identical to that of Subsection 2.2. Hence Theorem 1 applies, giving the result. □

Remark. Claim 2 applies also to the unique equilibrium $V(\cdot)$ function, proving Theorem 3 of Subsection 3.1.3.

This claim is quite powerful, because it applies not only to equilibria of the Harberger tax game, but to any candidate $q^*_{\mathcal{T}}\left(\gamma;\, \hat{V}(\gamma)\right)$ functions derived from an increasing candidate equilibrium value function $V(\cdot)$. This helps us control the behavior of the $\mathcal{T}$ operator, and is a core component in showing Claims 3, 4 and 5 below.

Consider the distribution of entering buyer values $F(\gamma)$. We will define $\rho-$*quantile truncations* of $F(\cdot)$ as follows:

Definition. $\tilde{F}(\gamma;\, \rho)$ is the $\rho-$*quantile truncation* of $F(\gamma)$, defined as:

$$\tilde{F}(\gamma;\ \rho) = \begin{cases} F(\gamma) & F(\gamma) \leqslant \rho \\ 1 & F(\gamma) > \rho \end{cases}$$

In words, $\tilde{F}(\gamma;\ \rho)$ takes all probability mass above the ρth quantile of F, and puts it on the ρth quantile. In the static Harberger tax game, sellers at quantiles below $1-\tau$ set quantile markups between their quantile $1-F^{-1}(\gamma)$ and $1-\tau$. Intuitively, then, the demand distribution at quantiles above $1-\tau$ should affect the behavior of these sellers; in particular, we can place all probability mass above buyer quantile $1-\tau$ at the $(1-\tau)$'th quantile, and this will not affect the behavior of sellers below the $(1-\tau)$'th quantile. This is formalized in the following claim.

Claim 3. (Truncation property) Suppose that $V(\cdot)$ is a stationary equilibrium value function for the Harberger tax game under tax τ, with entering buyer distribution F. Then, $V(\cdot)$ restricted to the interval $\gamma \in \left[0,\ F^{-1}(\rho)\right]$ is a stationary equilibrium value function for the Harberger tax game under tax τ, with entering buyer distribution $\tilde{F}(\gamma;\ \rho)$, for any $\rho \geqslant 1-\tau$.

Proof. An equilibrium of $V(\cdot)$ is a fixed point of the pseudo-Bellman operator $\mathcal{T}$, that is, it satisfies, for all γ:

$$V(\gamma) = \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]\right]$$

We want to show that, for any such V, we also have, for any $\rho \geqslant 1-\tau$,

$$V(\gamma) = \max_q (q-\tau)\, p_{V(\cdot),\tilde{F}(\gamma;\ \rho)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]\right]\ \forall \gamma \leqslant F^{-1}(\rho)$$

In other words, we want to show that the truncation of F and $V(\cdot)$ does not affect the optimization problem of any type with $\gamma < F^{-1}(\rho)$.

Recall the definition of the WTP function:

$$WTP(\gamma) = \gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]$$

Since we have assumed that $G(\gamma' \mid \gamma)$ satisfies $\gamma' \leqslant \gamma$ with probability 1, evaluating the WTP function at γ only requires evaluating V on the interval $[0,\gamma]$. Thus, for all $\gamma \in \left[0, F^{-1}(\rho)\right]$, we can still evaluate WTP using V truncated to the interval $\left[0, F^{-1}(\rho)\right]$. Under $\tilde{F}(\gamma;\ \rho)$, the inverse demand function becomes:

$$\tilde{p}(q;\ \rho) = \begin{cases} p(q) & 1-q \leqslant \rho \\ p(1-\rho) & 1-q > \rho \end{cases}$$

Thus, by construction, the modified inverse demand function agrees with p on the inverval $[0,\rho]$, that is:

$$\tilde{p}(q;\ \rho) = p(q)\ \ \forall\{q:\ 1-q \in [0,\rho]\}$$

From Claim 2, under any increasing candidate $\hat{V}$ function, any seller with quantile $F^{-1}(\gamma) \in [0,\ 1-\tau]$ chooses some $1-q \in [F(\gamma),\ 1-\tau]$. It must then be that the behavior of the inverse demand function $p(q)$ outside the range $1-q \in \left[F^{-1}(\gamma),\ 1-\tau\right]$ does not affect these sellers' optimization problem (as long as $p(q)$ is derived from an increasing candidate $\hat{V}$ function, i.e. is monotone). Likewise, from Claim 2, any seller quantile $F(\gamma) \in [0,\ 1]$ chooses some $1-q \in [1-\tau,\ F(\gamma)]$, hence the behavior of $p(q)$ outside the range $1-q \in [1-\tau,\ F(\gamma)]$ does not affect the optimization problem of seller value $F^{-1}(\gamma)$.

In the ρ-truncated problem, sellers with values $F(\gamma) \leqslant 1-\tau$ care about $p(q)$ in the range $[0, 1-\tau]$, and sellers with values $1-\tau \leqslant F(\gamma) \leqslant \rho$ care about $p(q)$ in the range $[1-\tau, \rho]$. Since $\tilde{p}(q;\ \rho) = p(q)$ on the interval $[0, \rho]$, $\tilde{p}(q;\ \rho)$ is identical to $p(q)$ from the perspective of all sellers types with quantiles $F^{-1}(\gamma) \in [0, \rho]$. Hence there is no seller type in the ρ-truncated problem whose optimization problem is affected by the truncation of $p(q)$. Thus, any optimal policy $q^*(\gamma)$ and value function $V(\gamma)$ in the original problem remains optimal in the truncated problem, proving the claim. □

We will now consider the most extreme possible truncation, $\rho = 1-\tau$. Under this truncation, there are no types with values strictly above the $1-\tau$th quantile; thus, all seller types in the truncated interval are net sellers. In the following claim, we use the net seller property to show that $\mathcal{T}$ is a contraction mapping on the $(1-\tau)$-truncated problem.

Claim 4. (Contraction property) For any $\tau, F(\cdot)$, consider the $(1-\tau)$-truncated problem, with entering buyer distribution $\tilde{F}(\gamma, 1-\tau)$. $\mathcal{T}$ is a contraction mapping on this problem, hence admits a unique fixed point. Moreover, the unique fixed point $V(\cdot)$ of $\mathcal{T}$ must be continuous.

Proof. Once again, $\mathcal{T}$ is:

$$\mathcal{T}[V](\gamma) = \max_q (q-\tau)\, p_{V(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right]\right]$$

Consider $V, \tilde{V}$ s.t. $\sup_\gamma \left|V(\gamma) - \tilde{V}(\gamma)\right| \leqslant a$. We want to bound the sup norm difference between $\mathcal{T}[V](\gamma)$ and $\mathcal{T}\left[\tilde{V}\right](\gamma)$. First, note that from the definition of $p_{V(\cdot),F(\cdot)}(\cdot)$,

$$p_{V(\cdot),F(\cdot)}(q) \equiv \left\{p:\ P_{V(\cdot),F(\cdot)}\left[\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right] > p\right] = q\right\}$$

we have that

$$\left|p_{V(\cdot),F(\cdot)}(q) - p_{\tilde{V}(\cdot),F(\cdot)}(q)\right| \leqslant$$

$$\left|\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right] - \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\tilde{V}(\gamma') \mid \gamma\right]\right| \leqslant \delta a$$

Now, writing $\mathcal{T}[V]$:

$$\mathcal{T}[V](\gamma) = [q^*_{\mathcal{T}}(\gamma;\ V) - \tau]\, p_{V(\cdot),F(\cdot)}(q^*_{\mathcal{T}}(\gamma;\ V)) + (1 - q^*_{\mathcal{T}}(\gamma;\ V))\left[\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right]\right]$$

We will show that, if under $\tilde{V}$ we fix the sale probability at $q^*_{\mathcal{T}}(\gamma;\ V)$, we lose at most δa. Hence the sup norm difference between $T[V]$, $T\left[\tilde{V}\right]$ is at most δa.

We can separately treat the "buyer" inverse demand and "seller" continuation value terms. For the seller's continuation value term, note that:

$$\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right] \leqslant \gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\tilde{V}(\gamma') \mid \gamma\right] + \delta a$$

Hence,

$$(1-q^*_{\mathcal{T}}(\gamma;V))\left[\gamma+\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right)\mid\gamma\right]\right]\leqslant$$

$$(1-q^*_{\mathcal{T}}(\gamma;V))\left[\gamma+\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\tilde{V}\left(\gamma'\right)\mid\gamma\right]\right]+(1-q^*_{\mathcal{T}}(\gamma;V))\,\delta a$$

Now for the buyer inverse demand term,

$$[q^*_{\mathcal{T}}(\gamma;V)-\tau]\,p_{V(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}(\gamma;V)\right)\leqslant[q^*_{\mathcal{T}}(\gamma;V)-\tau]\,p_{\tilde{V}(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}(\gamma;V)\right)+|q^*_{\mathcal{T}}(\gamma;V)-\tau|\,\delta a$$

Adding these inequalities, we have that:

$$\mathcal{T}[V](\gamma)=[q^*_{\mathcal{T}}(\gamma;V)-\tau]\,p_{V(\cdot),F(\cdot)}\left(q^*_{\mathcal{T}}(\gamma;V)\right)+(1-q^*_{\mathcal{T}}(\gamma;V))\left[\gamma+\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right)\mid\gamma\right]\right]\leqslant$$

$$\mathcal{T}\left[\tilde{V}\right](\gamma)+(1-q^*_{\mathcal{T}}(\gamma;V))\,\delta a+|q^*_{\mathcal{T}}(\gamma;V)-\tau|\,\delta a$$

Now by Claim 2, we know that all sellers in the truncated range are net sellers, that is, $1-q^*_{\mathcal{T}}(\gamma;V)\leqslant 1-\tau$, or $q^*_{\mathcal{T}}(\gamma;V)\geqslant\tau$. Hence $\left|q^*_V-\tau\right|<\left|q^*_V\right|$, hence we have:

$$(1-q^*_{\mathcal{T}}(\gamma;V))\,\delta a+|q^*_{\mathcal{T}}(\gamma;V)-\tau|\,\delta a\leqslant|(1-q^*_V)\,\delta a|+|q^*_V\delta a|\leqslant\delta a \tag{8}$$

Hence, we have shown that, for all γ,

$$\mathcal{T}[V](\gamma)\leqslant\mathcal{T}\left[\tilde{V}\right](\gamma)+\delta a$$

V and $\tilde{V}$ were arbitrary, so by switching their roles we get

$$\mathcal{T}\left[\tilde{V}\right](\gamma)\leqslant\mathcal{T}[V](\gamma)+\delta a$$

$$\implies\sup_{\gamma}\left[\mathcal{T}\left[\tilde{V}\right](\gamma)-\mathcal{T}[V](\gamma)\right]\leqslant\delta a$$

Hence $\mathcal{T}$ is a contraction mapping of modulus δ.

To show that the unique fixed point $V(\cdot)$ must be continuous, we will show that for an increasing but possibly discontinuous candidate value function $\hat{V}(\cdot)$, $\mathcal{T}\left[\hat{V}\right]$ must be continuous. Once again, $\mathcal{T}$ is:

$$\mathcal{T}\left[\hat{V}\right]=\max_q(q-\tau)\,p_{\hat{V}(\cdot),F(\cdot)}(q)+(1-q)\left[\gamma+\delta\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right)\mid\gamma\right]\right]$$

We need only to show that $\mathbb{E}_{G(\gamma'|\gamma)}[V(\gamma')\mid\gamma]$ is continuous in γ, since all other components of the maximand are continuous in γ. Since $\hat{V}$ is strictly increasing, the generalized inverse function $\hat{\Gamma}(\nu)\equiv\arg\min_{\gamma}|V(\gamma)-\nu|$ is everywhere well-defined. Since $\hat{V}(\cdot)$ is bounded, the "layer cake" representation of its expected value obtains. Letting $\bar{V} \equiv \max_{\gamma} \hat{V}(\gamma)$, we have:[22]

$$\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right] = \int_0^{\bar{V}} 1 - G\left(\hat{\Gamma}(\nu) \mid \gamma\right) d\nu$$

Since $G(\gamma' \mid \gamma)$ is continuous in γ for any γ', the integral $\int_0^{\bar{V}} G\left(\hat{\Gamma}(\nu) \mid \gamma\right) d\nu$ is also continuous in γ.[23] □

Remark. Equation 8 is the step where the contraction property fails in general, and is why we need this truncation argument. Suppose for example $\tau = 1$, so that $\tau > q$. Then the continuation value term has modulus $(1-q)\,\delta a$ and the buyer price term has modulus $|q-\tau|\,\delta a = |1-q|\,\delta a$. So the total modulus bound is $2(1-q)\,\delta a$, and we can't guarantee that $\mathcal{T}$ is a contraction.

Claim 4 shows that the equilibrium value function $V(\cdot)$ is uniquely pinned down in the $(1-\tau)$-truncated problem, and Claim 3 shows that the equilibrium $V(\cdot)$ functions from the original and truncated problems must agree. Hence we have shown that the equilibrium $V(\cdot)$ is unique at least in the truncated interval $\left[0,\ F^{-1}(1-\tau)\right]$. In Claim 5, we show that, in any ρ-truncated equilibrium, we can calculate the derivative of the value function $\frac{dV}{d\gamma}\mid_{\gamma=F^{-1}(\rho)}$ at the boundary type $F^{-1}(\rho)$.

Claim 5. (Envelope theorem) The envelope theorem applies to any equilibrium:

$$\frac{dV}{d\gamma} = \frac{\partial}{\partial\gamma}\left[\left(q^*(\gamma)-\tau\right) p_{V(\cdot),F(\cdot)}\left(q^*(\gamma)\right) + \left(1-q^*(\gamma)\right)\left[\gamma + \delta\mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]\right]\right]$$

$$= \left(1-q^*(\gamma)\right)\left[1 + \delta\frac{\partial\mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]}{\partial\gamma}\right] \tag{9}$$

Proof. Following Milgrom and Segal (2002), we need to show that the conjectured derivative:

$$(1-q)\left[1 + \delta\frac{\partial\mathbb{E}_{G(\gamma'|\gamma)}\left[V\left(\gamma'\right) \mid \gamma\right]}{\partial\gamma}\right]$$

is finite for any choice of q. Using the layer cake representation once again:

$$\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}\left(\gamma'\right) \mid \gamma\right] = \int_0^{\bar{V}} 1 - G\left(\hat{\Gamma}(\nu) \mid \gamma\right) d\nu$$

[22]This definition of the layer-cake integral is slightly wrong, failing if $\hat{V}(\cdot)$ and $G(\cdot)$ have discontinuities at the same value of γ. This can be fixed by redefining G such that that probability mass falls at the correct side of each $\hat{V}$ discontinuity.

[23]This follows even without assuming differentiability of G, from a "set excision" argument: for any ϵ, there is a δ satisfying continuity for $G\left(\hat{\Gamma}(\nu) \mid \gamma\right)$ except on a set of arbitrarily small ν-measure, and G is bounded between $[0,1]$ on the excised set.

Leibniz' formula implies that

$$\frac{\partial \mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right]}{\partial \gamma} = -\int_0^{\bar{V}} \frac{\partial G\left(\hat{\Gamma}(\nu) \mid \gamma\right)}{\partial \gamma} d\nu$$

We have assumed that $\frac{\partial G(\hat{\Gamma}(\nu)|\gamma)}{\partial \gamma}$ exists for any ν, hence this quantity is finite for any q, and thus the envelope theorem applies. □

Thus, in any ρ-truncated equilibrium, we can evaluate $q^*(\gamma)$ for the boundary type $\gamma = F^{-1}(\rho)$. Moreover, since the transition probability distribution G satisfies that $\gamma_{t+1} < \gamma_t$ $a.s.$, the expectation $\mathbb{E}_{G(\gamma'|\gamma)}\left[V(\gamma') \mid \gamma\right]$ puts positive probability only on values $\gamma' < \gamma$. Hence, in any ρ-truncated equilibrium, we can evaluate the derivative $\frac{dV}{d\gamma}$ for the boundary type $\gamma = F^{-1}(\rho)$ using only knowledge of the equilibrium V on the truncated interval $\left[0, F^{-1}(\rho)\right]$. Thus, after solving for V on the interval $\left[0, F^{-1}(1-\tau)\right]$ using the contraction mapping of Claim 4, we integrate the envelope formula 9 to recover the rest of the equilibrium $V(\cdot)$ function:

$$V(\gamma) = \int_{F^{-1}(1-\tau)}^{F^{-1}(\gamma)} (1 - q^*(\tilde{\gamma}))\left[1 + \delta \frac{\partial \mathbb{E}_{G(\tilde{\gamma}'|\tilde{\gamma})}\left[V(\tilde{\gamma}') \mid \tilde{\gamma}\right]}{\partial \tilde{\gamma}}\right] d\tilde{\gamma} \; \forall \gamma > F^{-1}(1-\tau)$$

We have thus proved Theorem 3: for any τ, $F(\gamma)$, $G(\gamma' \mid \gamma)$ satisfying our assumptions, there is a unique equilibrium of the dynamic Harberger taxation game.

When we computationally solve for equilibria in our calibrations, rather than explicitly applying this two-step procedure, we iterate the $\mathcal{T}$ operator until convergence. This effectively performs our two-step procedure, using the contraction property of $\mathcal{T}$ in the truncated value interval, and then "bootstrapping" upwards to solve for V on the remaining γ values.

A.4 Persistent investment

In order to accomodate investment, we need a nonstationary definition of equilibria in the Harberger tax game. Let $\zeta = (\zeta_0, \zeta_1, \zeta_2 \ldots)$ represent the path of common use values over time, and suppose that this is common knowledge. The use value for any agent A_t in any period is thus $\zeta_t + \gamma_t^{A_t}$. We will define the nonstationary value function $V_t(\gamma_t, \zeta)$ as the value of being a seller with type γ_t in period t, if the path of common use values is ζ. Analogously to above, we will define the inverse demand function in period t as:

$$p_{t,V_t(\cdot,\zeta),F(\cdot)}(q_t) = \left\{p_t : \; P_{V_{t+1}(\cdot,\zeta),F(\cdot)}\left[\gamma_t^{B_t} + \zeta_t + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V_{t+1}\left(\gamma_{t+1}^{B_t}, \zeta\right) \mid \gamma_t\right] > p_t\right] = q_t\right\}$$

Equilibrium then requires that, in each history,

$$V_t(\gamma_t, \zeta) = \max_{q_t}(q_t - \tau)\, p_{V_{t+1}(\cdot,\zeta),F(\cdot)}(q_t) + (1 - q_t)\left[\gamma_t + \zeta_t + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[V_{t+1}(\gamma_{t+1}, \zeta) \mid \gamma_t\right]\right] \quad (10)$$

We conjecture an equilibrium of this game of the following form:

$$V_t(\gamma_t, \zeta) = V(\gamma) + \sum_{t'=0}^{\infty} \delta^{t'} (1-\tau)^{t'+1} \zeta_{t+t'}$$

One can verify that if $V(\cdot)$ satisfies the "allocative" equilibrium Equation 7, then $V_t(\gamma_t, \zeta_t)$ satisfies Equation 10.

Intuitively, as in the static case, if the tax is τ agent A_t only owns $(1-\tau)$ of the asset in period t. But if the asset has some common value in period $t+t'$, agent A_t has to pay taxes t' times on the asset before enjoying its use value; hence she effectively only owns $(1-\tau)^{t'+1}$ of any common value of the asset in period t'.

For simplicity, we analyze the investment decision of the $t=0$ agent; the problem is additive and identical for all agents in all periods, hence all agents make the same choice of investment in each period. Suppose investment level η_0 produces common value $\zeta_t = H_t(\eta)$ in the future. Agents' FOC for investment is:

$$c'(\eta_0) = \frac{\partial V_0(\gamma_t, \zeta(\eta_0))}{\partial \eta_0}$$

Which implies

$$c'(\eta_0) = \sum_{t=0}^{\infty} \delta^t (1-\tau)^{t+1} H_t'(\eta_0) \tag{11}$$

B Calibration details

As we discuss in Appendix Subsection A.3, the equilibria of the dynamic Harberger taxation game are the unique fixed points of the pseudo-Bellman operator $\mathcal{T}$:

$$\mathcal{T}\left[\hat{V}\right] = \max_q (q-\tau)\, p_{\hat{V}(\cdot),F(\cdot)}(q) + (1-q)\left[\gamma + \delta \mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]\right]$$

While our proof of uniqueness in Section A.3 involves a procedure with multiple steps involving truncation of the buyer value distribution, in practice, for buyer value distributions $F(\cdot)$ supported on discrete grids, iteratively applying $\mathcal{T}$ rapidly converges to a fixed point V. Thus, we numerically solve our calibrations by iterating $\mathcal{T}$ on grid-supported F distributions.

B.1 Numerical procedures

We use gradient descent with our numerical equilibrium solver function to find moments σ, β to match the $sdmean$ and $saleprob$ moments, as we describe in Section 3.2.1 of the text. This gives us parameter values:

$$\sigma = 1.1493$$

$$\beta = 0.9716$$

For any given log standard deviation σ, we approximate the lognormal $F_\sigma(\cdot)$ using a discrete distribution $\tilde{F}_\sigma(\cdot)$ supported on a 1000-point grid

$$G_{1000}^{\sigma} = \left\{ F^{-1}(0), F^{-1}\left(\frac{1}{1000}\right), F^{-1}\left(\frac{2}{1000}\right), \ldots F^{-1}\left(\frac{999}{1000}\right) \right\}$$

of uniformly spaced quantiles of the lognormal. Given a candidate value function $\hat{V}$ and decay rate β, we can evaluate the continuation value $\mathbb{E}_{G(\gamma'|\gamma)}\left[\hat{V}(\gamma') \mid \gamma\right]$ for any type γ, and thus also the inverse demand function $p_{\hat{V}(\cdot),F(\cdot)}(q)$. Thus, we can find the optimal sale probability $q^*_{\mathcal{T}}(\gamma; \hat{V})$ for any $\hat{V}$, and thus calculate $\mathcal{T}(\hat{V})$. Starting from a linearly increasing $V(\cdot)$ function, we iteratively apply $\mathcal{T}$ until convergence, defined as:

$$\sup_{\gamma} |\mathcal{T}[V](\gamma) - V(\gamma)| < 10^{-4}$$

Once we have solved for $V(\gamma)$, this gives us equilibrium sale probability functions $q^*(\gamma, V)$ for every type γ. Together, the equilibrium $q^*(\gamma, V)$, the transition probability distribution $G(\gamma' \mid \gamma)$ and the distribution of entering buyer values $F(\gamma)$ define a ergodic discrete state Markov chain over values γ of the period-t owner of the asset S_t. We construct this transition probability matrix of this Markov chain, and solve for its unique stationary distribution, which we call $H_\tau(\gamma)$. We plot these stationary distributions for various values of τ in Figure 2.

Once we have solved for the equilibrium $V(\cdot)$, we can recover the equilibrium sale probability function $q^*(\gamma, V)$ and inverse demand function $p_{V(\cdot),F(\cdot)}(\cdot)$, and we can use these, together with $H_\tau(\gamma)$, to recover the stationary averages of various quantities that we plot in Figure 3. Specifically, the definitions of quantities plotted in Figure 3 are averages of the following quantities, with respect to $H_\tau(\gamma)$:

- **Use value**: $\gamma + \eta$
- **Sale probability**: $q^*(\gamma,\ V)$
- **Quantile markup**: $(1 - q^*(\gamma,\ V)) - (1 - F(\gamma))$
- **Tax revenue**: $\tau p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$

There is a minor accounting subtlety for stationary average use values: if sale occurs in period t, we should use the buyer's use value, not the seller's, in calculating the stationary average. This is accomodated by multiplying the stationary distribution $H_\tau(\gamma)$ by a "buyer transition" adjustment matrix, which reflects the probability that seller type γ "transitions" through sale of the good to any buyer type γ' with $p_{V(\cdot),F(\cdot)}(1 - F(\gamma')) > p_{V(\cdot),F(\cdot)}(q^*(\gamma, V))$.

For **asset prices**, we observe in the real world asset prices only for successful transactions; correspondingly, we would like to take an average of asset prices weighted by the probability of sale for each seller value γ. Thus, average asset prices in Figure 3, panel 2 are calculated as:

$$\frac{\int p_{V(\cdot),F(\cdot)}\left(q^*\left(\gamma, V\right)\right) q^*\left(\gamma, V\right) dH_\tau\left(\gamma\right)}{\int q^*\left(\gamma, V\right) dH_\tau\left(\gamma\right)}$$

We include investment value in the asset price by multiplying investment flow value by a factor $\frac{1}{1-\delta(1-\tau)}$. Note that, since taxes are collected regardless of sale, we do not weight offered prices by $q^*\left(\gamma, V\right)$ when we calculate average tax revenues.

Values labelled "NPV" are calculated by taking average flow values and multiplying by $\frac{1}{1-\delta}$. We solve for equilibria and calculate all average quantities on a grid of 500 equally spaced τ values in $[0, 1]$.

For the sensitivity graphs in Figure 4 for the sdmean moment, for a grid of values of σ, we search for a value of β which keeps $\mathit{saleprob}$ at its initial calibration value of $\frac{1}{13}$. Likewise, for the $\mathit{saleprob}$ graphs, we use a grid of β values, searching for matching σ values. Since the $\mathit{invfrac}$ moment is only affected by the investment total value parameter, for the $\mathit{invfrac}$ graphs we simply vary this parameter holding all others fixed. We use 500-point quantile grids throughout the sensitivity analysis. Note from Figure 3, panel 1 that welfare is very flat about the allocative and total welfare maximizing tax values. Thus, it is difficult to precisely pin down the values of optimal taxes, and some numerical error in the range of $\tau \pm 0.01$ or so is visible in Figure 4.

B.2 Persistent investment algebra

In our calibrations, we assume that investment decays geometrically at rate $\theta < 1$; that is, persistent investment η_0 generates period t value:

$$H_t\left(\eta_0\right) = \theta^t \eta_0$$

Hence, following Equation 11 in Appendix Subsection A.4, the present value of a unit of investment is:

$$\sum_{t=0}^{\infty} \eta_0 \delta^t \theta^t \left(1-\tau\right)^{t+1} = \frac{\eta_0\left(1-\tau\right)}{1-\delta\theta\left(1-\tau\right)}$$

and agents' investment FOCs are thus:

$$c'\left(\eta_0\right) = \frac{1-\tau}{1-\delta\theta\left(1-\tau\right)}$$

We will suppose that the cost function is:

$$c(\eta) = \frac{\eta^2}{2(1-\delta\theta)g}$$

for some parameter g. This is a convenient functional form which leads to a simple analytical solution. Total social investment welfare for investment level η_0 is

$$\frac{\eta_0}{(1-\delta\theta)} - \frac{\eta_0^2}{2(1-\delta\theta)g}$$

Since we measure allocative efficiency in terms of flow value, we will normalize by a factor $1-\delta$ to convert investment value into equivalent flow value:

$$Investment\ Welfare = \left(\eta_0 - \frac{\eta_0^2}{2g}\right)\left(\frac{1-\delta}{1-\delta\theta}\right) \tag{12}$$

The socially optimal level of investment is $\eta_0 = g$. The maximum possible investment NPV, in flow value terms, is thus:

$$\frac{g}{2}\frac{1-\delta}{1-\delta\theta}$$

As we discuss in Section 3.2.1, we choose g such that the maximum possible flow value of investment is some target fraction $invfrac$ of the average transaction price.

Given some tax level τ, constant for all time, the seller's FOC for investment is:

$$\frac{\eta}{(1-\delta\theta)g} = \frac{1-\tau}{1-\delta\theta(1-\tau)}$$

$$\implies \eta = g\frac{(1-\tau)(1-\delta\theta)}{1-\delta\theta(1-\tau)}$$

We can plug this into Equation 12 to calculate total investment welfare for any given value of τ.