Economics of Inequality

(Master PPD & APE, Paris School of Economics)

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Lecture 3: The dynamics of capital/income ratios: β=s/g

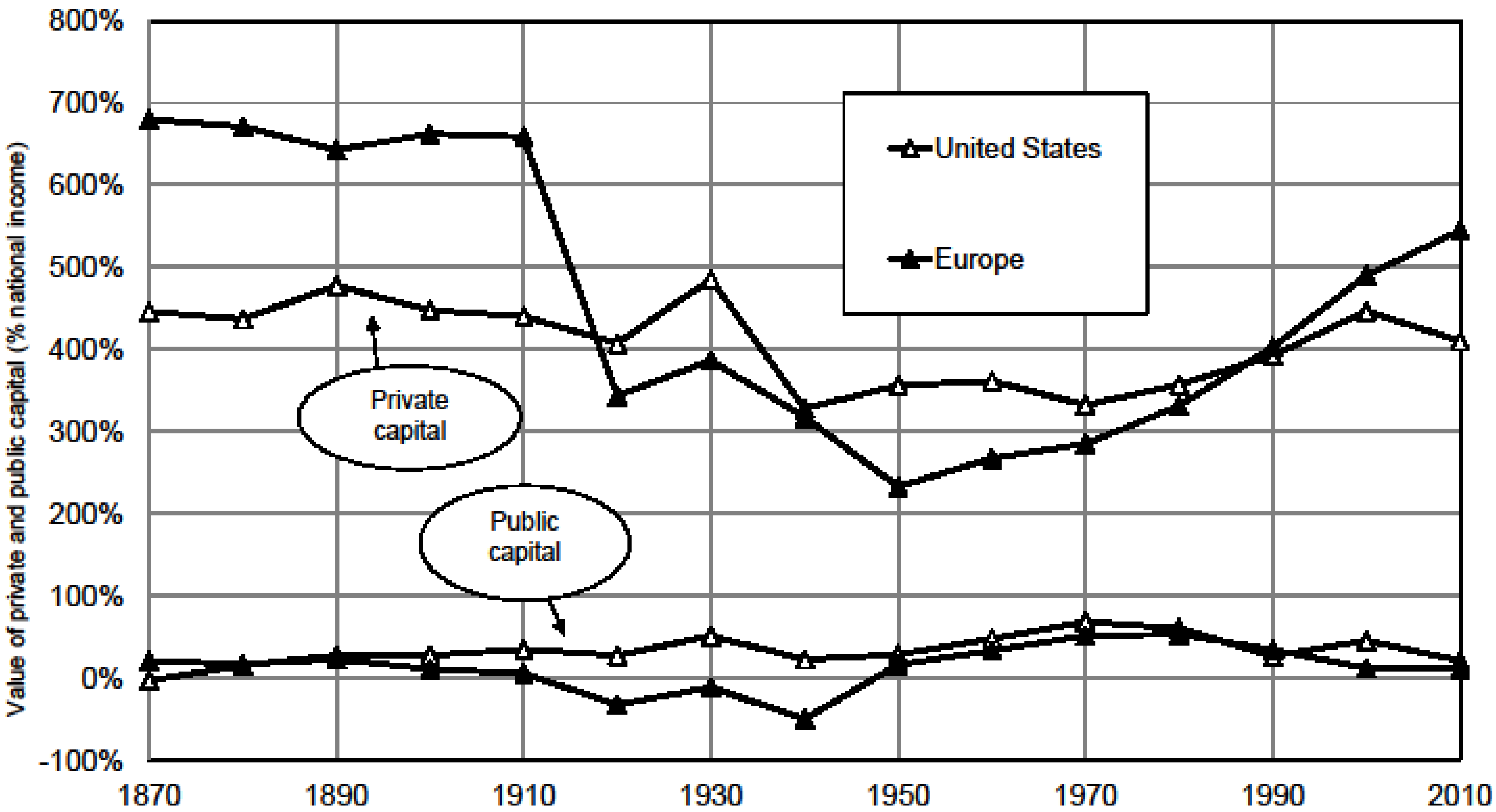
(Tuesday October 7th 2014)

(check on line for updated versions)

Summing up: what have we learned?

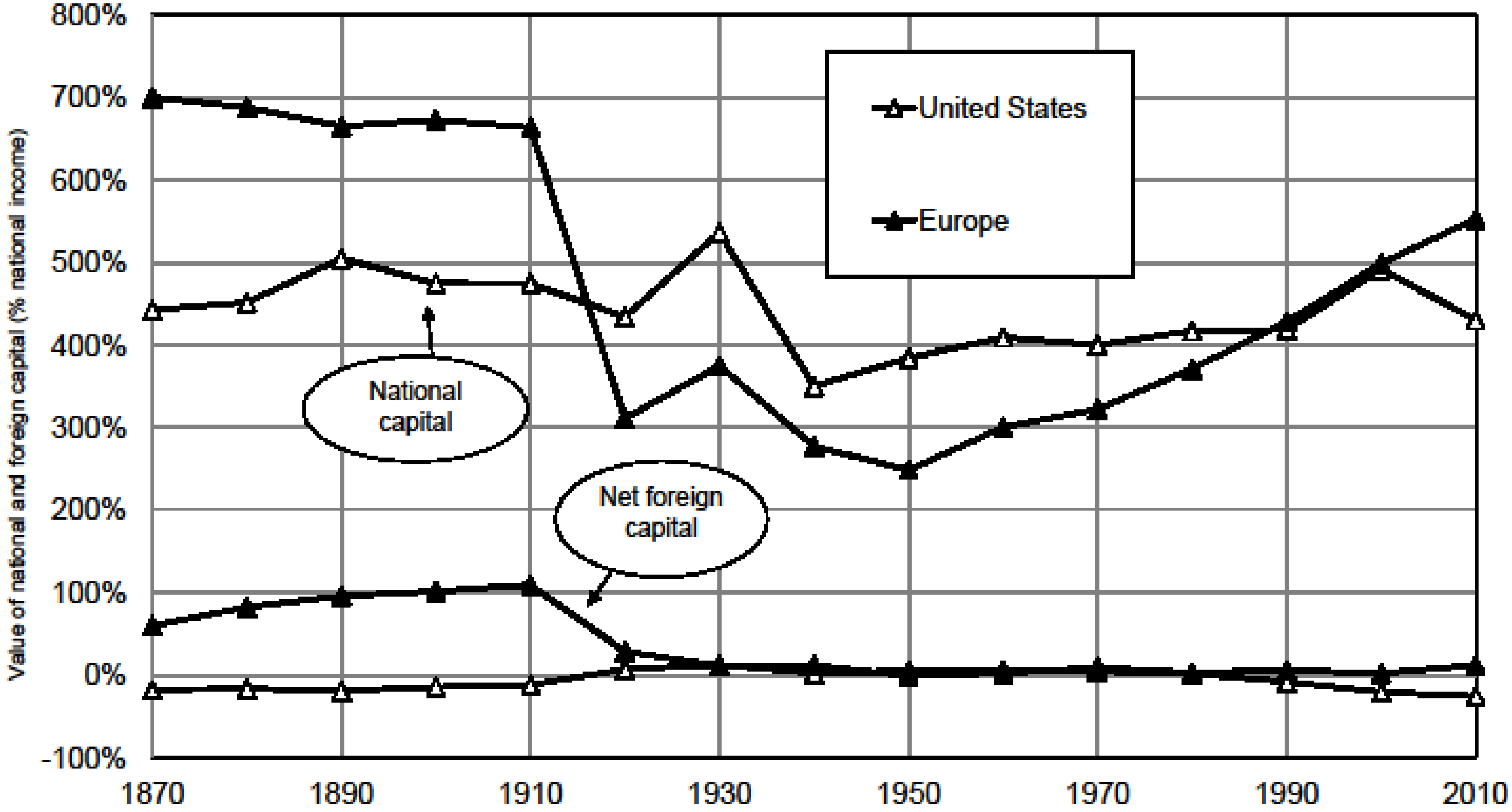
- National wealth-income ratios $\beta_n = W_n/Y$ followed a large U-shaped curve in Europe: 600-700% in 18c-19c until 1910, down to 200-300% around 1950, back to 500-600% in 2010
- U-shaped curve much less marked in the US
- Most of the long run changes in β_n are due to changes in the private wealth-income ratios $\beta = W/Y$
- But changes in public wealth-income ratios $\beta_g = W_g/Y$ (>0 or <0) also played an important role (e.g. amplified the β decline between 1910 and 1950)
- Changes in net foreign assets NFA (>0 or <0) also played an important role (e.g. account for a large part of the β decline between 1910 and 1950)

Figure 5.1. Private and public capital: Europe and America, 1870-2010



The fluctuations of national capital in the long run correspond mostly to the fluctuations of private capital (both in Europe and in the U.S.). Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 5.2. National capital in Europe and America, 1870-2010



National capital (public and private) is worth 6,5 years of national income in Europe in 1910, vs. 4,5 years in America.
Sources and series: see piketty.pse.ens.fr/capital21c.

Let’s move to theory: how can we explain capital-income ratios β=W/Y ?

- We first need a theory of why people own wealth: if economic agents only care about current consumption, then they should not own any wealth, i.e. β=0. So we need a dynamic model (at least two periods) where agents care about the future.
- OLG model: agents maximize $U(c_t, c_{t+1})$, where c_t = young-age consumption (working age) & c_{t+1} = old-age consumption
- Depending on utility function U(.,.) (rate of time preference, etc.), demographic parameters, etc., one obtains different saving rates and long run β (see « Modigliani triangle » formula in lecture 6)
- Pb with the pure life-cycle model: individuals are supposed to die with zero wealth; in the real world, inherited wealth is also important
- Models with utility for bequest: $U(c_t, w_{t+1})$, where c_t = lifetime consumption (young + old) & w_{t+1} = bequest (wealth) left to next generation
- Depending on the strength of the bequest motive in utility function U(c,w), one can obtain any saving rate and long run β

Harrod-Domar-Solow formula: $\beta=s/g$

- Exemple: if agents maximize $U(c_t,\Delta w_t=w_{t+1}-w_t)$, then with $U(c,\Delta)=c^{1-s}\,\Delta^s$, we get a fixed saving rate $s_t=s$, and $\beta_t \rightarrow \beta = s/g$
  (i.e. Max $U(c_t,\Delta w_t)$ under $c_t+\Delta w_t \leq y_t \rightarrow \Delta w_t = s\,y_t$)

- **More generally, this is what we get in any one-good capital accumulation model, whatever the saving motives and utility fonctions behind the saving rate s_t :**

- I.e. assume that: $W_{t+1} = W_t + s_t Y_t$
  $\rightarrow$ dividing both sides by Y_{t+1}, we get: $\beta_{t+1} = \beta_t\,(1+g_{wt})/(1+g_t)$

With $1+g_{wt} = 1+s_t/\beta_t$ = saving-induced wealth growth rate
$1+g_t = Y_{t+1}/Y_t$ = total income growth rate (productivity+population)

- If saving rate $s_t \rightarrow s$ and growth rate $g_t \rightarrow g$, then:

$$\boldsymbol{\beta_t \rightarrow \beta = s/g}$$

- Exemple: if s=10%, g=2%, $\beta_t \rightarrow \beta$ = 500%
- This is a pure accounting identity: β = 500% is the only wealth-income ratio such that a saving rate of 10% of income corresponds to a growth rate of 2% of the capital stock
- **Intuition: the more you save, the more you accumulate, especially in a slow-growth economy**

(on these models, see Piketty-Zucman 2013 section 3)

Another special case: the dynastic model

- Pure dynastic model = model with infinite horizon and fixed rate of time preference = individuals behave as if they were infinitely lived

- Discrete time version: $U_t = \sum_{t\geq 0} U(c_t)/(1+\theta)^t$ (θ = rate of time preference)
- Budget constraint: $\sum_{t\geq 0} c_t/(1+r)^t \leq \sum_{t\geq 0} y_t/(1+r)^t$

- r_t = rate of return = = $f'(k_t)$ = borrowing interest rate (perfect capital markets)
- Closed economy, representative agent: ind. wealth w_t = per capita capital stock k_t
- First-order condition: $U'(c_{t+1})/U'(c_t)=(1+\theta)/(1+r)$
- Assume $U(c)= c^{1-\gamma}/(1-\gamma)$, i.e. $U'(c)= c^{-\gamma}$, ($U(c)=\log(c)$ if $\gamma=1$)
- FO condition: $c_{t+1} =c_t \, [(1+r)/(1+\theta)]^{1/\gamma}$
- Intuition: if $r > \theta$, then agents want to postpone consumption to the future (conversely if $r < \theta$), and all the more so if γ close to 0, i.e. U(c) close to linear
- γ = curvature of U(c) (risk aversion coefficient), $1/\gamma$ = intertemporal elasticity of substitution
- Steady-state growth path: $y_t = y_0 (1+g)^t$, $k_t = k_0 (1+g)^t$, $c_t = c_0 (1+g)^t$

→ $1+r = (1+\theta) \times (1+g)^\gamma$ (with continuous time: $r = \theta + \gamma g$)

→ if $g=0$, then $r=\theta$ ($>g$) : rate of return is entirely determined by preferences

- With Cobb-Douglas production function $y=f(k)=k^{\alpha}$, then $r=f'(k)=\alpha k^{\alpha-1}$, so that capital income $rk=\alpha y$, i.e. capital-income ratio $\beta=k/y= \alpha/r$
- I.e. if $r = \theta + \gamma g$, then $\beta=\alpha/(\theta + \gamma g)$
- Exemple: if $g=0$, $\theta=5\%$ and $\alpha=25\%$, then $\beta= \alpha/\theta =500\%$
- I.e. if lower θ (more patient), higher β
- In effect, in the dynastic model, agents save a fraction g/r of their capital income rk, so that their wealth rises at rate g, like the rest of the economy (i.e. with $g=1\%$ and $r=5\%$, they save $1/5=20\%$ of their capital income, and eat the rest

→ saving rate $s = \alpha g/r$

capital-income ratio $\beta = s/g= \alpha/r$ =g=0

= special case of Harrod-Domar-Solow formula

Inequality in the dynastic model

- For simplicity, assume a two-point distribution of wealth.
- Dynasties can be of one of two types: either they own a large capital stock k_t^A, or they own a low capital stock k_t^B (with $k_t^A > k_t^B$).
- The proportion of high-wealth dynasties is equal to λ (and the proportion of low-wealth dynasties is equal to 1-λ), so that the average capital stock in the economy k_t is given by:

$$k_t = \lambda k_t^A + (1-\lambda) k_t^B$$

- Result: any distribution such that the average wealth k* satisfies $f'(k^*)=r= \theta + \gamma g$ can be a steady-state
- I.e. any wealth inequality is self-sustaining, both within countries and between countries, including with negative wealth $k_t^B <0$ for the poor (possibly extreme negative wealth = slavery)
- With shocks and mobility, steady-state wealth inequality is a rising function of the differential r-g **(see lecture 6)**

- **To summarize: Harrod-Domar-Solow formula $\beta = s/g$ is a pure accounting formula and is valid with any saving motive and utility function**

- **Wealth increase in the utility function**: Max $U(c_t,\Delta w_t=w_{t+1}-w_t)$

$\rightarrow$ if $U(c,\Delta)=c^{1-s}\Delta^s$, then fixed saving rate $s_t=s$, $\beta_t \rightarrow \beta = s/g$

(i.e. Max $U(c_t,\Delta w_t)$ under $c_t+\Delta w_t \leq y_t$ $\rightarrow$ $\Delta w_t = s\, y_t$)

- **Total wealth or bequest in the utility function:** Max $U(c_t,w_{t+1})$

$\rightarrow$ if $U(c,w)=c^{1-s}w^s$, then $w_{t+1}=s(w_t + y_t)$, $\beta_t \rightarrow \beta = s/(g+1-s) = s'/g$

(with $s'=s(1+\beta)-\beta$ = corresponding saving rate out of income)

- **Pure OLG lifecycle model**: saving rate s determined by demographic structure (more time in retirement $\rightarrow$ higher s), then $\beta_t \rightarrow \beta = s/g$

- **Dynastic utility**:

Max $\Sigma\, U(c_t)/(1+\theta)^t$, with $U(c)=c^{1-1/\gamma}/(1-1/\gamma)$

$\rightarrow$ unique long rate rate of return $r_t \rightarrow r = \theta + \gamma g > g$

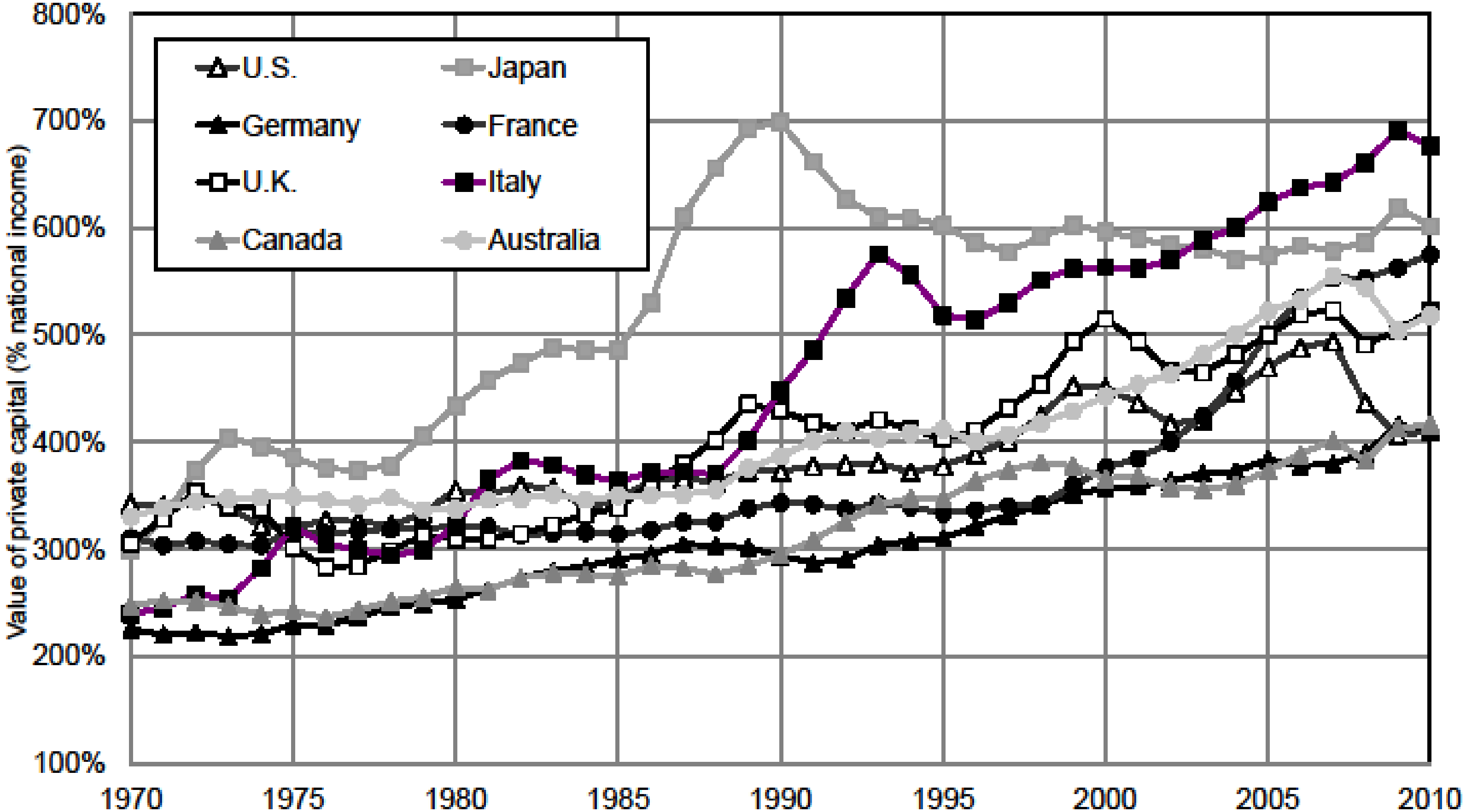
$\rightarrow$ long run saving rate $s_t \rightarrow s = \alpha g/r$, $\beta_t \rightarrow \beta = \alpha/r = s/g$

(on these models, see Piketty-Zucman 2013 section 3)

The rise of wealth-income ratios in rich countries 1970-2010: volume or price effects ?

- Over 1970-2010 period, the analysis can be extented to top 8 developed economies: US, Japan, Germany, France, UK, Italy, Canada, Australia
- Around 1970, $\beta \approx 200\text{-}350\%$ in all rich countries
- Around 2010, $\beta \approx 400\text{-}700\%$ in all rich countries
- Asset price bubbles (real estate and/or stock market) are important in the short-run and medium-run
- But the long-run evolution over 1970-2010 is more than a bubble: it happens in every rich country, and it is also consistent with the basic theoretical model $\beta = s/g$

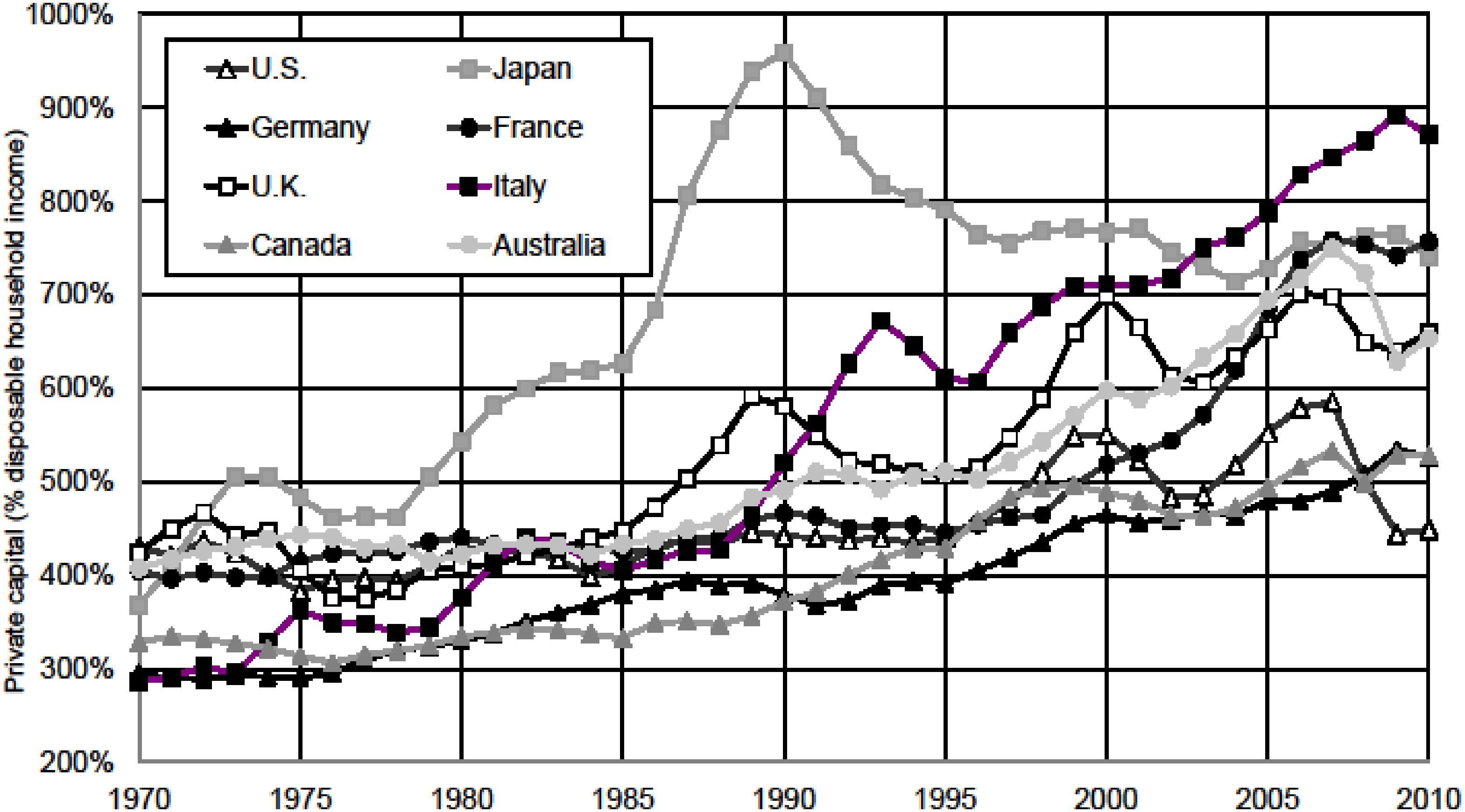
Figure 5.3. Private capital in rich countries, 1970-2010



Private capital is worth between 2 and 3,5 years of national income in rich countries in 1970, and between 4 and 7 years of national income in 2010. Sources and series: see piketty.pse.ens.fr/capital21c.

- The rise of β would be even larger is we were to divide private wealth W by disposable household income Y_h rather than by national income Y
- Y_h used to be ≈90% of Y until early 20c (=very low taxes and govt spendings); it is now ≈70-80% of Y (=rise of in-kind transfers in education and healh)
- $\beta_h = W/Y_h$ is now as large as 800-900% in some countries (Italy, Japan, France…)
- But in order to make either cross-country or time-series comparisons, it is better to use national income Y as a denominator (=more comprehensive and comparable income concept)

Figure 5.4. Private capital measured in years of disposable income

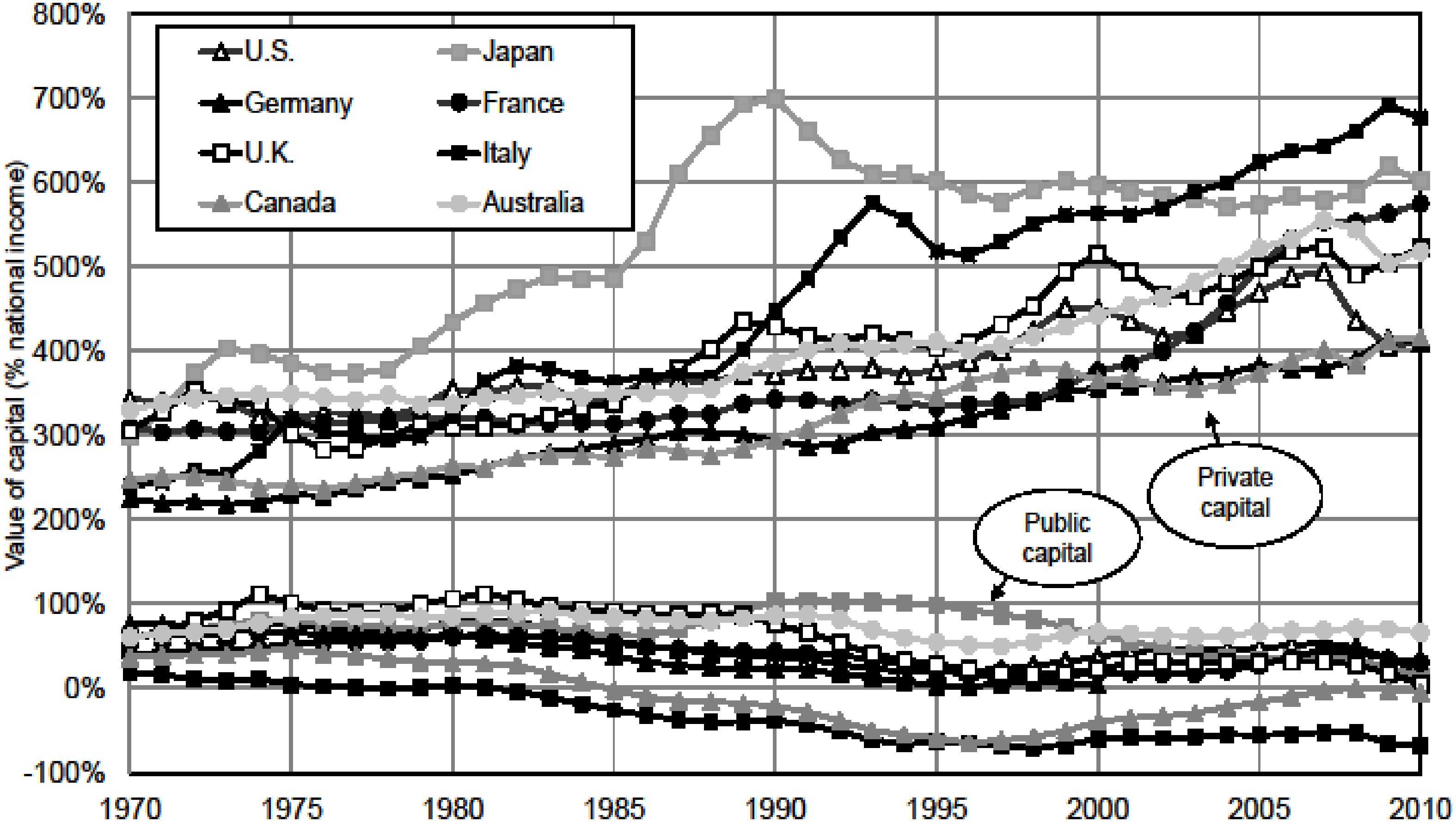


Expressed in years of household disposable income (about 70-80% of national income), the capital/income ratio appears to be larger than when it is expressed in years of national income.

Sources and series : see piketty.pse.ens.fr/capital21c.

- 1970-2010: rise of private wealth-income ratio β, decline in public wealth-inccome ratio β_g
- But the rise in β was much bigger than the decline in β_g, so that national wealth-income ratio $\beta_n=\beta+\beta_g$ rose substantially
- Exemple: Italy. β rose from 240% to 680%, β_g declined from 20% to -70%, so that β_n rose from 260% to 610%. I.e. at most 1/4 of total increase in β can be attributed to a transfer from public to private wealth (privatisation and public debt).

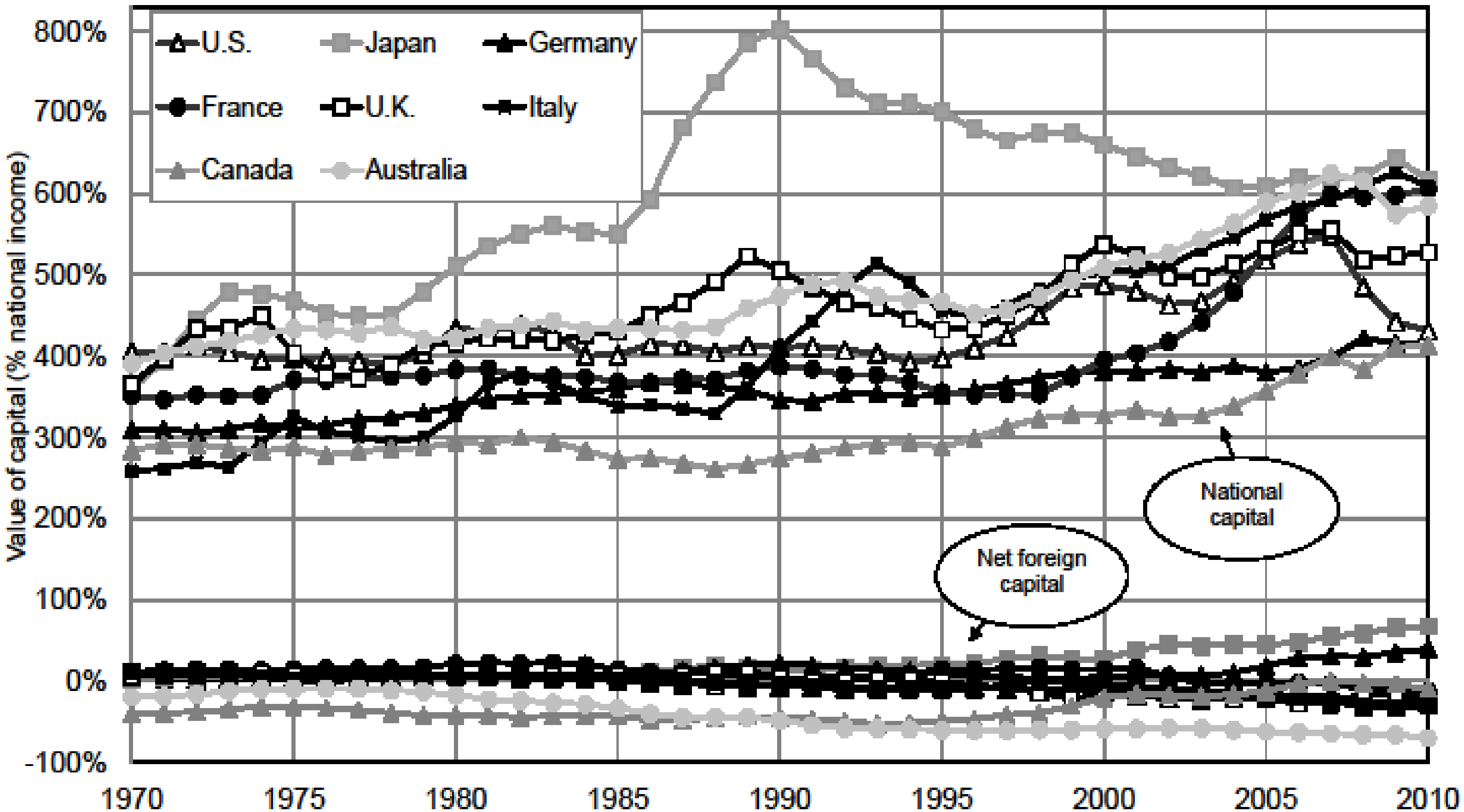
Figure 5.5. Private and public capital in rich countries, 1970-2010



In Italy, private capital rose from 240% to 680% of national income between 1970 and 2010, while public capital droptped from 20% to -70%. Sources and series: see piketty.pse.ens.fr/capital21c.

- In most countries, NFA ≈ 0, so rise in national wealth-income ratio ≈ rise in domestic capital-output ratio; in Japan and Germany, a non-trivial part of the rise in β_n was invested abroad (≈ 1/4)

Figure 5.7. National capital in rich countries, 1970-2010



Net foreign assets held by Japan and Germany are worth between 0,5 and 1 year of national income in 2010. Sources and series: see piketty.pse.ens.fr/capital21c.

- **Main explanation for rise in wealth-income ratio in the very long run: growth slowdown and $\beta = s/g$**
  (Harrod-Domar-Solow steady-state formula)

- **One-good capital accumulation model**: $\mathbf{W_{t+1} = W_t + s_t Y_t}$
  → dividing both sides by Y_{t+1}, we get: $\beta_{t+1} = \beta_t\,(1+g_{wt})/(1+g_t)$

With $1+g_{wt} = 1+s_t/\beta_t$ = saving-induced wealth growth rate

$1+g_t = Y_{t+1}/Y_t$ = total income growth rate (productivity+population)

- If saving rate $s_t \rightarrow s$ and growth rate $g_t \rightarrow g$, then:

$$\boldsymbol{\beta_t \rightarrow \beta = s/g}$$

- E.g. if s=10% & g=2%, then β = 500%: this is the only wealth-income ratio such that with s=10%, wealth rises at 2% per year, i.e. at the same pace as income
- If s=10% and growth declines from g=3% to g=1,5%, then the steady-state wealth-income ratio goes from about 300% to 600%

→ the large variations in growth rates and saving rates (g and s are determined by different factors and generally do not move together) explain the large variations in β over time and across countries

Table 5.1. Growth rates and saving rates in rich coutries, 1970-2010

	Growth rate of national income	Growth rate of population	Growth rate of per capita national income	**Private saving** (net of depreciation) (% national income)
U.S.	**2,8%**	1,0%	1,8%	**7,7%**
Japan	**2,5%**	0,5%	2,0%	**14,6%**
Germany	**2,0%**	0,2%	1,8%	**12,2%**
France	**2,2%**	0,5%	1,7%	**11,1%**
U.K.	**2,2%**	0,3%	1,9%	**7,3%**
Italy	**1,9%**	0,3%	1,6%	**15,0%**
Canada	**2,8%**	1,1%	1,7%	**12,1%**
Australia	**3,2%**	1,4%	1,7%	**9,9%**

Saving rates and demographic growth vary a lot within rich countries; growth rates of per capita national income vary much less.

Sources: see piketty.pse.ens.fr/capital21c

Table 5.2. Private saving in rich countries, 1970-2010

	Private saving (net of depreciation) (% national income)	incl. Household net saving	incl. Corporate net saving (net retained earnings)
U.S.	7,7%	4,6% ***60%***	3,1% ***40%***
Japan	14,6%	6,8% ***47%***	7,8% ***53%***
Germany	12,2%	9,4% ***77%***	2,8% ***23%***
France	11,1%	9,0% ***81%***	2,1% ***19%***
U.K.	7,4%	2,8% ***38%***	4,6% ***62%***
Italy	15,0%	14,6% ***97%***	0,4% ***3%***
Canada	12,1%	7,2% ***60%***	4,9% ***40%***
Australia	9,9%	5,9% ***60%***	3,9% ***40%***

A large part (variable across countries) of private saving comes from corporate retained earnings (undistributed profits).

Sources: see piketty.pse.ens.fr/capital21c

Table 5.3. Gross and net saving in rich countries, 1970-2010

	Gross private savings (% national income)	Minus: Capital depreciation	Equal: Net private saving
U.S.	18,8%	11,1%	**7,7%**
Japan	33,4%	18,9%	**14,6%**
Germany	28,5%	16,2%	**12,2%**
France	22,0%	10,9%	**11,1%**
U.K.	19,7%	12,3%	**7,3%**
Italy	30,1%	15,1%	**15,0%**
Canada	24,5%	12,4%	**12,1%**
Australia	25,1%	15,2%	**9,9%**

A large part of gross saving (generally about half) corresponds to capital depreciation; i.e. it is used solely to repair or replace used capital.

Sources: see piketty.pse.ens.fr/capital21c

Table 5.4. Private and public saving in rich countries, 1970-2010

	National saving (private + public) (net of depreciation) (% national income)	incl. Private saving	incl. Public saving
U.S.	**5,2%**	7,6%	-2,4%
Japan	**14,6%**	14,5%	0,1%
Germany	**10,2%**	12,2%	-2,0%
France	**9,2%**	11,1%	-1,9%
U.K.	**5,3%**	7,3%	-2,0%
Italy	**8,5%**	15,0%	-6,5%
Canada	**10,1%**	12,1%	-2,0%
Australia	**8,9%**	9,8%	-0,9%

A large part (variable across countries) of private saving is absorbed by public deficits, so that national saving (private + public) is less than private saving.

Sources: voir piketty.pse.ens.fr/capital21c

- **Two-good capital accumulation model**: one capital good, one consumption good
- Define $1+q_t$ = real rate of capital gain (or capital loss)

= excess of asset price inflation over consumer price inflation

- Then $\beta_{t+1} = \beta_t (1+g_{wt})(1+q_t)/(1+g_t)$

With $1+g_{wt} = 1+s_t/\beta_t$ = saving-induced wealth growth rate

$1+q_t$ = capital-gains-induced wealth growth rate (=residual term)

→ Main finding: **relative price effects (capital gains and losses) are key in the short and medium run and at local level, but volume effects (saving and investment) are probably more important in the long run and at the national or continental level**

See the detailed decomposition results for wealth accumulation into volume and relative price effects in Piketty-Zucman, Capital is Back: Wealth-Income Ratios in Rich Countries 1700-2010 , 2013, slides, data appendix

(see also Gyourko et al, « Superstar cities », AEJ 2013) (see also…)

Figure 7a: Observed vs. predicted national wealth / national income ratios (2010)

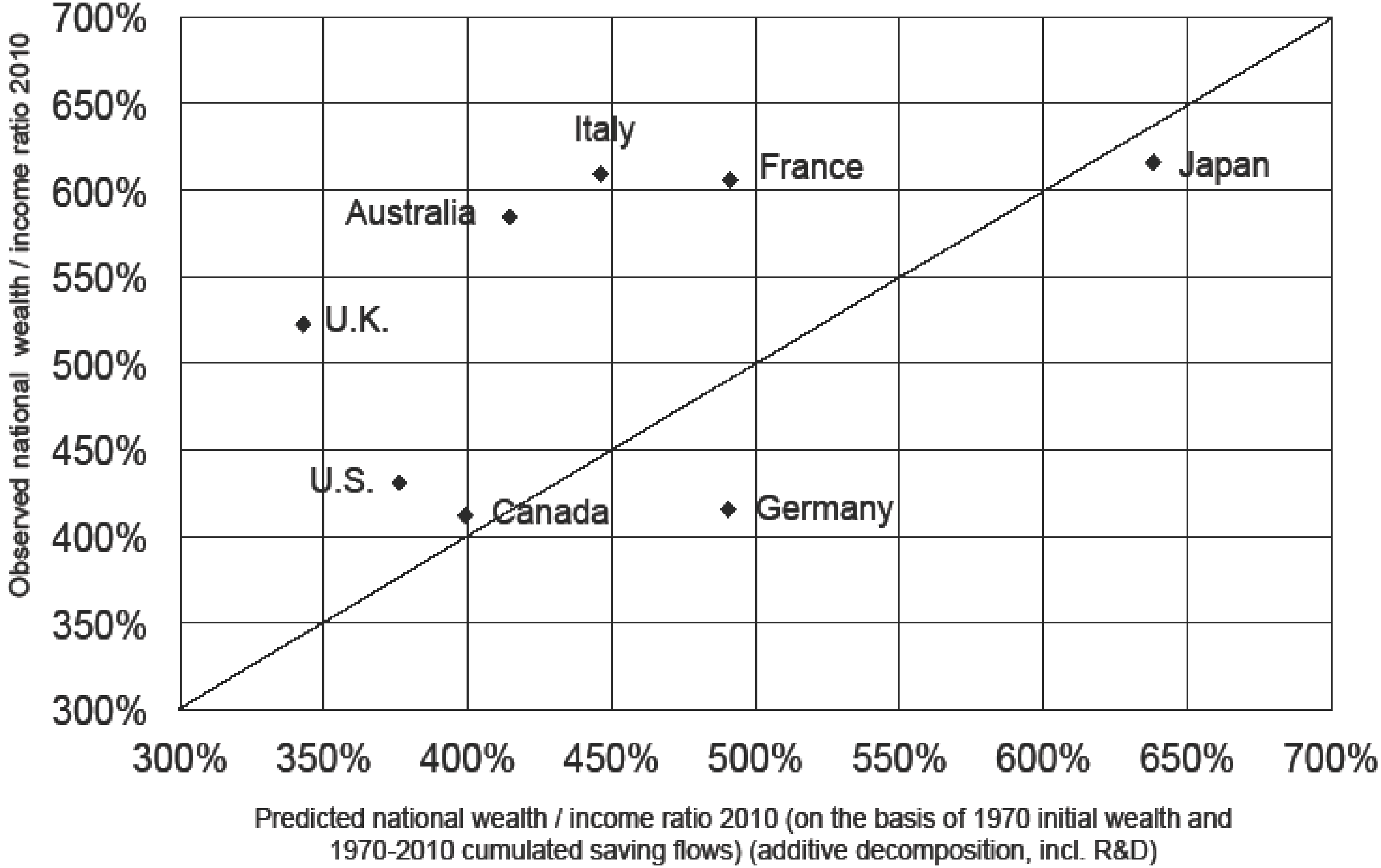


Figure 7b: Observed vs. predicted national wealth / national income ratios (2010)

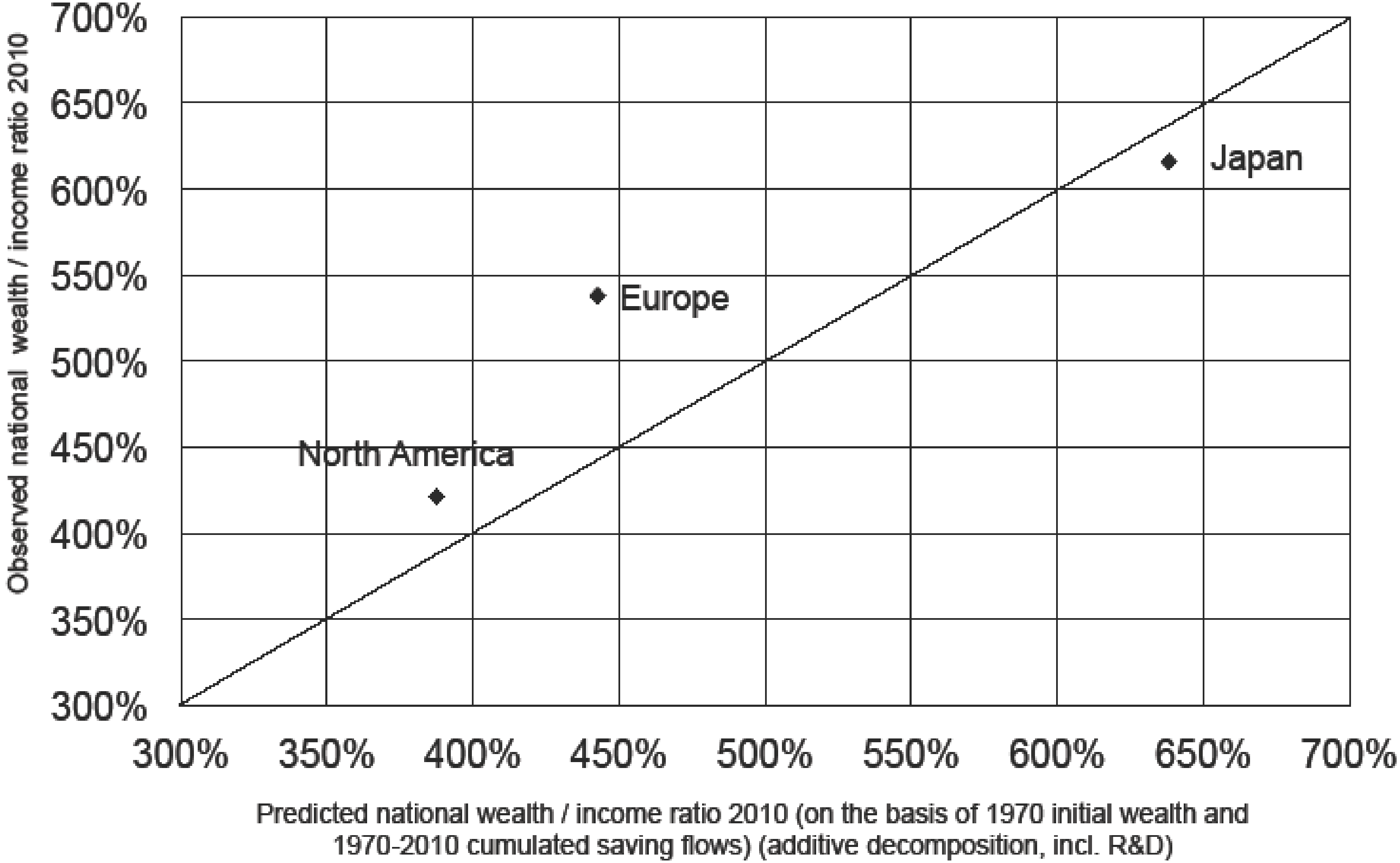


Table 10: Accumulation of national wealth in rich countries, 1910-1950

	National wealth-national income ratios		Decomposition of 1950 national wealth-national income ratio			
	β (1910)	β (1950)	Initial wealth effect	Cumulated new savings	Cumulated war destructions	Capital gains or losses
U.S.	469%	380%	132%	193%	0%	55%
Germany	637%	223%	400%	109% ***31%***	-120% ***29%***	-165% ***40%***
France	747%	261%	421%	144% ***38%***	-132% ***27%***	-172% ***35%***
U.K.	719%	208%	409%	75% ***46%***	-19% ***4%***	-256% ***50%***

Germany's national wealth-income ratio fell from 637% to 223% between 1910 and 1950. 31% of the fall can be attributed to insufficient saving, 29% to war destructions, and 40% to real capital losses.

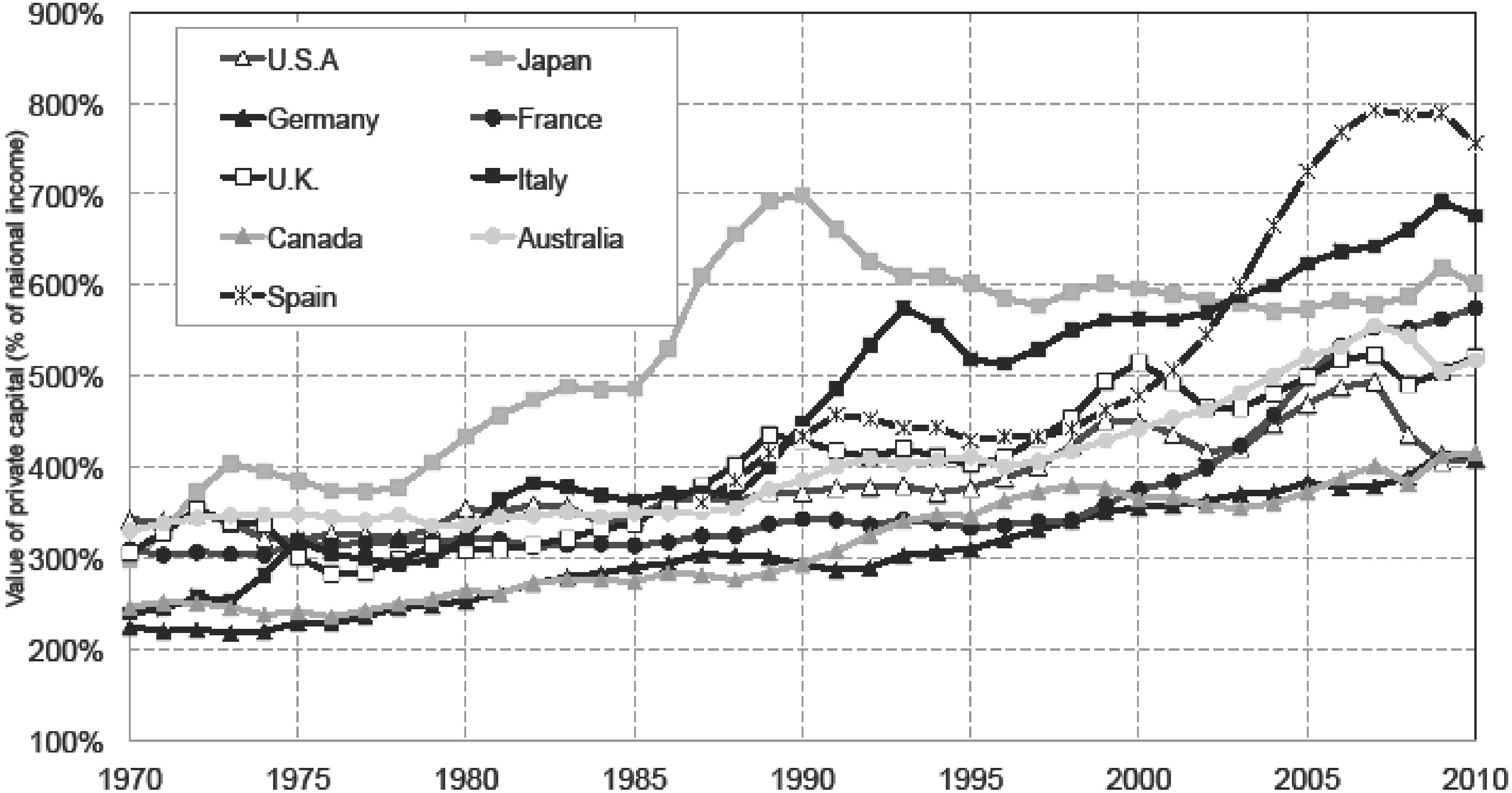
Can land and housing prices also matter in the very long run?

- Very difficult to identify pure land prices: hard to measure all past investment and improvment to the land, the local infrastructures, etc.
- There are good reasons to believe that price effects dominate in the short and medium run, but less so in the long run
- However one can also find mechanims explaining why land and housing prices might also matter in the very long run
- See e.g Gyourko et al, « Superstar cities », AEJ 2013
- See also Schularick et al 2014, « No price like home: global land prices 1870-2012 » : the speed of technical progress in transportation technology has been relatively faster in 1850-1960 than in 1960-2010 (relative to other sectors such as biotech, computer, etc.) (e.g. airplane speed unchanged in recent decades); this can potentially explain the rise of relative land prices in large capital cities in recent decades
- More generally, in models with n goods, different speed of technical change can explain any long-run change in relative prices: anything can happen

Gross vs net foreign assets: financial globalization in action

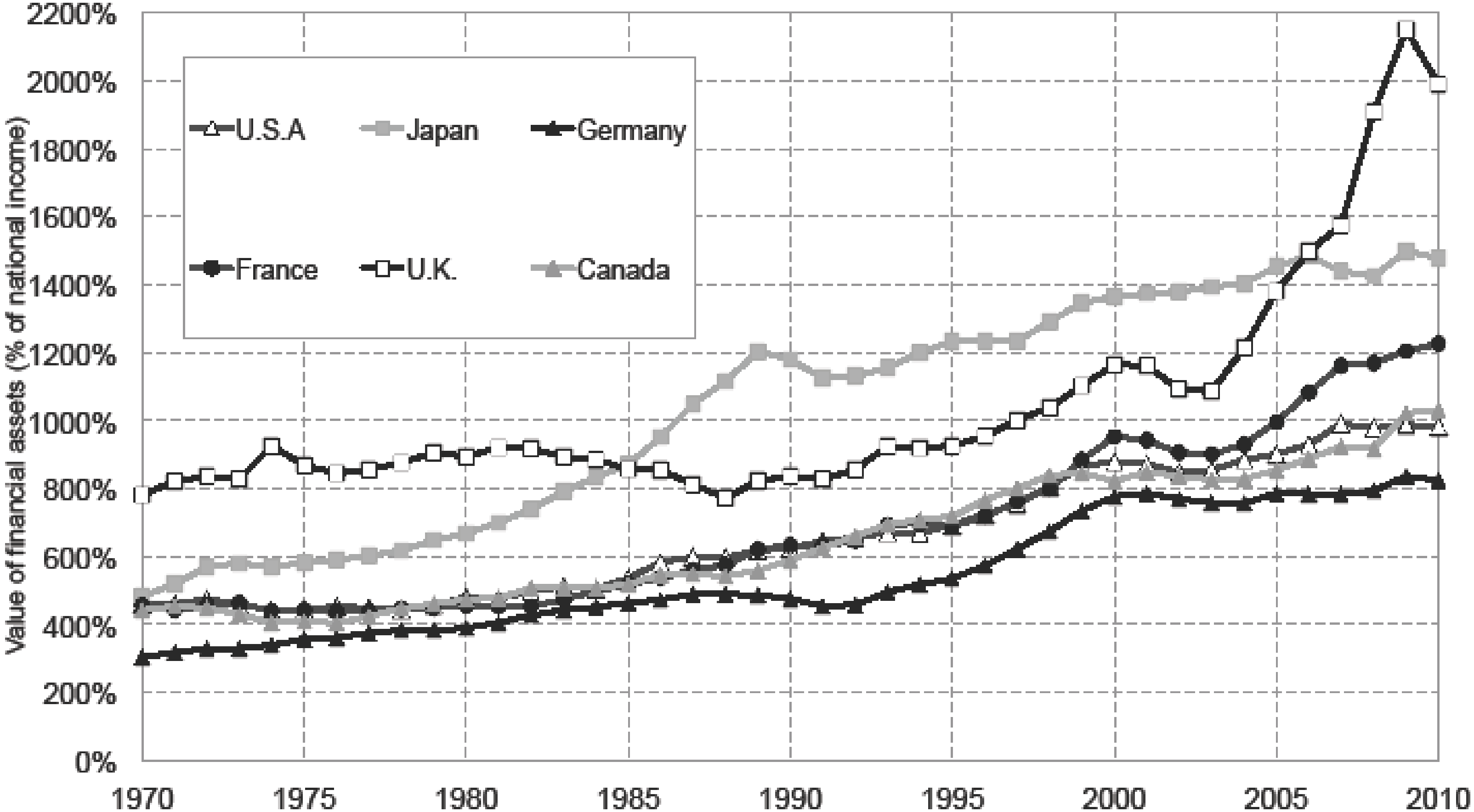
- Net foreign asset positions are smaller today than what they were in 1900-1910
- But they are rising fast in Germany, Japan and oil countries
- **And gross foreign assets and liabilities are a lot larger than they have ever been**, especially in small countries: about 30-40% of total financial assets and liabilities in European countries (even more in smaller countries)
- This potentially creates substantial financial fragility (especially if link between private risk and sovereign risk)
- This destabilizing force is probably even more important than the rise of top income shares (=important in the US, but not so much in Europe; see lecture 5 & PS, « Top incomes and the Great Recession », IMF Review 2013)

Figure S5.2. Private capital in rich countries: from the Japanese to the Spanish bubble



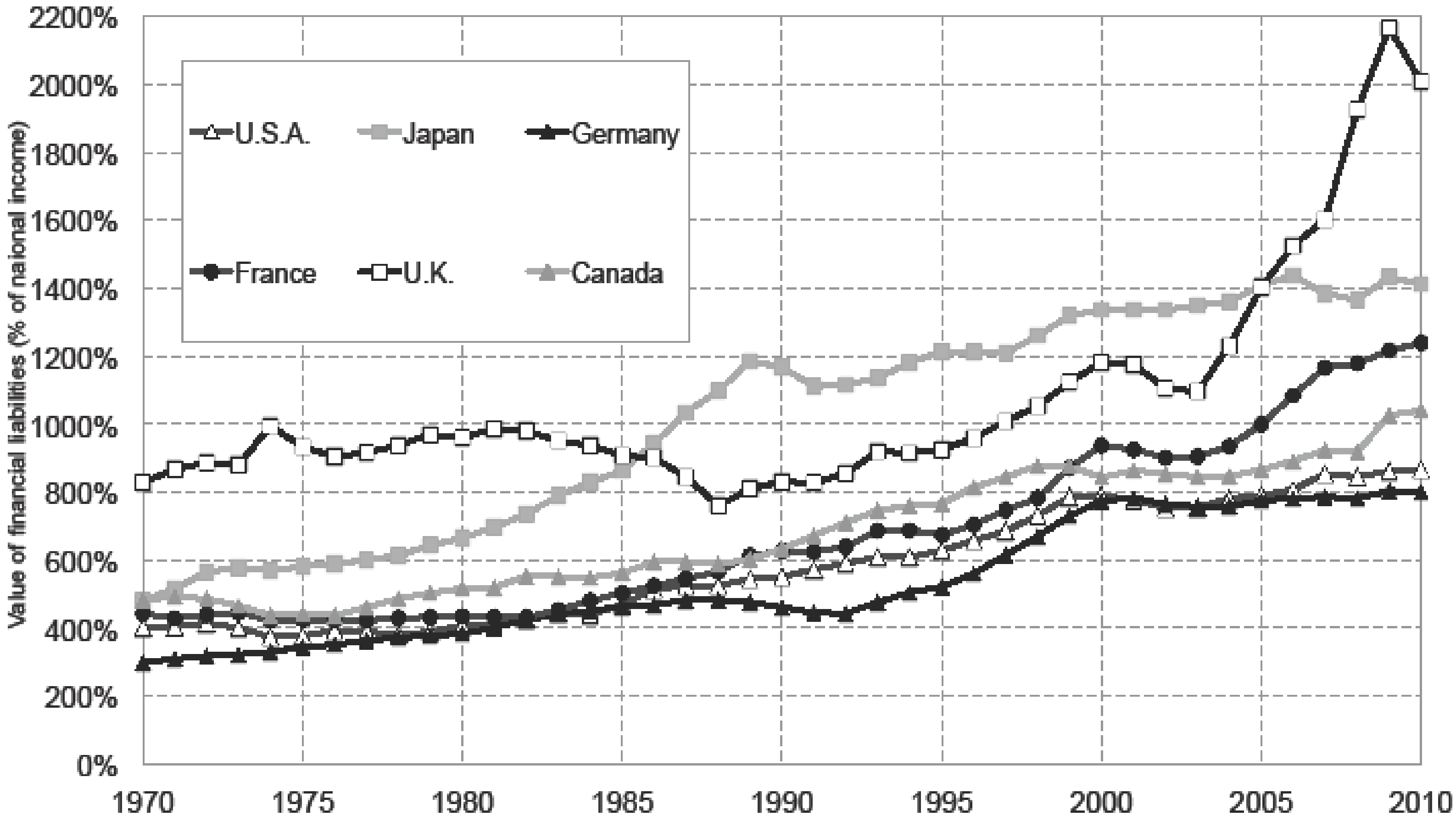
Private capital almost reached 8 years of national income in Spain at the end of the 2000s (ie. one more year than Japan in 1990). Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.3. Financial assets in rich countries



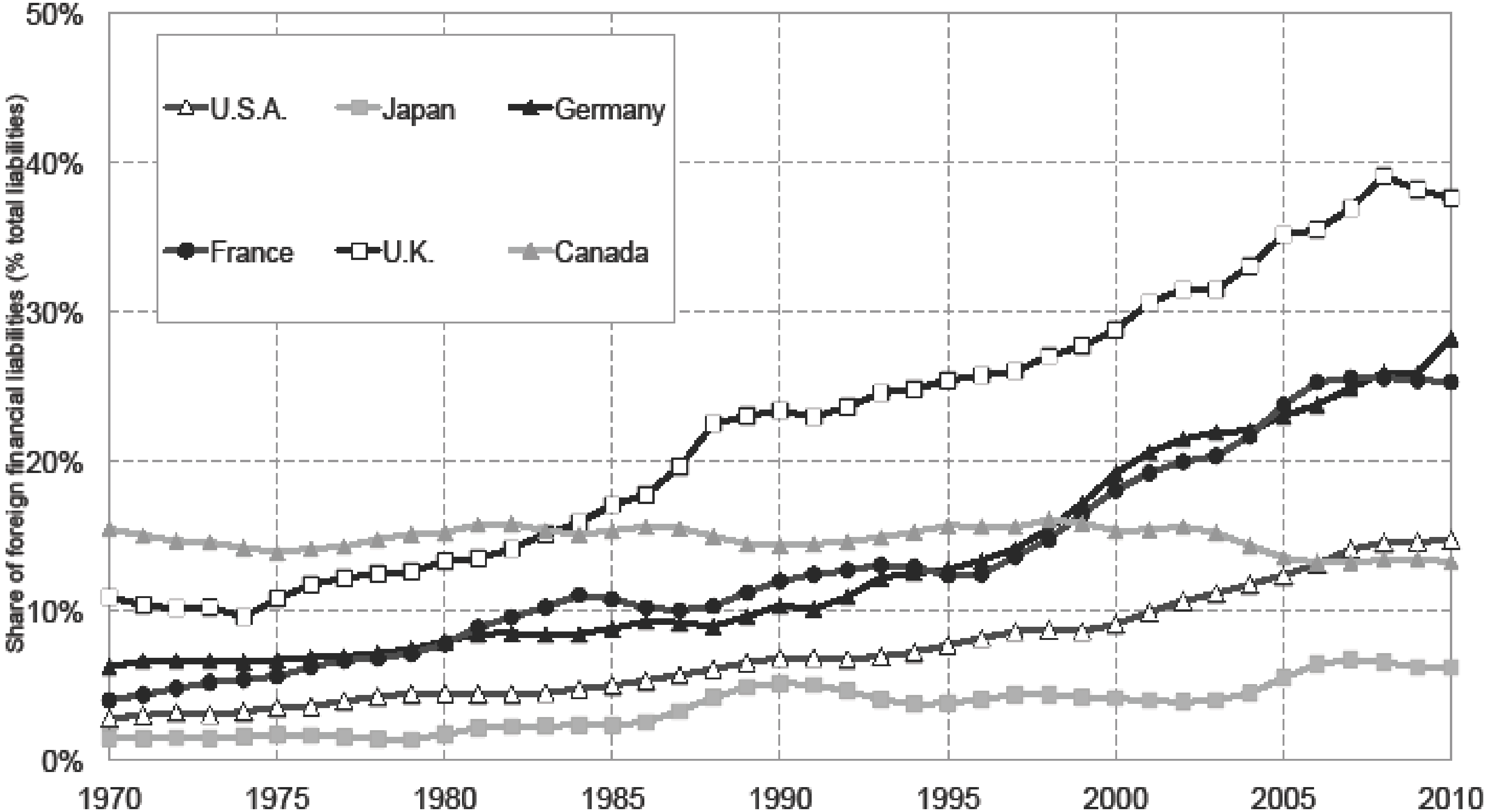
Total financial assets owned by the domestic sector (firms, households, administration) reached 20 years of national income in 2010 in the U.K.. Sources et series: voir piketty.pse.ens.fr/capital21c.

Figure S5.4. Financial liabilities in rich countries



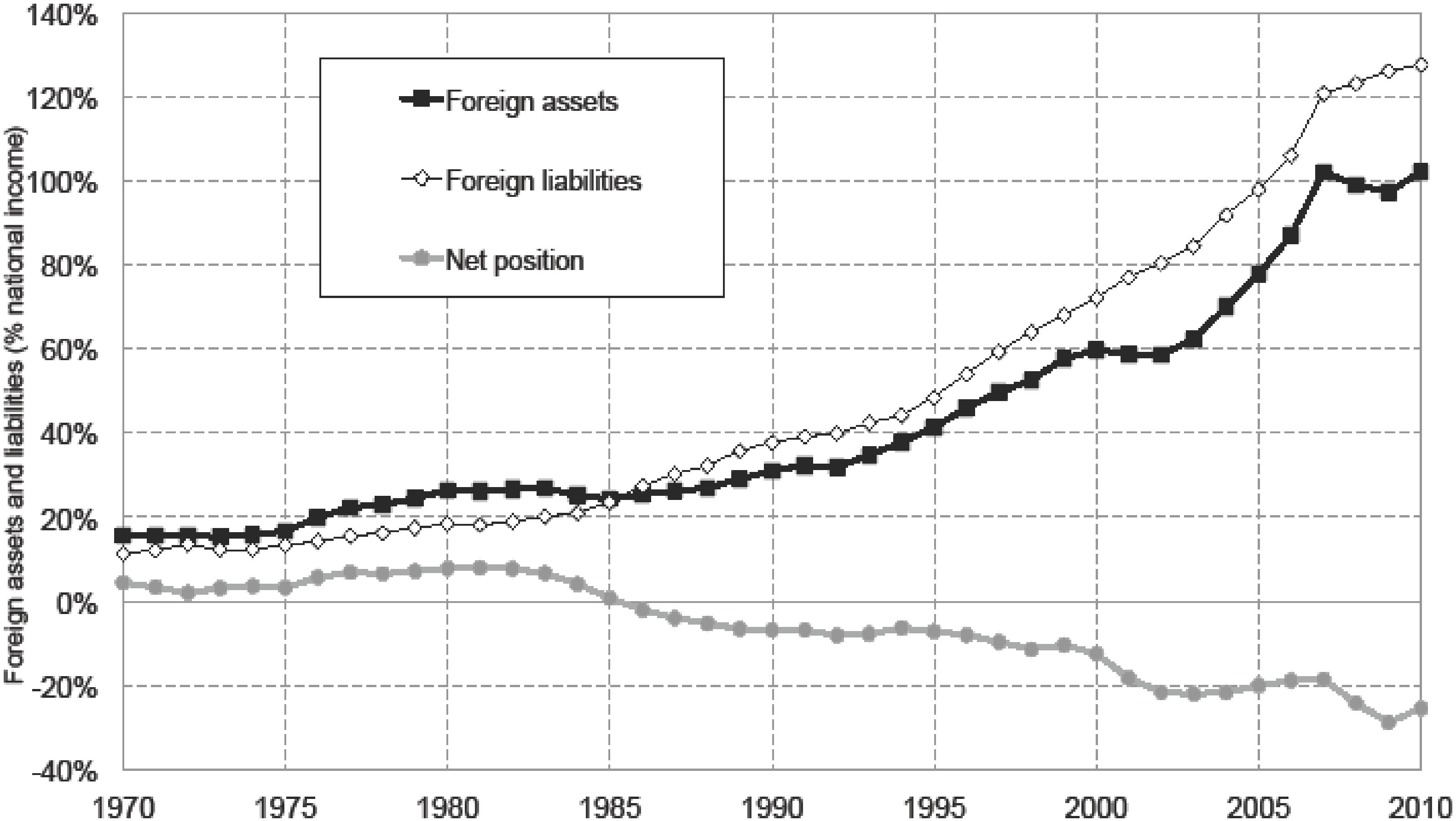
Total of financial liabilities owned by the domestic sector (firms, households, administration) reached 20 years of national income in 2010 in the U.K. Sources et series: voir piketty.pse.ens.fr/capital21c.

Figure S5.5. Share of foreign financial liabilites in the total financial liabilities in rich countries



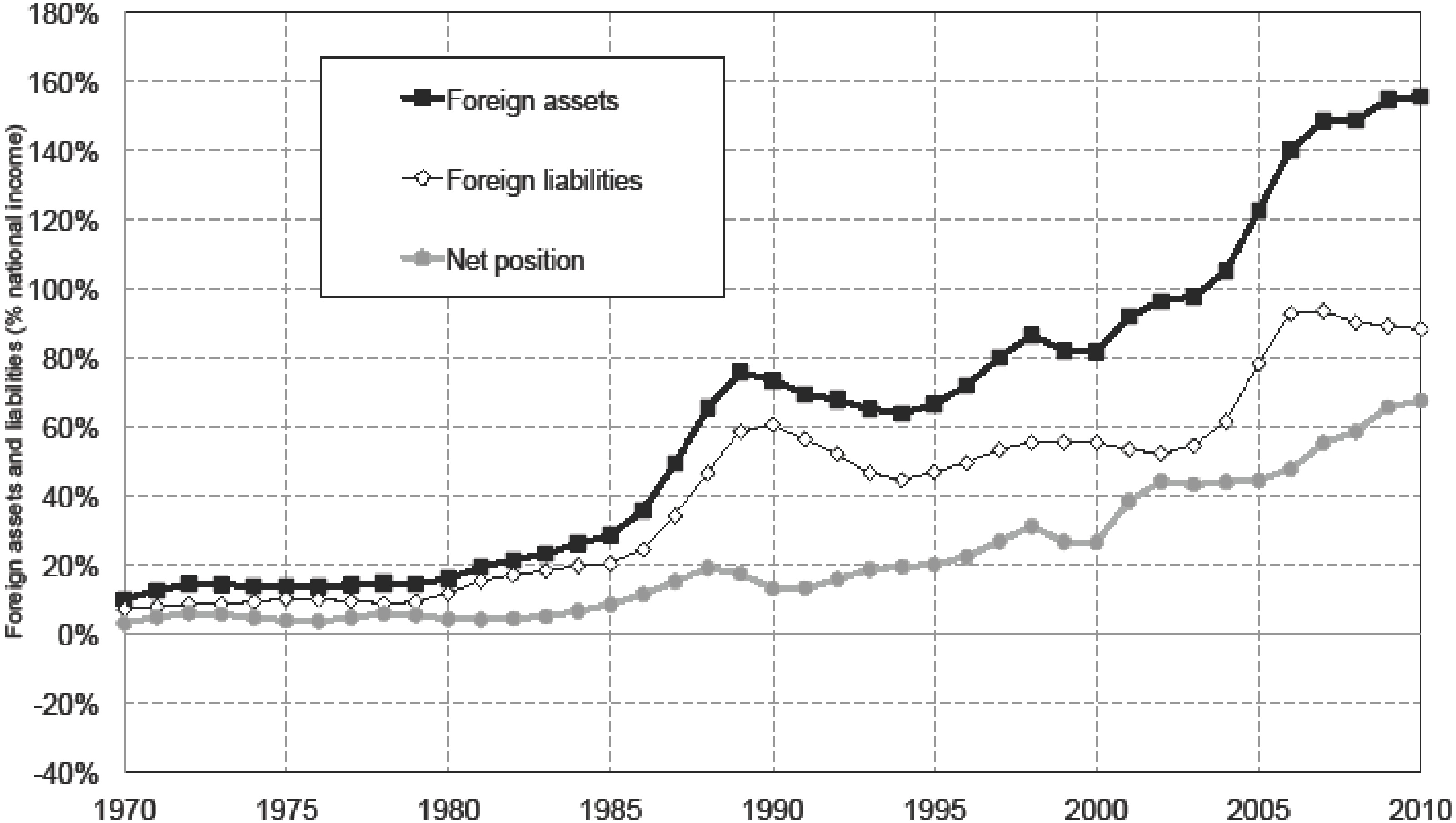
Total financial liabilities owned by the rest of the world amounts to around 40% of total financial liabilities of the domestic sector in the U.K. in 2010. Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.6. Foreign assets and liabilities in the U.S.A. 1970-2010



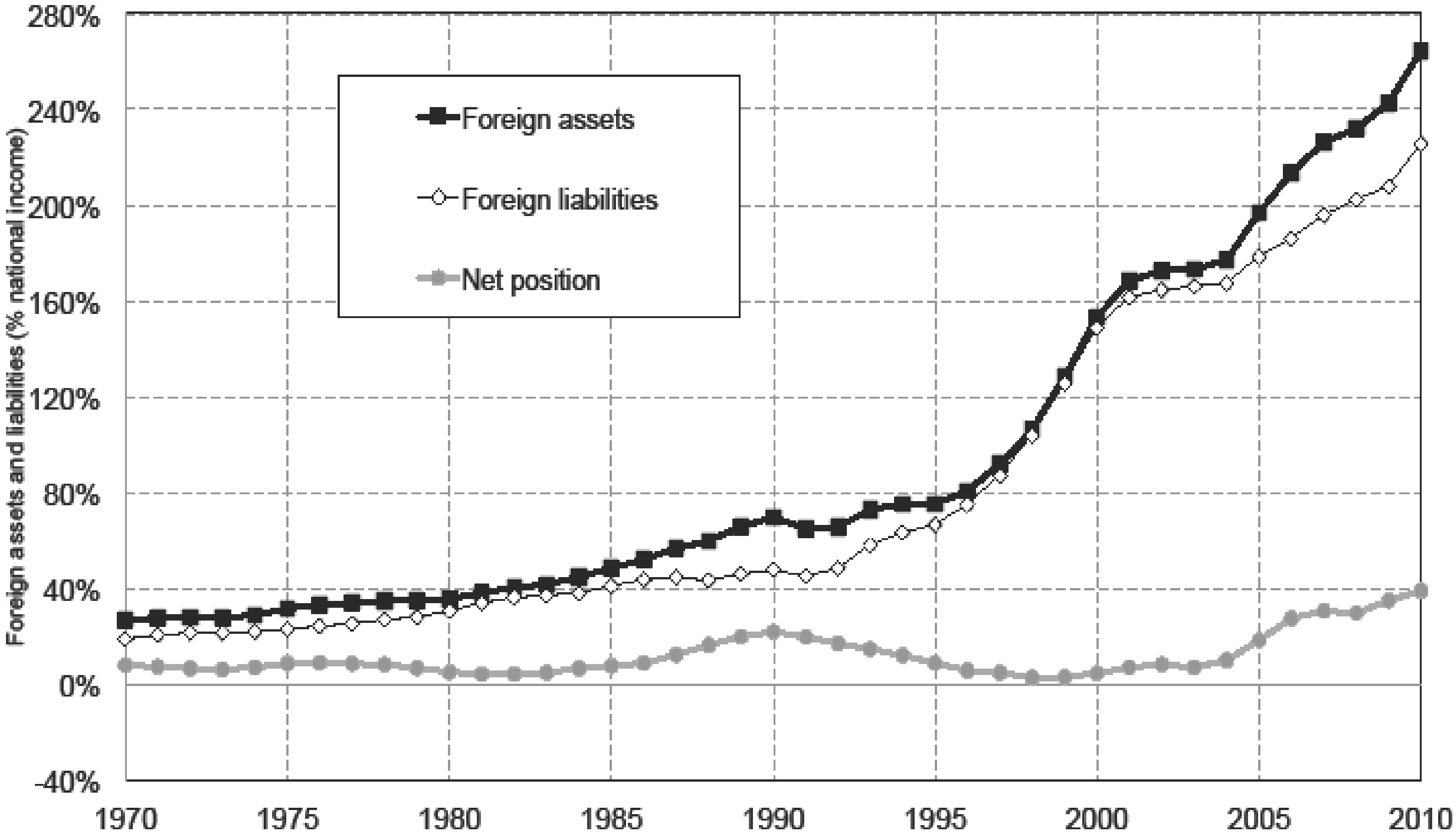
Foreign liabilities (what the rest of the world owns in the US) has outweighted foreign liabililes (what the US own in the rest of the world) since 1985-1986. Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.7. Foreign assets and liabilities in Japan 1970-2010



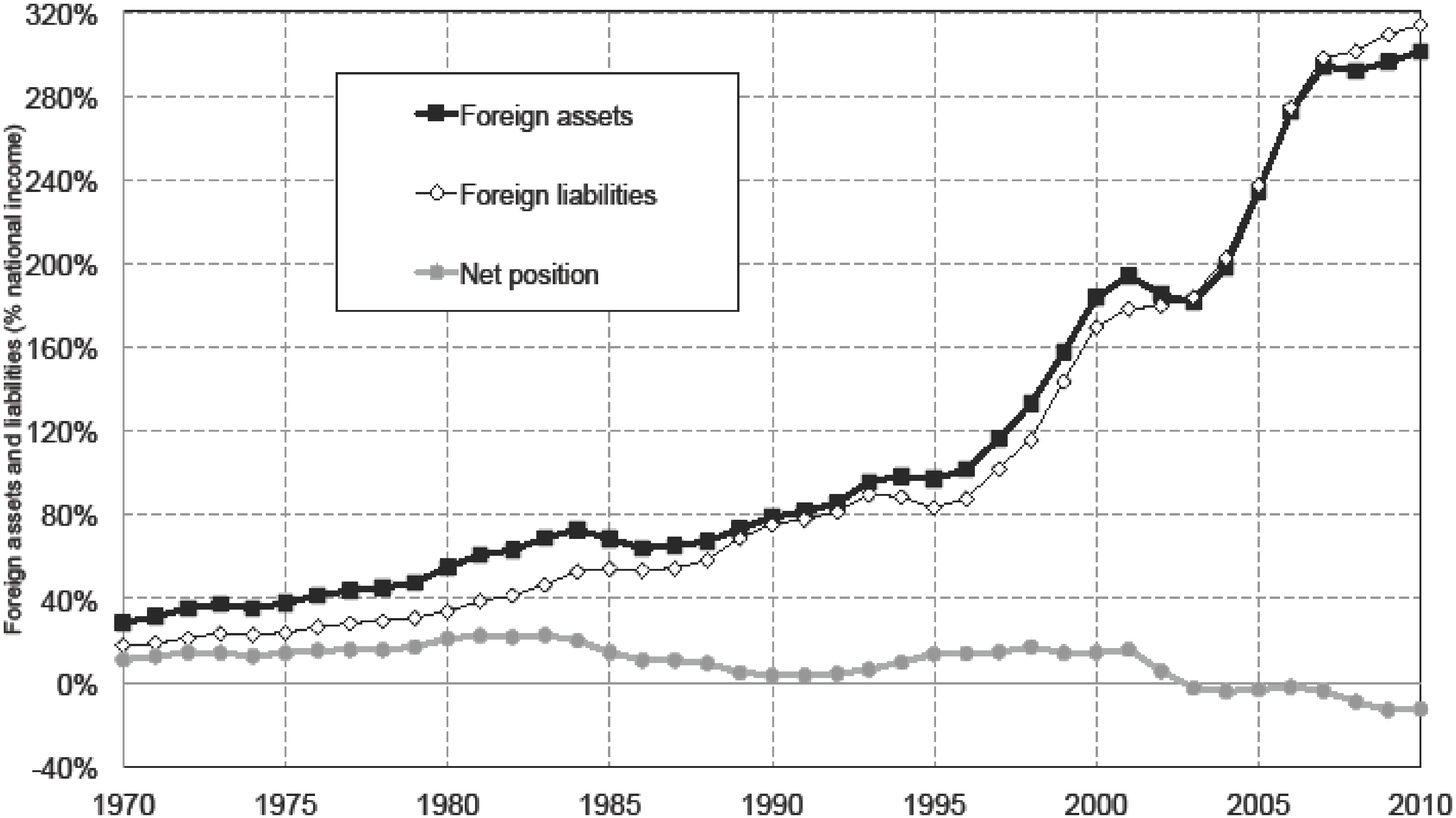
Foreign assets (what Japan owns in the rest of the world) are almost twice bigger than foreign liabilities (what the rest of the world owns in Japan) in 2010. Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.8. Foreign assets and liabilities in Germany, 1970-2010



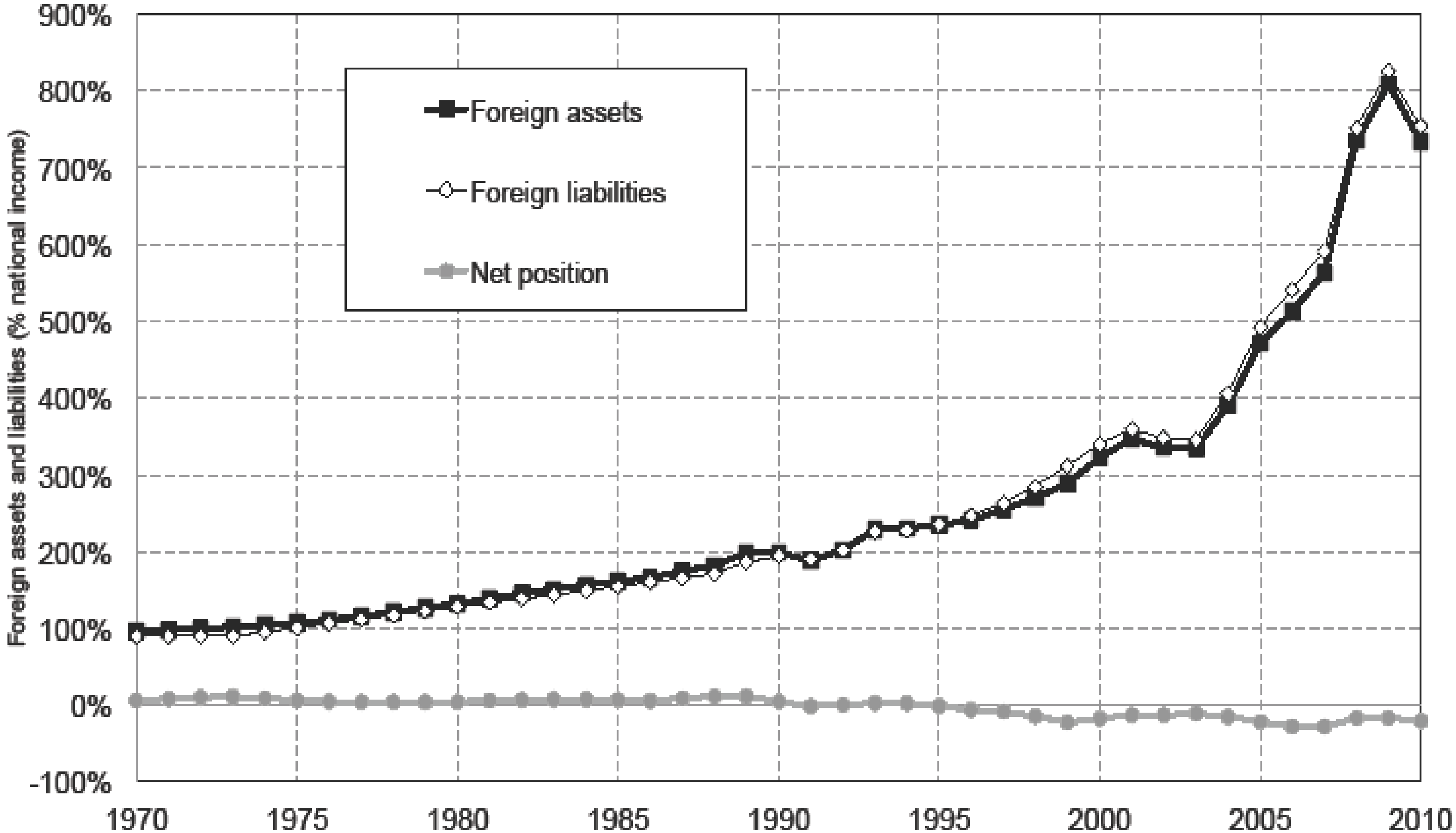
Foreign assets and liabilities in Germany have risen a lot since the 1980s-1990s.
Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.9. Foreign assets and liabilities in France, 1970-2010



Like in Germany, foreign assets and liabilities have risen a lot since 1980s-1990s (but with a negative net position at the end of the period. Sources et series: see piketty.pse.ens.fr/capital21c.

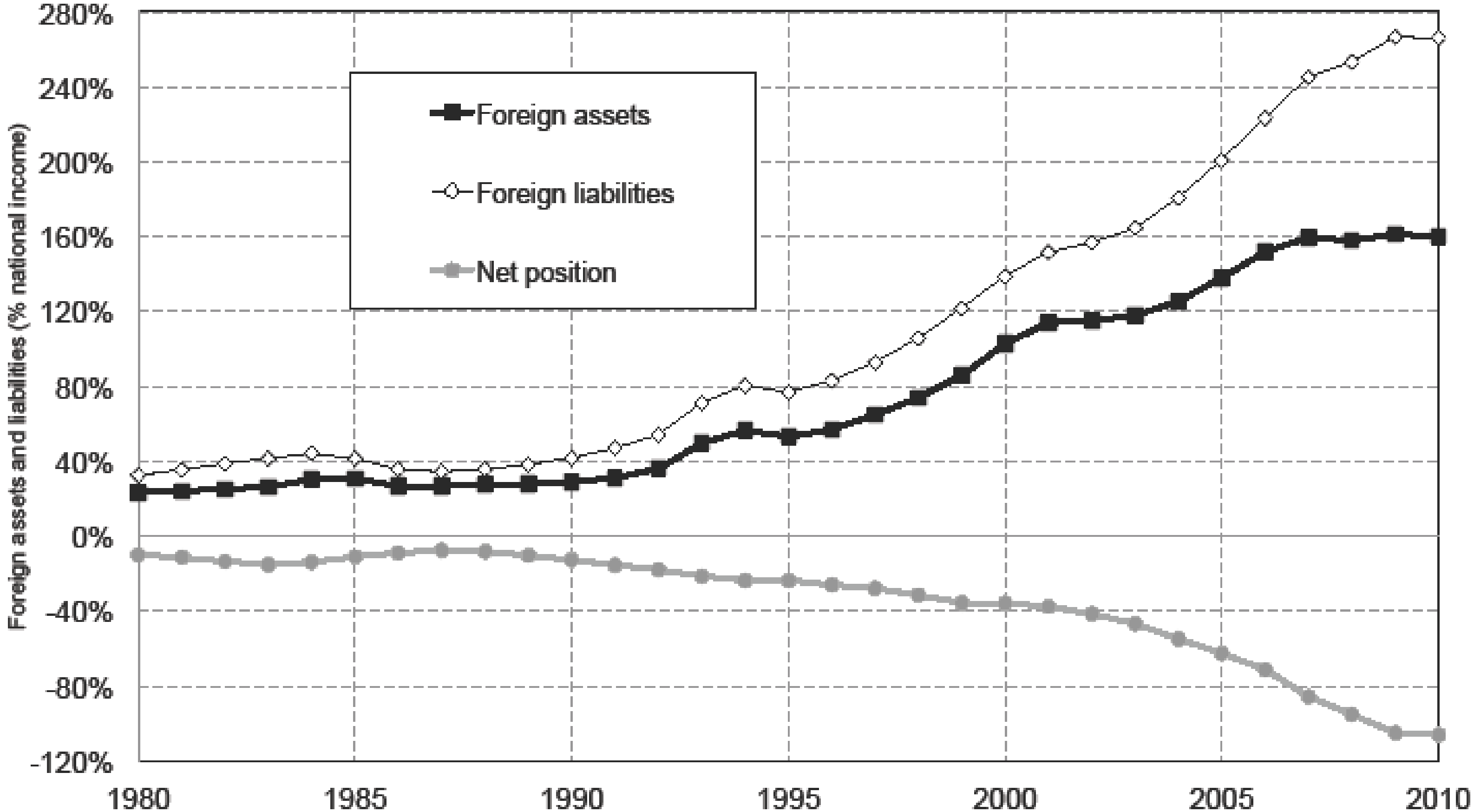
Figure S5.10. Foreign assets and liabilities in the U.K. 1970-2010



In the U.K., foreign assets and liabilities reached 7-8 years of national income at the end of the period

Sources et series: see piketty.pse.ens.fr/capital21c.

Figure S5.11. Foreign assets and liabilities in Spain, 1980-2010

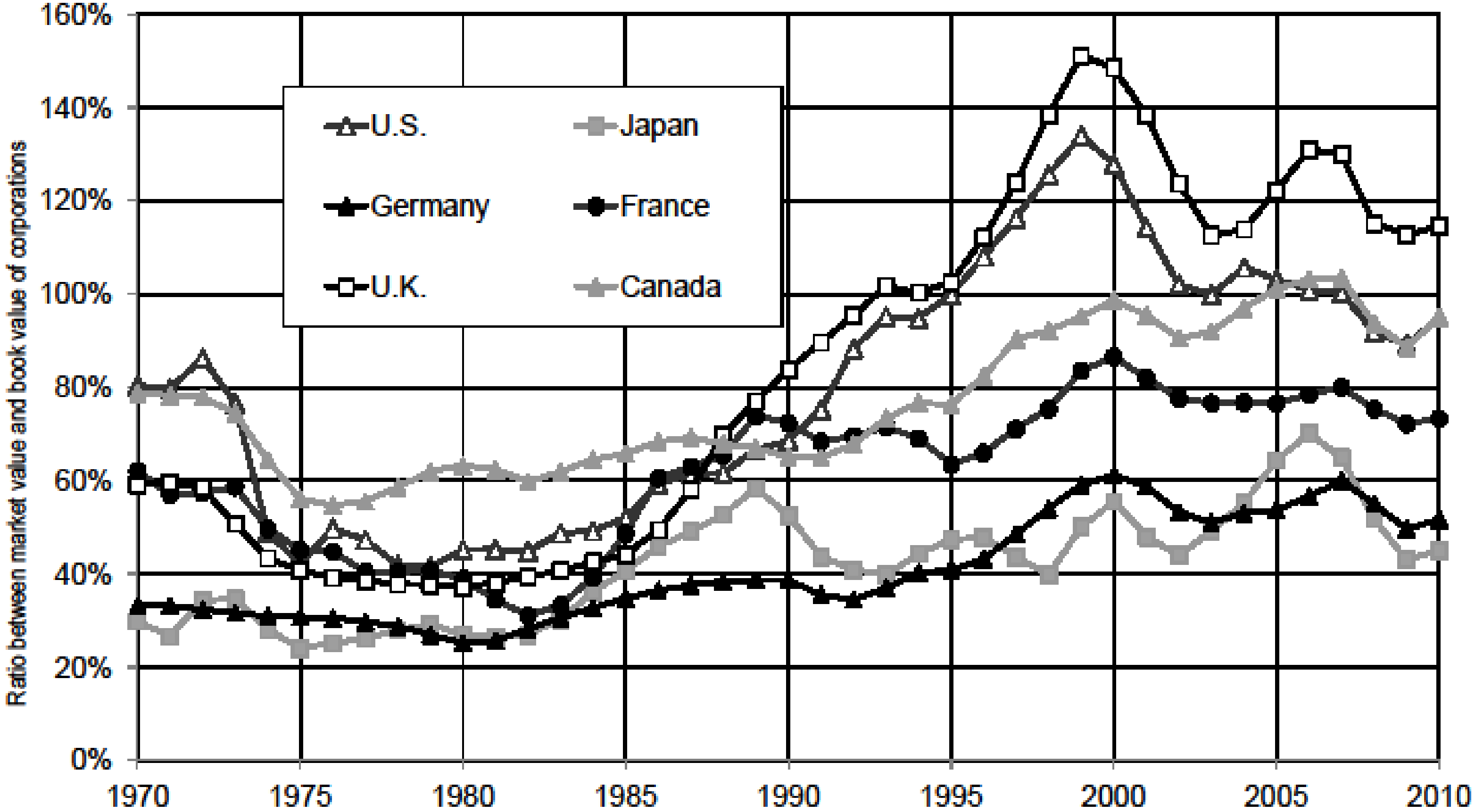


Net foreign debt of Spain exceeds a year of national income in 2010.
Sources et series: see piketty.pse.ens.fr/capital21c.

Market vs book value of corporations

- So far we used a market-value definition of national wealth W_n : corporations valued at stock market prices
- Book value of corporations = assets – debt
- Tobin's Q ratio = (market value)/(book value) (>1 or <1)
- Residual corporate wealth W_c = book value – market value
- Book-value national wealth $W_b = W_n + W_c$
- In principe, Q ≈ 1 (otherwise, investment should adjust), so that $W_c \approx 0$ and $W_b \approx W_n$
- But Q can be systematically >1 if immaterial investment not well accounted in book assets
- But Q can be systemativally <1 if shareholders have imperfect control of the firm (stakeholder model): this can explain why Q lower in Germany than in US-UK, and the general rise of Q since 1970s-80s
- From an efficiency viewpoint, unclear which model is best

Figure 5.6. Market value and book value of corporations

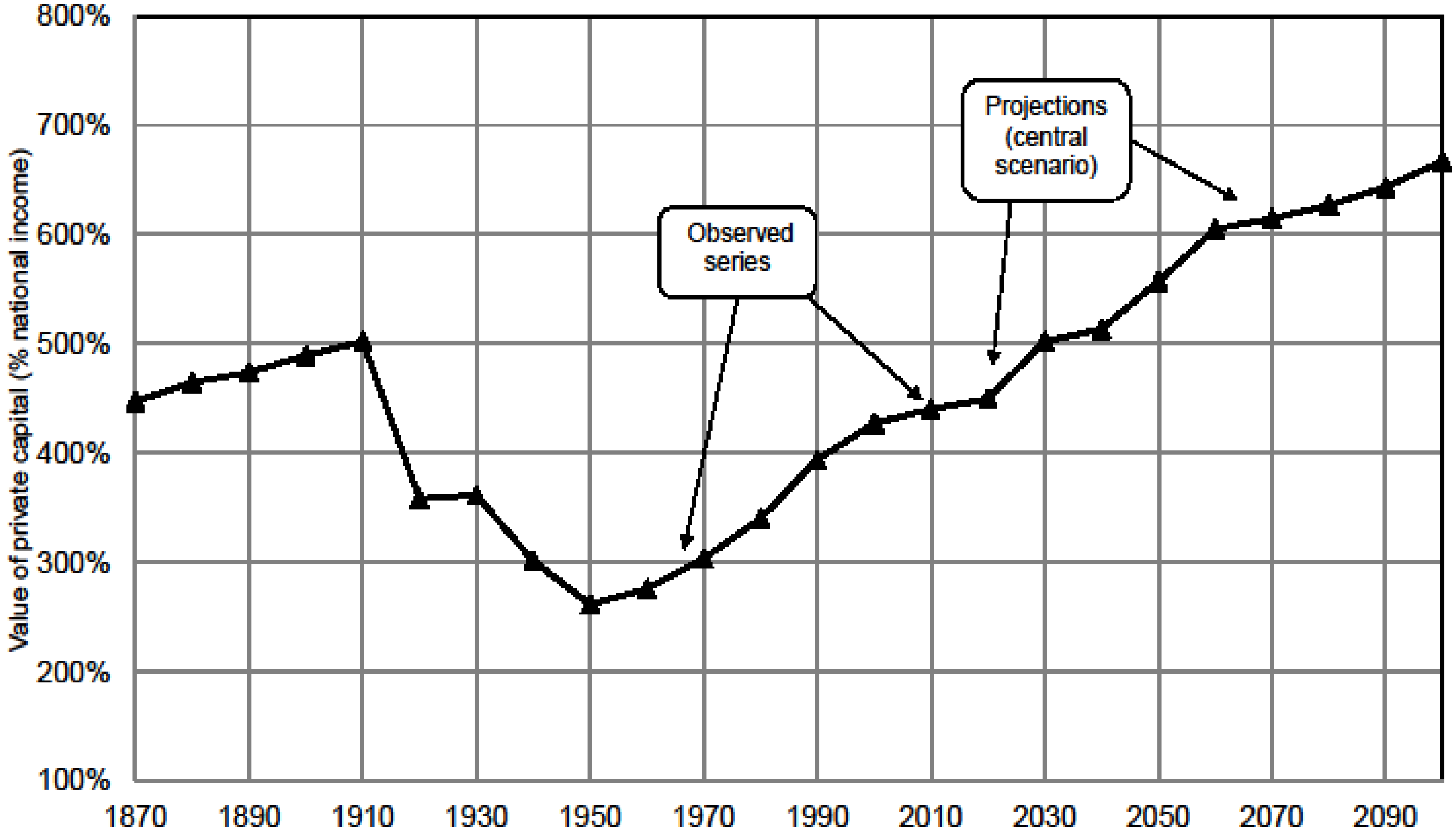


Tobin's Q (i.e. the ratio between market vaue and book value of corporations) has risen in rich countries since the 1970s-1980s. Sources and series: see piketty.pse.ens.fr/capital21c.

Summing up

- Wealth-income ratios β and β_n have no reason to be stable over time and across countries
- If global growth slowdown in the future (g≈1,5%) and saving rates remain high (s≈10-12%), then the global β might rise towards 700% (or more… or less…)
- Major issue: can depreciation of natural capital be stronger than the rise of private capital? See Barbier 2014a, 2014b
- What are the consequences for the share α of capital income in national income? See next lecture

Figure 5.8. The world capital/income ratio, 1870-2100



According to simulations (central scenario), the world capital/income ratio could be near to 700% by the end of the 21st century. Sources and series: see piketty.pse.ens.fr/capital21c.