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Interest Rate Risk and Other Determinants of Post-WWII US Government Debt/GDP Dynamics[†]

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This paper uses a sequence of government budget constraints to motivate estimates of returns on the US Federal government debt. Our estimates differ conceptually and quantitatively from the interest payments reported by the US government. We use our estimates to account for contributions to the evolution of the debt-GDP ratio made by inflation, growth, and nominal returns paid on debts of different maturities. (*JEL* E23, E31, E43, G12, H63)

This paper shows contributions that nominal returns, the maturity composition of the debt, inflation, and growth in real gross domestic product (GDP) have made to the evolution of the US debt-GDP ratio since World War II. Among the questions we answer are the following. Did the United States inflate away much of the debt by using inflation to pay negative real rates of return? Occasionally, but not usually. Did high net-of-interest deficits propel the debt-GDP ratio upward? Considerably during World War II, but not too much after that. How much did growth in GDP contribute to holding down the debt-GDP ratio? A lot. How much did variations in returns across maturities affect the evolution of the debt-GDP ratio? At times substantially, but, on average, not much since the end of World War II.

Of necessity, our answers to these questions rely on our own estimates of returns on government debt, not the series for interest payments reported by the US government.[1] The government budget constraint determines the evolution of the ratio of government debt to GDP. We propose an accounting scheme that emerges from a decomposition of the government's period-by-period budget constraint, to be described and justified in Section I. We use prices of indexed and nominal debt of each maturity to construct one-period holding period returns on government IOU's of various maturities. Multiplying the vector of returns by the vector of quantities

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[†] To comment on this article in the online discussion forum, or to view additional materials, visit the article page at http://www.aeaweb.org/articles.php?doi=10.1257/mac.3.3.192.

[1] Earlier researchers have also noticed the discrepancy between the concept underlying the government series on interest payments and the concept that appears in the government budget constraint. See Olivier Jean Blanchard and Jeffrey Sachs (1981), Michael Boskin (1982), Rudolph Penner (1982), Robert Eisner and Paul J. Pieper (1984), Congress of the United States (1985), and Henning Bohn (1992).

outstanding each period provides the measure of returns that appear in the government budget constraint.

The US government's interest payments series was not designed to measure the returns that appear in the government budget constraint. Instead, the government's series isolates the government's out-of-pocket, period-by-period cash dispersals used to service its debt.[2] The government's interest payments series answers the question "how many dollars must the Treasury devote to paying coupons on this period's outstanding Treasury notes and bonds while rolling over the nominal stock of Treasury bills?"[3]

The following observation indicates the essential difference in the questions being answered by the government's accounting system and ours.[4] Through a suitable debt-management policy, the Treasury could drive the government's measure of interest payments to zero every period, even though, from the point of view of the government budget constraint, it truly could be paying substantial interest that would propel the government debt-GDP ratio upward. The government could set officially measured interest payments to zero, for example, by issuing only zero-coupon 10-year bonds and perpetually rolling them over each year. These bonds would never pay coupons. They would never mature because each year they would be repurchased as nine year zero-coupon bonds and be replaced by newly issued 10-year zero-coupon bonds. Of course, although the government's accounts would put interest payments at zero, in truth the government would still pay interest in the form of the capital gains earned by the sellers of zero-coupon 9-year bonds (i.e., in the sense determined by its budget constraint).

I. Interest Payments in the Government Budget Constraint

Let Y_t be real GDP at t, and let B_t be the real value of IOU's from the government to the public. That least controversial equation of macroeconomics, the government budget constraint, accounts for how a nominal interest rate $r_{t-1,t}$, net inflation $\pi_{t-1,t}$, net growth in real GDP $g_{t-1,t}$, and the primary deficit def_t combine to determine the evolution of the government debt-GDP ratio

$$\frac{B_t}{Y_t} = (r_{t-1,t} - \pi_{t-1,t} - g_{t-1,t})\frac{B_{t-1}}{Y_{t-1}} + \frac{def_t}{Y_t} + \frac{B_{t-1}}{Y_{t-1}}. \tag{1}$$

The appropriate concept of a nominal return $r_{t-1,t}$ is one that verifies this equation.

The nominal return $r_{t-1,t}$ and the real stock of debt B_t in equation (1) are averages across terms to maturity. To bring out some of the consequences of interest rate risk and the maturity structure of the debt for the evolution of the debt-GDP

[2] The government figures reported by the Treasury and recorded in the National Income and Product Accounts (NIPA) are prepared by the Bureau of the Public Debt.

[3] As we document in the Appendix, the government's concept leaves out capital gains and losses on Treasury notes and bonds, a feature that is revealed by the absence of holding period returns for longer maturity government obligations in the government's formula for computing interest payments. Those capital gains and losses appear automatically in the law of motion for government debt, a.k.a. the government budget constraint.

[4] We describe the relationship between the government's accounting system and ours in detail in the Appendix.

ratio, we refine equation (1) to recognize that the government pays different nominal one-period holding period returns on the IOUs of different maturities that compose B_t. Let $\tilde{B}^j_{t-1}$ and $\overline{B}^j_{t-1}$ be the real values of nominal and inflation-indexed zero-coupon bonds of maturity j at $t-1$, while $\tilde{B}_{t-1} = \sum_{j=1}^{n} \tilde{B}^j_{t-1}$ and $\overline{B}_{t-1} = \sum_{j=1}^{n} \overline{B}^j_{t-1}$ are the total real values of nominal and indexed debt at $t-1$. Let $\tilde{r}^j_{t-1,t}$ be the net *nominal* holding period return between $t-1$ and t on nominal zero-coupon bonds of maturity j. Let $\overline{r}^j_{t-1,t}$ be the net *real* holding period return between $t-1$ and t on inflation-indexed zero-coupon bonds of maturity j.[5] Then the government budget constraint expresses the following law of motion for the debt-GDP ratio:

$$\begin{aligned}(2) \qquad \frac{\tilde{B}_t + \overline{B}_t}{Y_t} = {} & \sum_{j=1}^{n} \tilde{r}^j_{t-1,t} \frac{\tilde{B}^j_{t-1}}{Y_{t-1}} - (\pi_{t-1,t} + g_{t-1,t}) \frac{\tilde{B}_{t-1}}{Y_{t-1}} \\ & + \sum_{j=1}^{n} \overline{r}^j_{t-1,t} \frac{\overline{B}^j_{t-1}}{Y_{t-1}} - g_{t-1,t} \frac{\overline{B}_{t-1}}{Y_{t-1}} \\ & + \frac{\mathrm{def}_t}{Y_t} + \frac{\tilde{B}_{t-1} + \overline{B}_{t-1}}{Y_{t-1}}.\end{aligned}$$

Equation (2) distinguishes contributions to the growth of the debt-GDP ratio that depend on debt maturity j from those that don't. In particular, $\pi_{t-1,t}$ and $g_{t-1,t}$ don't depend on j and operate on the *total* real value of debt last period; but the holding-period returns $\tilde{r}^j_{t-1,t}$ and $\overline{r}^j_{t-1,t}$ do depend on maturity j and operate on the real values of the corresponding maturity j components $\tilde{B}^j_{t-1}$ and $\overline{B}^j_{t-1}$.

A. *Accounting Details*

At each date t, we compute the number of dollars the government has promised to pay at each date $t+j$, $j \geq 1$. A coupon bond is a stream of promised coupons plus an ultimate principal payment. We regard such a bond as a bundle of zero-coupon bonds of different maturities and price it by unbundling it into the underlying component zero-coupon bonds, one for each date at which a coupon or principal is due, valuing each promised payment separately, then adding up these values. In other words, we strip the coupons from each bond and price a bond as a weighted sum of zero-coupon bonds of maturities $j = 1, 2, \ldots, n$.[6]

We treat nominal bonds and inflation-indexed bonds separately. For nominal bonds, let s^t_{t+j} be the number of time $t+j$ dollars that the government has at time t promised to deliver. To compute s^t_{t+j} from historical data, we add up all of the dollar principal-plus-coupon payments that the government has at time t promised to deliver at date $t+j$. Because zero-coupon bond prices were not directly observable until prestripped coupon bonds were introduced in 1985, we extract the nominal

[5] In a *nonstochastic* version of the growth model that is widely used in macroeconomics and public finance, the net holding period return on debt is identical for zero-coupon bonds of all maturities (e.g., see Lars Ljungqvist and Sargent 2011, chapter 11). The presence of risk and possibly incomplete markets changes that.

[6] The market and the government already do this. Prestripped coupon bonds are routinely traded.

implicit forward rates from government bond price data. We then convert these nominal forward rates on government debt into prices of claims on future dollars. Let q^t_{t+j} be the number of time t dollars that it takes to buy a dollar at time $t+j$:

$$q^t_{t+j} = \frac{1}{(1 + \rho_{jt})^j},$$

where ρ_{jt} is the time t yield to maturity on bonds with j periods to maturity. The *yield curve* at time t is a graph of yield to maturity ρ_{jt} against maturity j. Let n be the longest maturity outstanding. The vector $\{q^t_{t+j}\}^n_{j=1}$ prices all nominal zero-coupon bonds at t. To convert t dollars to goods we use $v_t = 1/p_t$, where p_t is the price level in base year 2005 dollars, and v_t is the value of currency measured in goods per dollar.

For inflation-protected bonds (TIPS), let $\bar{s}^t_{t+j}$ be the number of time $t+j$ goods that the government at t promises to deliver. For indexed debt, when we add up the principal and coupon payments that the government has promised to deliver at date $t+j$ as of date t, we adjust for past realizations of inflation in ways consistent with the rules governing TIPS. We then compute $\bar{q}^t_{t+j}$, the number of time t goods that it takes to purchase a time $t+j$ good, by

$$\bar{q}^t_{t+j} = \frac{1}{(1 + \bar{\rho}_{jt})^j},$$

where $\bar{\rho}_{jt}$ is the time t yield to maturity on real bonds with j periods to maturity. The total real value of government debt outstanding in period t equals

$$v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j} + \sum_{j=1}^{n} \bar{q}^t_{t+j} \bar{s}^t_{t+j}.$$

The first term is the real value of the nominal debt, computed by multiplying the number of time $t+j$ dollars that the government has sold, s^t_{t+j}, by their price in terms of time t dollars, q^t_{t+j}, summing over all outstanding bonds, $j = 1, \ldots, n$, and then converting from dollars to goods by multiplying by v_t. The second term is the value of the inflation-protected debt, computed by multiplying the number of time $t+j$ goods that the government has promised, $\bar{s}^t_{t+j}$, by their price in terms of time t goods, $\bar{q}^t_{t+j}$, and then summing over $j = 1, \ldots, n$.

With def_t denoting the government's real net-of-interest budget deficit, measured in units of time t goods, the government's time t budget constraint is

$$(3)\quad v_t \sum_{j=1}^{n} q^t_{t+j} s^t_{t+j} + \sum_{j=1}^{n} \bar{q}^t_{t+j} \bar{s}^t_{t+j} = v_t \sum_{j=1}^{n} q^t_{t+j-1} s^{t-1}_{t+j-1} + \sum_{j=1}^{n} \bar{q}^t_{t+j-1} \bar{s}^{t-1}_{t+j-1} + def_t,$$

where it is to be understood that $q^t_t = 1$ and $\bar{q}^t_t = v_t$.

The left-hand side of equation (3) is the real value of the interest bearing debt at the end of period t. The right-hand side of equation (3) is the sum of the real value of the primary deficit and the real value of the outstanding debt that the government owes at the beginning of the period, which, in turn, is simply the real value this period of outstanding promises to deliver future dollars s_{t-1+j}^{t-1} and goods $\overline{s}_{t-1+j}^{t-1}$ that the government issued last period.

To attain the government budget constraint in the form of equation (2), we simply rearrange (3) to get

$$(4)\quad \frac{\sum_{j=1}^{n} v_t q_{t+j}^t s_{t+j}^t + \sum_{j=1}^{n} \overline{q}_{t+j}^t \overline{s}_{t+j}^t}{Y_t} = \sum_{j=1}^{n} \left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^t}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \frac{v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1}}{Y_{t-1}}$$
$$+ \sum_{j=1}^{n} \left(\frac{\overline{q}_{t+j-1}^t}{\overline{q}_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \frac{\overline{q}_{t+j-1}^{t-1} \overline{s}_{t+j-1}^{t-1}}{Y_{t-1}}$$
$$+ \frac{\mathrm{def}_t}{Y_t}$$
$$+ \frac{\sum_{j=1}^{n} v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1} + \sum_{j=1}^{n} \overline{q}_{t+j-1}^{t-1} \overline{s}_{t+j-1}^{t-1}}{Y_{t-1}}.$$

To recognize that this equation is equivalent with (2), use the definitions

$$(5)\quad v_{t-1} q_{t+j-1}^{t-1} s_{t+j-1}^{t-1} = \tilde{B}_{t-1}^j$$

$$(6)\quad \overline{q}_{t+j-1}^{t-1} \overline{s}_{t+j-1}^{t-1} = \overline{B}_{t-1}^j$$

$$(7)\quad \tilde{B}_{t-1} = \sum_{j=1}^{n} \tilde{B}_{t-1}^j$$

$$(8)\quad \overline{B}_{t-1} = \sum_{j=1}^{n} \overline{B}_{t-1}^j$$

$$(9)\quad \left(\frac{v_t}{v_{t-1}} \frac{q_{t+j-1}^t}{q_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \approx \tilde{r}_{t-1,t}^j - \pi_{t-1,t} - g_{t-1,t}$$

$$(10)\quad \left(\frac{\overline{q}_{t+j-1}^t}{\overline{q}_{t+j-1}^{t-1}} \frac{Y_{t-1}}{Y_t} - 1 \right) \approx \overline{r}_{t-1,t}^j - g_{t-1,t}.$$

To implement budget constraint (4), it is important to recognize that Federal debt has both marketable and nonmarketable components. For the marketable components, it is straightforward to measure the appropriate prices, q_{t+j}^t, $\overline{q}_{t+j}^t$, and the associated returns, $q_{t+j-1}^t / q_{t+j-1}^{t-1}$, $\overline{q}_{t+j-1}^t / \overline{q}_{t+j-1}^{t-1}$, but for the nonmarketable components, implicit returns must somehow be synthesized. In Section IB, we describe in detail how we did that.

B. *Data*

Our data are end-of-year observations from 1941 to 2009. As described in online Appendix A, the total outstanding debt held by the public is the sum of the marketable (i.e., Treasury bills, notes, bonds, and TIPS) and nonmarketable (i.e., savings bonds, and special issues to state and local governments) debt. We obtained prices and quantities of marketable nominal bonds held by the public from the Center for Research in Security Prices (CRSP) Monthly US Treasury Database (2010). Since CRSP only reports the quantity-held data back to 1960, we extended this series using data from the *Treasury Bulletin* (1940–2009). The quantities outstanding of the TIPS are from December issues of the US Treasury's *Monthly Statement of the Public Debt* (1940–2009). For the pre-1970 period, we fit a zero-coupon forward curve from the coupon bond price data via Daniel F. Waggoner's (1997) cubic spline method. For the period of 1970 to 2009, we use the nominal and real zero-coupon yield curves computed by Refet S. Gurkaynak, Brian Sack, and Jonathan H. Wright (2007, 2010).[7]

Our analysis focuses solely on Treasury debt. We exclude agency securities (e.g., the Tennessee Valley Authority and government sponsored enterprises) that are implicitly or explicitly guaranteed by the federal government, since the revenue streams that back these securities are not included in the primary deficit series of our government budget constraint, equation (1). Nor do we include in the analysis government assets such as gold reserves and real estate holdings. Finally, our analysis does not incorporate unfunded future government liabilities.

While marketable securities today represent the lion's share of the debt held by the public ($7.2 out of $7.8 trillion, or a little less than 93 percent in 2009), this has not always been the case. In Figure 1, we plot the debt-GDP ratio for three different measures of the debt: the marketable debt held by the public, the sum of the marketable and the nonmarketable debt held by the public, and the total outstanding debt. Over the entire period, marketable debt has averaged about 80 percent of the total debt held by the public (i.e., the ratio of the solid line to the dashed line). Early in the sample, this ratio was about two-thirds, and it has steadily increased over time. Nonmarketable savings bonds and Victory loans played a much larger role in Treasury borrowing during World War II and the Korean War than they do today.

The Federal Government reports its receipts, expenditures, and interest payments in two places: the annual budget issued by the Treasury and the The National Income and Product Accounts (NIPA 2010). For two reasons, we use fiscal data from the NIPA Table 3.2 to compute the primary deficit rather than budget data from the Treasury. First, the Treasury reports data for the fiscal year, which runs from October to September, while we measure returns on a calendar year basis. Second, NIPA interest payments (NIPA Table 3.2, line 28) exclude interest paid to other government trust funds, such as the Social Security trust fund. Interest on the public debt reported by the Treasury includes interest paid to these trust funds. NIPA interest payments include interest paid to the military and civil service retirement funds. We net out these payments using data on NIPA Table 3.18B, line 24. We compute

[7] These yield curves are available from http://www.federalreserve.gov/econresdata/researchdata.htm.

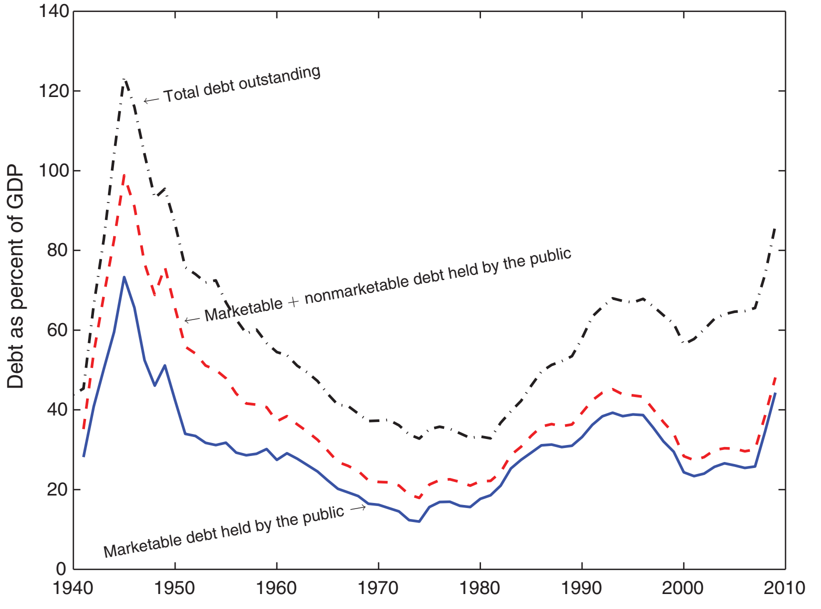

Figure 1. Marketable Debt Held by the Public, Total Debt Held by the Public, and Total Debt Outstanding as Percentages of GDP

Notes: The solid line is the ratio of the par value of marketable Treasury securities held by the public to GDP. The dashed line is ratio of the par value of all Treasury securities (marketable and nonmarketable) held by the public to GDP. The dot-dashed line is ratio of the par value of total outstanding debt to GDP.

output growth rates using real GDP from the NIPA. For the value of currency, v_t, we take the inverse of the fourth quarter observation of the GDP price deflator.

The left side of equation (3) is the real *value* of the interest bearing debt held by the public at the end of period t. To compute the contribution that marketable debt makes to this sum, instead of estimating quantities of zero-coupon bonds and their prices, as we do (i.e., computing the s_{t+j}^t sequences and estimating a zero-coupon yield curve), we could just multiply the vector of market prices by the vector of the quantities outstanding for each security. These alternative calculations yield nearly identical debt series. Of course, an advantage of our computation that uses estimates of $\{q_{t+j}^t\}$ and $\{s_{t+j}^t\}$ is that we can decompose returns by maturity.

Ipso facto, market prices for the nonmarketable portion of the debt are unavailable. Therefore, we proceeded as follows. We obtained the par value of the total nonmarketable debt held by the public from Table OFS-1 of the *Treasury Bulletin* (1940–2009) and from the *Monthly Statement of the Public Debt* (1940–2009). To estimate a market value of the nonmarketable portion of the debt, we multiplied its par value by the ratio of the market value to the par value of marketable debt held by the public.[8]

[8] See Figure III in online Appendix A for a graph of this ratio.

C. *Previous Work*

It is useful to relate our market value of debt and return series to previous estimates. John J. Seater (1981), W. Michael Cox and Eric Hirschhorn (1983), James L. Butkiewicz (1983), Eisner and Pieper (1984), Cox (1985), and Bohn (1992) have calculated series on the market value of the Treasury's portfolio. Our debt series most closely aligns with Seater's (1981) MVPRIV3 series (see his table 1) and Cox and Hirschhorn's (1983) series "Market value of privately held treasury debt" (see their table 6).[9]

Since we compute the returns on the marketable debt directly, our estimates of these returns are not sensitive to how either the primary deficit or the value of the nonmarketable debt is measured. By way of contrast, estimates that some other authors have created are sensitive to how the primary deficit and the value of nonmarketable debt are measured. Eisner and Pieper (1984), Eisner (1986), and Bohn (1992) computed measures of the government's interest payments that are conceptually similar to ours. But instead of computing the terms on the left side of (3) directly, they used the intertemporal budget constraint (1) to compute total returns $\tilde{r}_{t-1,t}\tilde{B}_{t-1}$ as the change in the market value of debt minus the primary deficit. An advantage of that alternative approach is that it avoids using data on pricing kernels q_{t+j}^{t} and promised payments s_{t+j}^{t}. Instead, the market value of the debt can be computed directly from the observed prices and quantities outstanding of government bonds.[10]

However, while in theory the government budget identity (2) should hold exactly, with measured series this equation carries residuals that have several sources. Early in the sample, much of the data from the NIPA are reported to just two (and in some cases just one!) significant digits. While we have tried to minimize discrepancies, there are still small differences between the NIPA fiscal data and the Treasury's accounting.[11] Further, the change in the market value of the debt is sensitive to the definition of the debt (e.g., should the monetary base be included or not? How should debt from government corporations and agencies or government assets such as gold be treated?). The computed return series will be a weighted average of returns on the securities included. Further, the primary deficit series should be consistent with the choice of securities. Discrepancies between the debt and deficit series will corrupt any measure of returns computed as a residual.[12]

We prefer our calculations because they avoid some (but not all) of these measurement error issues. Furthermore, our calculations also allow us to account for holding-period returns on obligations of different maturities and thereby form the

[9] The Cox and Hirschhorn (1983) series has been updated and is available from http://www.dallasfed.org/data/data/natdebt.tab.htm.

[10] We will employ a similar strategy in Section II as one of two ways to estimate the returns on the nonmarketable portion of the debt.

[11] For example, NIPA interest payments include interest paid by the IRS on certain tax refunds.

[12] Nevertheless, the two approaches lead to similar results quantitatively. For the period in which our study overlaps with Bohn's (1948–1989), his return series and ours move together, although ours is more volatile, particularly during the 1980s; the mean and standard deviation of our value-weighted return series is 1.64 and 4.28, compared with 2.42 and 3.19, respectively, for Bohn's return series. The correlation coefficient between his return series and ours is 0.76.

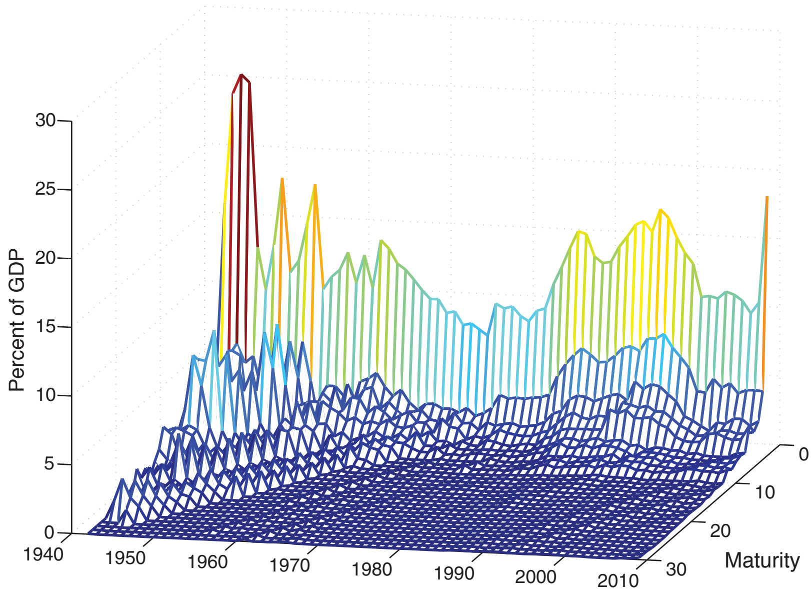

Figure 2. Promised Nominal Marketable Payments, s^t_{t+j}, by Date and Maturity

decompositions of interest payments in Table 2 and Figure 4, execute counterfactual debt management experiments, and dissect the difference between our estimates of the interest costs and those reported by the Treasury. We turn to this last task in the Appendix.

II. Contributions to the Evolution of the US Debt-GDP Ratio

To set the stage for the role that interest rate risks will play in our story, Figure 2 shows the evolution of the government's promised nominal marketable payments s^t_{t+j} over time and across maturities measured in years. Throughout the post-war period, the largest share of the promised payments are due within one year. The size of these promised payments diminishes quickly as the term to maturity increases. During the 1940s, long-term debt made up a large share of government borrowing. The share of long-term obligations steadily declined over the 1950s and 1960s. By the 1970s, the government had very few promised payments more than 15 years out. Finally, note the sharp increase in debt due within one year in 2009.

Figure 3 displays the one-period holding period returns by maturity over time. During the first half of the sample, the returns are relatively flat across maturities and stable across years. During the later part of the sample, post-1980, the returns become considerably more volatile across maturities and years.[13]

Figure 4 shows contributions to the propulsion of $(\tilde{B} + \overline{B})/Y$ in formula (2) from nominal interest payments $\tilde{r}^j_{t-1,t}\, \tilde{B}^j_{t-1}/Y_{t-1}$ for various maturities j. The figure

[13] See online Appendix C for simple calculations that provide intuition behind the large capital gains and losses on pure discount bonds.

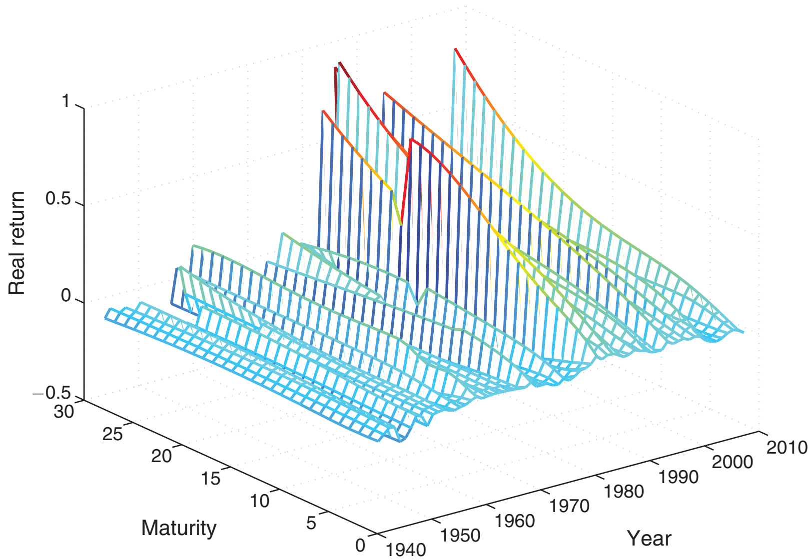

Figure 3. One Period Holding Period Returns for Marketable Debt

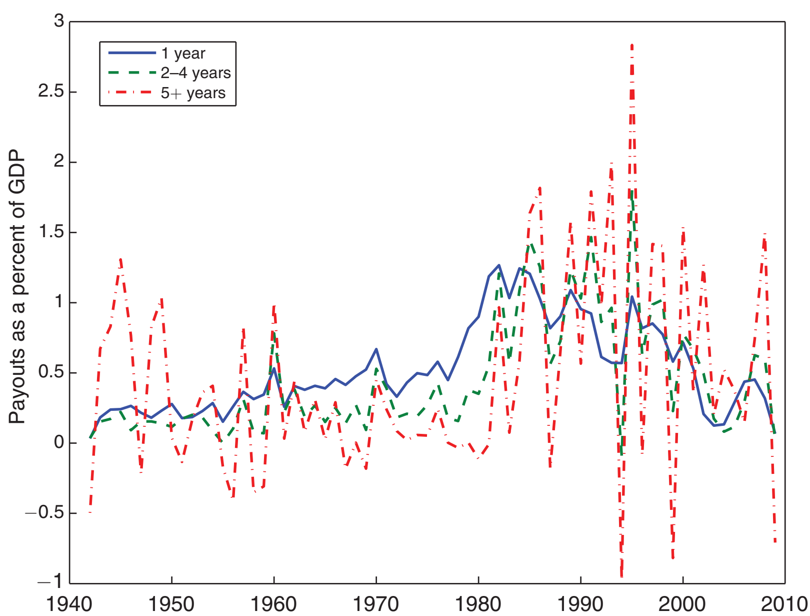

Figure 4. Decomposition of the Nominal Payouts by Maturity of Obligation

Notes: The line labeled "1 year" is $100 \times \tilde{r}^{1}_{t-1,t}\tilde{B}^{1}_{t-1}/Y_{t-1}$; the line labeled "2–4 years" is $100 \times \sum_{j=2}^{4} \tilde{r}^{j}_{t-1,t}\tilde{B}^{j}_{t-1}/Y_{t-1}$; and the line labeled "5+ years" is $100 \times \sum_{j=5}^{n} \tilde{r}^{j}_{t-1,t}\tilde{B}^{j}_{t-1}/Y_{t-1}$.

shows that volatility of nominal interest rate payments has been larger for longer horizons. For the period 1942–2009, Figure 5 plots the mean and standard deviation of one-year real holding period returns by maturity for the nominal, marketable

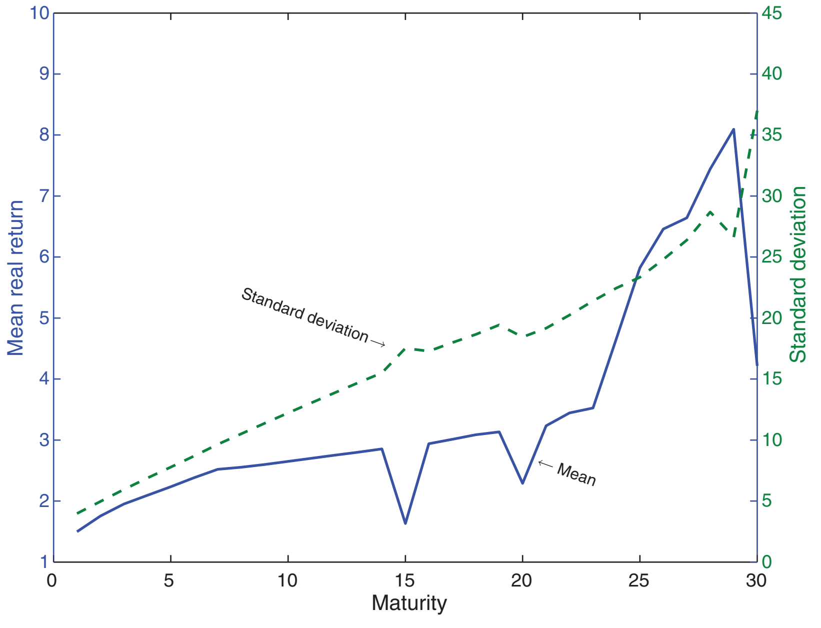

Figure 5. Mean and Standard Deviation of Real Holding Period Returns by Maturity

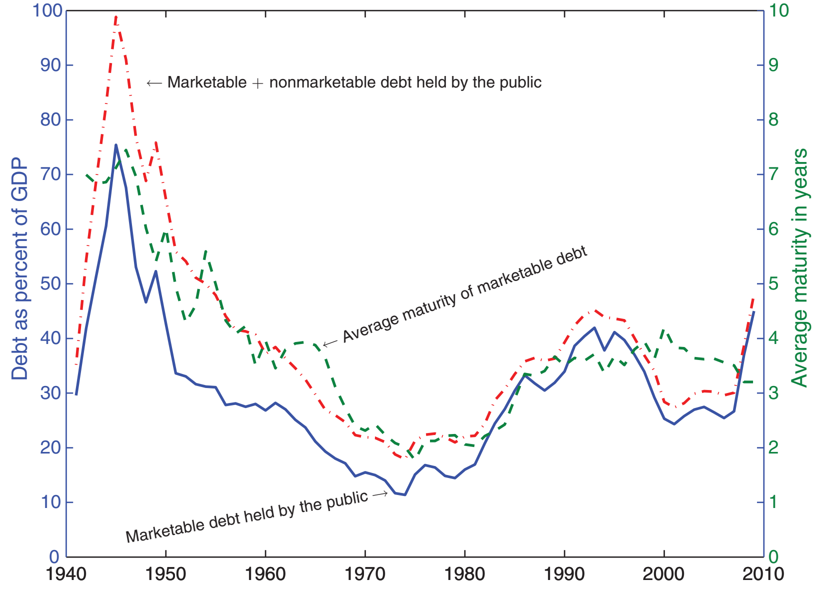

Figure 6. Ratio of Marketable and Total Debt Held by the Public to GDP and Average Maturity of the Marketable Debt Held by the Public

portion of the debt.[14] Figure 5 reveals that while longer maturities have generally

[14] A principal aim of stochastic discount factor models like the one proposed by Monica Piazzesi and Martin Schneider (2006) is to capture how means and standard deviations of one-period holding-period returns depend on maturity.

been associated with higher and more volatile returns, returns on bonds maturing in 15, 20, and 30 years were, on average, lower than those for adjacent maturities. We suspect that this outcome partly, but not entirely, reflects investors' preferences for newly issued or so-called "on-the-run" securities.

Figure 6 plots the average maturity, in years, of the marketable Treasury debt held by the public along with the ratios of the marketable and total debt held by the public to GDP from 1941 to 2009. The average maturity moves with the debt-GDP ratio. Immediately after World War II, the average maturity of the government's marketable portfolio was approximately seven years. As can be anticipated from the time path of promised payments in Figure 2, over the next three decades it fell steadily, reaching a trough in the mid-1970s at around two years. During the 1960s and early 1970s, this fall was partly the consequence of federal legislation, repealed in 1975, that had prevented the Treasury from issuing securities paying interest above 4.25 percent—a threshold that was below long-term market rates during that period. As we shall see, by causing the Treasury to shorten the average maturity of its debt during the high inflation years of the 1970s, this law prevented the Treasury from fully benefiting from the negative implicit real interest it managed to pay through inflation. Since the repeal of this restriction, the Treasury has lengthened the average maturity to between three and four years.

In 1941, the ratio of the market value of the total debt held by the public to GDP was 37.0 percent. By 1945 this ratio had risen to 97.2 percent. It fell steadily over the next three decades, reaching a trough in 1974 at 16.9 percent. After the deficits of the 1980s, it peaked again in 1993 at 48.2 percent. It fell below 30 percent during the Clinton administration, but by December 2009, it had climbed back to 48.8 percent.

What contributions did inflation, growth, and compound interest make to the evolution of the debt-GDP ratio depicted in Figure 6? To answer this question, we take $\tilde{B}_{t-\tau} + \overline{B}_{t-\tau}/Y_{t-\tau}$ as an initial condition at time $t - \tau$ and iterate on (2) to arrive at the following useful decomposition:

$$(11)\quad \frac{\tilde{B}_t + \overline{B}_t}{Y_t} - \frac{\tilde{B}_{t-\tau} + \overline{B}_{t-\tau}}{Y_{t-\tau}} = \sum_{s=0}^{\tau-1}\Bigg[\sum_{j=1}^{n}\left(\tilde{r}^{j}_{t-s-1,t-s} - \pi_{t-s-1,t-s} - g_{t-s-1,t-s}\right)\frac{\tilde{B}^{j}_{t-s-1}}{Y_{t-s-1}} + \sum_{j=1}^{n}\left(\overline{r}^{j}_{t-s-1,t-s} - g_{t-s-1,t-s}\right)\frac{\overline{B}^{j}_{t-s-1}}{Y_{t-s-1}} + \frac{def_{t-s}}{Y_{t-s}}\Bigg].$$

Before describing the results of applying decomposition (11), we briefly describe how we addressed issues associated with the presence of nonmarketable government debt within $\tilde{B}_t + \overline{B}_t$. To compute the return on the nonmarketable portion of the debt, we use two alternative methods summarized in Table 1. Unlike for the marketable portion of the debt, we do not have security-level data for the nonmarketable debt, so we cannot construct nonmarketable counterparts to the s^{t}_{t+j} series. Thus, the returns we compute are average returns for the entire stock of nonmarketable debt. Under the heading "Nominal return I" (row 8), we report the average

Table 1—Contributions to Changes in the Debt-GDP Ratio

	Period							
	1941–1945	1945–1974	1974–1981	1981–1993	1993–2001	2001–2009	1941–2009	1945–2009
Debt/GDP								
(1) End	97.2	16.9	19.9	48.2	28.5	48.8	48.8	48.8
(2) Start	37.0	97.2	16.9	19.9	48.2	28.5	37.0	97.2
(3) Change	60.2	−80.3	3.0	28.3	−19.7	20.3	11.8	−48.4
Marketable debt								
(4) Nominal return	3.6	21.8	7.5	36.5	17.7	8.9	96.0	92.4
(5) Inflation	−7.3	−35.0	−8.1	−11.6	−5.2	−5.0	−72.4	−65.0
(6) Real return	−3.5	−11.6	−0.6	24.1	12.2	3.8	24.4	27.9
(7) GDP growth	−15.6	−21.3	−3.3	−10.9	−10.7	−3.3	−65.1	−49.5
Nonmarketable debt								
(8) Nominal return I	−0.8	11.0	4.9	1.1	−3.1	0.4	13.4	14.2
(9) Nominal return II	1.7	9.1	2.5	6.4	2.5	1.6	23.7	22.0
(10) Difference	−2.5	1.9	2.4	−5.3	−5.6	−1.2	−10.3	−7.8
(11) Inflation	−2.4	−15.2	−2.9	−1.9	−0.7	−0.8	−23.9	−21.5
(12) Real return	−3.2	−4.2	2.0	−0.8	−3.8	−0.4	−10.9	−7.3
(13) GDP growth	−4.9	−10.5	−1.1	−1.7	−1.5	−0.6	−20.2	−15.3
(14) Deficit/GDP	84.7	−34.7	5.8	17.8	−15.9	20.8	78.5	−6.2

Notes: Marketable debt include both the marketable nominal bonds and the TIPS. Nominal Return I is the return component of the nonmarketable debt computed as a residual to equation (2). Nominal Return II is the return component of the nonmarketable debt assuming the return on the nonmarketable debt is equal to the return on the marketable debt.

return on the entire stock of nonmarketable debt that makes equation (2) hold with equality. Under the heading "Nominal return II" (row 9), we report the return computed by assuming that the average return on the nonmarketable portion of the debt is the same as the average return on the marketable portion of the debt. When using the row 9 method, equation (2) will not necessarily hold with equality. In row 10, we report the size of the residual in equation (2) left under this row 9 way of computing the return. Contributions from the marketable debt and the primary deficit are computed independently from the nonmarketable debt and so are unaffected by any assumptions made about the nonmarketable debt. Reassuringly, the two methods deliver similar contributions for four of the six subperiods. The two subperiods in which the two contributions diverge (1981–1993 and 1993–2001) were periods in which long-term bondholders did particularly well. If the maturity structure across the marketable and nonmarketable debt differs substantially, the row 9 way of computing the return will be biased during periods in which the slope of the yield curve is changing dramatically.

Tables 1 and 2 report elements of a decomposition based on equation (11).[15] In particular, for various values of t and τ, Table 1 reports decompositions of the debt-GDP increments $\big((\tilde{B}_t + \overline{B}_t)/Y_t\big) - \big((\tilde{B}_{t-\tau} + \overline{B}_{t-\tau})/Y_{t-\tau}\big)$ by components attributable to nominal interest payments, inflation, GDP growth, and the primary deficit for both the marketable and the nonmarketable portions of the debt. Table 2 then decomposes the nominal interest payments, inflation, and GDP growth components for the marketable debt by maturity.

[15] We report plots of these decomposed series in online Appendix B.

Table 2—Contributions to Changes in the Debt-GDP Ratio Decomposed by Maturity of Marketable Debt

	Period							
	1941–1945	1945–1974	1974–1981	1981–1993	1993–2001	2001–2009	1941–2009	1945–2009
Debt/GDP								
End	97.2	16.9	19.9	48.2	28.5	48.8	48.8	48.8
Start	37.0	97.2	16.9	19.9	48.2	28.5	37.0	97.2
Change	60.2	−80.3	3.0	28.3	−19.7	20.3	11.8	−48.4
Marketable debt								
Nominal returns								
Nominal bonds	3.6	21.8	7.5	36.5	17.6	8.5	95.5	91.9
$j \leq 1$	0.7	10.1	5.0	11.7	5.9	2.0	34.4	34.7
$2 \leq j \leq 4$	0.6	6.1	2.3	12.4	6.0	2.4	29.8	29.3
$j \geq 5$	2.3	5.5	0.2	12.5	5.7	4.1	30.2	27.9
TIPS					0.1	0.4	0.5	0.5
Inflation								
Nominal bonds	−7.3	−35.0	−8.1	−11.6	−5.2	−5.0	−72.4	−65.0
$j \leq 1$	−1.3	−11.4	−4.4	−4.9	−2.0	−2.0	−26.4	−24.7
$2 \leq j \leq 4$	−4.4	−7.4	−2.6	−4.0	−1.8	−1.6	−18.7	−17.4
$j \geq 5$	3.6	−16.2	−1.1	−2.7	−1.4	−1.5	−27.3	−22.9
GDP growth								
Nominal bonds	−15.6	−21.3	−3.3	−10.9	−10.6	−3.2	−64.9	−49.3
$j \leq 1$	−3.3	−8.6	−1.8	−4.6	−4.0	−1.2	−23.4	−20.2
$2 \leq j \leq 4$	−2.7	−6.5	−1.0	−3.8	−3.8	−1.1	−18.8	−16.2
$j \geq 5$	−9.7	0.9	−0.4	−2.6	−2.9	−1.0	−22.6	−12.9
TIPS					−0.1	−0.1	−0.2	−0.2
Nonmarketable debt	−8.0	−14.7	−0.6	−2.4	−5.4	−1.0	−30.6	−22.6
Deficit/GDP	84.7	−34.7	5.8	17.8	−15.9	20.8	78.5	−6.2

Note: The nominal bonds are decomposed into three groups: bonds maturing within one year, $j \leq 1$; bonds maturing between two and four years, $2 \leq j \leq 4$; bonds maturing in five years or more, $j \geq 5$.

Figure 8 plots the inflation rate, the growth rate of real GDP, and the value weighted return on the government's debt portfolio. For the first half of the sample, the growth rate of GDP exceeded the return on the debt, while in the second half of the sample, the return on the government debt exceeded the growth rate.

Tables 1 and 2 and Figures 7 and 8 reveal the following patterns in the way that the United States grew, inflated, and paid its way toward higher or lower debt-GDP ratios:

1. From 1945 to 1974, the debt-GDP ratio fell from 97.2 to 16.9. Of this 80.3 percentage drop,

   (a) 15.8 was due to negative real returns on both the marketable and nonmarketable debt via inflation (Table 1). For the marketable portion of the debt, we see in Table 2 this largely (approximately 10.7 out 11.6) hit the long-term bondholders.[16, 17] The average maturity of the debt was around seven years immediately after WWII.

[16] To compute the real return contribution on the long term (5+) bonds, the real return is approximately equal to the nominal return component minus the inflation component or $5.5 - 16.2 = 10.7$.

[17] Between 1946 and 1955, inflation pushed the price level up by 37.8 percent. Joshua Aizeman and Nancy Marion (2009) divide the initial debt-GDP ratio by 1.378 to estimate the reduction in the debt-GDP ratio contributed by inflation from 1946–1955. During that period, from equation (11), we compute a smaller role for inflation, about 23 percent.

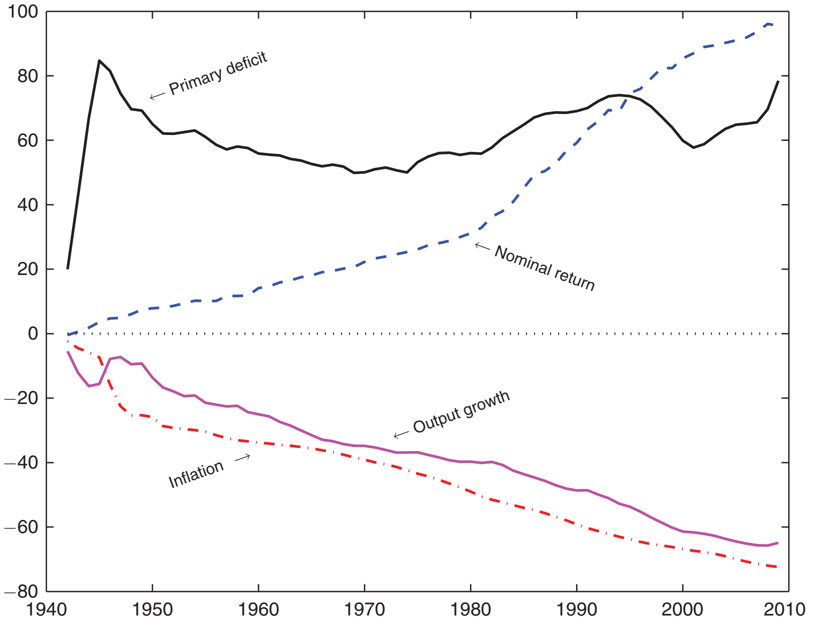

FIGURE 7. CUMULATIVE SUM OF THE COMPONENTS OF THE CHANGE IN THE RATIO OF MARKETABLE DEBT TO GDP

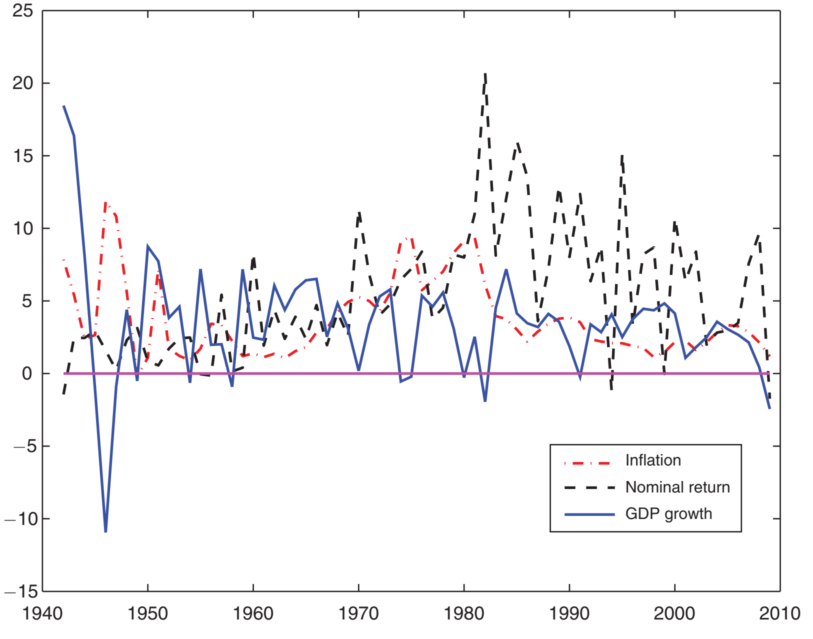

FIGURE 8. RETURN ON GOVERNMENT DEBT, INFLATION, AND GDP GROWTH RATE

Note: The solid line is the growth rate in real GDP, the dot-dashed line is the inflation rate, and the dashed line is the value-weighted nominal return on the government's portfolio of debt.

   (b) 31.8 was due to growth in real GDP.

   (c) 34.7 was due to running primary surpluses.

2. During the 1970s, the United States continued to inflate away part of the debt, but the magnitudes were small.

   (a) Long-term bondholders received negative real returns, but since there was not much debt outstanding (B/Y was less than 0.2), and the average maturity of the debt was low (around 2 years), the government was unable to nail the long-term bondholders as it had done immediately after WWII.

   (b) B/Y continued to grow during the 1970s in spite of the government inflating away part of the debt. The causes were insufficiently rapid real GDP growth and primary deficits.

3. During the President Ronald Reagan-Vice President George H. W. Bush years (1981–1993), the debt-GDP ratio grew from 19.9 in 1981 to 48.2 in 1993, an increase of 28.3 percent.

   (a) Over half of this increase (17.8) came from primary deficits.

   (b) Despite strong GDP growth, B/Y grew by more than the primary deficits due to large real returns paid to bondholders. Returns to long-term bondholders account for 9.8 (12.5–2.7) of the 28.3 increase. Thus, while long-term bondholders were heavily taxed by inflation after WWII, they did very well when Federal Reserve Chairman Paul Volcker brought inflation down during the early 1980s.

4. The reduction in B/Y that occurred during the President Bill Clinton years (1993–2000) was largely driven by primary surpluses. Real returns to bondholders approximately offset the contribution from GDP growth.

5. During the President George W. Bush and President Barack Obama years (2001–2009), primary deficits largely fueled growth in B/Y. As in the previous decade, real returns to bond holders approximately offset GDP growth.

Over the entire postwar period from 1945 to 2009, the debt-GDP ratio fell from 97.2 percent to 48.8 percent. During these 64 years, nominal returns to government creditors of marketable debt exceeded inflation. While the government has at times inflated away its debt, on average, holders of Treasury bills, notes, and bonds were paid positive returns. These returns pushed up the debt-GDP ratio 27.9 percentage points. The government ran primary surpluses in 32 of these 64 years. By this accounting, 1/8 (i.e., 6.2/48.4) of the drop in indebtedness is due to the government simply paying off the debt. But far and away the largest contributor to holding down the debt-GDP ratio was economic growth.

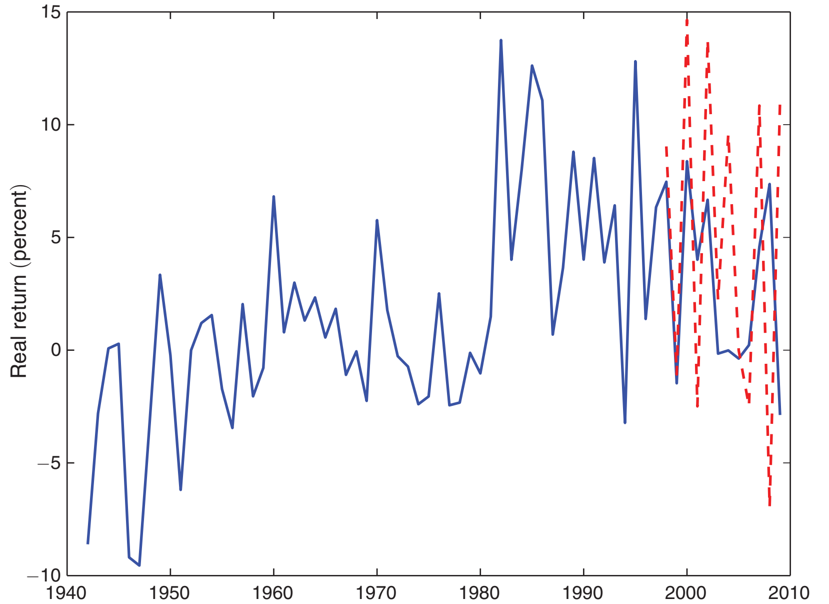

Figure 9. One-Year Holding Period Real Value-Weighted Returns of Nominal Debt and Inflation-Protected Debt

Notes: The solid line is the value-weighted average return on the nominal portion of the debt, namely, $\sum_{j=1}^{n}(\tilde{r}^{j}_{t-1,t}-\pi_{t-1,t})\tilde{B}^{j}_{t-1}/\sum_{j=1}^{n}\tilde{B}^{j}_{t-1}$. The dashed line is the value-weighted average return on the TIPS portion of the debt, namely, $\sum_{j=1}^{n}\overline{r}^{j}_{t-1,t}\overline{B}^{j}_{t-1}/\sum_{j=1}^{n}\overline{B}^{j}_{t-1}$.

Figure 9 plots, $\sum_{j=1}^{n}(\tilde{r}^{j}_{t-1,t}-\pi_{t-1,t})\tilde{B}^{j}_{t-1}/\sum_{j=1}^{n}\tilde{B}^{j}_{t-1}$ and $\sum_{j=1}^{n}\overline{r}^{j}_{t-1,t}\overline{B}^{j}_{t-1}/\sum_{j=1}^{n}\overline{B}^{j}_{t-1}$, which are the value-weighted real one-year holding-period returns on the government's portfolio of nominal and inflation-protected debt, respectively. These two series are quite volatile. The average annual return on the nominal portion of the debt over the entire time period from 1942 to 2009 was 1.6 percent with a standard deviation of 4.9 percent.[18] Figure 9 reveals three especially striking outcomes:

- There were large negative returns immediately after World War II.
- There were large positive returns in the early 1980s after Volcker brought down inflation.[19]
- Annual real returns were considerably more volatile in the period between 1980 and 2006—a period of low volatility in GDP growth often described as the Great Moderation.

[18] In online Appendix C, we report simple calculations to provide some intuition behind the large capital gains and losses on pure discount bonds.

[19] It is interesting to compare these outcomes with predictions of Robert E. Lucas Jr. and Nancy L. Stokey's (1983) model of tax smoothing, according to which government debt pays low returns when there are high government expenditure shocks. See Antje Berndt, Hanno Lustig, and Sevin Yeltekin (2010) for an empirical study and also Hanno Lustig, Christopher Sleet, and Yeltekin (2008).

Table 3—Means and Standard Deviations of Components to Debt-GDP Dynamics: 1942–2009

Variable	Mean	SD
Nominal return on nominal debt	5.36	4.58
Real return on nominal debt	1.63	4.86
Inflation	3.73	2.67
Nominal GDP growth	6.98	3.87
Real GDP growth	3.22	3.47
100 × Deficit to GDP ratio	0.15	2.97
Real return on TIPS (1998–2009)	4.84	7.40
Real return on nominal debt (1998–2009)	2.82	4.01

We see in Table 3 that the average growth rate of real GDP exceeds the sum of the average real return paid to the government's creditors and the average deficit-to-GDP ratio. Finally, it is interesting to note that since the introduction of TIPS, their returns have, on average, exceeded those of the nominal debt. For the TIPS, the real return for the period from 1998 to 2009 is 4.8 percent with a standard deviation of 7.4. For the nominal portion of the debt over this 10-year period, the real return was 2.8 percent with a standard deviation of 4.0.

III. Concluding Remarks

The Congressional Budget Office estimates that the US debt-GDP ratio will return to World War II levels by the end of 2011 as a consequence of recent large primary deficits and drops in GDP growth.[20] This has reawakened concerns that rising government interest payments could eventually unleash inflation or other painful fiscal readjustments via "unpleasant monetarist arithmetic" (Sargent and Neil Wallace 1981).[21] Growing interest payments play a key role in that unpleasant arithmetic. So to frame the tradeoffs and risks facing the United States, it is important to account appropriately for the interest that the US government pays to the public and the abundant interest rate risks that the government shares with its creditors. To account for these payments and risks and to measure their contributions to the evolution of the debt-GDP ratio accurately, we advocate computing the real returns on government debts of each maturity.

Finally, we indicate how the government's way of accounting for interest payments and the quantity of debt might explain a peculiar preference long expressed by experts who are responsible for designing the term structure of coupon payments of US Treasury bonds.[22] The authorities have sought to set the coupon rate on a long-term Treasury bond in a way that makes the initial market value of a bond equal to its

[20] See table 1.1 on page 2 of "The Budget and Economic Outlook: An Update," *Congressional Budget Office*, August 2009.

[21] See, for example, Edward Andrews' article in the November 22, 2009 *New York Times* "Payback Time: Wave of Debt Payments Facing US Government," and Michael Kinsley's column in the April 2010 issue of *The Atlantic* "My Inflation Nightmare: Am I Crazy, or is the Commentariat Ignoring our Biggest Threat."

[22] Today, the Treasury leans heavily on the advice of experts from the financial community who are members of the Treasury Borrowing Advisory Committee.

par value.[23] It is impossible to understand such a preference by using, for example, the theory of optimal debt management provided by Lucas and Stokey (1983).

But consider the following imperfect rationalization based on the government's *reported* measure of interest payments and also its practice of reporting the par value rather than the market value of its debt.[24] Recall that when a coupon bond sells at par, its yield to maturity equals its coupon rate. Assuming an approximately flat term structure of interest rates, if the coupon rate is set so that the market value is near the par value, then, at least initially, the government's accounting methods do a good job of approximating both the market value and the interest payments that belong in the government budget constraint.

Appendix: Reconciling Our Estimates with the Government's

As documented earlier by Hall and Sargent (1997), our estimates of the interest paid on US government debt differ substantially from those reported by the government. In this Appendix, we isolate the differences between our way of accounting for interest and the government's. Since they give different answers, these two accounting systems must be asking different questions. Our series answers the question "what returns appear in the law of motion over time of real government indebtedness?"[25] What question does the government's interest payment series answer? And how can we compute it in terms of the objects $q_{t+j}^{t}, \overline{q}_{t+j}^{t}, s_{t+j}^{t}, \overline{s}_{t+j}^{t}$ defined in Section II?

According to the *Code of Federal Regulation*,[26]

> Interest on bills consists of the difference between the discounted amount paid by the investor at original issue and the par value we pay to the investor at maturity. Interest on notes and bonds accrues from the dated date. Interest is payable on a semiannual basis on the interest payment dates specified in the auction announcement through the maturity date. If any principal or interest payment date is a Saturday, Sunday, or other day on which the Federal Reserve System is not open for business, we will make the payment (without additional interest) on the next business day.

Thus, the government computes interest expenses by adding next year's coupon payments on Treasury notes and bonds to the product of the stock of Treasury bills and the associated one-period holding-period return on those bills. To cast the government's computations in terms of our notation, it is useful to define the decomposition

[23] See Davis Rich Dewey (1902, chapter XIII) for an account of how these preferences played a significant role in controversies surrounding the design of bonds by the US Congress during the Civil War. In particular, Dewey discusses the failure of debt issues in 1862 and 1864 due to the Treasury's refusal to sell bonds below their par value. In 1864, Treasury Secretary Salmon Chase insisted on lowering the government's interest payments by issuing 5 percent coupon bonds (i.e., the ten-forties) in the place of 6 percent coupon bonds (i.e., the five-twenties). By insisting that these new bonds be sold at par, despite current market interest rates that could not support that price, the initial issue of the ten-forties was (in Dewey's words) "a disaster."

[24] See the discussion of Figure II in online Appendix A.

[25] The law of motion of real government indebtedness is also known as the government budget constraint.

[26] US Government Printing Office, *Code of Federal Regulations*, Volume 2, Title 31—Money and Finance: Treasury, Chapter II—Fiscal Service, Department of the Treasury, Part 356 Sale and Issue of Marketable Book-Entry Treasury Bills, Notes and Bonds, Subpart D—Miscellaneous Provisions, Section 356.30, Revised July 1, 2009. See http://edocket.access.gpo.gov/cfr_2009/julqtr/pdf/31cfr356.30.pdf See Appendix B of Part 356 of the *Code of Federal Regulations* for mathematical examples of the computation of interest on Treasury bonds and notes.

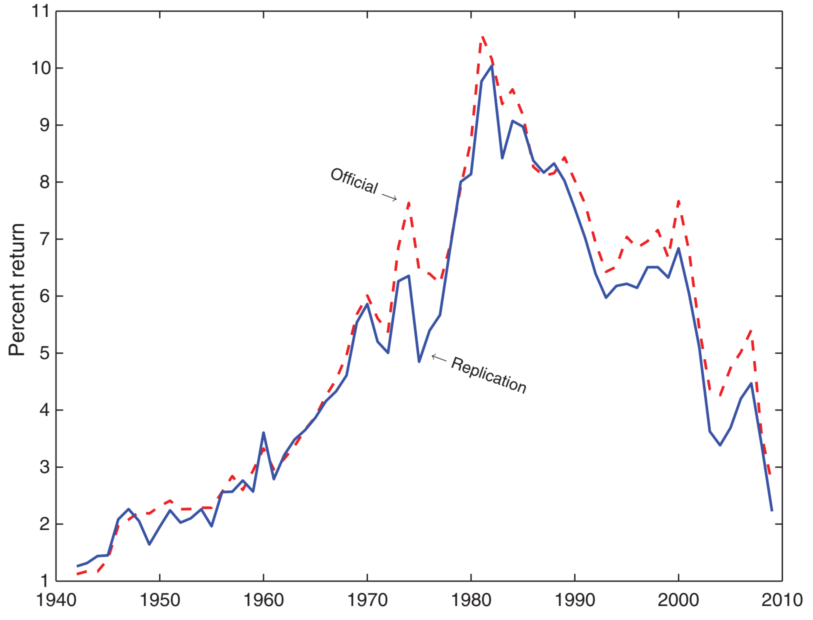

Figure 10. Replicating the Government's Reported Interest Payments

Notes: The dashed line is the officially reported interest costs divided by the outstanding value of the total debt held by the public. The solid line is equation (12) divided by the outstanding value of the marketable debt held by the public.

$s_t^{t-1} = s_t^{t-1}(tb) + s_t^{t-1}(p) + s_t^{t-1}(c)$, where $s_t^{t-1}(tb)$ represents the par value of one-period pure discount Treasury bills; $s_t^{t-1}(p)$ denotes the contribution to s_t^{t-1} coming from principal due on longer term notes and bonds that mature at t; and $s_t^{t-1}(c)$ represents coupon payments on notes and bonds accruing at time t.

The government reports the following object as its nominal interest payments at time t:

$$s_t^{t-1}(c) + v_t^{-1}\overline{s}_t^{t-1}(c) + (1 - q_t^{t-1})s_t^{t-1}(tb). \tag{12}$$

The term $s_t^{t-1}(c)$ in the first expression is the nominal value of the coupon payments on nominal bonds, while $v_t^{-1}\overline{s}_t^{t-1}(c)$ is the nominal value of coupon payments on indexed bonds. The term $(1 - q_t^{t-1})s_t^{t-1}(tb)$ is the government's estimate of the nominal payments on Treasury bills. Thus, the government's estimate answers the following accounting question: "How many dollars must the government come up with this period to pay the coupons due on its debt while rolling over its stock of Treasury bills?" It is worthwhile to have an answer to this interesting question, but it is not the question that our alternative concept of returns seeks to answer.

In Figure 10, we plot the government's official interest payments series and our concept (12). Since the government's series includes interest payments on both the marketable and nonmarketable debt held by the public, while our data covers just the marketable debt held by the public, we divide our concept by the outstanding value of the marketable debt held by the public. The two series track each other quite closely. The correlation coefficient for the two series is 0.99.

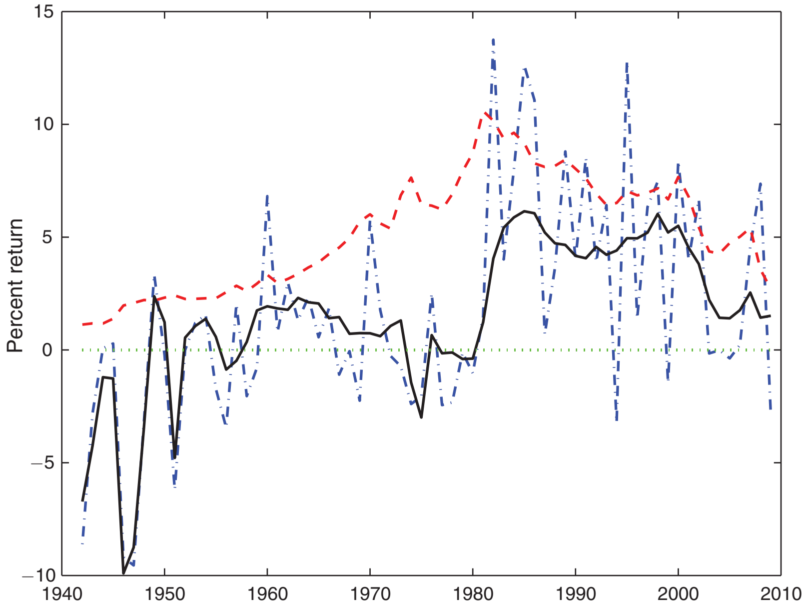

Figure 11. A Comparison of the Government's Reported Interest Payments and Our Estimates of Interest Payments

Notes: The dot-dashed line is our computed value-weighted return on the marketable debt. The dashed line is the government's reported interest payments divided by the total debt held by the public. The solid line is the dashed line minus the inflation rate.

Table 4—Means and Standard Deviations of Percent Returns: 1942–2009

Variable	Mean	SD
Official interest/debt	5.20	2.54
Inflation	3.73	2.67
Official interest/debt—inflation	1.47	3.31
Real return on nominal debt	1.63	4.86

In Figure 11, we contrast the Federal Government official interest payment series with our interest payment series using annual end of the year data from 1941 to 2009. In this graph, we report both our measure of interest paid (dashed-dotted line) and the government's reported interest payments (dashed line) as percentages of the market value of debt. As can be seen in this figure, our series is lower, on average, and considerably more volatile than the government's. As we report in Table 4, the official interest payments average 5.20 percent of the debt, while our measure of the real return on the debt averages 1.47. We then subtract the inflation rate from officially reported interest payments (solid line). The two series have roughly the same mean (1.47 versus 1.63). Until the 1980s, it appears that much of the difference between the reported series and our series was due to inflation.[27] Post-1980

[27] Whether or not the two series resemble each other after adjusting for inflation depends partly on debt management policy. For example, as mentioned in the introduction, there exist debt management policies that can set the government's interest payment series always to be identically zero.

something else was going on, namely, nominal interest rate risk that, in a lower and less volatile inflation environment, translated into real interest rate risk.

Pinpointing Discrepancies between the Government's Interest Payments Estimates and Ours.—Rewriting the government's concept of interest payments (12) as

$$\left\{ s_t^{t-1}(c) + v_t^{-1} \overline{s}_t^{t-1}(c) \right\} + \tilde{r}_{t-1,t}^1 s_t^{t-1}(tb) \tag{13}$$

isolates the sources of the discrepancies between the government's way of accounting for interest payments and ours. This expression reveals the following differences between the two accounting systems:

- The term in braces is total coupon payments. But coupon payments should not be viewed purely as interest payments because they are partly principal repayments, partly interest payments. Our accounting method takes that into account, but the government's does not.
- The term $\tilde{r}_{t-1,t}^1 s_t^{t-1}(tb)$ correctly measures a *part* of government interest payments according to our budget-constraint-driven definition (1) or (2), namely, the capital gains or losses that the government pays on its *one-period* zero coupon bonds; but …
- Expression (13) evidently omits the capital gains or losses that the government pays on its zero-coupon bonds of maturities longer than one period. One period holding-period returns $\tilde{r}_{t-1,t}^j, \overline{r}_{t-1,t}^j$ and promised coupon payments for maturities j exceeding 1 do not appear in (13), but they do appear in (2).

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