Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond

By ODED GALOR AND DAVID N. WEIL*

This paper develops a unified growth model that captures the historical evolution of population, technology, and output. It encompasses the endogenous transition between three regimes that have characterized economic development. The economy evolves from a Malthusian regime, where technological progress is slow and population growth prevents any sustained rise in income per capita, into a Post-Malthusian regime, where technological progress rises and population growth absorbs only part of output growth. Ultimately, a demographic transition reverses the positive relationship between income and population growth, and the economy enters a Modern Growth regime with reduced population growth and sustained income growth. (*JEL* J13, O11, O33, O40)

This paper analyzes the historical evolution of the relationship between population growth, technological change, and the standard of living. It develops a unified model that encompasses the transition between three distinct regimes that have characterized the process of economic development: the "Malthusian Regime," the "Post-Malthusian Regime," and the "Modern Growth Regime." We view the unified modeling of this long transition process, from thousands of years of Malthusian stagnation through the demographic transition to modern growth, as one of the most significant research challenges facing economists interested in growth and development.

The analysis focuses on the two most important differences between these regimes from a macroeconomic viewpoint: first, in the behavior of income per capita; and second, in the relationship between the level of income per capita and the growth rate of population.

The Modern Growth Regime is characterized by steady growth in both income per capita and the level of technology. In this regime there is a negative relationship between the level of output and the growth rate of population: the highest rates of population growth are found in the poorest countries, and many rich countries have population growth rates near zero.

At the other end of the spectrum is the Malthusian Regime in which technological progress and population growth were glacial by modern standards, and income per capita was roughly constant. Further, the relationship between income per capita and population growth was the opposite of that which exists in the Modern Growth Regime: "The most decisive mark of the prosperity of any country," observed Adam Smith (1776), "is the increase in the number of its inhabitants."

The Post-Malthusian Regime, which occurred between the Malthusian and Modern

* Galor: Department of Economics, Brown University, Providence, RI 02912, and Department of Economics, Hebrew University, Jerusalem 91905, Israel (e-mail: Oded_Galor@brown.edu); Weil: Department of Economics, Brown University, Providence, RI 02912 (e-mail: David_Weil@brown.edu). The authors thank the referees and seminar participants at Bar-Ilan University; Boston College; Boston University; the Board of Governors of the Federal Reserve; Brandeis, Brown, Columbia, Cambridge, Harvard, and Hebrew Universities; the Institute of International Economic Studies, Stockholm University; the London School of Economics; the University of Michigan; Michigan State University; the Federal Reserve Bank of Minneapolis; the Massachusetts Institute of Technology; Pennsylvania State University; the Universities of Rochester and Rome; Tel Aviv University; the University of Toulouse; University College, London; Uppsala and Yale Universities; the 1997 European Science Foundation Growth conference, Castelvecchio Pascoli, Italy; the 1998 Aiyagari's Memorial Conference, Haifa, Israel; the 1999 Technology, Human Capital, and Growth conference, Hebrew University, Jerusalem; the 1999 Dynamics, International Trade, and Growth conference, Tilburg University; the 2000 Theories and Method in Macroeconomics Conference, Paris; and the National Bureau of Economic Research Summer Institute, Cambridge, 1999, for useful comments. Galor's research is supported by National Science Foundation Grant No. SBR-9709941.

Growth Regimes, shared one characteristic with each of them. Income per capita grew during this period, although not as rapidly as it would during the Modern Growth Regime. At the same time, the Malthusian relationship between income per capita and population growth was still in place. Rising income was reflected in rising population growth rates.

The most basic description of the relation between population growth and income was proposed by Thomas R. Malthus (1798). The Malthusian model has two key components. The first is the existence of some factor of production, such as land, which is in fixed supply, implying decreasing returns to scale for all other factors. The second is a positive effect of the standard of living on the growth rate of population.

According to Malthus, when population size is small, the standard of living will be high, and population will grow as a natural result of passion between the sexes. When population size is large, the standard of living will be low, and population will be reduced by either the "preventive check" (intentional reduction of fertility) or by the "positive check" (malnutrition, disease, and famine).

The Malthusian model implies that, in the absence of changes in technology or in the availability of land, the size of the population will be self-equilibrating. Further, increases in available resources will, in the long run, be offset by increases in the size of the population. Countries with superior technology will have denser populations, but the standard of living will not be related to the level of technology, either over time or across countries.

The Malthusian model's predictions are consistent with the evolution of technology, population, and output per capita for most of human history. For thousand of years, the standard of living was roughly constant and did not differ greatly across countries. As depicted in Figure 1, Angus Maddison (1982) estimates that the growth rate of GDP per capita in Europe between 500 and 1500 was zero. Furthermore, Ronald D. Lee (1980) reports that the real wage in England was roughly the same in 1800 as it had been in 1300. According to Kang Chao's (1986) analysis, real wages in China were lower at the end of the eighteenth century than they had been at the beginning of the first century. Joel Mokyr (1990), Lant Pritchett (1997), and Robert E. Lucas, Jr. (1999) argue that even in the richest countries, the phenomenon of sustained growth in living standards is only a few centuries old.

Similarly, the pattern of population growth is consistent with the predictions of the Malthusian model. Population growth was nearly zero, reflecting the slow pace of technological progress. As depicted in Figure 1, the rate of population growth in Europe between the years 500 and 1500 was 0.1 percent per year. Furthermore, Massimo Livi-Bacci (1997) estimates the growth rate of world population from the year 1 to 1750 at 0.064 percent per year.

Fluctuations in population and wages also bear out the predictions of the Malthusian model. Lee (1997) reports positive income elasticities of fertility and negative income elasticities of mortality from studies examining a wide range of preindustrial countries. Similarly, Edward A. Wrigley and Roger S. Schofield (1981) find that there was a strong positive correlation between real wages and marriage rates in England over the period 1551–1801. Negative shocks to population, such as the Black Death, were reflected in higher real wages and faster population growth (Livi-Bacci, 1997).

Finally, the prediction of the Malthusian model that differences in technology should be reflected in population density but not in standards of living is also borne out. As argued by Richard Easterlin (1981), Pritchett (1997), and Lucas (1999), prior to 1800 differences in standards of living among countries were quite small by today's standards; yet there did exist wide differences in technology. China's sophisticated agricultural technologies, for example, allowed high per-acre yields, but failed to raise the standard of living above subsistence. Similarly in Ireland a new productive technology—the potato—allowed a large increase in population over the century prior to the Great Famine without any improvement in standards of living (Livi-Bacci, 1997). Using this interpretation, Michael Kremer (1993) argues that changes in the size of population can be taken as a direct measure of technological improvement.

Ironically, it was only shortly before the time that Malthus wrote that humanity began to emerge from the trap that he described. As is apparent from Figure 1 the process of emergence from the Malthusian trap was a slow one. The figure shows the growth rate of total

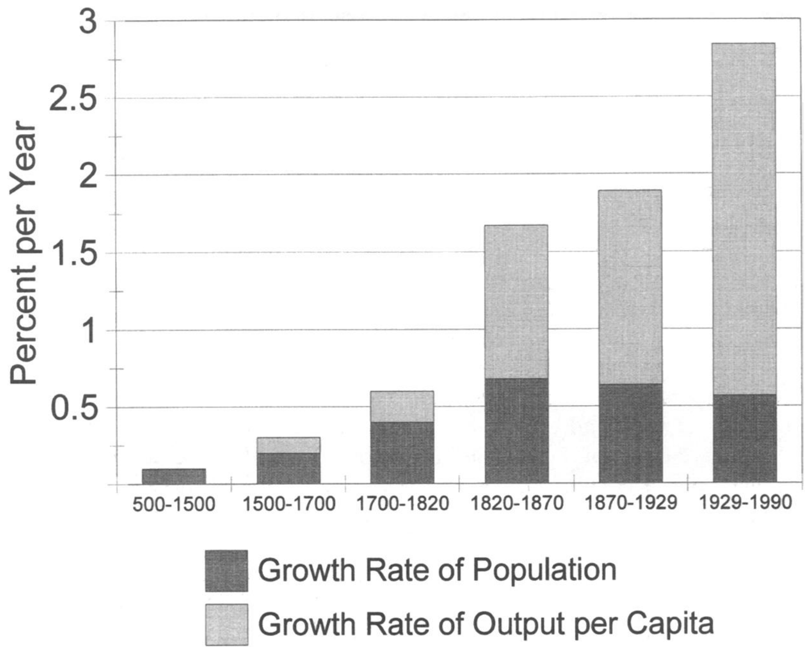

FIGURE 1. OUTPUT GROWTH IN WESTERN EUROPE, 500–1990

Notes: Data from 500–1820 are from Angus Maddison (1982) and apply to Europe as a whole. Data for 1820–1990 are from Maddison (1995), Table G, and apply to Western Europe.

output in Western Europe between the years 500 and 1990, as well as the breakdown between growth of output per capita and growth of population. The growth rate of total output in Europe was 0.3 percent per year between 1500 and 1700, and 0.6 percent per year between 1700 and 1820. In both periods, two-thirds of the increase in total output was matched by increased population growth, so that the growth of income per capita was only 0.1 percent per year in the earlier period and 0.2 percent in the later one. In the United Kingdom, where growth was the fastest, the same rough division between total output growth and population growth can be observed: total output grew at an annual rate of 1.1 percent in the 120 years after 1700, whereas population grew at an annual rate of 0.7 percent over that period.

Thus the initial effect of faster income growth in Europe was to increase population. Income per capita rose much more slowly than did total output, and as income per capita rose, population grew ever more quickly. Only the fact that output growth accelerated allowed income per capita to continue rising. During this Post-Malthusian Regime, the Malthusian mechanism linking higher income to higher population growth continued to function, but the effect of higher population on diluting resources per capita, and thus lowering income per capita, was counteracted by technological progress, which allowed income to keep rising.

Both population and income per capita continued to grow after 1820, but increasingly the growth of total output was expressed as growth of income per capita. Indeed, whereas the rate of total output growth increased, the rate of growth of population peaked in the nineteenth century and then began to fall. Population growth was 40 percent as large as total output growth over the period 1820–1870, but only 20 percent as large as total output growth over the period 1929–1990. Over the next several decades much of Western Europe is forecast to have negative population growth.

The dynamics of population growth reflected both changes in constraints and qualitative changes in household behavior induced by the economic environment. The Malthusian demographic regime had been characterized by high levels of both fertility and mortality. As living standards rose, mortality fell. Between the 1740's and the 1840's, life expectancy at birth rose from 33 to 40 in England and from 25 to 40 in France (Livi-Bacci, 1997). Robert Fogel (1997) estimates that 85 percent of the decline in mortality in France between 1785 and 1870 was simply the result of better nutrition. Mortality reductions led to growth of the population both because more children reached breeding age and because each person lived for a greater number of years.

The initial effect of higher income was also to raise fertility directly, primarily by raising the propensity to marry. Fertility rates increased in most of Western Europe until the second half of the nineteenth century, peaking in England and Wales in 1871 and in Germany in 1875 (Tim Dyson and Mike Murphy, 1985; Ansley J. Coale and Roy Treadway, 1986). Thus, in Malthusian terms, the positive check was being weakened and the preventive check was being less assiduously enforced. But as income continued to rise, population growth fell further below the maximum rate that could be sustained given the mortality regime. The reduction in fertility was most rapid in Europe around the turn of the twentieth century. In England, for example, live births per 1,000 women aged 15–44 fell from 153.6 in 1871–1880 to 109.0 in 1901–1910 (Wrigley, 1969). Notably, the reversal of the Malthusian relation between income and population growth corresponded to an increase in the level of resources invested in each child. For example, the average number of years of schooling in England and Wales rose from 2.3 for the cohort born between 1801 and 1805 to 5.2 for the cohort born 1852–1856 and 9.1 for the cohort born 1897–1906 (Robert C. O. Matthews et al., 1982).

This historical evidence suggests that the key event that separates the Malthusian and Post-Malthusian Regimes is the acceleration in the pace of technological progress, whereas the event that separates the Post-Malthusian and Modern Growth eras is the demographic transition that followed the industrial revolution. The emergence from the Malthusian trap and the onset of the demographic transition raise intriguing questions. How was the link between income per capita and population growth, which had for so long been a constant of human existence, so dramatically severed? How does one account for the sudden spurt in growth rates? Is there a unified framework of analysis that can account for this intricate evolution of population, technology, and growth throughout human history?

Neoclassical growth models with exogenous population clearly are unable to capture this intricate transition process. Further, the existing literature on the relation between population growth and output has tended to focus on only one of the regimes described earlier. The majority of the literature has been oriented toward the modern regime, trying to explain the negative relation between income and population growth, either cross-sectionally or within a single country over time (e.g., Robert J. Barro and Gary S. Becker, 1989). Among the mechanisms highlighted in this literature are that higher returns to child quality in developed economies induce a substitution of quality for quantity (Becker et al., 1990); that developed economies pay higher relative wages of women, thus raising the opportunity cost of children (Galor and Weil, 1996); and that the net flow of transfers from parents to children grows (and possibly switches from negative to positive) as countries develop (John W. Caldwell, 1976).[1] The negative effect of high income on fertility is often examined in conjunction with a model in which high fertility has a negative effect on income as a result of capital dilution. Recent papers that are concerned with the Malthusian Regime are Kremer (1993) and Lucas (1999). Lucas presents a Malthusian model in which households optimize over fertility and consumption, whereas in Kremer (1993) a feedback loop between technology and population generates a transition from the proximity of a Malthusian equilibrium to the Post-Malthusian Regime.[2]

[1] See Isaac Ehrlich and Fracis Lui (1997), James A. Robinson and T. N. Srinivasan (1997), and T. Paul Schultz (1997) for surveys of the literature in this area, and Richard R. Nelson (1956) and Momi Dahan and Daniel Tsiddon (1998) for an alternative mechanism.

[2] To generate a demographic transition, Kremer assumes that population growth increases with income at low levels of income and then decreases with income at high levels of income. Another strand of literature (Marvin Goodfriend and John McDermott, 1995; Daron Acemoglu and Fabrizio Zilibotti, 1997) has attempted to model the acceleration of

This paper accounts for the transition from the Malthusian Regime, through the Post-Malthusian Regime and the demographic transition, to the Modern Growth Regime in a unified model. At the heart of our model is a novel explanation for the reduction in fertility that has allowed income per capita to rise so far above subsistence. Most studies of the demographic transition focus on the effect of a high *level* of income in inducing parents to switch to having fewer, higher-quality children. In our model, parents also switch out of quantity and into quality, but do so not in response to the level of income but rather in response to technological progress. The "disequilibrium" brought about by technological change raises the rate of return to human capital, and thus induces the substitution of quality for quantity.

The argument that technological progress itself raises the return to human capital was most clearly stated by Theodore W. Schultz (1964). Examining agriculture, Schultz argued that when productive technology has been constant for a long period of time, farmers will have learned to use their resources efficiently. Children will acquire knowledge of how to deal with this environment directly from observing their parents, and formal schooling will have little economic value. But when technology is changing rapidly, the knowledge gained from observing the previous generation will be less valuable and the trial-and-error process, which led to a high degree of efficiency under static conditions, will not have had time to function. New technology will create a demand for the ability to analyze and evaluate new production possibilities, which will raise the return to education.[3] Such an effect would be a natural explanation for the dramatic rise in schooling in Europe over the course of the nineteenth century.

The effect of technology on the return to human capital in which we are most interested is the short-run impact of a new technology. In the long run, technologies may be "skill biased" or "skill saving." But we would argue that the introduction of new technologies is mostly skill biased.[4] If technological changes are skill biased in the long run, then the effect on which we focus will be enhanced, whereas if technology is skill saving it will be diluted.

The second piece of the model is more straightforward: the choice of parents regarding the education level of their children affects the speed of technological progress. Children with high levels of human capital are, in turn, more likely to advance the technological frontier or to adopt advanced technologies.[5]

The third piece of the model links the size of the population to the rate of technological progress and to the take-off from the Malthusian Regime. Holding the level of education constant, the speed of technological progress is also a positive function of the overall size of the population. For a given level of education, higher population generates a larger supply, larger demand, and more rapid diffusion of new ideas.

The final piece of the model is the most Classical. The economy is characterized by the existence of a fixed factor of production, land, and a subsistence level of consumption below which individuals cannot survive. If technological progress permits output per worker to exceed the subsistence level of consumption, population rises, the land-labor ratio falls, and, in the absence of further technological progress, wages fall back to the subsistence level of consumption. Income per capita is therefore self-equilibrating. Sustained technological progress, however, can overcome the offsetting effect of population growth, allowing sustained income growth.

The model produces a Malthusian "pseudo steady state" that is stable over long periods of time, but vanishes endogenously in the long run. In this Malthusian regime output per capita is stationary. Technology progresses only slowly, and is reflected in proportional increases in output

output growth at the time of the Industrial Revolution without considering the determinants of population growth. See also Assaf Razin and Uri Ben-Zion (1975), Zvi Eckstein et al. (1988), John Komlos and Mark Artzrouni (1990) and Lakshmi K. Raut and T. N. Srinivasan (1994).

[3] Schultz (1975) cites a wide range of evidence in support of this theory. Similarly, Andrew D. Foster and Mark R. Rosenzweig (1996) find that technological change during the green revolution in India raised the return to schooling, and that school enrollment rates responded positively to this higher return.

[4] See Galor and Tsiddon (1997) and Claudia Goldin and Lawrence F. Katz (1998).

[5] This link between education and technological change was first proposed by Nelson and Edmund S. Phelps (1966). For supportive evidence see Easterlin (1981) and Mark Doms et al. (1997).

and population. Shocks to the land to labor ratio will induce temporary changes in the real wage and fertility, which will in turn drive income per capita back to its stationary equilibrium level. Because technological progress is slow, the return to human capital is low, and parents have little incentive to substitute child quality for quantity. The Malthusian pseudo steady state vanishes in the long run because of the impact of population size on the rate of technological progress. At a sufficiently high level of population, the rate of population-induced technological progress is high enough that parents find it optimal to provide their children with some human capital. At this point, a virtuous circle develops: higher human capital raises technological progress, which in turn raises the value of human capital.

Increased technological progress initially has two effects on population growth. On the one hand, improved technology eases households' budget constraints, allowing them to spend more resources on raising children. On the other hand, it induces a reallocation of these increased resources toward child quality. In the Post-Malthusian Regime, the former effect dominates, and so population growth rises. Eventually, however, more rapid technological progress resulting from the increase in the level of human capital triggers a demographic transition: wages and the return to child quality continue to rise, the shift away from child quantity becomes more significant, and population growth declines. In the Modern Growth Regime, technology and output per capita increase rapidly, whereas population growth is moderate.

The rest of this paper is organized as follows. Section I formalizes the assumptions about the determinants of fertility and relative wages presented earlier, and incorporates them into an overlapping generations model. Section II derives the dynamical system implied by the model, and analyzes the evolution of the economy along transitions to the steady state. Section III concludes.

I. The Basic Structure of the Model

Consider an overlapping-generations economy in which activity extends over infinite discrete time. In every period the economy produces a single homogeneous good, using land and efficiency units of labor as inputs. The supply of land is exogenous and fixed over time. The number of efficiency units of labor is determined by households' decisions in the preceding period regarding the number and level of human capital of their children.

A. *Production of Final Output*

Production occurs according to a constant-returns-to-scale technology that is subject to endogenous technological progress. The output produced at time t, Y_t, is

$$Y_t = H_t^{\alpha}(A_t X)^{1-\alpha}, \tag{1}$$

where X and H_t are the quantities of land and efficiency units of labor employed in production at time t, $\alpha \in (0, 1)$, and $A_t > 0$ represents the endogenously determined technological level at time t. The multiplicative form in which technology (A_t) and land (X) appear in the production function implies that the relevant factor for the output produced is the product of the two, which we define as "effective resources."

Output per worker produced at time t, y_t, is

$$y_t = h_t^{\alpha} x_t^{(1-\alpha)} \equiv y(h_t, x_t), \tag{2}$$

where $y_h(h_t, x_t) > 0$ and $y_x(h_t, x_t) > 0 \; \forall (h_t, x_t) \geqslant 0$, $h_t \equiv H_t/L_t$ is the number of efficiency units of labor per worker and $x_t \equiv (A_t X)/L_t$ is the amount of effective resources per worker at time t.

Suppose that there are no property rights over land. The return to land is therefore zero, and the wage per efficiency unit of labor is therefore equal to its average product:

$$w_t = (x_t/h_t)^{1-\alpha} \equiv w(h_t, x_t), \tag{3}$$

where $w_h(h_t, x_t) < 0$ and $w_x(h_t, x_t) > 0$, $\forall (h_t, x_t) \geqslant 0$.

We base the modeling of the production side on two simplifying assumptions. First, capital is not an input in the production function; second, the return to land is zero. Alternatively we could have assumed that the economy is small and open to a world capital market in which the interest rate is constant. In this case, the quantity of capital will be set to equalize its marginal product to the interest rate, whereas the price of land will follow a path such that the total return on land (rent plus net price appreciation) is also equal to the interest rate. This is the case presented in Galor and Weil (1998). Capital, however, has no role in the mechanism that we examine, and the qualitative results would not be affected if the supply of capital were endogenously determined.[6] Allowing for capital accumulation and property rights over land would complicate the model to the point of intractability.

B. *Preferences and Budget Constraints*

In each period t a generation that consists of L_t identical individuals joins the labor force. Each individual has a single parent. Members of generation t live for two periods. In the first period of life (childhood), $t - 1$, individuals consume a fraction of their parent's time. The required time increases with children's quality. In the second period of life (parenthood), t, individuals are endowed with one unit of time, which they allocate between child-rearing and labor force participation. They choose the optimal mixture of quantity and quality of children and supply their remaining time in the labor market, consuming their wages.

The preferences of members of generation t are defined over consumption above a subsistence level $\tilde{c} > 0$, as well as over the potential aggregate income of their children. They are represented by the utility function[7]

$$u^t = (c_t)^{(1-\gamma)}(w_{t+1}n_t h_{t+1})^{\gamma}, \tag{4}$$

where n_t is the number of children of individual t, h_{t+1} is the level of human capital of each child, and w_{t+1} is the wage per efficiency unit of labor at time $t + 1$.

The utility function is strictly monotonically increasing and strictly quasi-concave, satisfying the conventional boundary conditions that ensure that, for sufficiently high income, there exists an interior solution for the utility maximization problem. However, for a sufficiently low level of income the subsistence consumption constraint is binding and there is a corner solution with respect to the consumption level.[8]

Following the standard model of household fertility behavior (Becker, 1960) the household chooses the number of children and their quality in the face of a constraint on the total amount of time that can be devoted to child-raising and labor-market activities. We further assume that the only input required to produce both child quantity and child quality is time.[9] Since all members of a generation are identical in their endowments, the budget constraint is not affected if child quality is produced by professional educators rather than by parents.

Let $\tau^q + \tau^e e_{t+1}$ be the time cost for a member of generation t of raising a child with a level of education (quality) e_{t+1}. That is, τ^q is the fraction of the individual's unit time endowment that is required to raise a child, regardless of quality, and τ^e is the fraction of the individual's unit time endowment that is required for each unit of education for each child.

Consider members of generation t who are endowed with h_t efficiency units of labor at time t. Define potential income z_t as the amount that they would earn if they devoted their entire time endowment to labor-force participation: $z_t \equiv w_t h_t$. Potential income is divided between expenditure on child-rearing (quantity as well

[6] An alternative mechanism to deal with land in the model would be to assume that land is owned by a small fraction of the population, which consumes the rents that it receives and which has a negligible impact on the evolution of population.

[7] The second component of the utility function may represent either intergenerational altruism or implicit concern about potential support from children in old age. The interpretation that emphasizes intergenerational altruism reflects an implicit bounded rationality on the part of the parent. Alternative formulations, according to which individuals generate utility from the utility of their children or from the *actual* aggregate income of their offspring, would require parental predictions about fertility choices of their dynasty. These approaches would greatly complicate the model and we conjecture that they would not affect the qualitative results.

[8] As will become clear below, the presence of a subsistence consumption constraint provides the Malthusian piece of our model. The formulation that we use implicitly stresses a "demand" explanation for the positive income elasticity of population growth at low-income levels, since higher income will allow individuals to afford more children. However, one could also cite "supply" factors, such as declining infant mortality and increased natural fertility, to explain the same phenomenon. See Nancy Birdsall (1988) and Randall J. Olsen (1994).

[9] If both time and goods are required to produce child quality, the process we describe would be intensified. As the economy develops and wages increase, the relative cost of a quality child will diminish and individuals will substitute quality for quantity of children.

as quality), at an opportunity cost of $w_t h_t[\tau^q + \tau^e e_{t+1}]$ per child, and consumption c_t.

Hence, in the second period of life (parenthood), the individual faces the budget constraint

$$(5) \quad w_t h_t n_t(\tau^q + \tau^e e_{t+1}) + c_t \le w_t h_t.$$

C. *The Production of Human Capital*

An individual's level of human capital is determined by his/her quality (education) as well as by the technological environment. Incorporating the insight of Schultz (1964), discussed earlier, technological progress is assumed to raise the value of education in producing human capital. The level of human capital of children of members of generation t, h_{t+1}, is an increasing function of their education e_{t+1}, and a decreasing function of the rate of progress in the state of technology from period t to period $t + 1$, $g_{t+1} \equiv (A_{t+1} - A_t)/A_t$. The higher the children's quality, the smaller the adverse effect of technological progress.

$$(6) \qquad h_{t+1} = h(e_{t+1}, g_{t+1}),$$

where $h(e_{t+1}, g_{t+1}) > 0$, $h_e(e_{t+1}, g_{t+1}) > 0$, $h_{ee}(e_{t+1}, g_{t+1}) < 0$, $h_g(e_{t+1}, g_{t+1}) < 0$, $h_{gg}(e_{t+1}, g_{t+1}) > 0$, and $h_{eg}(e_{t+1}, g_{t+1}) > 0 \; \forall (e_{t+1}, g_{t+1}) \ge 0$.

Hence, the individual's level of human capital is an increasing, strictly concave function of education, and a decreasing strictly convex function of the rate of technological progress.[10] Furthermore, education lessens the adverse effect of technological progress. That is, technology complements skills in the production of human capital.

Moreover, although the number of efficiency units of labor per worker is diminished during the transition from one technological state to another—the "erosion effect"—the *effective* number of the efficiency units of labor per worker, which is the product of the workers' level of human capital and the economy's technological state (reflected in the wage per efficiency unit of labor), is assumed to be higher as a result of technological progress. That is, $\partial y(h_t, x_t)/\partial g_t > 0$.

D. *Optimization*

Members of generation t choose the number and quality of their children, and therefore their own consumption, so as to maximize their intertemporal utility function. Substituting (5) and (6) into (4), the optimization problem of a member of generation t is

$$(7) \quad \{n_t, e_{t+1}\} = \operatorname{argmax}\{w_t h_t[1 - n_t(\tau^q + \tau^e e_{t+1})]\}^{1-\gamma} \times \{(w_{t+1} n_t h(e_{t+1}, g_{t+1})\}^{\gamma}$$

subject to

$$w_t h_t[1 - n_t(\tau^q + \tau^e e_{t+1})] \ge \tilde{c};$$

$$(n_t, e_{t+1}) \ge 0.$$

The optimization with respect to n_t implies that, as long as potential income at time t is sufficiently high so as to ensure that $c_t > \tilde{c}$, the time spent by individual t raising children is γ, whereas $1 - \gamma$ is devoted for labor-force participation. However, for low levels of potential income, the subsistence constraint binds. The individual consumes the subsistence level $\tilde{c}$, and uses the rest of the time endowment for child-rearing.

Let $\tilde{z}$ be the level of potential income at which the subsistence constraint is just binding; that is, $\tilde{z} \equiv \tilde{c}/(1 - \gamma)$). It follows that for $z_t \ge \tilde{c}$

$$(8) \qquad n_t[\tau^q + \tau^e e_{t+1}] = \begin{cases} \gamma & \text{if } z_t \ge \tilde{z} \\ 1 - [\tilde{c}/w_t h_t] & \text{if } z_t \le \tilde{z}. \end{cases}$$

As long as the potential income of a member of generation t, $z_t \equiv w_t h_t$, is below $\tilde{z}$, then the fraction of time necessary to ensure subsistence consumption $\tilde{c}$ is larger than $1 - \gamma$, and the fraction of time devoted for child-rearing is therefore below γ. As the wage per efficiency unit of labor increases, the individual can

[10] Strict convexity with respect to g_{t+1} is not essential. It is designed to ensure that the level of human capital will not become zero at high rates of technological progress. Alternative assumptions will not affect the qualitative analysis.

generate the subsistence consumption with smaller labor force participation and the fraction of time devoted to child-rearing increases.[11]

Figure 2 shows the effect of an increase in potential income z_t on the individual's choice of total time spent on children and consumption. The income expansion path is vertical until the level of income passes the critical level that permits consumption to exceed the subsistence level. Thereafter, the income expansion path becomes horizontal at a level γ in terms of time devoted for child-rearing.[12]

Regardless of whether potential income is above or below $\tilde{z}$, increases in wages will not change the division of child-rearing time between quality and quantity. What *does* affect the division between time spent on quality and time spent on quantity is the rate of technological progress, which changes the return to education.

Specifically, using (8), the optimization with respect to e_{t+1} implies that, independently of the subsistence consumption constraint, the implicit functional relationship between e_{t+1} and g_{t+1}, as depicted in Figures 3–5 and derived in Lemma 1, is given by

$$(9)\qquad G(e_{t+1}, g_{t+1}) \equiv (\tau^q + \tau^e e_{t+1})h_e(e_{t+1}, g_{t+1}) - \tau^e h(e_{t+1}, g_{t+1}) \begin{cases} = 0 & \text{if } e_{t+1} > 0 \\ \leq 0 & \text{if } e_{t+1} = 0, \end{cases}$$

where $G_e(e_{t+1}, g_{t+1}) < 0$ and $G_g(e_{t+1}, g_{t+1}) > 0\ \forall g_{t+1} \geq 0$ and $\forall e_{t+1} > 0$.

To ensure the existence of a positive level of g_{t+1} such that the chosen level of education is 0, it is assumed that

$$(A1)\qquad G(0, 0) = \tau^q h_e(0, 0) - \tau^e h(0, 0) < 0.$$

LEMMA 1: *If* $(A1)$ *is satisfied, then the level of education chosen by members of generation t for their children is a nondecreasing function of* g_{t+1}.

$$e_{t+1} = e(g_{t+1}) \begin{cases} = 0 & \text{if } g_{t+1} \leq \hat{g} \\ > 0 & \text{if } g_{t+1} > \hat{g}, \end{cases}$$

where, $\hat{g} > 0$, *and*

$$e'(g_{t+1}) > 0 \qquad \forall\, g_{t+1} > \hat{g}.$$

PROOF:

As follows from (6) and (9), $G(0, g_{t+1})$ is monotonically increasing in g_{t+1}. Furthermore, (6) implies that $\lim_{g_{t+1}\to\infty} G(0, g_{t+1}) > 0$, whereas $(A1)$ implies that $G(0, 0) < 0$. Hence, there exists $\hat{g} > 0$ such that $G(0, \hat{g}) = 0$, and therefore, as follows from (9) $e_{t+1} = 0$ for $g_{t+1} \leq \hat{g}$. Furthermore, it follows from (9) that e_{t+1} is a single-valued function of g_{t+1}, where $e'_{t+1}(g_{t+1}) = -G_g(e_{t+1}, g_{t+1})/G_e(e_{t+1}, g_{t+1}) > 0$.

As is apparent from (9), $e''(g_{t+1})$ depends on the third derivatives of the production function of human capital. A concave reaction of the level of education to the rate of technological progress appears plausible economically, hence it is assumed that[13]

$$(A2)\qquad e''(g_{t+1}) < 0 \qquad \forall\, g_{t+1} > \hat{g}.$$

[11] John D. Durand (1975) and Goldin (1994) report that, across a large sample of countries, the relationship between women's labor-force participation and income is U-shaped. The model presented here explains the negative effect of income on labor-force participation for poor countries, and further predicts that this effect should no longer be operative once potential income has risen sufficiently high; it does not, however, explain the positive effect of income on participation for richer countries. See, however, Galor and Weil (1996) for a model that does explain this phenomenon.

[12] An alternative way of generating a qualitatively similar result would be to assume a Stone-Geary utility function of the form $u^t = (c_t - \tilde{c})^{(1-\gamma)}(w_{t+1}n_t h_{t+1})^{\gamma}$. In this case the income expansion path would be nearly vertical for low levels of potential income and asymptotically horizontal for high levels of potential income. Adopting this formulation would raise the dimensionality of the system, however.

[13] Alternatively, if $e(g_{t+1})$ is strictly convex we may assume that for physiological or other reasons, the maximum amount of education that a child can receive is bounded from above. In the model we ignore integer constraints on the number of children, so that absent a constraint on the quality per child, parents might choose to have an infinitesimally small number of children with infinitely high quality. Thus the existence of integer constraints may be taken as one justification for an upper bound on level of education.

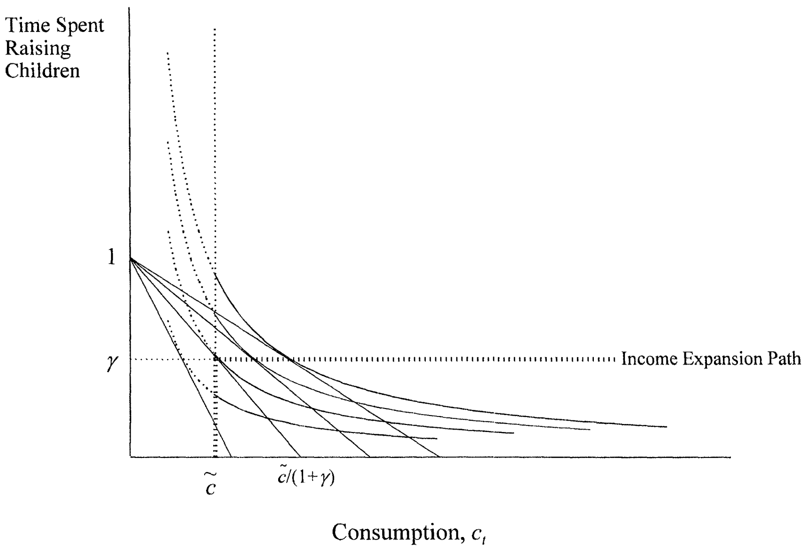

FIGURE 2. PREFERENCES, CONSTRAINTS, AND INCOME EXPANSION PATH

Notes: The figure depicts the household's indifference curves, budget constraints, as well as the subsistence consumption constraint $c \geq \tilde{c}$. The income expansion path, as derived in Proposition 1, is vertical as long as the subsistence consumption constraint is binding and horizontal at a level γ once the subsistence consumption constraint is not binding.

Furthermore, substituting $e_{t+1} = e(g_{t+1})$ into (8), it follows that n_t is

$$(10)\quad n_t = \begin{cases} \dfrac{\gamma}{\tau^q + \tau^e e(g_{t+1})} \equiv n^b(g_{t+1}) & \text{if } z_t \geq \tilde{z} \\ \dfrac{1 - [\tilde{c}/z_t]}{\tau^q + \tau^e e(g_{t+1})} \equiv n^a(g_{t+1}, z_t) & \text{if } z_t \leq \tilde{z}. \end{cases}$$

As follows from (3), (6), and the definition of z_t,

$$(11)\quad z_t = w_t h_t = h_t^{\alpha} x_t^{(1-\alpha)} \equiv z(e_t, g_t, x_t),$$

where $z_e(e_t, g_t, x_t) > 0$, $z_x(e_t, g_t, x_t) > 0$, and $z_g(e_t, g_t, x_t) < 0\ \forall (e_t, g_t, x_t) \gg 0$.[14]

The following proposition summarizes the properties of the functions $e(g_{t+1})$, $n^a(z_t, g_{t+1})$, and $n^b(g_{t+1})$ and their significance for the evolution in the substitution of quality for quantity in the process of development.

[14] It should be noted that, whereas the partial derivative of z_t with respect to g_t is negative (holding x_t and thus A_t constant), the total derivative of z_t with respect to g_t (holding A_{t-1} constant) is positive.

PROPOSITION 1: *Under (A1)–(A2)*

(*a*) *Technological progress that is expected to occur between the first and second periods of children's lives results in a decline in the parents' chosen number of children and an increase in their quality, i.e.,*

$$\partial n_t / \partial g_{t+1} \leq 0 \quad \textit{and} \quad \partial e_{t+1} / \partial g_{t+1} \geq 0.$$

(*b*) *If parental potential income is below $\tilde{z}$ (i.e., if the subsistence consumption constraint is binding), an increase in parental potential income raises the number of children, but has no effect on their quality, i.e.,*

$$\partial n_t / \partial z_t > 0 \quad \textit{and} \quad \partial e_{t+1} / \partial z_t = 0 \quad \textit{if } z_t < \tilde{z}.$$

(*c*) *If parental potential income is above $\tilde{z}$, an increase in parental potential income does not change the number of children or their quality, i.e.,*

$$\partial n_t / \partial z_t = \partial e_{t+1} / \partial z_t = 0 \qquad \textit{if } z_t > \tilde{z}.$$

PROOF:

Follows directly from Lemma 1, (8)–(10), and assumptions (A1)–(A2).

It follows from Proposition 1 that if the subsistence consumption constraint is binding, an increase in the effective resources per worker raises the number of children, but has no effect on their quality, whereas if the subsistence constraint is not binding, an increase in the effective resources per worker does not change the number of children or their quality.

E. *Technological Progress*

Suppose that technological progress g_{t+1}, which takes place between periods t and $t + 1$, depends on the education per capita among the working generation in period t, e_t, and the population size of the working generation in period t, L_t.[15]

$$(12) \qquad g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t} = g(e_t, L_t),$$

where for $L_t \gg 0$ and $e_t \geq 0$, $g(0, L_t) > 0$, $g_i(e_t, L_t) > 0$, and $g_{ii}(e_t, L_t) < 0$, $i = e_t, L_t$.[16]

Hence, for a sufficiently large population size, the rate of technological progress between time t and $t + 1$ is a positive, increasing, strictly concave function of the size and level of education of the working generation at time t. Furthermore, the rate of technological progress is positive even if labor quality is zero.

As will become apparent, the dynamical system of the described economy is rather complex; hence, to simplify the exposition, the dynamical system is analyzed *initially* under the assumption that an increase in population size has no effect on technological progress. In particular, it is initially assumed that

$$(A3) \qquad g_L(e_t, L_t) = 0 \qquad \forall\, L_t > 0.$$

In later stages of the analysis the effect of the size of population on the relationship between technological progress and the level of education as specified in (12) is fully incorporated into the analysis.

F. *The Evolution of Population, Technology, and Effective Resources*

The size of the working population at time $t + 1$, L_{t+1}, is

$$(13) \qquad L_{t+1} = n_t L_t,$$

where L_t is the size of the working population at time t, n_t is the number of children per person, and $n_t - 1$ is the rate of population growth. The size of the working population at time 0 is historically given at a level L_0.

The state of technology at time $t + 1$, A_{t+1}, as derived from (12), is

$$(14) \qquad A_{t+1} = (1 + g_{t+1})A_t,$$

where the state of technology at time 0 is historically given at a level A_0.

The evolution of effective resources per worker, $x_t \equiv (A_t X)/L_t$, depends on the evolution in the technological level and the rate of population growth:

$$(15) \qquad x_{t+1} = \frac{1 + g_{t+1}}{n_t} x_t,$$

where $x_0 \equiv A_0 X/L_0$ is historically given.

Substituting (11) and (12) into (10), and (10) and (12) into (15),

$$(16) \quad x_{t+1} = \begin{cases} \dfrac{[1 + g(e_t, L_t)][\tau^q + \tau^e e(g(e_t, L_t))]}{\gamma} x_t \\ \qquad \equiv \phi^b(e_t, L_t)x_t \quad \text{if } z_t \geq \tilde{z} \\ \dfrac{[1 + g(e_t, L_t)][\tau^q + \tau^e e(g(e_t, L_t))]}{1 - [\tilde{c}/z(e_t, g_t, x_t)]} x_t \\ \qquad \equiv \phi^a(e_t, g_t, x_t, L_t)x_t \quad \text{if } z_t \leq \tilde{z}, \end{cases}$$

[15] We consider a modification of equation (12) along the lines suggested by Jones (1995) in Section II.D.

[16] It should be noted that we assume that for a sufficiently small population the rate of technological progress is strictly positive only every several periods. That is, for a sufficiently small $L_t > 0$, $g(0, L_t) \geq 0$, $g_i(e_t, L_t) \geq 0$, for all t, and $g(0, L_t) > 0$, $g_i(e_t, L_t) > 0$, for some t. Furthermore, the number of periods that pass between two episodes of technological improvement declines with the size of population. These assumptions ensure that in early stages of development the economy is indeed in a Malthusian steady state. Clearly, if technological progress occurred in every time period at a pace that increased with the size of population, the growth rate of output per capita would always be positive, despite the adjustment in the size of population.

where, as follows from Lemma 1, (11), and (12), $\phi_e^b(e_t, L_t) > 0$, and $\phi_x^a(e_t, g_t, x_t, L_t) < 0 \ \forall e_t \geq 0$.

II. The Dynamical System

The development of the economy is characterized by the evolution of output per worker, population, technological level, education per worker, human capital per worker, and effective resources per worker. The evolution of the economy, is fully determined by a sequence $\{e_t, g_t, x_t, L_t\}_{t=0}^{\infty}$ that satisfies (12)–(16) and Lemma 1 in every period t.

The dynamical system is characterized by two regimes. In the first regime the subsistence consumption constraint is binding and for a given population size L, the evolution of the economy is governed by a *three*-dimensional nonlinear first-order autonomous system:[17]

$$(17) \quad \begin{cases} x_{t+1} = \phi^a(e_t, g_t, x_t; L)x_t \\ e_{t+1} = e(g(e_t; L)) & \text{if } z_t \leq \tilde{z} \\ g_{t+1} = g(e_t; L) \end{cases}$$

where the initial conditions e_0, g_0, and x_0 are historically given. In the second regime the subsistence consumption constraint is not binding and, for a given population size L, the evolution of the economy is governed by a *two*-dimensional system:

$$(18) \quad \begin{cases} x_{t+1} = \phi^b(e_t, x_t; L)x_t \\ e_{t+1} = e(g(e_t; L)). \end{cases} \quad \text{if } z_t \geq \tilde{z}$$

In both regimes, however, the analysis of the dynamical system is greatly simplified by the fact that, as follows from Lemma 1, (12), and ($A3$), the joint evolution of e_t and g_t is determined independently of the x_t. Furthermore, the evolution of e_t and g_t is independent of whether the subsistence constraint is binding, and is therefore independent of the regime in which the economy is located. The education level of workers in period $t + 1$ depends only on the level of technological progress expected between period t and period $t + 1$, whereas technological progress between periods t and $t + 1$ depends only on the level of education of workers in period t. Thus for a given population size L, we can analyze the dynamics of technology and education independently of the evolution of resources per capita.

A. *The Evolution of Technology and Education*

The evolution of technology and education, given ($A3$), is characterized by the sequence $\{g_t, e_t\}_{t=0}^{\infty}$ that satisfies in every period t the equations $g_{t+1} = g(e_t; L)$ and $e_{t+1} = e(g_{t+1})$. Although this dynamical subsystem consists of two independent one dimensional, nonlinear first-order difference equations, it is more revealing to analyze them jointly.

In light of the properties of the functions $e(g_{t+1})$ and $g(e_t; L)$ given in Lemma 1, ($A2$), ($A3$), and (12), it follows that if population size *does* play a role in technological progress, this dynamical subsystem is characterized by three qualitatively different configurations, which are depicted in Figures 3–5. The economy shifts endogenously from one configuration to another as population increases and the curve $g(e_t; L)$ shifts upward to account for the effect of an increase in population.

In Figure 3, for a range of small population sizes, the dynamical system is characterized by globally stable steady state equilibria. For a given population size in this range, the steady-state equilibrium is $(\bar{e}, \bar{g}) = (0, g^l)$. As implied by (12), the rate of technological change in a temporary steady state increases monotonically with the size of population, whereas the level of education remains unchanged.

In Figure 4, for a range of moderate population sizes, the dynamical system is characterized by three steady-state equilibria. For a given population size in this range, there exist two locally stable steady-state equilibria: $(\bar{e}, \bar{g}) = (0, g^l)$ and $(\bar{e}, \bar{g}) = (e^h, g^h)$, and an interior unstable steady state $(\bar{e}, \bar{g}) \equiv (e^u, g^u)$. The steady-state equilibria (e^h, g^h) and g^l increase monotonically with the size of population.

Finally, in Figure 5, for a range of large population sizes, the dynamical system is characterized by globally stable steady state equilibria. For a given population size in this range, there exists a

[17] For a given population, the entire dynamical system can be represented by the sequence $(g_t, x_t)_{t=0}^{\infty}$. However, since $e(g_t)$ is not invertible, the sequence $(e_t, x_t)_{t=0}^{\infty}$ does not represent the dynamical system, and a dynamical system that incorporates the evolution of e_t is necessarily three-dimensional in the first regime.

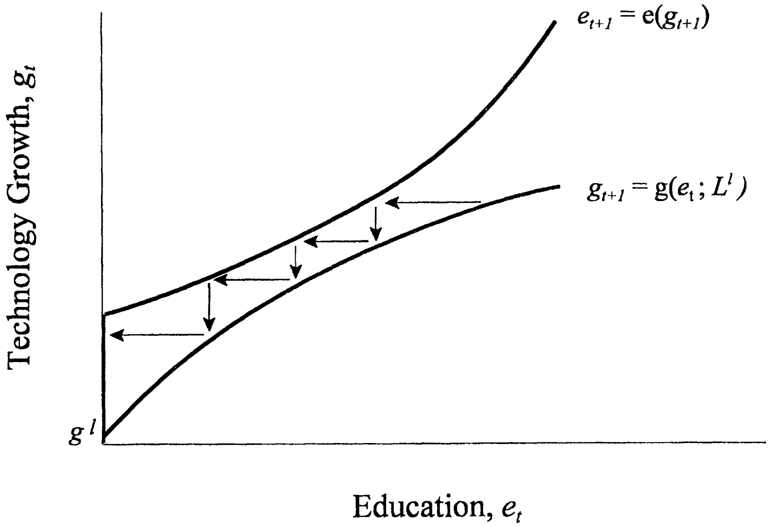

FIGURE 3. THE EVOLUTION OF TECHNOLOGY AND EDUCATION FOR A SMALL POPULATION

Notes: The figure describes the evolution of education e_t and the rate of technological change g_t for a constant small population L^l. The curve labeled $g_{t+1} = g(e_t; L^l)$ shows the effect of education on the growth rate of technology as presented in equation (12). The curve labeled $e_{t+1} = e(g_{t+1})$ shows the effect of expected technological change on optimal education choices derived in Lemma 1. The point of intersection between the two curves is the globally stable steady-state equilibrium $(0, g^l)$. In early stages of development, the economy is in the vicinity of this steady state in which education is zero and the rate of technological progress is slow.

unique globally stable steady-state equilibrium: $(\bar{e}, \bar{g}) = (e^h, g^h)$. These temporary steady-state levels increase with population.

B. *Global Dynamics*

This section analyzes the evolution of the economy from the Malthusian Regime, through the Post-Malthusian Regime, to the demographic transition and Modern Growth. The global analysis is based on a sequence of phase diagrams that describe the evolution of the system within each regime and the transition between the different regimes in the plain (e_t, x_t). The phase diagrams, depicted in Figures 6–8 contain three elements: the Malthusian Frontier, which separates the regions in which the subsistence constraint is binding from those where it is not; the *XX* locus, which denotes the set of all pairs (e_t, x_t) for which effective resources per worker are constant; and the *EE* locus, which denotes the set of all pairs for which the level of education per worker is constant.

The Malthusian Frontier.—As was established in (17) and (18) the economy exits from the subsistence consumption regime when potential income z_t exceeds the critical level $\tilde{z}$. This switch of regime changes the dimensionality of the dynamical system from three to two.

Let the *Malthusian Frontier* be the set of all triplets of (e_t, x_t, g_t) for which individuals' incomes equal $\tilde{z}$.[18] Using the definitions of z_t and $\tilde{z}$, it follows from (6) and (11) that the Malthusian Frontier *MM* is

$$(19)\quad MM \equiv \{(e_t, x_t, g_t) : x_t^{(1-\alpha)} h(e_t, g_t)^\alpha = \tilde{c}/(1-\gamma)\}.$$

[18] As was shown in Proposition 1, below the Malthusian Frontier, the effect of income on fertility will be positive, whereas above the frontier there will be no effect of income on fertility. Thus the Malthusian Frontier separates the Malthusian and Post-Malthusian Regimes, on the one hand, from the Modern Growth Regime, on the other.

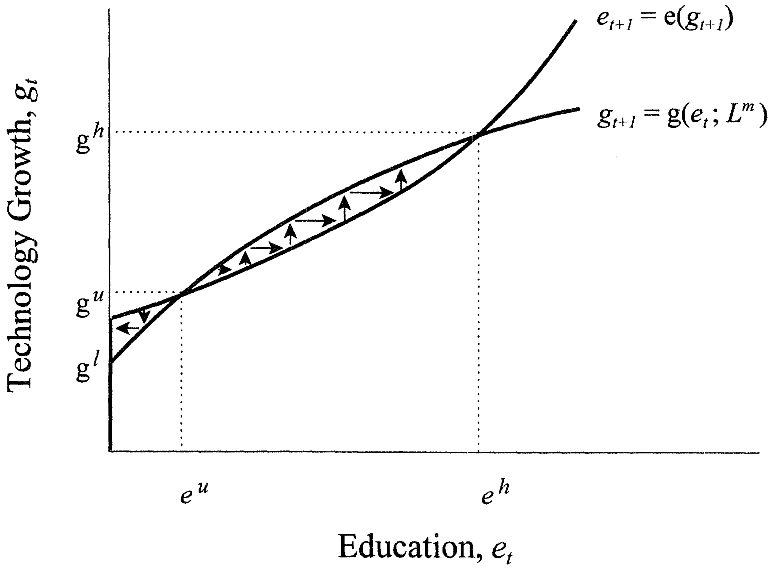

FIGURE 4. THE EVOLUTION OF TECHNOLOGY AND EDUCATION FOR A MODERATE POPULATION

Notes: The figure describes the evolution of education e_t and the rate of technological change g_t once the size of the population has grown to reach a moderate size, L^m. The system is characterized by multiple steady state equilibria. The steady- state equilibria $(0, g^l)$ and (e^h, g^h) are locally stable, whereas (e^u, g^u) is unstable. Given the initial conditions, in the absence of large shocks the economy remains in the vicinity of the low steady-state equilibrium $(0, g^l)$, in which education is still zero but the rate of technological progress is moderate.

Let the *Conditional Malthusian Frontier* be the set of all pairs (e_t, x_t) for which, conditional on a given technological level g_t, individuals' incomes equal $\tilde{z}$. Following the definitions of z_t and $\tilde{z}$, equations (6) and (11) imply that the Conditional Malthusian Frontier $MM_{|g_t}$, as depicted in Figures 6–8, is

$$(20)\quad MM_{|g_t} \equiv \{(e_t, x_t) : x_t^{(1-\alpha)} h(e_t, g_t)^{\alpha} = \tilde{c}/(1-\gamma) | g_t\}.$$

LEMMA 2: *If* $(e_t, x_t) \in MM_{|g_t}$ *then* x_t *is a decreasing strictly convex function of* e_t.

PROOF:

The lemma follows from (6) and (20).

Hence, the Conditional Malthusian Frontier, as depicted in Figures 6–8, is a strictly convex, downward sloping, curve in the (e_t, x_t) space. Furthermore, it intersects the x_t axis and asymptotically approaches the e_t axis as x_t approaches infinity. The frontier shifts upward as g_t increases in the transition to a Modern Growth regime.

The XX Locus.—Let XX be the locus of all triplets (e_t, g_t, x_t), such that for a given population size the effective resources per worker, x_t, are in a steady state:

$$XX \equiv \{(e_t, x_t, g_t) : x_{t+1} = x_t\}.$$

Along the XX locus the growth rates of population and technology are equal. Above the Malthusian Frontier, the fraction of time devoted to child-rearing is not dependent on the level of effective resources per worker. In this case, the growth rate of population will just be a negative function of the growth rate of technology, since for higher technology growth, parents will spend more of their resources on child quality and thus less on child quantity. Thus there will be a particular level of technological progress that induces an equal rate of population growth. Since the growth rate of technology is, in turn, a positive function of the level of education, this rate of technology growth

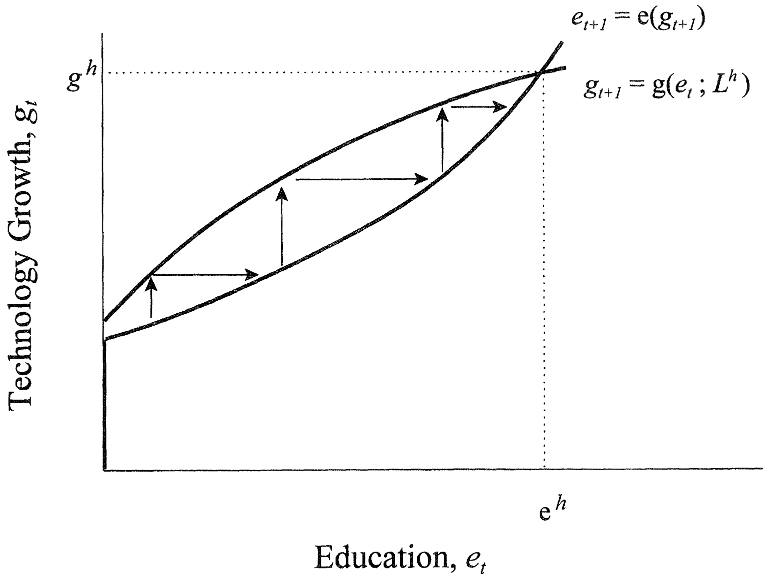

FIGURE 5. THE EVOLUTION OF TECHNOLOGY AND EDUCATION FOR A LARGE POPULATION

Notes: The figure describes the evolution of education e_t and the rate of technological change g_t once the size of the population grows to a high level, L^h. The system is characterized by a unique globally stable steady-state equilibrium (e^h, g^h). In mature stages of development, the economy converges monotonically to this steady state with high levels of education and technological progress.

will correspond to a particular level of education, denoted $\hat{e}$. Below the Malthusian Frontier, the growth rate of population depends on the level of effective resources per capita x, as well as on the growth rate of technology. The lower the value of x, the smaller the fraction of the time endowment devoted to child-rearing, and so the lower the population growth. Thus, below the Malthusian Frontier, a lower value of effective resources per capita will mean that lower values of technology growth (and thus education) will be consistent with population growth being equal to technology growth. Thus, as drawn in Figures 6–8, lower values of x will be consistent with lower values of e on the part of the XX locus that is below the Malthusian Frontier.

Lemmas 3, 4, and 5, derive the properties of this locus. To simplify the exposition without affecting the qualitative nature of the dynamical system, the parameters of the model are restricted so as to ensure that the XX locus is nonempty when $z_t \geq \tilde{z}$; that is,

$$(A4) \qquad \hat{g} < (\gamma/\tau^q) - 1 < g(e^h(L_0), L_0).$$

LEMMA 3: *If (A1)–(A4) are satisfied, then for $z_t \geq \tilde{z}$, there exists a unique value $0 < \hat{e} < e^h$, such that $x_t \in XX$. Furthermore, for $z_t \geq \tilde{z}$*

$$x_{t+1} - x_t \begin{cases} > 0 & \text{if } e_t > \hat{e} \\ = 0 & \text{if } e_t = \hat{e} \\ < 0 & \text{if } e_t < \hat{e}. \end{cases}$$

PROOF:

For $z_t \geq \tilde{z}$, it follows from (16) that

$$x_{t+1} = x_t \qquad \text{if and only if } \phi^b(e_t; L)$$
$$\equiv [1 + g(e_t; L)][\tau^q + \tau^e e(g(e_t; L))]/$$
$$\gamma = 1.$$

Since $\phi^b(e_t; L)$ is strictly monotonically increasing in e_t and since (A4) implies that for all $L_t > 0$, $\phi^b(0; L) < 1$ and $\phi^b(e^h; L) > 1$, there exists a unique value $0 < \hat{e} < e^H$, such that $\phi^b(\hat{e}; L) = 1$ and hence $x_t \in XX$. Furthermore, since $\phi^b(e_t; L)$ is strictly monotonically increasing in e_t, it follows from (16) that $x_{t+1} > x_t$ if and only if $\phi^b(e_t; L) > 1$ and hence $e_t > \hat{e}$, whereas $x_{t+1} < x_t$ if and only if $\phi^b(e_t; L) < 1$ and hence $e_t < \hat{e}$.

Hence, the XX locus, as depicted in Figures 6–8 in the space (e_t, x_t), is a vertical line above the Conditional Malthusian Frontier at a level $\hat{e}$. This critical level decreases with the size of the population.

Lemma 3 holds as long as consumption is above subsistence. In the case where the subsistence constraint is binding, the evolution of x_t, as determined by equation (16), is based on the rate of technological change g_t, the effective resources per worker x_t, as well as the quality of the labor force e_t.

Let $XX_{|g_t}$ be the locus of all pairs (e_t, x_t), such that $x_{t+1} = x_t$ for a given level of g_t; that is,

$$XX_{|g_t} \equiv \{(e_t, x_t) : x_{t+1} = x_t | g_t\}.$$

LEMMA 4: *If (A1)–(A4) are satisfied, then for* $z_t \leq \tilde{z}$ *and for* $0 \leq e_t \leq \hat{e}$, *there exists a single-valued function* $x_t = x(e_t)$, *such that* $(x(e_t), e_t) \in XX_{|g_t}$. *Furthermore, for* $z_t \leq \tilde{z}$,

$$x_{t+1} - x_t \begin{cases} <0 & \text{if } (e_t, x_t) > (e_t, x(e_t)) \\ & \text{and } 0 \leq e_t \leq \hat{e}, \\ =0 & \text{if } x_t = x(e_t) \\ & \text{and } 0 \leq e_t \leq \hat{e}, \\ >0 & \text{if } [(e_t, x_t) < (e_t, x(e_t)) \\ & \text{and } 0 \leq e_t \leq \hat{e}], \text{ or } [e_t > \hat{e}]. \end{cases}$$

PROOF:

As follows from (16), $x_{t+1} = x_t$ if and only if

$$\phi^a(e_t, g_t, x_t) = [1 + g(e_t; L)] \times [\tau^q + \tau^e e(g(e_t; L))] \div \{1 - [\tilde{c}/z(e_t, g_t, x_t)]\} = 1.$$

Since $\phi^a(e_t, g_t, x_t; L)$ is strictly monotonically *decreasing* in x_t, there exists a single-valued function $x_t = x(e_t)$, such that $\phi^a(e_t, x_t|g_t) = 1$ and therefore $(e_t, x(e_t)) \in XX_{|g_t}$. Moreover, since $\phi^a_e(e_t, g_t, x_t; L)$ is not necessarily monotonic, $x'(e_t)$ is not necessarily monotonic as well. Furthermore, since $\phi^a(e_t, x_t|g_t)$ is strictly monotonically decreasing in x_t, it follows from (16) that for $0 \leq e_t \leq \hat{e}$ and for $z_t \leq \tilde{z}$

$$x_{t+1} > x_t$$

$$\text{if and only if } x_t < \max[x(e_t), x_t^M],$$

$$\text{where } (e_t, x_t^M) \in MM_{|g_t}$$

and

$$x_{t+1} < x_t \qquad \text{if and only if } x_t > x(e_t).$$

Hence, without loss of generality, the locus $XX_{|g_t}$ is depicted in Figures 6–8, as an upward-sloping curve in the space (e_t, x_t), defined for $e_t \leq \hat{e}$. $XX_{|g_t}$ is strictly below the Conditional Malthusian Frontier for the value of $e_t < \hat{e}$, and the two coincide at $\hat{e}$.

LEMMA 5: *Let* $(\hat{e}, \hat{x}) \in MM_{|g_t}$. *If (A4) is satisfied, then*

$$(\hat{e}, \hat{x}) = XX_{|g_t} \cap MM_{|g_t} \cap XX.$$

PROOF:

Let $(\hat{e}, \hat{x}) \in MM_{|g_t}$. It follows from the definition of $MM_{|g_t}$ that $z(\hat{e}, \hat{x}|g_t) = \tilde{z}$. Hence, Lemma 2 implies that $(\hat{e}, \hat{x}) \in XX$. Furthermore, since Lemmas 2 and 3 are both valid for $z_t = \tilde{z}$, it follows that $x(\hat{e}) = \hat{x}$ and hence $(\hat{e}, \hat{x}) \in XX_{|g_t}$.

Hence, the Conditional Malthusian Frontier, the XX locus, and the $XX_{|g_t}$ locus, as depicted in Figures 6–8 in the (e_t, x_t) space, coincide at $(\hat{e}, \hat{x})$.

The EE Locus.—Let EE be the locus of all triplets (e_t, g_t, x_t), such that the quality of labor e_t is in a steady state:

$$EE \equiv \{(e_t, x_t, g_t) : e_{t+1} = e_t\}.$$

As follows from the analysis in Section II, subsection A for a given population size, the steady-state values of e_t are independent of the values of x_t and g_t. The locus EE evolves through three phases in the process of development, corresponding to the three phases that

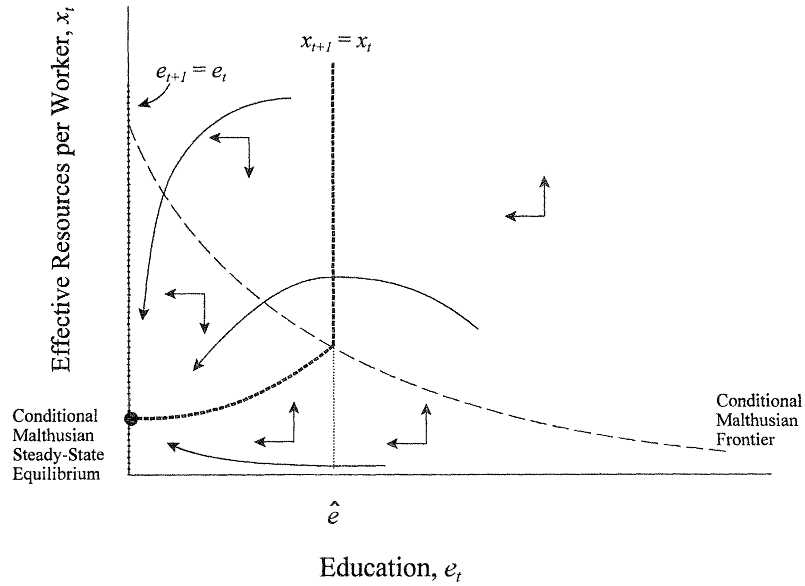

FIGURE 6. THE CONDITIONAL DYNAMICAL SYSTEM FOR A SMALL POPULATION

Notes: This figure describes the evolution of education e_t and effective resource per worker x_t for a constant small population L^l. The curve $e_{t+1} = e_t$ is the set of all pairs (e_t, x_t), for which education is constant over time. The curve $x_{t+1} = x_t$ is the set of all pairs (e_t, x_t), given g_t, for which effective resource per worker is constant over time (Lemmas 3, 4, and 5). The point of intersection between the two curves is the unique globally stable steady-state equilibrium. In early stages of development, the system is in the vicinity of this conditional Malthusian steady-state equilibrium. The Conditional Malthusian Frontier as defined in equation (20) is the set of all pairs (e_t, x_t), given g_t, below which the subsistence consumption constraint is binding.

describe the evolution of education and technology depicted in Figures 3, 4, and 5.

In the early stages of development, when population size is sufficiently small, the joint evolution of education and technology is characterized by a globally stable temporary steady-state equilibrium, $(\bar{e}, \bar{g}) = (0, g^l)$, as depicted in Figure 3. The corresponding *EE* locus, depicted in the space (e_t, x_t) in Figure 6, is vertical at the level $e = 0$, for a range of small population sizes. Furthermore, for this range, the global dynamics of e_t in this configuration are given by

$$(21) \qquad e_{t+1} - e_t \begin{cases} = 0 & \text{if } e_t = 0 \\ < 0 & \text{if } e_t > 0. \end{cases}$$

In later stages of development, as population size increases sufficiently, the joint evolution of education and technology is characterized by multiple locally stable temporary steady-state equilibria, as depicted in Figure 4. The corresponding *EE* locus, depicted in the space (e_t, x_t) in Figure 7, consists of three vertical lines corresponding to the three steady-state equilibria for the value of e_t—that is, $e = 0$, $e = e^u$, and $e = e^h$. The vertical lines $e = e^u$ and $e = e^h$ shift rightward as population size increases. Furthermore, the global dynamics of e_t in this configuration are given by

$$(22) \qquad e_{t+1} - e_t \begin{cases} < 0 & \text{if } 0 < e_t < e^u \quad \text{or} \quad e_t > e^h \\ = 0 & \text{if } e_t \in \{0, e^u, e^h\} \\ > 0 & \text{if } e^u < e_t < e^h. \end{cases}$$

In mature stages of development, when population size is sufficiently large, the joint evolution of education and technology is characterized by globally stable steady-state equilibrium at the point $(\bar{e}, \bar{g}) = (e^h, g^h)$, as depicted in Figure 5. The corresponding *EE* locus, as depicted in Figure 8 in the space (e_t, x_t), is vertical at the level $e = e^h$. This vertical line shifts rightward as population size increases. Furthermore, the

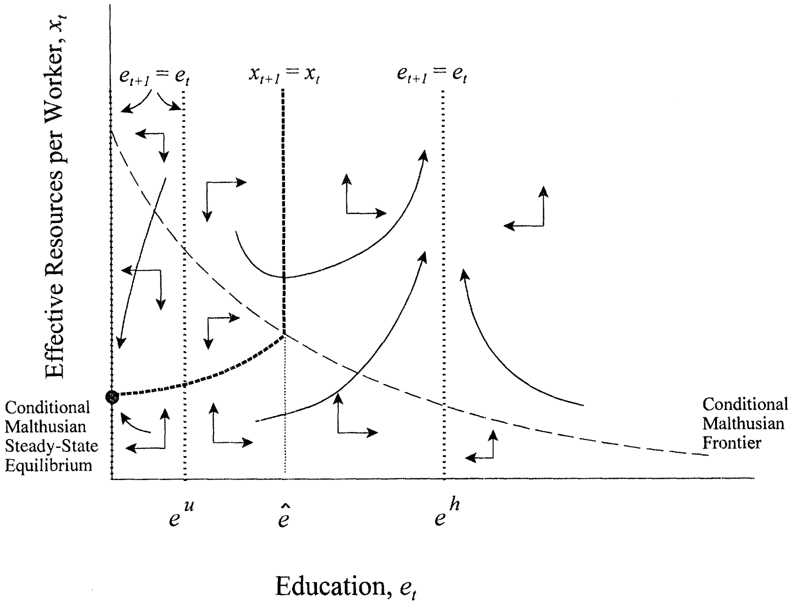

FIGURE 7. THE CONDITIONAL DYNAMICAL SYSTEM FOR A MODERATE POPULATION

Notes: This figure describes the evolution of education e_t and effective resource per worker x_t, once the size of the population has grown to reach a moderate size, L^m. The system is characterized by multiple steady state equilibria. Given the initial conditions, in the absence of large shocks, the economy remains in the vicinity of the conditional Malthusian steady state equilibrium.

global dynamics of e_t in this configuration are given by

$$(23)\quad e_{t+1} - e_t \begin{cases} > 0 & \text{if } 0 \le e_t < e^h \\ = 0 & \text{if } e_t = e^h. \\ < 0 & \text{if } e_t > e^h. \end{cases}$$

C. *Conditional Steady-State Equilibria*

In early stages of development, when population size is sufficiently small, the dynamical system, as depicted in Figure 6 in the space (e_t, x_t), is characterized by a unique and globally stable conditional steady-state equilibrium.[19] It is given by a point of intersection between the *EE* locus and the *XX* locus. That is, conditional on a given rate of technological progress g_t and a given population size, the Malthusian steady state $(0, \bar{x}(g_t))$ is globally stable.[20] In later stages of development, as population size increases sufficiently, the dynamical system as depicted in Figure 7 is characterized by two conditional steady-state equilibria. The Malthusian conditional steady-state equilibrium is locally stable, whereas the conditional steady-state equilibrium (e^u, x^u) is a saddlepoint.[21] In addition, for education levels above e^u the system converges to a stationary level of education e^h and possibly to a steady-state *growth rate* of x_t,

[19] Since the dynamical system is discrete, the trajectories implied by the phase diagrams do not necessarily approximate the actual dynamic path, unless the state variables evolve monotonically over time. As shown in Section II, subsection A, the evolution of e_t is monotonic, whereas the evolution and convergence of x_t may be oscillatory. Nonmonotonicity may arise only if $e < \hat{e}$. Nonmonotonicity in the evolution of x_t does not affect the qualitative description of the system. Furthermore, if $\phi_x^a(e_t, g_t, x_t; L)x_t > -1$ the conditional dynamical system is locally nonoscillatory. The phase diagrams in Figures 6–8 are drawn under the assumptions that ensure that there are no oscillations.

[20] The local stability of the steady-state equilibrium $(0, \bar{x}(g_t))$ can be derived formally. The eigenvalues of the Jacobian matrix of the conditional dynamical system evaluated at the conditional steady-state equilibrium are both smaller than 1 (in absolute value) under (*A*1)–(*A*3).

[21] Convergence to the saddlepoint takes place only if the level of education is e^u. That is, the saddlepath is the entire vertical line that corresponds to $e_t = e^u$.

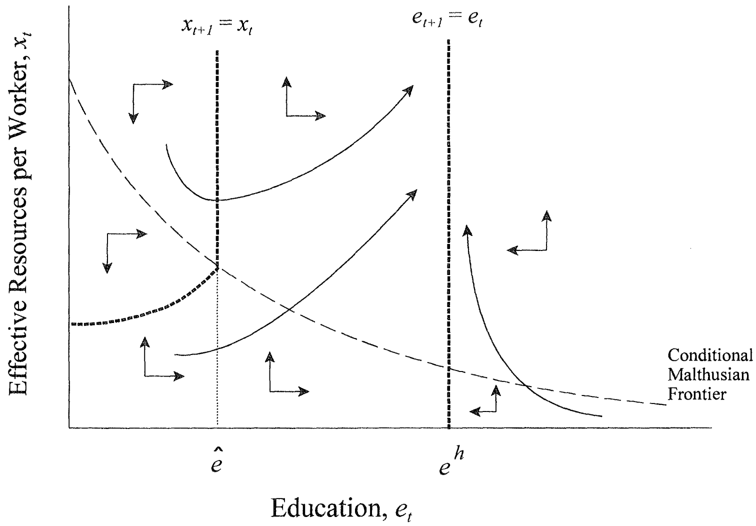

FIGURE 8. THE CONDITIONAL DYNAMICAL SYSTEM FOR A LARGE POPULATION

Notes: The figure describes the evolution of education e_t and the rate of technological change x_t, once the size of the population has reached a high level, L^h. The dynamical system changes qualitatively and the conditional Malthusian steady state vanishes. The economy evolves through a Post-Malthusian Regime until it crosses the Conditional Malthusian Frontier and enters the Modern Growth Regime.

given the population size. In mature stages of development when population size is sufficiently large, the system converges globally to an educational level e^h and possibly to a steady-state *growth rate* of x_t, given the population size.

D. *Analysis*

The transition from the Malthusian regime through the Post-Malthusian regime to the demographic transition and a Modern Growth regime emerges from Proposition 1 and Figures 2–8. Consider an economy in the early stages of development. Population is low enough that the implied rate of technological change is very small, and parents have no incentive to provide education to their children. As depicted in Figure 3 in the space (e_t, g_t), the economy is characterized by a single temporary steady-state equilibrium in which technological progress is very slow and children's level of education is zero. This temporary steady-state equilibrium corresponds to a globally stable conditional Malthusian steady-state equilibrium, drawn in Figure 6 in the space (e_t, x_t). For a given rate of technological progress, effective resources per capita, as well as the level of education, are constant and hence, as follows from (2) and (6), output per capita is constant as well. Moreover, shocks to population or resources will be undone in a classic Malthusian fashion. Population will be growing slowly, in parallel with technology.

As long as the size of the population is sufficiently small, no qualitative changes occur in the dynamical system described in Figures 3 and 6. The temporary steady-state equilibrium depicted in Figure 3 gradually shifts vertically upward, reflecting small increments in the rate of technological progress as the size of the population increases, while the level of education remains constant at zero. Similarly, the conditional Malthusian steady-state equilibrium, drawn in Figure 6 for a constant rate of technological progress, shifts upward vertically. However, output per capita remains constant at the subsistence level.

Over time, the slow growth in population that takes place in the Malthusian regime will raise the rate of technological progress and shift the $g(e_{t+1}; L^l)$ locus in Figure 3 upward so that it has the configuration shown in Figure 4. At this point, the dynamical system of education and technology will be characterized by multiple, history-dependent steady states. One of these steady states will be Malthusian, characterized by constant resources per capita, slow technological progress, and no education. The other will be characterized by a high level of education, rapid technological progress, growing income per capita, and moderate population growth.

For the deterministic description of a take-off from the Malthusian equilibrium that is emphasized in this paper, however, the existence of multiple steady states turns out not to be relevant. Since the economy starts out in the Malthusian steady state, it will remain there at this intermediate stage. If we were to allow for stochastic shocks to education or technological progress, it would be possible for an economy in the Malthusian steady state of Figure 4 to jump to the Modern Growth steady state, but we do not pursue this possibility.

Figure 5 shows that the increasing size of the population continues to raise the rate of technological progress, reflected in a further upward shift of the $g(e_{t+1}; L_t)$ locus. At a certain level of population, the steady-state Malthusian vanishes, and the economy transitions out of the Malthusian Regime. Increases in the rate of technological progress and the level of education feed back on each other until the economy converges to the unique, stable steady state.

Although both the evolution of education and technological progress traced in Figure 5 are monotonic once the Malthusian steady state has been left behind, the evolution of population growth and the standard of living, which can be seen in Figure 8, are more complicated. The reason for this complication is that technological progress has two effects on the evolution of population, as shown in Proposition 1. First, by inducing parents to give their children more education, technological progress will ceteris paribus lower the rate of population growth. But, second, by raising potential income, technological progress will increase the fraction of their time that parents can afford to devote to raising children. Initially, while the economy is in the Malthusian region of Figure 8, the effect of technology on the parent's budget constraint will dominate, and so the growth rate of the population will increase. This is the Post-Malthusian Regime.[22] The positive income effect of technological progress on fertility functions only in the Malthusian region of Figure 8, however; as the figure shows, the economy eventually crosses the Malthusian frontier. Once this has happened, further improvements in technology no longer have the effect of changing the amount of time devoted to child-rearing, whereas faster technological change will continue to raise the quantity of education that parents give each child. Thus once the economy has crossed the Malthusian Frontier, population growth will fall as education and technological progress rise.

In the Modern Growth Regime, resources per capita will rise, as technological progress outstrips population growth. Figure 5 shows that the levels of education and technological progress will be constant in the steady state, provided that population size is constant (i.e., population growth is zero). This implies that the growth rate of resources per capita, and thus the growth rate of output per capita, will also be constant. However, if population growth is positive in the Modern Growth Regime and if its effect on technological progress remains positive, then education and technological progress will continue to rise, and, similarly, if population growth is negative they will fall. In fact, the model makes no firm prediction about what the growth rate of population will be in the Modern Growth Regime, other than that population growth will fall once the economy exits from the Malthusian region. It may be the case that population growth will be zero, in which case the Modern Growth Regime would constitute a global steady state, in which e and g were constant. Alternatively, population growth could be either positive or negative in the Modern Growth Regime, with e and g behaving accordingly if the effect of population size on the rate of technological progress remains positive.[23]

[22] Literally, income per capita does not change during the Post-Malthusian Regime. It remains fixed at the subsistence level. This is an artifact of the assumption that the only input into child quality is parental time, and that this time input does not produce measured output. If child-rearing, especially the production of quality, requires goods or time supplied through a market (e.g., schooling), the shift toward higher child quality that takes place during the Post-Malthusian Regime would be reflected in higher market expenditures (as opposed to parental time expenditures) and rising measured income.

[23] Jones (1995) has argued for a model of technology creation in which the steady-state growth rate of technology is related to the growth rate of population, rather than to its level. Under such a specification, our model would have a steady-state modern growth regime, in which the growth rates of population and technology would be constant. Further, such a

III. Concluding Remarks

This paper develops a unified endogenous growth model in which the evolution of population, technology, and output growth is largely consistent with the process of development in the last millennia. The model generates an endogenous takeoff from a Malthusian Regime, through a Post-Malthusian Regime, to a demographic transition and a Modern Growth Regime. In early stages of development—the Malthusian Regime—the economy remains in the proximity of a Malthusian trap, where output per capita is nearly stationary and episodes of technological change bring about proportional increases in output and population. In the intermediate stages of development—the Post-Malthusian Regime—the intensified pace of technological change that is caused by the increase in the size of population during the Malthusian Regime permits the economy to take off. Production takes place under a state of technological disequilibrium in which the relative return to skills rises, inducing the household to shift its spending on children toward quality and away from quantity. Output per capita increases along with an increase in the rate of population growth and human-capital accumulation. Eventually, rapid technological progress, which results from high human-capital accumulation, triggers a demographic transition in which fertility rates permanently decline.

One of the significant components of the model is the effect of technological change on the return to education. Specifically, technological transitions, in and of themselves, are assumed to raise the return to education. An alternative assumption that would produce many of the same results is that the return to education rises with the *level* of technology, so that, for example, a technologically stagnant economy with a high level of technology would have a higher return to education than a similarly stagnant economy with a low level of education. A model incorporating this assumption would produce a technological takeoff that was not related to the size of population: even if population were constant, technological progress would eventually raise the rate of return to education sufficiently to induce parents to give their children more schooling, and this would in turn feed back to raise the rate of technological progress. Making this assumption, however, would be equivalent to assuming that changes in technology were skill biased throughout human history. Although, on average, technological change may have been skilled biased, our mechanism allows us to consider those periods in which technological change was unskilled biased in the long run (most notably, elements of the industrial revolution).

The model abstracts from several factors that are relevant for economic growth. Differences between countries in the determination of population growth or in the process of technological change (as a result of institutions and cultural factors, for example) would be reflected in their ability to escape the Malthusian trap and in the speed of their takeoff. Similarly, differences in policies, such as the public provision of education, would change the dynamics of the model. One interesting possibility that the model suggests is that the inflow of grain and other commodities as well as the outflow of migrants during the nineteenth century may have played a crucial role in Europe's development. By easing the land constraint at a crucial point—when income per capita had begun to rise rapidly, but before the demographic transition had gotten under way—the "ghost acres" of the New World provided a window of time, which allowed Europe to pull decisively away from the Malthusian equilibrium (Kenneth Pomeranz, 1999).

Even though the model presents a unified description of the development process followed by Europe and its offshoots, it is clearly not fully applicable to countries that are developing today. For currently developing countries, a large stock of preexisting technology is available for import, and so the relationship between population size and technology growth, which helped trigger the demographic transition in Europe, is no longer relevant. Similarly, the relationship between income and population growth has changed dramatically, resulting from the import of health technologies. Countries that are poor, even by the standards of nineteenth-century Europe, are experiencing growth rates of population far higher than those ever experienced in Europe.

We end by stressing the importance of the construction of unified models of population and development that encompass the endogenous transition between the three fundamental regimes that have characterized the process of develop-

steady state would be stable: if population growth fell, the rate of technological progress would also fall, inducing a rise in fertility.

ment.[24] Imposing the constraint that a single model account for the entire process of development is a discipline that would improve the understanding of the underlying phenomena and generate superior testable predictions and more accurate analysis of the effects of policy interventions.

REFERENCES

Acemoglu, Daron and Zilibotti, Fabrizio. "Was Prometheus Unbound by Chance? Risk, Diversification, and Growth." *Journal of Political Economy*, August 1997, *105*(4), pp. 709–51.

Barro, Robert J. and Becker, Gary S. "Fertility Choice in a Model of Economic Growth." *Econometrica*, March 1989, *57*(2), pp. 481–501.

Becker, Gary S. "An Economic Analysis of Fertility," in Ansley J. Coale, ed., *Demographic and economic change in developed countries*. Princeton, NJ: Princeton University Press, 1960, pp. 209–40.

Becker, Gary S.; Murphy, Kevin M. and Tamura, Robert F. "Human Capital, Fertility, and Economic Growth." *Journal of Political Economy*, October 1990, *98*(5), Pt. 2, pp. S12–S37.

Birdsall, Nancy. "Economic Approaches to Population Growth," in Hollis Chenery and T. N. Srinivasan, eds., *Handbook of development economics*. Amsterdam: North-Holland, 1988, pp. 477–542.

Caldwell, John C. "Toward a Restatement of Demographic Transition Theory." *Population and Development Review*, September–December 1976, *2*(3/4), pp. 321–66.

Chao, Kang. *Man and land in Chinese history: An economic analysis*. Stanford, CA: Stanford University Press, 1986.

Coale, Ansley J. and Treadway, Roy. "A Summary of the Changing Distribution of Overall Fertility, Marital Fertility, and the Proportion Married in the Provinces of Europe," in Ansley J. Coale and S. Watkins, eds., *The decline of fertility in Europe*. Princeton, NJ: Princeton University Press, 1986, pp. 31–181.

Dahan, Momi and Tsiddon, Daniel. "Demographic Transition, Income Distribution, and Economic Growth." *Journal of Economic Growth*, March 1998, *3*(1), pp. 29–52.

Doms, Mark; Dunne, Timothy and Troske, Kenneth R. "Workers, Wages and Technology." *Quarterly Journal of Economics*, February 1997, *112*(1), pp. 253–90.

Durand, John D. "The Labor Force in Economic Development and Demographic Transition," in Leon Tabah, ed., *Population growth and economic development in the Third World*. Dolhain, Belgium: Ordina Editions, 1975, pp. 47–78.

Dyson, Tim and Murphy, Mike. "The Onset of Fertility Transition." *Population and Development Review*, September 1985, *11*(3), pp. 399–440.

Easterlin, Richard. "Why Isn't the Whole World Developed?" *Journal of Economic History*, March 1981, *41*(1), pp. 1–19.

Eckstein, Zvi; Stern, Steven and Wolpin, Kenneth. "Fertility Choice, Land, and the Malthusian Hypothesis." *International Economic Review*, May 1988, *29*(2), pp. 353–61.

Ehrlich, Isaac and Lui, Francis. "The Problem of Population and Growth: A Review of the Literature from Malthus to Contemporary Models of Endogenous Population and Endogenous Growth." *Journal of Economic Dynamics and Control*, January 1997, *21*(1), pp. 205–42.

Fogel, Robert. "New Findings on Secular Trends in Nutrition and Mortality: Some Implications for Population Theory," in Mark R. Rosenzweig and Oded Stark, eds., *Handbook of population and family economics*, Vol. 1A. Amsterdam: North-Holland, 1997, pp. 433–81.

Foster, Andrew D. and Rosenzweig, Mark R. "Technical Change and Human-Capital Returns and Investments: Evidence from the Green Revolution." *American Economic Review*, September 1996, *86*(4), pp. 931–53.

Galor, Oded and Tsiddon, Daniel. "Technological Progress, Mobility, and Growth." *American Economic Review*, June 1997, *87*(3), pp. 363–82.

Galor, Oded and Weil, David N. "The Gender Gap, Fertility, and Growth." *American Economic Review*, June 1996, *86*(3), pp. 374–87.

______. "Population, Technology and Growth: From the Malthusian Regime to the Demographic Transition." National Bureau of Economic Research (Cambridge, MA) Working Paper No. 6811, October 1998.

______. "From Malthusian Stagnation to Modern Growth." *American Economic Review*, May 1999 (*Papers and Proceedings*), *89*(2), pp. 150–54.

Goodfriend, Marvin and McDermott, John. "Early Development." *American Economic Review*, March 1995, *85*(1), pp. 116–33.

[24] For a description of alternative unified models see Galor and Weil (1999).

Goldin, Claudia. "The U-Shaped Female Labor Force Function in Economic Development and Economic History." National Bureau of Economic Research (Cambridge, MA) Working Paper No. 4707, April 1994.

Goldin, Claudia and Katz, Lawrence F. "The Origins of Technology-Skill Complementary." *Quarterly Journal of Economics*, August 1998, *113*(3) pp. 693–732.

Jones, Charles I. "R&D-Based Models of Economic Growth." *Journal of Political Economy*, August 1995, *103*(4), pp. 759–84.

Komlos, John and Artzrouni, Mark. "Mathematical Investigations of the Escape from the Malthusian Trap." *Mathematical Population Studies*, December 1990, *2*(4), pp. 269–87.

Kremer, Michael. "Population Growth and Technological Change: One Million B.C. to 1990." *Quarterly Journal of Economics*, August 1993, *108*(3), pp. 681–716.

Lee, Ronald D. "A Historical Perspective on Economic Aspects of the Population Explosion: The Case of Preindustrial England," in Richard A. Easterlin, ed., *Population and economic change in developing countries*. Chicago: University of Chicago Press, 1980, pp. 517–66.

______. "Population Dynamics: Equilibrium, Disequilibrium, and Consequences of Fluctuations," in Mark Rosenzweig and Oded Stark, eds., *Handbook of population and family economics*, Vol. 1B. Amsterdam: North-Holland, 1997, pp. 1063–115.

Livi-Bacci, Massimo. *A concise history of world population*. Oxford: Blackwell, 1997.

Lucas, Robert E., Jr. "The Industrial Revolution: Past and Future." Mimeo, Department of Economics, University of Chicago, 1999.

Maddison, Angus. *Phases of capitalist development*. New York: Oxford University Press, 1982.

______. *Monitoring the world economy, 1820–1992*. Paris: OECD, 1995.

Malthus, Thomas R. *An essay on the principle of population*. Cambridge: Cambridge University Press, 1826.

Matthews, Robert C. O.; Feinstein, Charles H. and Odling-Smee, John C. *British economic growth 1856–1973*. Stanford, CA: Stanford University Press, 1982.

Mokyr, Joel. *The lever of riches*. New York: Oxford University Press, 1990.

Nelson, Richard R. "A Theory of Low-Level Equilibrium Trap in Underdeveloped Economies." *American Economic Review*, December 1956, *46*(5), pp. 894–908.

Nelson, Richard R. and Phelps, Edmund S. "Investment in Humans, Technological Diffusion, and Economic Growth." *American Economic Review*, May 1966 (*Papers and Proceedings*), *56*(2), pp. 69–75.

Olsen, Randall J. "Fertility and the Size of the U.S. Labor Force." *Journal of Economic Literature*, March 1994, *32*(1), pp. 60–100.

Pomeranz, Kenneth. "East Asia, Europe, and the Industrial Revolution." Mimeo, Department of History, University of California-Irvine, 1999.

Pritchett, Lant. "Divergence, Big Time." *Journal of Economic Perspectives*, Summer 1997, *11*(3), pp. 3–17.

Raut, Lakshmi K. and Srinivasan T. N. "Dynamics of Endogenous Growth." *Economic Theory*, 1994, *4*(5), pp. 777–90.

Razin, Assaf and Ben-Zion, Uri. "An Intergenerational Model of Population Growth." *American Economic Review*, December 1975, *65*(5), pp. 923–33.

Robinson James A. and Srinivasan, T. N. "Long-Term Consequences of Population Growth: Technological Change, Natural Resources, and the Environment," in Mark R. Rosenzweig and Oded Stark, eds., *Handbook of population and family economics*, vol. 1B. Amsterdam: North-Holland, 1997, pp. 1175–298.

Schultz, T. Paul. "Demand for Children in Low Income Countries," in Mark R. Rosenzweig and Oded Stark, eds., *Handbook of population and family economics*. Amsterdam: North-Holland, 1997, pp. 349–430.

Schultz, Theodore W. *Transforming traditional agriculture*. New Haven: Yale University Press, 1964.

______. "The Value of the Ability to Deal with Disequilibria." *Journal of Economic Literature*, September 1975, *13*(3), pp. 827–46.

Smith, Adam. *The wealth of nations*. London: W. Strahan and T. Cadell, 1776. [*An inquiry into the nature and causes of the wealth of nations*].

Wrigley, Edward A. *Population and history*. New York: McGraw-Hill, 1969.

Wrigley, Edward A. and Schofield, Roger S. *The population history of England 1541–1871: A reconstruction*. Cambridge, MA: Harvard University Press, 1981.