

Economics of Inequality

(Master PPD & APE, Paris School of Economics)

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Lecture 4: From capital/income ratios to capital shares

(Tuesday December 17th 2013)

(check [on line](#) for updated versions)

Capital-income ratios β vs. capital shares α

- Capital/income ratio $\beta = K/Y$
- Capital share $\alpha = Y_K/Y$
with $Y_K =$ capital income (=sum of rent, dividends, interest, profits, etc.: i.e. all incomes going to the owners of capital, independently of any labor input)
- I.e. $\beta =$ ratio between capital stock and income flow
- While $\alpha =$ share of capital income in total income flow

- By definition: **$\alpha = r \times \beta$**
With $r = Y_K/K =$ average real rate of return to capital

- If $\beta = 600\%$ and $r = 5\%$, then $\alpha = 30\% =$ typical values

- In practice, the average rate of return to capital r (typically $r \approx 4-5\%$) varies a lot across assets and over individuals (more on this in Lecture 6)
- Typically, rental return on housing = 3-4% (i.e. the rental value of an apartment worth 100 000€ is generally about 3000-4000€/year) (+ capital gain or loss)
- Return on stock market (dividend + k gain) = as much as 6-7% in the long run
- Return on bank accounts or cash = as little as 1-2% (but only a small fraction of total wealth)
- Average return across all assets and individuals $\approx 4-5\%$

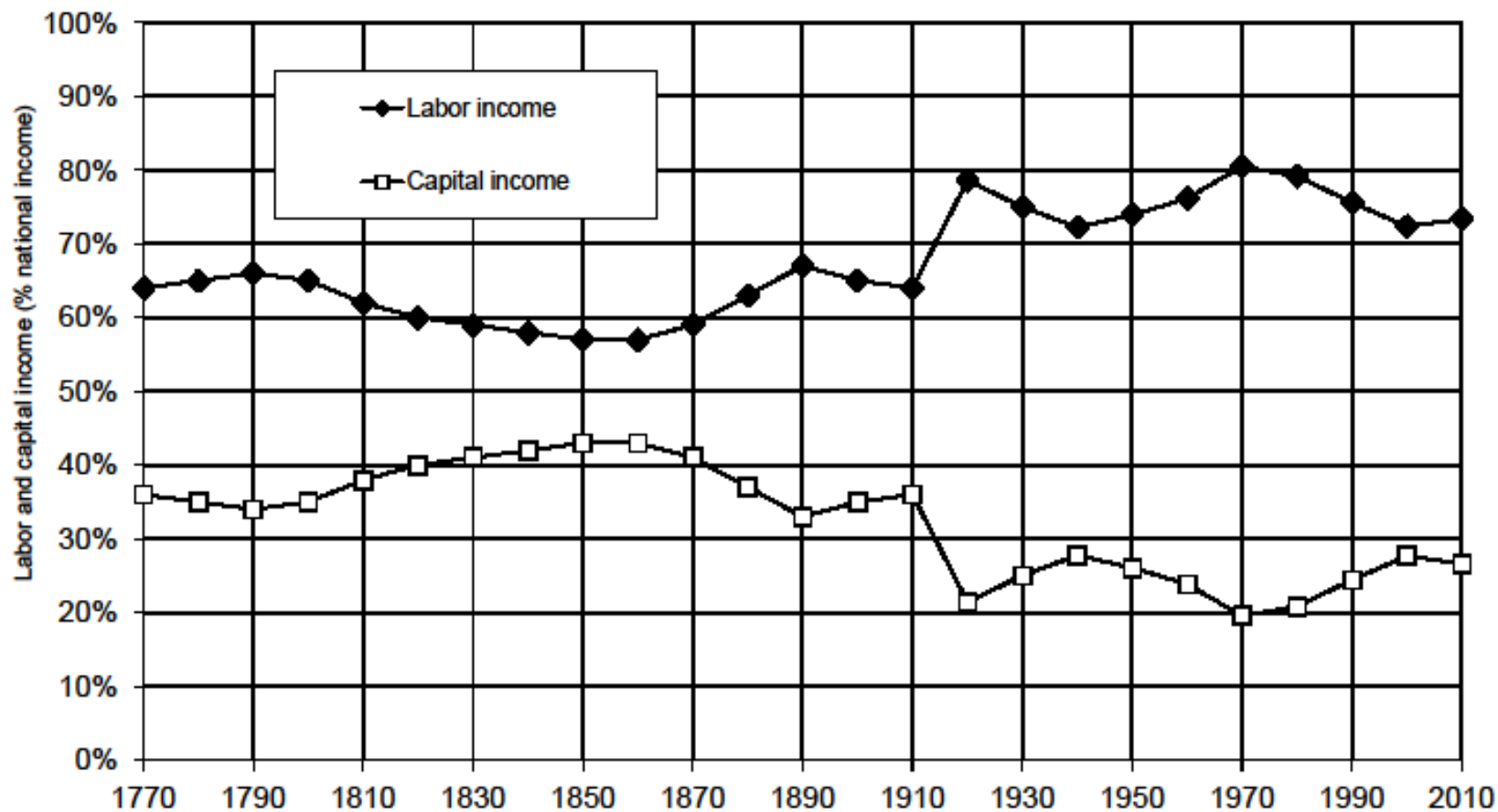
The Cobb-Douglas production function

- Cobb-Douglas production function: $Y = F(K,L) = K^\alpha L^{1-\alpha}$
- With perfect competition, wage rate $v =$ marginal product of labor, rate of return $r =$ marginal product of capital:
$$r = F_K = \alpha K^{\alpha-1} L^{1-\alpha} \quad \text{and} \quad v = F_L = (1-\alpha) K^\alpha L^{-\alpha}$$
- Therefore capital income $Y_K = r K = \alpha Y$
& labor income $Y_L = v L = (1-\alpha) Y$
- I.e. capital & labor shares are entirely set by technology (say, $\alpha=30\%$, $1-\alpha=70\%$) and do not depend on quantities K, L
- Intuition: Cobb-Douglas \leftrightarrow elasticity of substitution between K & L is exactly equal to 1
- I.e. if v/r rises by 1%, $K/L = \alpha/(1-\alpha) v/r$ also rises by 1%. So the quantity response exactly offsets the change in prices: if wages \uparrow by 1%, then firms use 1% less labor, so that labor share in total output remains the same as before

The limits of Cobb-Douglas

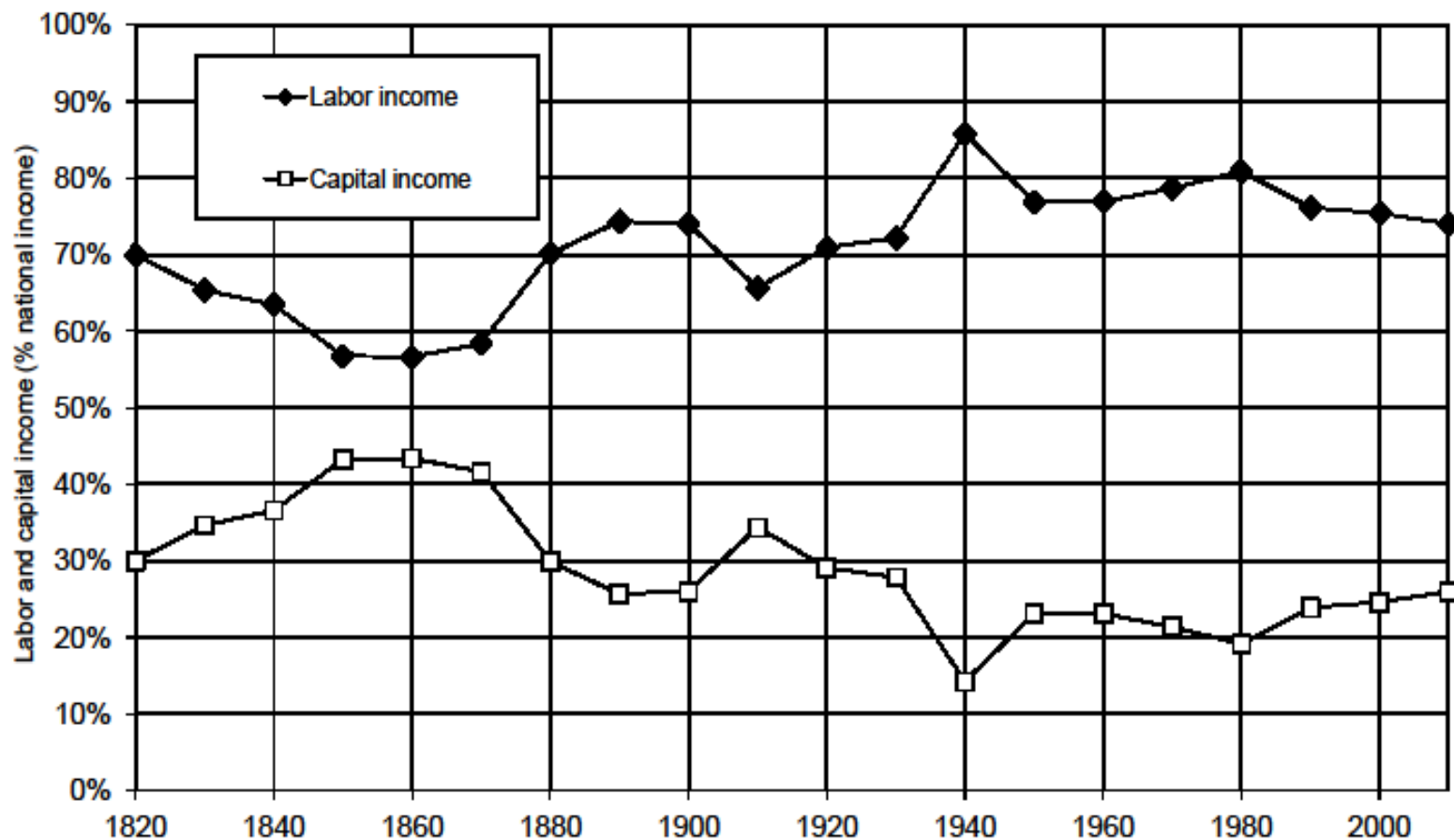
- Economists like Cobb-Douglas production function, because stable capital shares are approximately stable
- However it is only an approximation: in practice, capital shares α vary in the 20-40% range over time and between countries (or even sometime in the 10-50% range)
- In 19c, capital shares were closer to 40%; in 20c, they were closer to 20-30%; structural rise of human capital (i.e. exponent $\alpha \downarrow$ in Cobb-Douglas production function $Y = K^\alpha L^{1-\alpha}$?), or purely temporary phenomenon ?
- Over 1970-2010 period, capital shares have increased from 15-25% to 25-30% in rich countries : very difficult to explain with Cobb-Douglas framework

Figure 6.1. The capital-labor split in the United Kingdom, 1770-2010



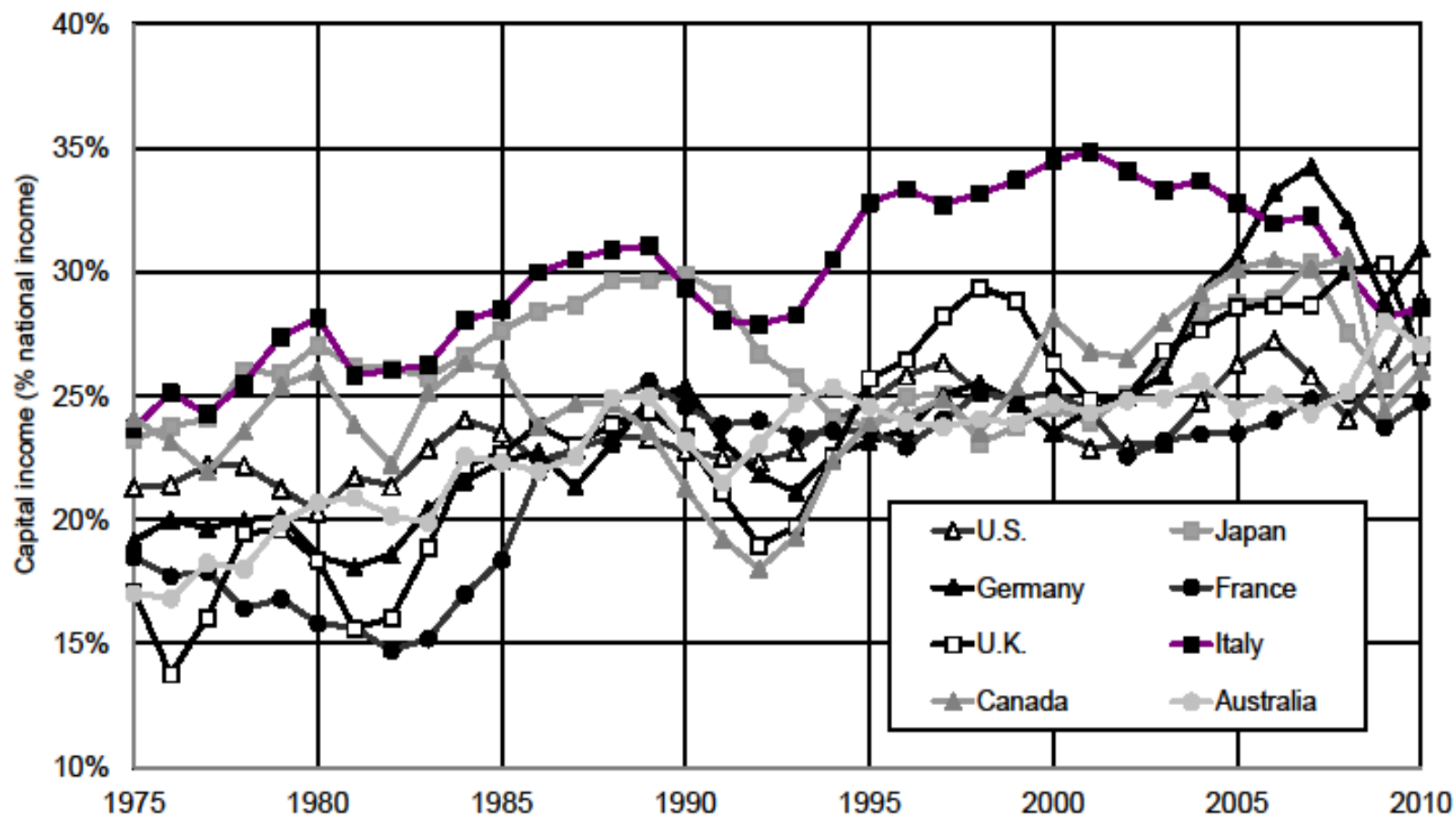
During the 19th century, capital income (rent, profits, dividends, interest,...) absorbed about 40% of national income, vs. 60% for labor income (salaried and non salaried). Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 6.2. The capital-labor split in France, 1820-2010



In the 21st century, capital income (rent, profits, dividends, interest,...) absorbs about 30% of national income, vs. 70% for labor income (salaried and non salaried). Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 6.5. The capital share in rich countries, 1975-2010



Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c

The CES production function

- CES = a simple way to think about changing capital shares
- CES : $Y = F(K,L) = [a K^{(\sigma-1)/\sigma} + b L^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$
with $a, b = \text{constant}$
- $\sigma = \text{constant elasticity of substitution between K and L}$
- $\sigma \rightarrow \infty$: linear production function $Y = r K + v L$
(infinite substitution: machines can replace workers and vice versa, so that the returns to capital and labor do not fall at all when the quantity of capital or labor rise) (= robot economy)
- $\sigma \rightarrow 0$: $F(K,L) = \min(rK, vL)$ (fixed coefficients) = no substitution possibility: one needs exactly one machine per worker
- $\sigma \rightarrow 1$: converges toward Cobb-Douglas; but all intermediate cases are also possible: Cobb-Douglas is just one possibility among many
- Compute the first derivative $r = F_K$: the marginal product to capital is given by

$$r = F_K = a \beta^{-1/\sigma} \quad (\text{with } \beta = K/Y)$$

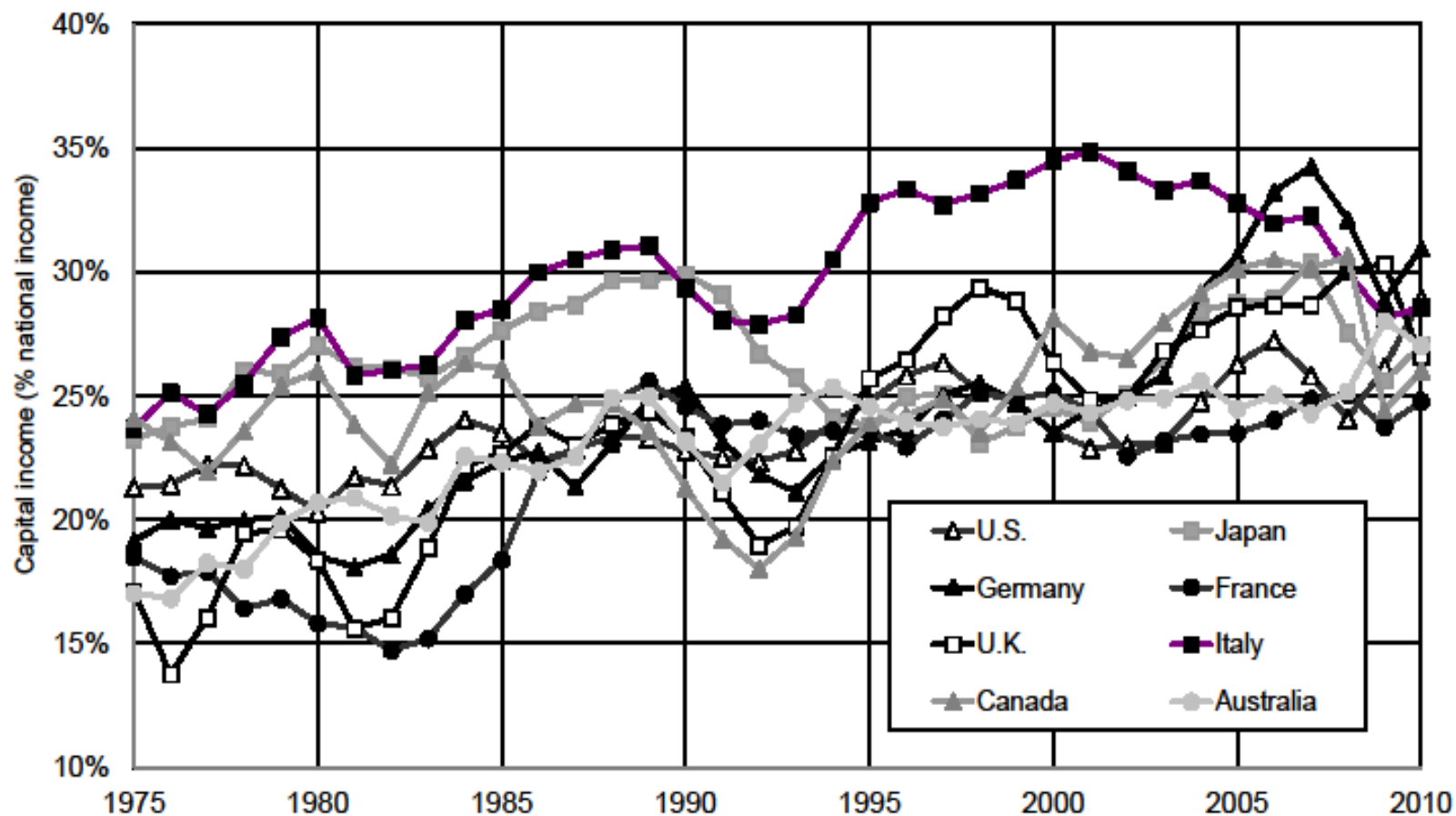
i.e. $r \downarrow$ as $\beta \uparrow$ (more capital makes capital less useful),

but the important point is that the speed at which $r \downarrow$ depends on σ

- With $r = F_K = a \beta^{-1/\sigma}$, the capital share α is given by:

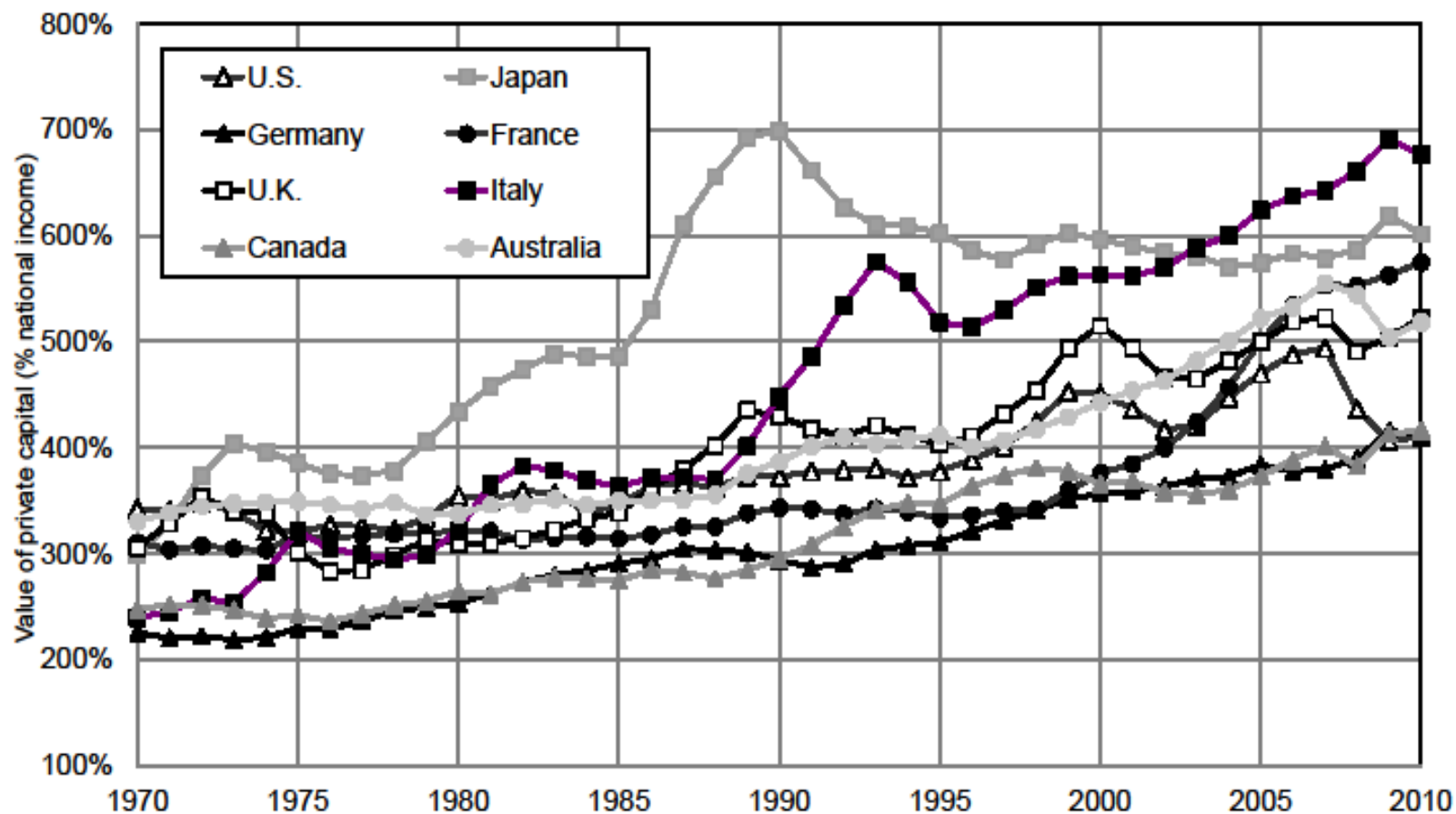
$$\alpha = r \beta = a \beta^{(\sigma-1)/\sigma}$$
- I.e. α is an increasing function of β if and only if $\sigma > 1$ (and stable iff $\sigma = 1$)
- The important point is that with large changes in the volume of capital β , small departures from $\sigma = 1$ are enough to explain large changes in α
- If $\sigma = 1.5$, capital share rises from $\alpha = 28\%$ to $\alpha = 36\%$ when β rises from $\beta = 250\%$ to $\beta = 500\%$
 = more or less what happened since the 1970s
- In case β reaches $\beta = 800\%$, α would reach $\alpha = 42\%$
- In case $\sigma = 1.8$, α would be as large as $\alpha = 53\%$

Figure 6.5. The capital share in rich countries, 1975-2010



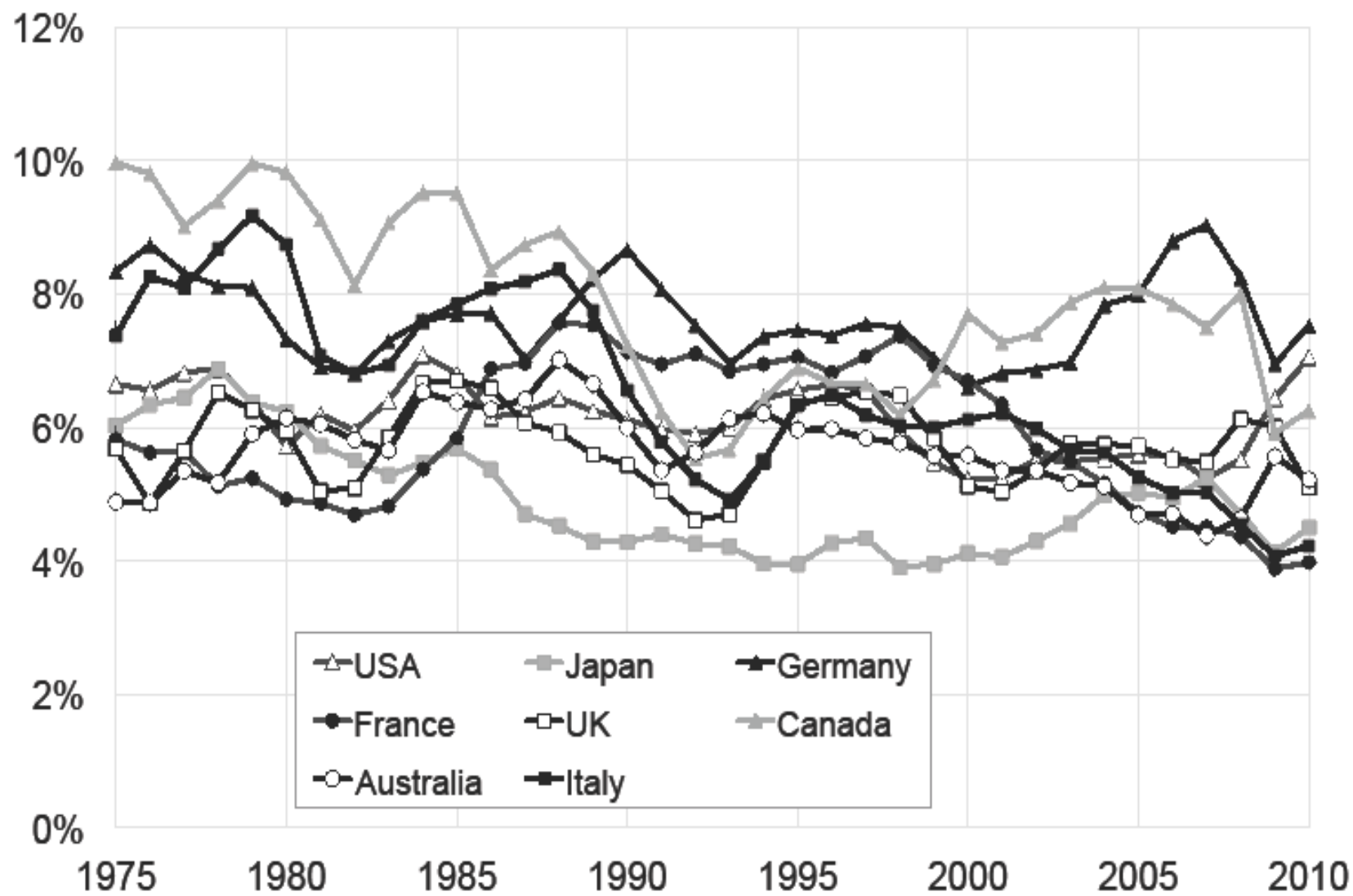
Capital income absorbs between 15% and 25% of national income in rich countries in 1970, and between 25% and 30% in 2000-2010. Sources and series: see piketty.pse.ens.fr/capital21c

Figure 5.3. Private capital in rich countries, 1970-2010



Private capital is worth between 2 and 3,5 years of national income in rich countries in 1970, and between 4 and 7 years of national income in 2010. Sources and series: see piketty.pse.ens.fr/capital21c.

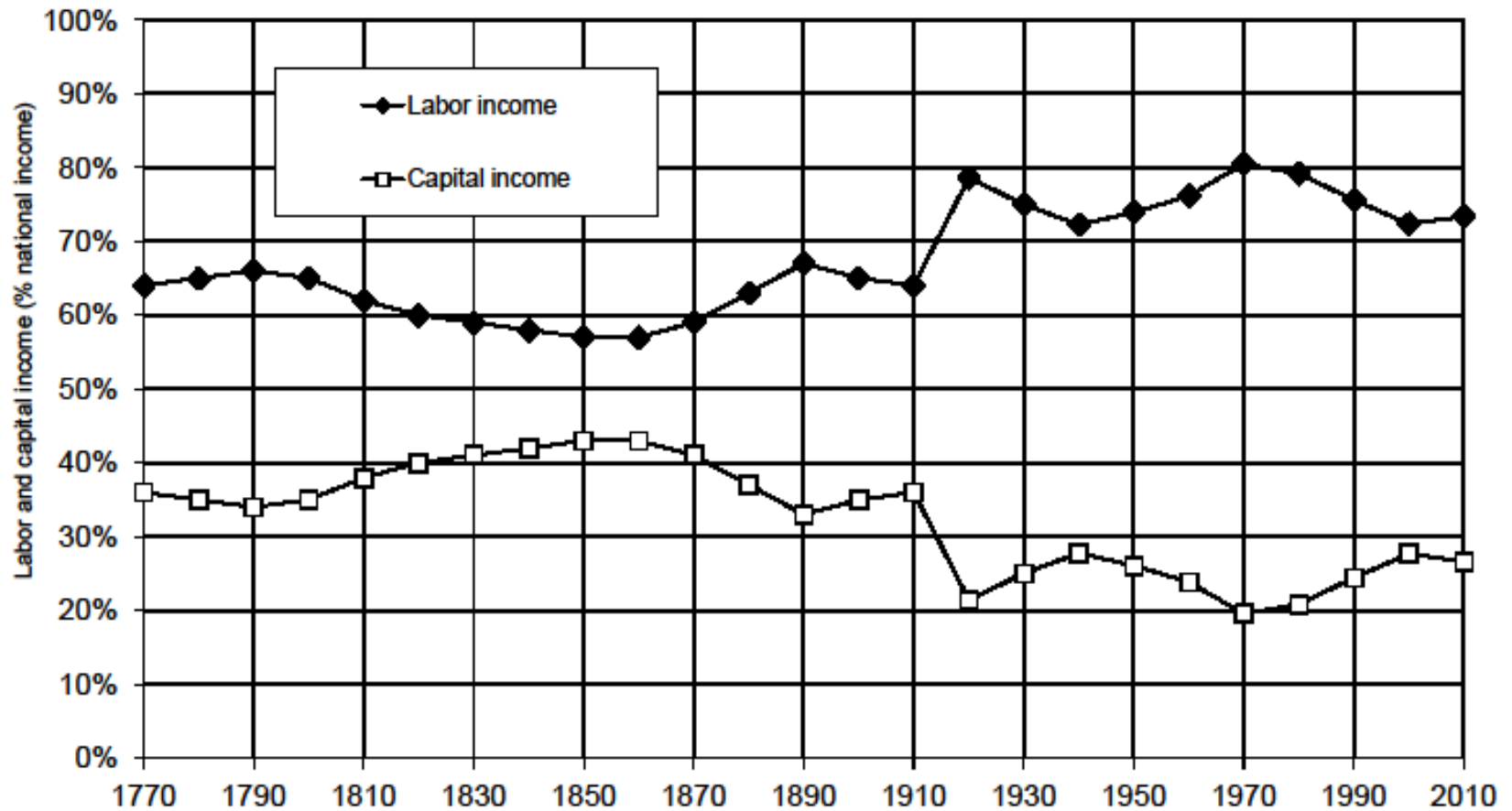
Figure 14: Average return on private wealth 1975-2010



Measurement problems with capital shares

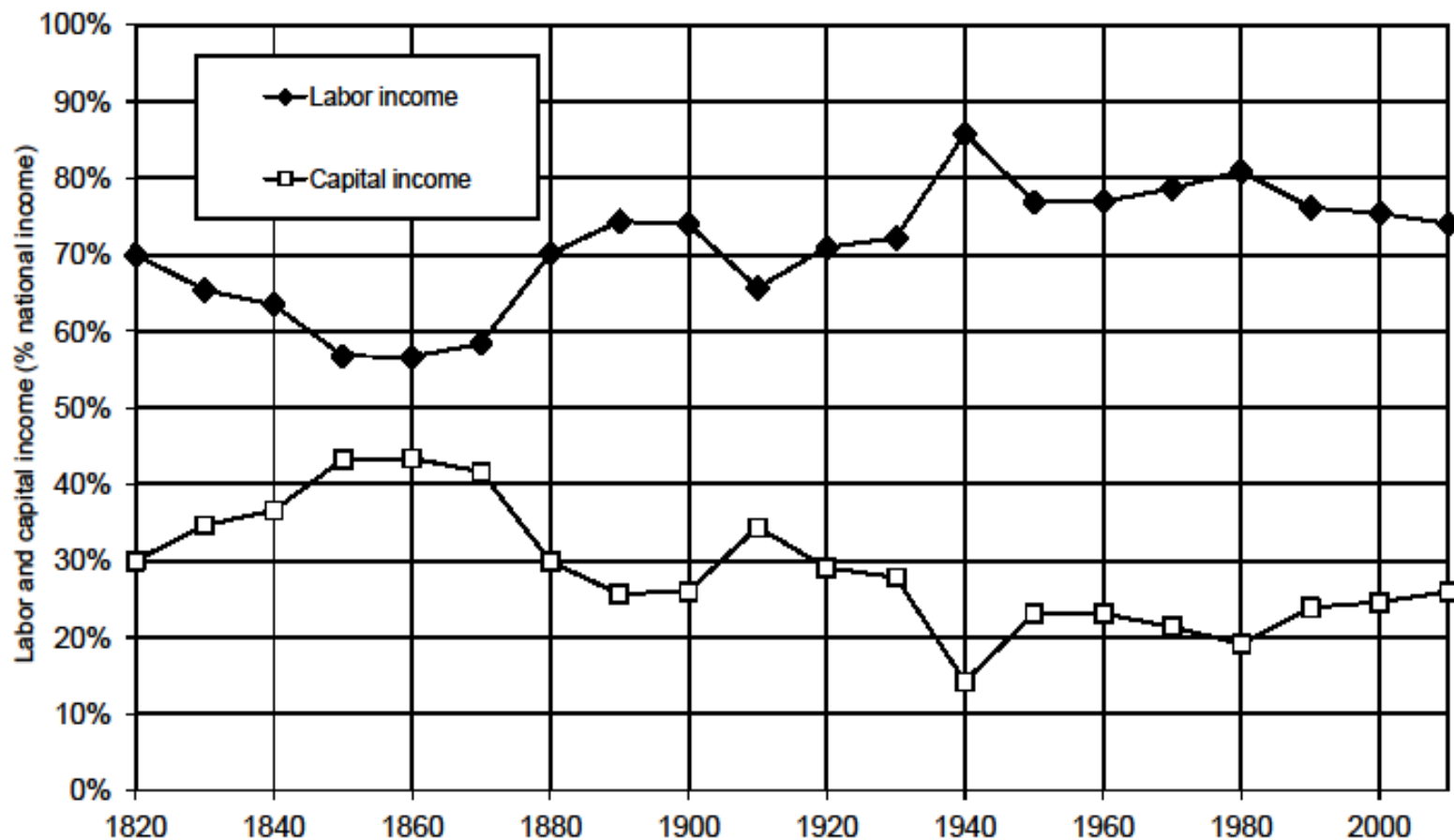
- In many ways, β is easier to measure than α
- In principle, capital income = all income flows going to capital owners (independantly of any labor input); labor income = all income flows going to labor earners (independantly of any capital input)
- But in practice, the line is often hard to draw: family firms, self-employed workers, informal financial intermediation costs (=the time spent to manage one's own portfolio)
- If one measures the capital share α from national accounts (rent+dividend+interest+profits) and compute average return $r=\alpha/\beta$, then the implied r often looks very high for a pure return to capital ownership: it probably includes a non-negligible entrepreneurial labor component, particularly in reconstruction periods with low β and high r ; the pure return might be 20-30% smaller (see estimates)
- Maybe one should use two-sector models $Y=Y_h+Y_b$ (housing + business); return to housing = closer to pure return to capital

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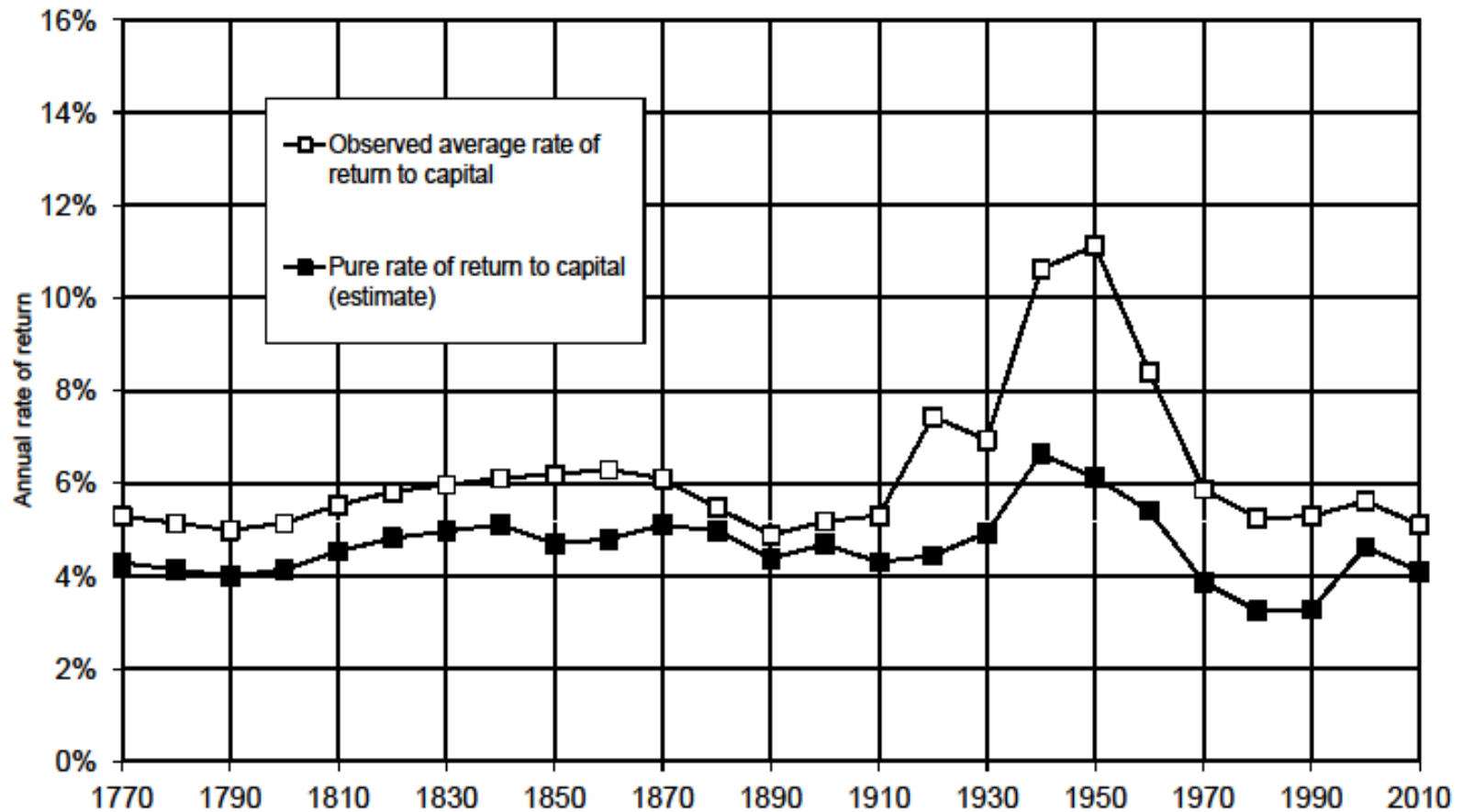
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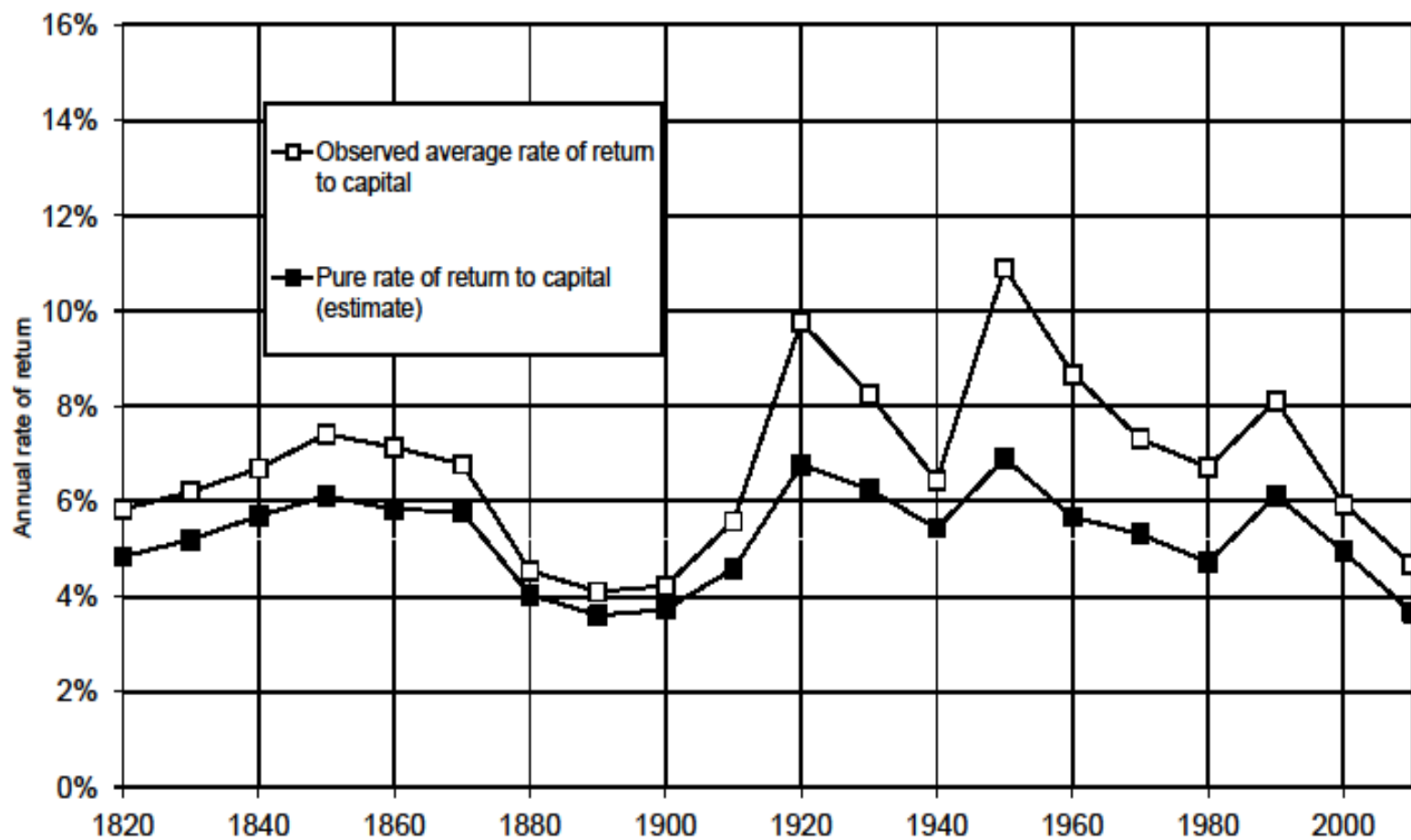
Figure 6.3. The pure return to capital in the United Kingdom, 1770-2010



The pure rate of return to capital is roughly stable around 4%-5% in the long run.

Sources and series: see piketty.pse.ens.fr/capital21c.

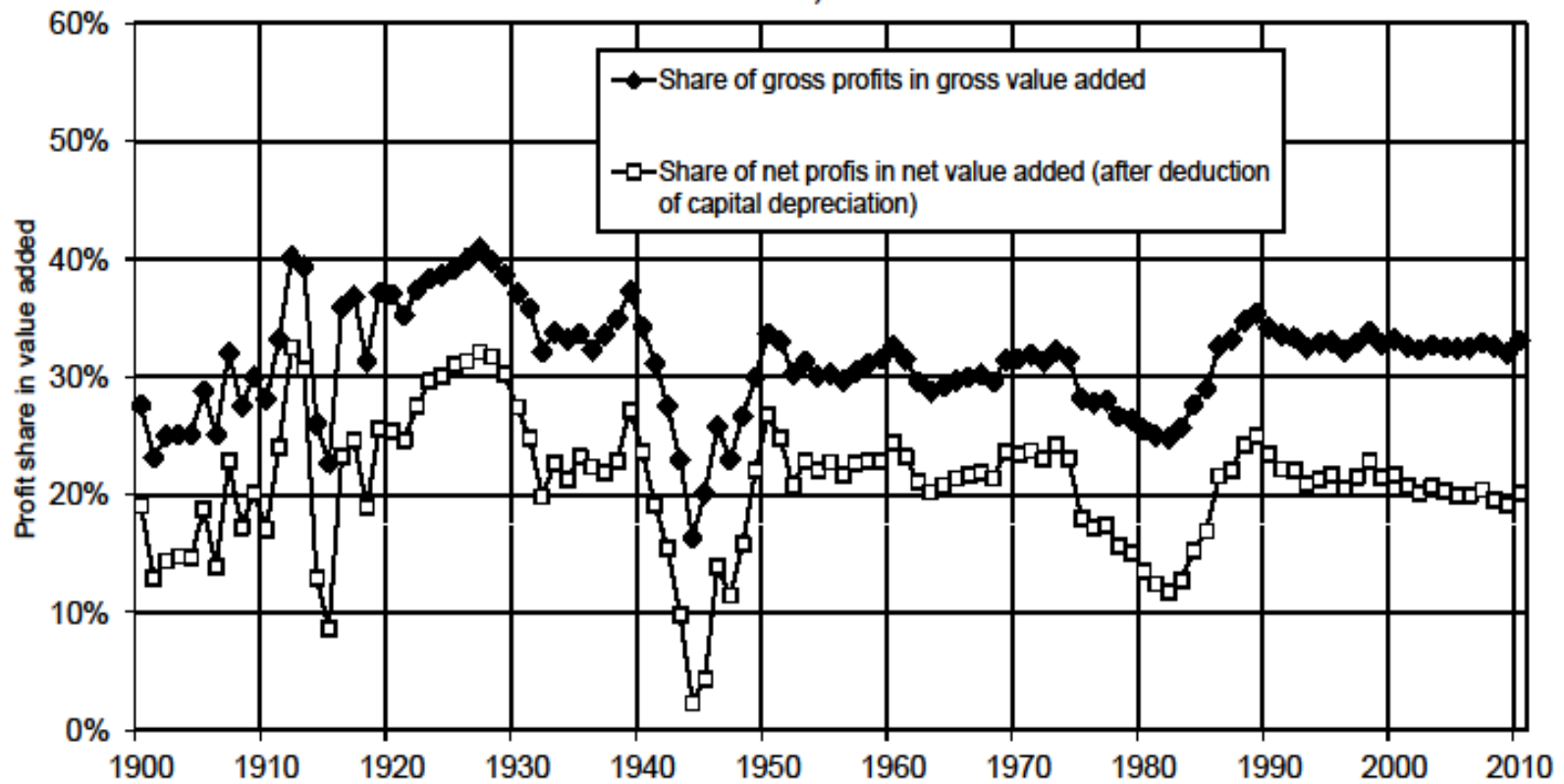
Figure 6.4. The pure rate of return to capital in France, 1820-2010



The observed average rate of return displays larger fluctuations than the pure rate of return during the 20th century.

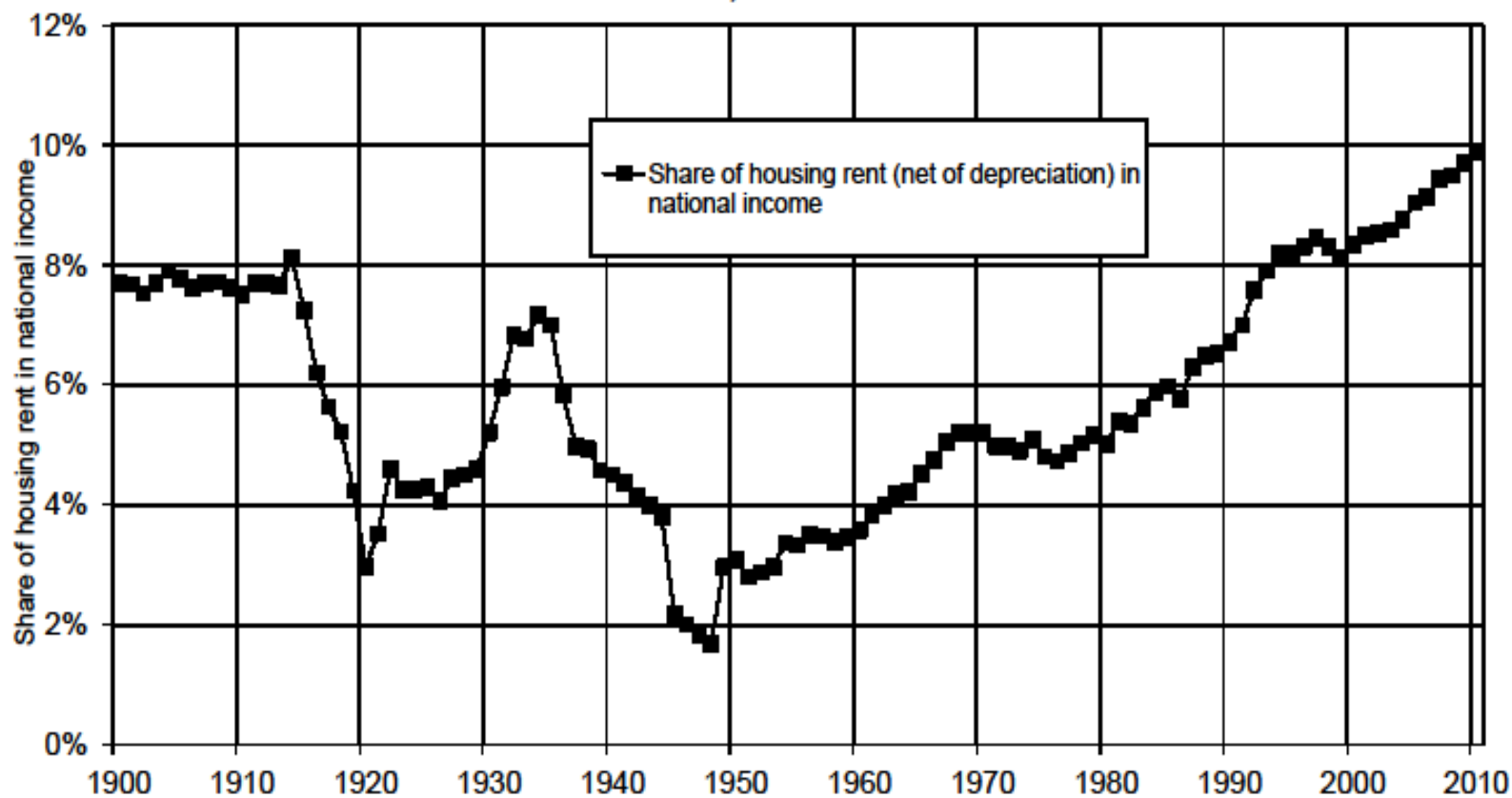
Sources and series: see piketty.pse.ens.fr/capital21c.

Figure 6.6. The profit share in the value added of corporations in France, 1900-2010



The share of gross profits in gross value added of corporations rose from 25% in 1982 to 33% in 2010; the share of net profits in net value added rose from 12% to 20%. Sources and series: see piketty.pse.ens.fr/capital21c

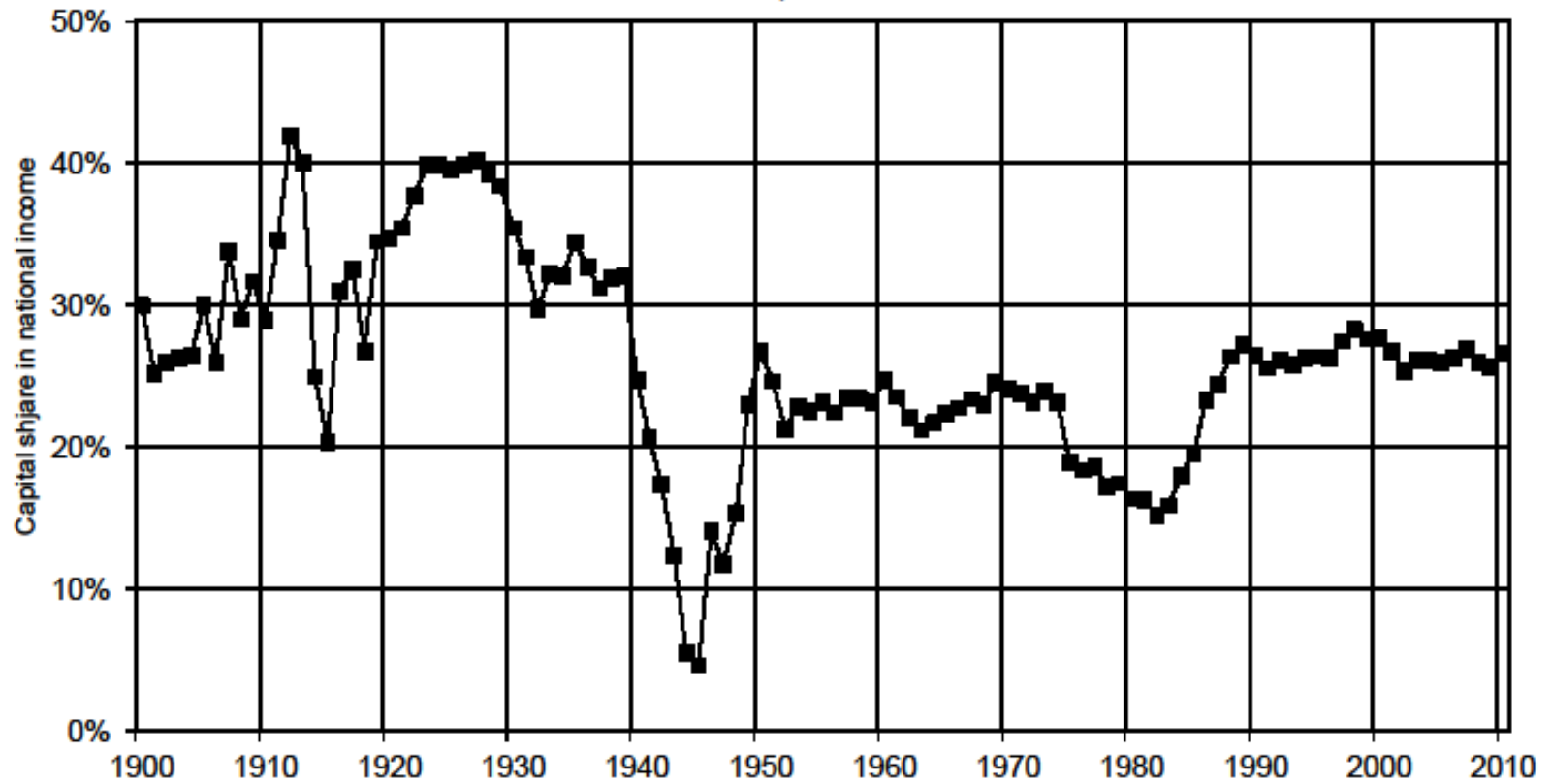
Figure 6.7. The share of housing rent in national income in France, 1900-2010



The share of housing rent (rental value of dwellings) rose from 2% of national income in 1948 to 10% in 2010.

Sources and series: see piketty.pse.ens.fr/capital21c.

**Figure 6.8. The capital share in national income
in France, 1900-2010**



The share of capital income (net profits and rents) rose from 15% of national income in 1982 to 27% in 2010.

Sources and series: see piketty.pse.ens.fr/capital21c.

Recent work on capital shares

- Imperfect competition and globalization: see [Karabarmounis-Neiman 2013](#) , « The Global Decline in the Labor Share »
- Public vs private firms: see [Azmat-Manning-Van Reenen 2011](#), « Privatization and the Decline of the Labor Share in GDP: A Cross-Country Aanalysis of the Network Industries »
- Capital shares and CEO pay: see [Pursesey 2013](#), « CEO Pay and Factor shares: Bargaining effects in US corporations 1970-2011 »

Summing up

- The rate of return to capital r is determined mostly by technology: $r = F_K =$ marginal product to capital, elasticity of substitution σ
- The quantity of capital β is determined by saving attitudes and by growth (=fertility + innovation): $\beta = s/g$
- The capital share is determined by the product of the two: $\alpha = r \times \beta$
- Anything can happen

- Note: the return to capital $r=F_K$ is determined not only by technology but also by psychology, i.e. saving attitudes $s=s(r)$ might vary with the rate of return
- In models with wealth or bequest in the utility function $U(c_t, w_{t+1})$, there is zero saving elasticity with $U(c, w)=c^{1-s} w^s$, but with more general functional forms one can get any elasticity
- In pure lifecycle model, the saving rate s is primarily determined by demographic structure (more time in retirement \rightarrow higher s), but it can also vary with the rate of return, in particular if the rate of return becomes very low (say, below 2%) or very high (say, above 6%)

- In the dynastic utility model, the rate of return is entirely set by the rate of time preference (=psychological parameter) and the growth rate:

$$\text{Max } \Sigma U(c_t)/(1+\delta)^t, \text{ with } U(c)=c^{1-1/\xi}/(1-1/\xi)$$

→ unique long rate rate of return $r_t \rightarrow r = \delta + \xi g > g$
($\xi > 1$ and transversality condition)

This holds both in the representative agent version of model and in the heterogeneous agent version (with insurable shocks); more on this in Lecture 6