

# **Public Economics**

*(Master PPD & APE)*

*(EHESS & Paris School of Economics)*

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## **Lecture 8: Normative and intertemporal theories of social and fiscal justice**

*(check [on line](#) for updated versions)*

- “Public Economics”: see complete Syllabus [here](#)
- Lectures 1-7 and 11-12 by A. Bozio and J. Grenet
- You have already covered lots of topics, including optimal taxation of labor income. Now we’re going to focus on intertemporal issues and capital taxation:
- **Lecture 8: Normative and intertemporal theories of fiscal and social justice** (Tuesday Nov. 2<sup>nd</sup> 2021)
- **Lecture 9: Capital income, inheritance & wealth taxes over time & across countries** (Tuesday November 9<sup>th</sup> 2021)
- **Lecture 10: Optimal taxation of capital income, inheritance & wealth** (Tuesday November 23<sup>rd</sup> 2021)

# Roadmap of lecture 8

- Do conflicts about inequality and the role of government come from different interests, different values/objectives, or different beliefs systems about the the economy is working?
- A simple model of inequality and beliefs
- The dimensions of political conflict and beliefs systems about inequality: taxation vs education vs globalization
- The problem of intertemporal justice: how much capital and debt should we leave to our children?
- Aggregating different interests/values/beliefs: Arrow's impossibility theorem, Condorcet's paradox & majority cycles
- Condorcet's jury theorem & the constructive view of political institutions

- **Q.:** Do conflicts about inequality and the role of government come from different interests, different values/objectives, or different beliefs systems about the society and the economy are working?
- **A.:** Probably from all three dimensions. But the « different beliefs systems » view appears to be more powerful to explain observed conflicting attitudes toward inequality. And it provides a more optimistic and constructive view of human nature and democratic political institutions.
- **Different interests.** E.g. poor vs rich. Obviously important, but not enough. Not all poor vote alike, not all rich vote alike. Income or wealth are correlated with political attitudes, but have never been perfect predictors of the different possible attitudes.

- **Different values/objectives.** All individuals care about justice and fairness. Or at least they feel the need to present their most-preferred policy as determined by universal objectives & principles (rather their self-interest).
- E.g. in all civilisations, rich people usually say something like « if you cut my taxes, then in the end this is also going to be good for the poor » (rather than « cut my taxes so that I am better off and the poor worst off »)
- Even if they are not always entirely sincere (problem of self-serving beliefs), these statements illustrate the need to refer to universal moral values and objectives
- This in itself is interesting, and it does put constraints on political discourses and outcomes.
- Therefore the discussion about normative theories of justice is critical if we want to understand actual policies

- Do different individuals have different values/objectives?
- Yes to some extent: different life objectives, religious values, world views etc.
- But in order to explain different attitudes toward inequality, it does not seem to be very useful to characterize different individuals as having radically different objective functions, in the sense of different views on utilitarianism vs Rawlsianism, or more generally in the sense of different concavity parameters in a general social welfare function. Nobody seems to express his or her views on social justice in this manner.
- On social welfare functions, utilitarianism, concavity parameters, Rawls, maximin, Sen, capabilities, etc., see slides from Lecture 1 (by A. Bozio)

- **Different beliefs systems.** It seems more useful and relevant to characterize different individuals as having the same general objective function (at least in part), but different - legitimate, though possibly self-serving - beliefs systems about the society and the economy are working.
- « Cut my taxes, and in the end it will be also be good for the poor ». Ok, why not, but where do we stop exactly? A proper answer to this question requires a lot of empirical and historical knowledge, and will always be relatively uncertain. How do different individuals form different beliefs systems about these difficult issues?

# A simple model of inequality and beliefs

- The aim of this simple model is to illustrate the discussion about values vs beliefs
- Simplified version of the model presented in T. Piketty, “Social Mobility & Redistributive Politics”, [QJE 1995](#)
- Initial objective of this paper: explain why intergenerational income mobility – and not only current income – seem to matter to explain voting preferences
- I.e. upwardly mobile individuals (moving from low parental income to high current income) and downwardly mobile individuals (moving from high to low) have a probability to vote for left-wing parties that is intermediate between those of permanently low-income and permanently high-income groups



**TABLE I**  
**PERCENTAGE OF VOTES FOR LEFT-WING PARTIES AS A FUNCTION OF INDIVIDUAL**  
**MOBILITY EXPERIENCE**

		Respondent's income	
		Low income	High income
Parents income	Low income	72%	38%
	High income	49%	24%

(Average matrix for six countries: Germany 1953, Britain 1962, United States, 1953, Finland 1949, France 1966, Norway 1957. Standard deviation = 5.78%.)

*Source.* Cherkaoui [1992, p. 189].

- Two possible income levels:  $y_0 < y_1$
- $y_0$  = low-paid job;  $y_1$  = high-paid job
- Probability ( $y_i=y_1$ ) =  $\pi_0 + \theta e_i$  if parental income =  $y_0$
- Probability ( $y_i=y_1$ ) =  $\pi_1 + \theta e_i$  if parental income =  $y_1$

With  $e_i$  = effort,  **$\theta$  = index of how much individual effort matters,  $\Delta\pi = \pi_1 - \pi_0$  = index of how much inequality in social origins matters (better access to education, connexions to find jobs, etc.)**

- Redistributive taxation:  $c_0 = (1-t)y_0 + ty$ ,  $c_1 = (1-t)y_1 + ty$ ,  
With  $t$  = income tax rate,  $y = (1-p)y_0 + py_1$  = average pre-tax income,  
 $p$  = pop. fraction getting high income
  - Per capita tax revenue  $ty$  used to pay lump-sum cash transfer  
(or to finance equal access to education or other public services)
  - $t=0\%$  : no redistribution;  $t=100\%$ : full redistribution
  - Individual  $i$  has utility  $U_i = c_i - C(e_i)$ , with  $C(e) = e^2/2a$ :
  - Max  $(1-p_i)c_0 + p_i c_1 - C(e_i)$ , with  $p_i = \pi_i + \theta e_i$
  - FO condition:  $e_i = a\theta(1-t)(y_1 - y_0)$
- more redistribution leads to less effort → how much this matters depends on relative importance of  $\theta$  vs  $\Delta\pi$

- Assume everybody agrees about some form of maximin objective:

$$\text{Max } (1-p)c_0 + pc_1 - C(e), \text{ with } p = \pi_0 + \theta e$$

(i.e. expected welfare of individuals with low parental income)

- Then one can show that optimal  $t^*$  is given by:

$$t^* = H\Delta\pi/a(y_1 - y_0)\theta^2$$

( $H$  = pop fraction of indiv. with high-income parents)

- I.e. optimal tax  $t^* \uparrow$  if  $\Delta\pi \uparrow$  or  $\theta \downarrow$ , i.e. if parental origins more important & role of effort less important
- **But this does not mean that everybody agrees about  $t^*$  : different beliefs about  $\Delta\pi$  and  $\theta$  can lead to different  $t^*$  → politics as a conflict over beliefs**

- Why different beliefs?
- Because it is difficult to learn about  $\Delta\pi$  and  $\theta$
- Self-serving beliefs also play a role: high-income individuals have a clear incentive to pretend that  $\theta$  matters more than  $\Delta\pi$ , and to try to spread their views in the media & political parties
- But even if all individuals have fully sincere, and start with same initial beliefs, one can show that different families will end up with different beliefs: e.g. if you put a lot of effort experience an upward mobility experience, you will tend to believe that effort works and update your beliefs accordingly
- This can explain why mobility experience and not only current income determines political attitudes

- In the long run, high-income individuals tend to be more right-wing on average than low-income individuals (they want less redistribution), even if they are not selfish at all

(in effect, right-wing dynasties believe more in effort and end up with higher average incomes, whether their beliefs are right or wrong)

(see [QJE 1995](#))

- Other, more sophisticated ways to learn about optimal  $t^*$ : study the comparative history of inequality and taxation, read econometric estimates of labor supply elasticities...
- But there will always be a lot of uncertainty about the conclusions one can draw from historical or econometric evidence, so the political conflict will continue, and democratic institutions will do what they can to aggregate these different beliefs

- This simple model also illustrates the difficulties to define the proper social objective
- Assume  $\Delta\pi=0$ , i.e. family origins do not matter, everybody faces the same probability  $\pi + \theta e$  to have high income  $y_1 > y_0$
- With risk aversion ( $U_i = V(c_i) - C(e_i)$  and  $V(\cdot)$  concave), one still wants redistributive taxation
- But with  $U_i = c_i - C(e_i)$ , do we really want  $t^*=0$ ?
- If  $y_1$  vs  $y_0$  = long-term, lifetime inequality, then risk aversion might not be the relevant way to determine the socially desirable level of redistribution (with short-term risk, this is questionable as well: public unemployment insurance)
- Assume social objective =  $\text{Max } c_0 - C(e)$

- Max  $c_0 - C(e) = (1-t)y_0 + ty - C(e)$   
with  $e = a\theta(1-t)(y_1 - y_0)$ ,  $y = (1-p)y_0 + py_1$ ,  $p = \pi + \theta e$
- Then one can show that

$$t^* = 2/3 + \pi/3a(y_1 - y_0)\theta^2$$

(formula to be checked; please do it for next time!)

→ This leads to much more redistribution than with expected welfare maximization

**→ Discussion about objective function matters, together with the discussion about beliefs and elasticities**



# The dimensions of political conflict and beliefs systems about inequality: taxation vs education vs globalization

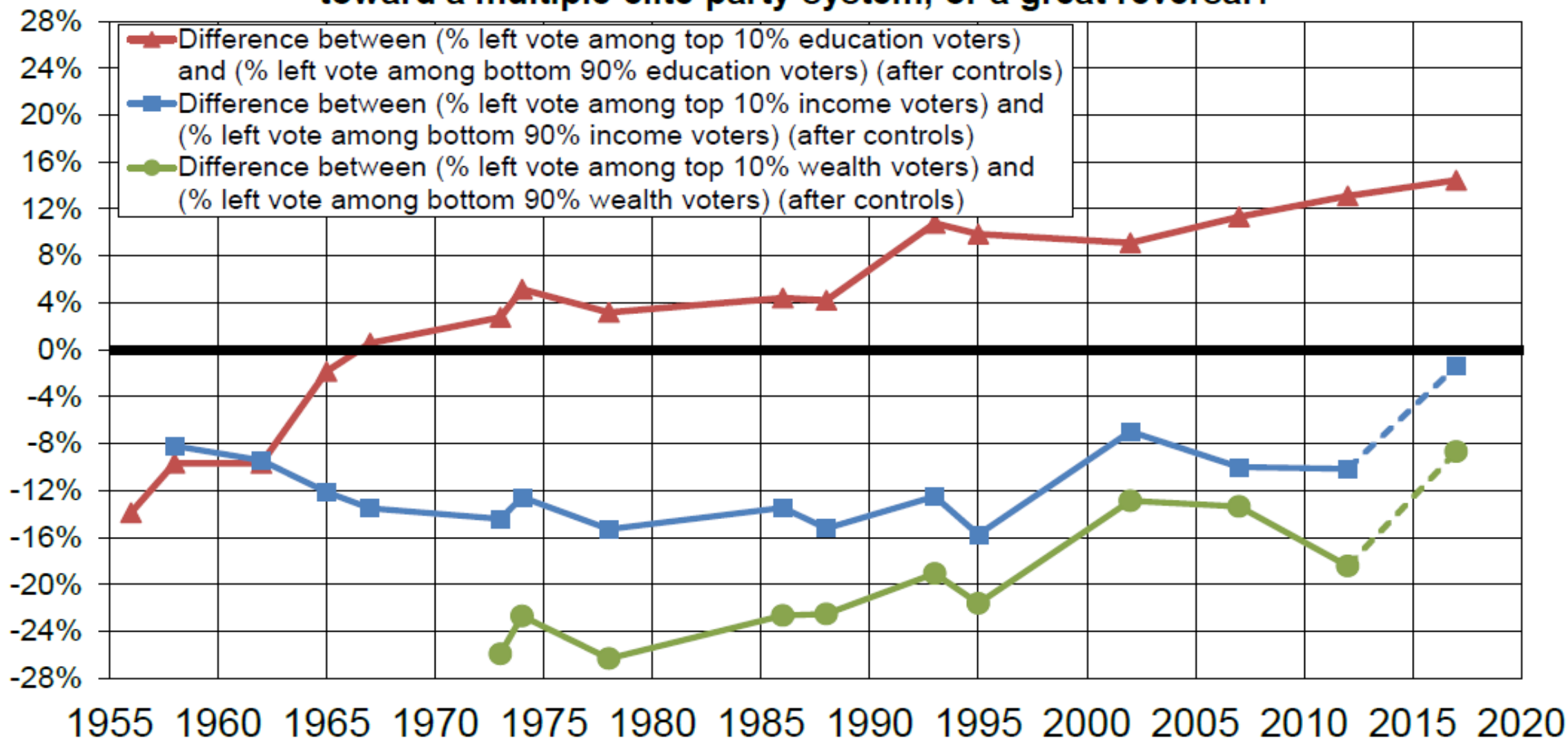
- There is extensive evidence showing that beliefs about luck vs effort and about income mobility prospects matter to explain attitudes towards inequality.
- See e.g. [Alesina-LaFerrara 2005](#) ; [A-Angeletos 2005](#); [A-Stantcheva-Tevo 2017](#); [Alesina et al 2016](#); [Alesina et al 2015](#) on beliefs in luck vs effort and mobility in US vs Europe
- On mobility prospects, inequality and beliefs, see also models by [Benabou-Ok 2001](#); [Benabou 2000](#), [Benabou-Tirole 2006](#), [Benabou 2008](#), [Benabou et al 2015](#)
- But the basic model about the role of luck and effort is a bit simplistic: it captures only a small part of the issues on which people disagree regarding inequality; in practice, key conflicts involve the role of education, globalization, etc.

- Basic model on luck & effort: left = pessimistic about mobility (beliefs in luck and redistributive taxation), right = optimistic about mobility (beliefs in effort)
- In practice this can be much more complicated. E.g. one can have « pessimistic right » by introducing individual ability parameters  $\beta_i$ :
- Probability ( $y_i=y_1$ ) =  $\pi_0 + \theta\beta_i e_i$  if parental income =  $y_0$
- Probability ( $y_i=y_1$ ) =  $\pi_1 + \theta\beta_i e_i$  if parental income =  $y_1$
- With  $e_i$  = effort,  $\theta$  = index of how much effort matters,  $\Delta\pi = \pi_1 - \pi_0$  = index of how much parental inequality matters,  $\beta_i$  = individual ability parameter
- If one believes in different average  $\beta_1 > \beta_0$  depending on family origin, then one can easily get a « conservative right »: beliefs in low tax and low mobility

- If one believes in high  $\text{var}(\beta_i)$ , but with limited correlation with parental origins, then one can generate an « eugenist right »: beliefs in low tax and innate talent
- For a model with beliefs in luck, effort and talent, see T. Piketty, « Self-fulfilling models about social status », [JPubEc 1998](#)  
(with no correlation of  $\beta_i$  with parental origins)

- Other major limitation of basic model of luck and effort: the only policy instrument is redistributive taxation
- Historically, **left beliefs focus on the role of education** (tax revenues can be used to finance education and reduce parental-origin inequality  $\Delta\pi = \pi_1 - \pi_0$ ), unions/worker co-determination/power sharing, and other policies affecting primary inequality (rather than just doing secondary ex-post redistribution) (**vs right beliefs in private property**)
- Depending on the beliefs on the efficiency of education spending, one can generate « optimistic left »: high mobility and high taxation
- Beliefs in education can naturally be influenced by individual trajectories. Can also explain the rise of high-education & to some extent high-income left vote
- Beliefs in education, family and gender roles are also key to explain changing political preferences of women (right>left)
- **On changing effect of income, wealth, education, gender on left and right vote**, see « [Brahmin Left vs Merchant Right: Rising Inequality and the Changing Structure of Political Conflict](#) »: long-run series on changing political cleavages in FR-US-UK 1948-2017, + two-dimensional models of redistributive politics (see also [World Political Cleavages and Inequality Database](#), WPID.world)

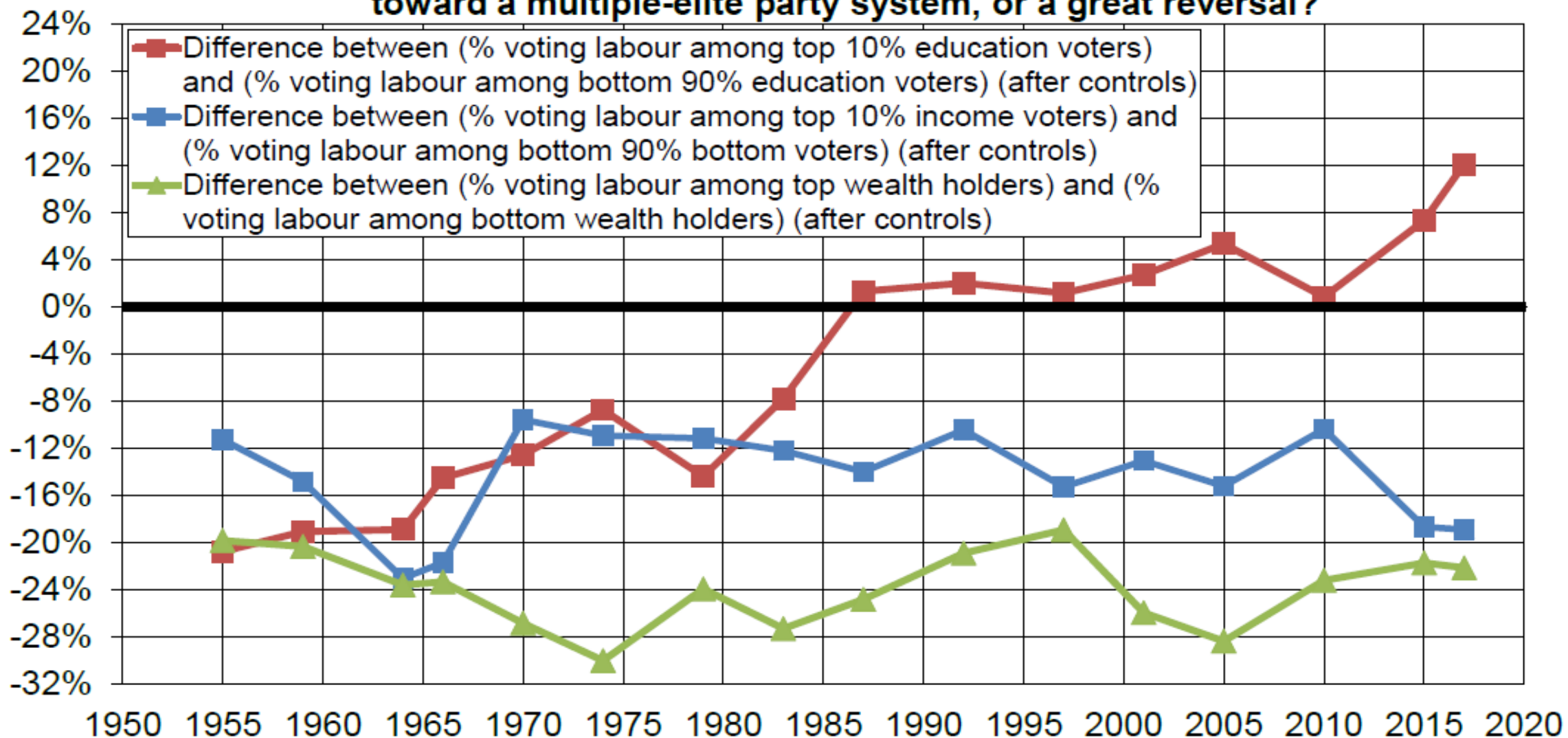
## Political conflict in France, 1956-2017: toward a multiple-elite party system, or a great reversal?



**Source:** author's computations using French post-electoral surveys 1956-2017 (see [piketty.pse.ens.fr/conflict](http://piketty.pse.ens.fr/conflict))

**Reading:** the left vote used to be associated with lower education and lower income voters; it has gradually become associated with higher education voters, giving rise to a "multiple-elite" party system (education vs wealth); it might also become associated with high-income voters in the future, giving rise to a great reversal or realignment of the party system.

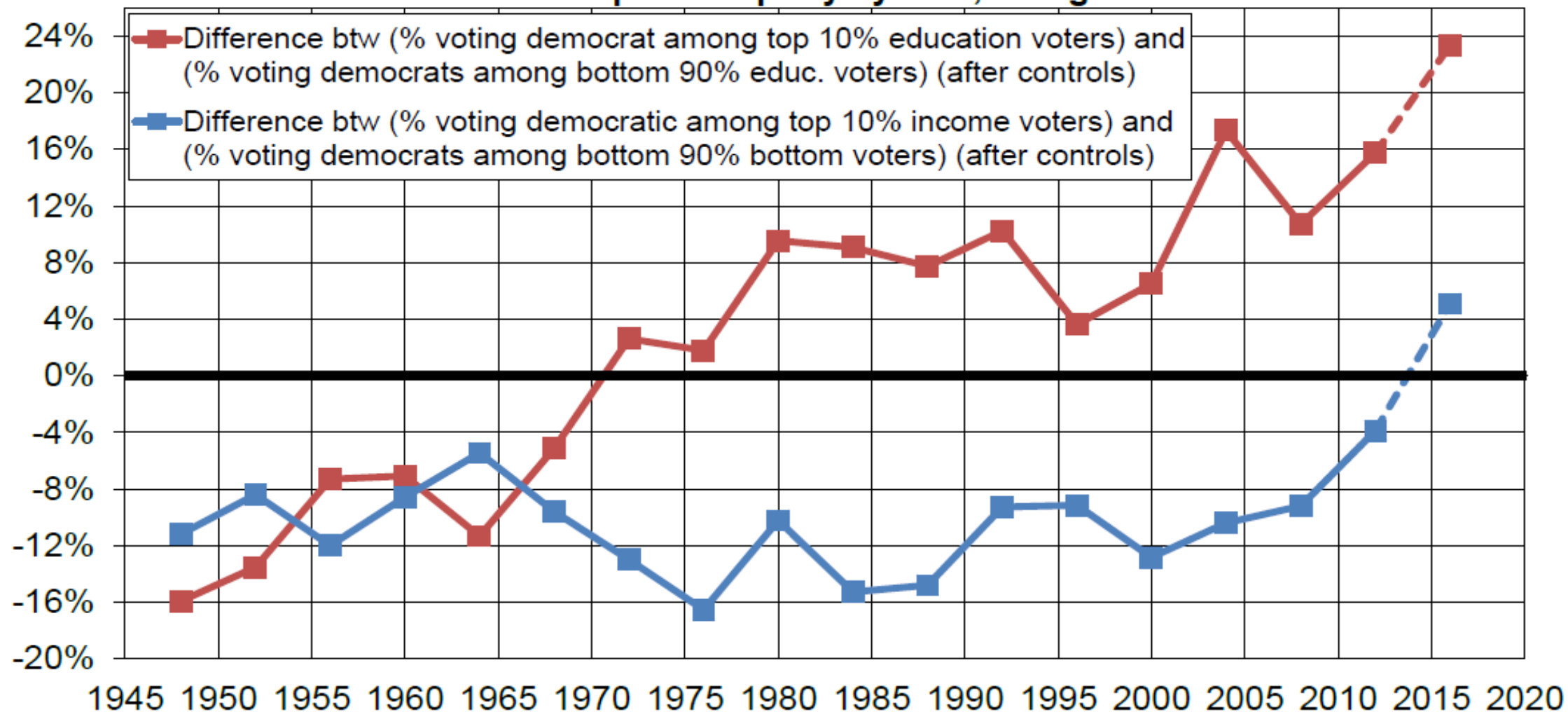
## Political conflict in Britain, 1955-2017: toward a multiple-elite party system, or a great reversal?



**Source:** author's computations using British post-electoral surveys 1963-2017 (BES) (see [piketty.pse.ens.fr/conflict](http://piketty.pse.ens.fr/conflict))

**Reading:** the labour vote used to be associated with low education and low income voters; it has gradually become associated to high education voters, giving rise to a "multiple-elite" party system (education vs income); it might also become associated with high income voters in the future, but at this stage this scenario seems less likely in Britain than in France or the US.

## Political conflict in the US, 1948-2017: toward a multiple-elite party system, or a great reversal?



Source: author's computations using US post-electoral surveys 1948-2016 (ANES) (see [piketty.pse.ens.fr/conflict](http://piketty.pse.ens.fr/conflict))

Reading: the democratic vote used to be associated with low education and low income voters; it has gradually become associated to high education voters, giving rise to a "multiple-elite" party system (education vs income); it might also become associated with high income voters in the future, giving rising to great reversal and complete realignment of the party system.

## Two-dim model 1: domestic inequality vs external inequality

- Introducing globalization: in addition to policy dimension  $T_L$  vs  $T_R$  (redistributive domestic tax rate between rich and poor), assume there's also some other dimension: openness/migration with  $O_L > O_R$
- As conflict about  $O_L > O_R$  becomes more salient (rise of extra-European migration in Europe, rise of civil rights/latinos in US), some the poor start to vote for the right, assuming preferences for  $O_L > O_R$  are correlated with education and income
- Further assume that globalization makes it easier to evade taxes: by putting dissimulation effort  $f$  then high-income taxpayers can manage with proba  $\omega f$  to pretend that they have  $y_0$  instead of  $y_1$
- With  $\omega$  large enough, then the policy conflict about redistribution vanishes: both  $T_L$  and  $T_R$  close to 0  $\rightarrow$  the political conflict gradually focuses on  $O_L > O_R \rightarrow$  « **globalists** » vs « **nativists** » **party system**



## Two-dim model 2: education inequality vs wealth inequality

- Introducing educational expansion: with rise of higher education, not possible to provide everyone with same education; depending on educational effort  $f_i$ , one face different chances to be admitted to selective higher education (education  $x_1$  rather than  $x_0$ )
- Probability ( $x_i=x_1$ ) =  $\alpha_0 + \phi f_i$  if parental education =  $x_0$
- Probability ( $x_i=x_1$ ) =  $\alpha_1 + \phi f_i$  if parental education =  $x_1$
- Higher education increases probability to access a high-paid job:
- Probability ( $y_i=y_1$ ) =  $\pi_0 + \theta e_i + \mu s$  if  $x_i=x_1$  (& parental income =  $y_0$ )

→ « **multiple-elite** » party system: Brahmin left believes more in education-related effort parameter  $\phi$ , while Merchant right believes more in business-related effort parameter  $\theta$

- Two key dimensions of political conflict: attitudes on vertical redistribution dimension (poor vs rich) vs attitudes on foreigners/migrants/minorities/openness
- See Roemer-Lee-Van der Straeten, *Racism, Xenophobia, and Distribution: Multi-Issue Politics in Advanced Democracies*, HUP 2007 (see also journal articles: [JPubE 2006](#); [JEEA 2006](#); [JE 2005](#) ; [SJE 2006](#); ) the xenophobia dimension substantially reduces the equilibrium level of vertical redistribution.
- See also Kuziemko-Norton-Saez-Stantcheva, « How Elastic are Preferences for Redistribution? », [AER 2015](#) : inequality dimension vs trust-in-government dimension.

- See also Alesina-Glaeser-Sacerdote, « Why doesn't the US have a European style Welfare », [BPEA 2001](#); Alesina-Glaeser, *Fighting Poverty in the US and Europe: a word of difference*, OUP 2004
- Main explanation: less demand for redistribution because more racial prejudice in the US (also: stronger US beliefs in effort and mobility, but difficult to separate from racial prejudice); negative cross-country correlation between racial fractionalisation and social transfers
- One pb with Roemer et al/Alesina et al: lack of historical perspective on how party systems and inequality change over time; US was in some ways more equal than Europe in 19c and invented steeply progressive taxation during 20c; historical changes are more interesting to study than supposedly permanent differences (see [Advanced Econ History](#) course)

# The problem of intertemporal justice: how much capital and debt should we leave to our children?

- Redistributive taxation of income involves serious conflicts of beliefs about the functioning of the economy and the way inequality is generated, the role of effort, education, etc.
- Redistributive taxation of wealth and capital involves equally complex (or arguably more complex) issues related to the large concentration of property, the dynamic impacts of taxation, the different forms of taxation (flows vs stocks). See lectures 9-10.
- But even if there was no wealth inequality at all, i.e. assuming everybody owns an equal share of the world capital stock, the issue of capital accumulation would raise serious problems of intertemporal justice: how much should we leave to our children? **Inequality between generations (rather than within generations) = arguably the most pressing issue, especially if we include natural capital.**

- [Phelps AER 1961](#), « The Golden rule of capital accumulation: a fable for growthmen »; [AER 1965](#), « Second essay on the Golden rule of accumulation » = one of the first mathematical models of Golden rule = « **re-invest the full capital share, until the point where the marginal product capital  $r$  equals the economy's growth rate  $g$**  ».

But as we will see this simple  $r=g$  rule makes sense only under very special assumptions (zero productivity growth + exogenous & permanently positive population growth)

- « Golden rule » = generic name often used to refer to intertemporal rules aimed at preparing the future; rules about long term investment, deficits, etc.
- In political debates, « Golden rules » (in a vague and general sense) are often invoked to limit the power of today's electoral majorities, sometime with good reasons: otherwise today's generations might eat up the future
- The pb is that these rules are often instrumentalized in ways that are not so good for the future...

- E.g. European budgetary rules can be viewed as a sort of « Golden rule »: 3% maximal deficit rule in Euro zone (Maastricht Treaty 1992), reduced in 2011-2012 to 0.5% structural deficit (i.e. potentially huge primary surplus)
- It is unclear however whether they've had a positive impact on Europe's welfare: e.g. Euro recession 2011-13 (not in US); most importantly, **these rules entirely ignore investment in education/human capital, natural capital, public wealth, etc., so it is not sure at all that they prepare the future**  
(see e.g. [Jackson et al 2013](#), [2015](#), [2020](#) GPI series: **Genuine Progress Indicator**)
- More generally, constitutional provisions protecting private property have often been imposed by invoking long-run public interest (e.g. to justify censitary vote: only the rich should vote, because the poor are not patient!)
- **Preparing the future is a complex multi-dimensional task, and no simple mathematical formula is going to solve the problem for us** : we need permanent & extensive democratic deliberation; be careful with automatic rules
- **That being said, it's interesting to look at what these mathematical formulas look like, starting with Phelps 1961**

- **Phelps 1961's Golden rule formula**

- Assume population growth rate  $n \geq 0$ :  $N_t = N_0 e^{nt}$

- Assume productivity growth rate  $h$ , so that efficient labour supply grows at rate  $g = n + h$  (with  $h =$  productivity growth rate):

$$L_t = L_0 e^{gt} = L_0 e^{(n+h)t}$$

( $L_0 = \lambda N_0$ , with  $\lambda =$  labour participation rate)

- Assume production function  $Y_t = F(K_t, L_t)$

with  $Y_t =$  output,  $K_t =$  capital input,  $L_t =$  labour input

- For instance  $F(K, L) = K^\alpha L^{1-\alpha}$  (with  $0 < \alpha < 1$ )

- Or any other functional form, assuming constant returns to scale (i.e.

$$F(aK, aL) = aF(K, L)), \text{ so that : } y = Y/L = F(K, L)/L = F(K/L, 1) = f(k)$$

( $y = Y/L =$  output per labor unit,  $k = K/L =$  capital per lab. unit)

- If  $Y = F(K, L) = K^\alpha L^{1-\alpha}$  then  $y = f(k) = k^\alpha$  :  $y$  is a rising concave function  $k$  →  
 **$k$  is useful, but where should we stop?**

- Output can be used to consume (food, i-phones etc.) or to invest (more machines, buildings, equipment to produce food and i-phones, or more investment for the future in education, environment, etc.):

- I.e.  $Y_t = C_t + I_t$ , with  $C_t = (1-s_t)Y_t$ ,  $I_t = s_t Y_t$ ,  $s_t$  = saving rate

- Capital accumulation is determined by saving:

$$dK_t/dt = I_t = s_t Y_t$$

- Q.: If we care about « long-run welfare » (to be defined), how much capital  $K_t$  should we accumulate, i.e. how should we choose the saving rate  $s_t$  ?
- Here we entirely ignore inequality: everybody is assumed the same average capital stock (representative agent approach); but this is already a difficult question to solve
- A.: With zero productivity growth ( $h=0$ ) and exogenous, permanently positive population growth ( $g=n>0$ ), then you should save until  $f'(k^*) = r^* = g$
- If  $h>0$ , or  $n=0$ , or  $n$  endogenous, it's difficult to say anything



- **Proof.** In the long run, it will generally be optimal to follow a balanced growth path, i.e. capital input  $K_t$  will grow at the same rate as labor input  $L_t$ : i.e.  $K_t = K_0 e^{gt}$
- Long-run saving equation:  $dK_t/dt = g K_t = I_t = sY_t$  (with  $s_t \rightarrow s =$  long-run saving rate).  
i.e. capital-output ratio  $\beta_t = K_t/Y_t \rightarrow \beta = s/g$
- Long-run capital per labor unit  $k_t \rightarrow k$   
Long-run output per labor unit  $y_t \rightarrow y = f(k)$ , with  $k/y = \beta = s/g$
- Assume  $h=0, n>0$ . No long-run productivity growth implies that per capita long run output and consumption will be stationary (zero growth). **With zero growth, a reasonable long-run welfare objective is simply to maximize long-run per capita consumption.**
- Long-run per capita consumption  $c = (1-s)f(k) = f(k) - sf(k) = f(k) - nk$
- **Max  $c = f(k) - nk \rightarrow f'(k^*) = n$**
- i.e. Long-run optimal capital stock  $k^*$  is such that marginal product of capital  $r^* = f'(k^*) = n$  (i.e.  $r^* = g$ , since  $g = n + h = n$  if  $h = 0$ )
- **Intuition:** with  $r = n$ , then long-run saving  $sy = nk = rk =$  capital share. **There's no point saving more than the capital share.** Otherwise, what's the point of accumulating  $k$ ?  $k$  is supposed to increase  $c$ . **Golden rule: stop accumulating  $k$  if maintaining this per capita capital level requires you to save more than the capital share.**

- Exemple:  $f(k)=k^\alpha$ , so that  $r=f'(k)=\alpha k^{\alpha-1}$
  - I.e.  $r=\alpha f(k)/k$ , so that  $rk=\alpha f(k)=\alpha y$ , i.e. capital share = fixed fraction  $\alpha$  of output  $y=f(k)$
  - With Cobb-Douglas production function, the marginal product of capital  $r=f'(k)$  declines exactly in the same proportions as the increase in  $k/y$ , so that the two effects exactly cancel each other and the capital share is constant (elasticity of substitution = 1)
  - Assume population growth rate  $n=1\%$  and capital share  $\alpha=30\%$ . Then  $r^*=f'(k^*)=1\%$  implies  $\beta^*=k^*/y^*=\alpha/r^*=3000\%$ .
  - I.e. we should accumulate 30 years of income in capital.
- This is a lot of capital: in practice  $\beta=500-600\%$  (5-6 years).
- So here the Golden rule is telling us that we should accumulate a lot more capital than we do.

- Intuition. **Why should we stop accumulating capital at  $\beta^*=3000\%$ ?**
- Because with a population growth rate  $n=1\%$ , then in order to maintain a per capita capital/income ratio higher than this, then we would need to save more than 30% of output each year. Given that the production function is such that the capital share is 30%, this would mean saving more than the capital share.
- E.g. in order to maintain  $\beta=4000\%$ , we would need to save at rate  $s=40\%$ . There would be so much capital that the rate of return would be less than 1%: it would be equal to  $r=\alpha/\beta=0.75\%$ . Everybody would be better off by reducing saving (i.e. by eating some of the capital stock): the current generation and the future generations.

- **Pb: if  $n \rightarrow 0\%$  (=UN demographic projections for 21c), then there is no limit to capital accumulation:  $r^*=n \rightarrow 0\%$ ,  $\beta^*=\alpha/r^* \rightarrow \infty$ , i.e. we should accumulate infinite capital & postpone consumption entirely to the future!**
- **Pb with simple Golden rule  $r^*=n$  : it does not put enough weight on current generations, which given the existence of productivity growth is particularly problematic**  
 **$\rightarrow$  « modified Golden rule »  $r^* = \delta + \gamma g$**

(other pb: population growth is a choice, so it is strange to treat population growth rate as given; and what do we do if  $n < 0$ ?

Complicated welfare considerations on optimal population size)

- « **Modified Golden rule** »:  $r^* = \delta + \gamma g$

with  $\delta$  = pure social rate of time preference

$g$  = economy's growth rate:  $Y_t = e^{gt} Y_0$

$\gamma$  = concavity of social welfare function

- **Same model as before**, except that we now assume positive long-run productivity growth  $h > 0$  (and zero long-run population growth  $n = 0$ , so that  $g = n + h = h$ ), and except that we now specify explicitly the intertemporal social welfare function
- $r^* = f'(k^*) = \delta + \gamma g$  is the optimal long-run level of capital accumulation that should be set by a social planner choosing saving rates so as to maximize  $V = \int_{t>0} e^{-\delta t} U(c_t)$   
with  $U(c) = c^{1-\gamma} / (1-\gamma)$  (i.e.  $U'(c) = c^{-\gamma}$ )
- $\gamma \geq 0$  measures the speed at which the marginal social utility of consumption goes to zero  
= how useful is it to have another i-phone if you already have 100000 i-phones?  
( $\gamma = 0$ : linear utility  $U(c) = c$ ;  $\gamma = 1$ : log utility  $U(c) = \log(c)$ ;  
 $\gamma > 1$ : utility function more concave than log function)

- **The modified Golden rule  $r^* = \delta + \gamma g$  is used extensively in policy debates, e.g. in the global warming debate.** The pb is that there is no clear way to pick parameters  $\delta$  and especially  $\gamma$ .
- The choice of parameters has a strong impact on the social discount rate  $r^*$ : are future generations going to be so rich and so productive that they will be able to clean up our pollution?
- [Stern 2006 Report](#) on the costs of global warming
- An important part of the controversy was due to differences in the social discount rate
- I.e. assume that we agree that global warming will cause catastrophies that are equivalent to a loss equal to  $\lambda\%$  of world GDP in T years
- Say  $\lambda=10\%$ , and T=70 years (sea will rise around 2080)
- Q.: How much welfare should we ready to sacrifice today in order to avoid this? Should we stop using cars entirely?
- A.: We should be able to sacrifice  $\mu Y_0 = e^{-r^*T} \lambda Y_T$ ,  
with  $r^* = \delta + \gamma g =$  social discount rate = rate at which an ideal social planner should discount the future

- Intuition behind  $r^* = \delta + \gamma g$
- If  $g=0$ , then  $r^*=\delta$  : social rate of time preference
- From an ethical viewpoint, everybody agrees that  $\delta$  should be close to 0%: it is difficult to justify why we should put a lower welfare weight on future generations
- Both Stern & Nordhaus pick  $\delta=0,1\%$  (Stern mentions estimates of meteorit crash: the probability that earth disappears is  $<0,1\%/yr$ )  
 → with zero growth, everybody agrees that  $\mu \approx \lambda$   
 (of course, private rate of time preference – i.e. how private individuals behave in their own life – are a different matter: they can be a lot larger, typically private  $\delta =$  at least 1-2%, and private  $\gamma > 1$ )

- With  $g > 0$ , one has to compute the impact on social welfare of reducing consumption by  $dc_T < 0$  at time  $t=T$  and raising it by  $dc_0 > 0$  at time  $t=0$ :
- Social welfare:  $V = \int_{t>0} e^{-\delta t} U(c_t)$   
with  $U(c) = c^{1-\gamma} / (1-\gamma)$  (i.e.  $U'(c) = c^{-\gamma}$ )
- $dV = U'(c_0) dc_0 + e^{-\delta t} U'(c_T) dc_T$
- $c_T = e^{gT} c_0 \rightarrow dV = 0$  iff  $dc_0 = e^{-(\delta+\gamma g)t} dc_T$   
 $\rightarrow$  MGR:  $r^* = \delta + \gamma g$
- Intuition:  $\gamma$  very large means that extra consumption not so useful for future generations, because they will be very rich anyway  $\rightarrow$  very large  $r^*$ , even if  $g$  is quite small and uncertain



- Stern vs Nordhaus controversy: both agree with the modified Golden rule formula  $r^* = \delta + \gamma g$  and about the long-run productivity growth prospects ( $g=1,3\%$ ) but disagree about parameters (especially  $\gamma$ )
- Stern 2006 :  $\delta=0,1\%$ ,  $g=1,3\%$ ,  $\gamma=1$ , so  $r^*=1,4\%$   
(see Stern 2006 report, [chapter 2A](#))
- Nordhaus 2007:  $\delta=0,1\%$ ,  $g=1,3\%$ ,  $\gamma=3$ , so  $r^*=4,0\%$   
(see Nordhaus, "Critical Assumptions in the Stern Review on Climate Change", [Science 2007](#); see also [JEL 2007](#) symposium)

- Whether one adopts  $r^*=1,4\%$  or  $r^*=4,0\%$  (for a given growth rate  $g=1,3\%$ ) makes a huge difference:
- We should spend:  $\mu Y_0 = e^{-r^*T} \lambda Y_T$ , i.e.  $\mu = e^{-(r^*-g)T} \lambda$   
(since  $Y_T = e^{gt} Y_0$ )
- According to Stern  $r^*-g=0,1\%$ , so with  $T=70$ ,  $e^{(r^*-g)T}=1,07$  : it is worth spending about 9% of GDP in 2010 in order to avoid a 10% GDP loss in 2080: we need to reduce emissions right now & to finance large green investments
- But  $e^{(r^*-g)T}=6,61$  according to Nordhaus ( $r^*-g=2,7\%$ ): it is worth spending only 1,5% of GDP in 2010 in order to avoid a 10% GDP loss in 2080: don't worry too much, growth will clean up the mess
- $\approx$  EU vs US position

- What is strange in this controversy is that both Stern and Nordhaus take opposite sides on concavity parameter  $\gamma$  as compared to the parameters that they usually favor for cross-sectional redistribution purposes: Stern would usually favor high  $\gamma$  (high redistribution) and Nordhaus low  $\gamma$  (low redistribution)
- If future growth was certain (i.e. future generations will be more productive, whatever they do), then it might indeed make sense to have high  $\gamma$  or even infinite  $\gamma$  = Rawlsian objective: we should only care about maximizing the lowest welfare or consumption level, i.e. the level of the current generation

- Two pb with this intergenerational Rawlsian reasoning:
- (1) growth is endogenous: if we leave infinite pollution (or debt) to future generations, maybe  $g$  will not be so large
- (2) one-good models are not well suited to study these issues: in the long run the relative price of the environment might be infinite (i.e. if we all have 100000 i-phones, but unbreathable air, maybe the relative value of having a little bit clean air will be quite large)

See J. Sterner, "An Even Sterner Review: Introducing Relative Prices into the Discounting Debate", [JEP 2008](#)

See also Drupp et al, « Discounting disentangled », [2015](#); Nordhaus, « Climate Change: the Ultimate challenge for economics », [AER 2019](#); Sterner et al, « Climate Economics Support for UN Mandate », [Nature and Climate Change 2020](#)

- **Summing up:** the interesting intuition behind the Golden rule is that we should never accumulate so much  $k$  that the return rate  $r$  falls below the growth rate  $g$ , i.e. so much that the capital share  $rk$  falls below the required saving  $gk$  to keep  $k/y$  constant
- We always want  $r > g$ , so that the return to capital  $rk$  is higher than what we need to save each year to maintain the capital stock.
- E.g. with  $r=5\%$ ,  $g=1\%$ , we only need to reinvest 1/5th of the return to capital, and we can consume the other 4/5th.
- The point of capital accumulation is to be able to consume more in the future, not less.
- **The Golden rule  $r=g$  puts a maximum on capital accumulation** : in case we accumulate more than this, then we are in a situation of « dynamic inefficiency », in which case it is Pareto-improving to reduce the capital stock (e.g. by setting up a pay-as-you-go pension system, as in [Diamond 1965](#)'s OLG model)
- **But it does not say that we should accumulate so much:** it provides an upper bound on capital accumulation (and a lower bound on rate of return), not a target.
- **Modified Golden rule  $r^* = \delta + \gamma g$  is not so useful:** this is trying to put too much into a simple mathematical formula.

- **Computing optimal capital accumulation for the future is a complex multi-dimensional task, and no simple mathematical formula is going to solve the problem for us.**
- We first need to make sure that we properly measure the different dimensions of capital accumulation: e.g. we need to take into account public assets and not only public debt (if we take a complete view, current obsession with private debt not justified); we need to take into account human capital (investment in education) and natural capital (which existing balance sheets don't do well at this stage).
- Some economic computations can be useful, but not all: e.g. Stern's estimates of lost GDP (fall in economic activity) if sea levels rise can be useful; but discount rate controversy not so useful
- In order to properly analyze optimal public policies regarding capital accumulation, one needs to introduce inequality into the picture. See lectures 9-10.

# Condorcet paradox & majority cycles

- Because of different interests, values and beliefs systems, different individuals have different preferences about policies, and in particular about inequality, capital accumulation, etc.
- Q.: Which mechanisms and political institutions should be used to combine these preferences and take a collective decision? = the pb of « social choice » (or « preference aggregation »)
- K. Arrow, *Social Choice and Individual Values*, 1951
- Arrow's Impossibility theorem = if we rule out interpersonal comparisons of utilities, then there is no consistent collective rule to aggregate individual preferences and take collective decisions (i.e. we need minimal agreement about common values; if all disagreements are allowed, then chaos prevails)
- In particular, the “majority rule” doesn't work
- In general, one can find policies A,B,C such that a majority prefers A to B, a majority prefers B to C & a majority prefers C to A = “**majority cycle**”

- **Condorcet paradox 1785:** with multi-dimensional political conflicts, majority cycles are pervasive (→ **democracy needs to be organized, constitution design is important**)
- Concrete example of multi-dimensional political conflict: attitudes toward redistribution (e.g. level of progressive taxation, size of govt) vs attitudes toward foreigners/migrants
- Then one can easily find 3 candidates A,B,C such that we have a cycle.
- E.g. A=Le Pen, B=Melenchon, C=Macron?
- A defeats B, B defeats C, C defeats A?



- Arrow's impossibility theorem = simple generalization of Condorcet majority paradox : majority rule doesn't work, and no simple political rule can work
- Arrow 1951 = negative results about political institutions = often viewed as the negative equivalent of Debreu 1959 *Theory of Value – An axiomatic analysis of economic equilibrium* = positive results about economic institutions (two welfare theorems on market efficiency)
- In brief: economics is about creating new value, politics is about dividing the pie in a more or less chaotic manner  
= “public choice” school, nihilist view of politics
- But in fact there are many ways out of Arrow 1951's negative result: we need minimal agreement on common values and goods, we need to talk about constitution design... and we should not forget about Condorcet's jury theorem = the positive, constructive side of politics

# Condorcet jury theorem and the constructive view of political institutions

- **Condorcet 1785 jury theorem.** Assume that everybody has the same objective function (same values and preferences), but has different beliefs and information about what policy is optimal (given these values and preferences).
- Further assume that we have to choose between two policies A and B, that everybody receives a signal providing information as to whether A or B is the optimal policy, and that everybody has the same probability  $p > 0.5$  to receive the right signal.
- Then with a large population, the probability that the majority rule leads to the right decision approaches 1. Nobody wants to be dictator, everybody prefers majority-rule democracy. Simple, but powerful: democracy is based upon the idea that more than half of the people are right more than half of the time, & on the law of large numbers.

- With different signal qualities, more than 2 policies, etc. then one may prefer indirect democracy, etc.: constitution design matters
- See Condorcet 1785, *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix* ; and Condorcet's contribution to revolutionary debates on ideal constitution
- Condorcet jury theorem = basic positive result about democratic institutions (aggregation of information through voting). Equivalent to Arrow-Debreu positive result about economic institutions (aggregation of information through the markets)
- The jury theorem and the majority-cycle paradox should be viewed as complementary: democracy can work, but it needs to be organized

- If political conflict is about different beliefs and information (and not simply about conflicting interests and preferences), then different electoral & political systems allow for different levels of aggregation of information: trade-off between the decision-making and political-stability dimensions of the electoral system and the deliberative dimension
- See T. Piketty “The Information-Aggregation Approach to Political Institutions”, [EER 1999](#) ; “Voting as Communicating”, [RES 2000](#)
- More generally, if politics is about beliefs and information, then public deliberation and communication via medias, political parties, books etc. play a critical role.

- In particular, deliberation can reduce the dimensionality of political conflict. See D. Spector, “Rational Debate Leads to One-Dimensional Conflict”, [QJE 2000](#)
- Intuition: as long as there are different dimensions of conflict, one can find ways to solve credibility problems.
- Politics is both about sincere beliefs/ideology and about conflicting interests and self-serving beliefs. So it is important to regulate political finance, access to the media, etc. See Bonica-Rosenthal, « Why Hasn't Democracy Slowed Rising Inequality », [JEP 2013](#) ; T. Kuhner, *Capitalism vs Democracy: Money in Politics and the Free Market Constitution*, SUP 2014
- More on political parties, electoral conflict & inequality in [Advanced Economic History](#) course