Master PPD - Public Economics: Tax & Transfer Policies

Final Exam, March 10, 2015 - 17h30-19h30

The exam is 2 hours long and can be done either in French or English. ***No*** *document whatsoever is allowed.*

Exercise 1 : Global warning and carbon taxes (7 points)

Part A : Pigouvian Corrective Taxation (5 points)

Consider an economy with a continuum of agents i in $[0, 1]$. There are two goods : a non-energy good (c) and an energy good (x). Each agent has the same utility function :

$$U_i = U_i(c_i, x_i, X) = c_i^{1-\alpha} \cdot x_i^{\alpha} \cdot X^{-\lambda}$$

where :

- c_i is the individual consumption level of the non-energy good,
- x_i is the individual consumption level of the energy good,
- X is the aggregate consumption level of the energy good,
- $0 < \alpha < 1$ and $0 < \lambda < 1$

In this framework, earnings of individuals are equal to y_i and the prices of energy and non-energy goods are set to 1.

1) Why do x and X affect differently the utility of individuals ? (1 point)

Answer : Individuals benefit positively from their personal energy consumption (transport, heating. . .). The aggregate consumption of the energy good have a negative impact on environnement and air quality, which affect negatively all individuals.

2) Compute the optimal levels of c_i and x_i in a laissez-faire economy. (1 point)

Answer : Program of the individual i :

$$\begin{cases} \text{Max } U(c,x) = c_i^{1-\alpha} \cdot x_i^{\alpha} \cdot X^{-\lambda} \\ \text{s.t } c_i + x_i = y_i \end{cases}$$

FOC :

$$\begin{cases} x_i^\star = \frac{\alpha}{1-\alpha} c_i^\star \\ c_i + x_i = y_i \end{cases} \Rightarrow c_i^\star = (1-\alpha) y_i \text{ and } x_i^\star = \alpha y_i$$

3) In a planned economy, a social planner wants now to maximize social welfare of individuals. Define the social welfare function and the aggregate budget constraint of the social planner.(0.5 point)

Notes : C and X are respectively the aggregate levels of consumption of the non-energy good and of the energy good. Y is the national income, i.e aggregate earnings of all individuals.

Answer : Social welfare function of the government :

$$\begin{aligned} SWF = \quad & \text{Max} \int_i U_i(c_i, x_i) = U(C,X) \\ & \qquad = C^{1-\alpha} \cdot X^{\alpha-\lambda} \\ \text{s.t} \quad & \qquad C + X = Y \end{aligned}$$

4) Compute the socially optimal values of C and X in the planned economy. (0.5 point)

Answer : FOC :

$$\begin{cases} X^\star = \frac{\alpha-\lambda}{1-\alpha} C^\star \\ C + X = Y \end{cases} \Rightarrow C^\star = \frac{1-\alpha}{1-\lambda} Y \text{ and } X^\star = \frac{\alpha-\lambda}{1-\lambda} Y$$

5) Show and explain briefly why the consumption of energy good is lower in the planned economy than in the laissez-faire economy. (0.5 point)

Answer : In the laissez-faire economy, the aggregate consumption of energy good is $\int_0^1 e_i d_i = \alpha Y$. Since $0 < \alpha < 1$ and $0 < \lambda < 1$, $\alpha Y > \frac{\alpha-\lambda}{1-\lambda} Y$. The government internalizes the fact that energy good consumption introduces a negative externality on individual welfare. Therefore, the consumption of energy good is reduced.

In order to implement the social optimum, the social planner introduces a corrective tax τ on energy consumption. For simplicity, we assume that the revenues from the tax are not redistributed to individuals.

6) Define the new budget constraint of individuals and compute the optimal levels of c_i and x_i.(1 point)
Answer : The new budget constraint becomes $c_i + (1+\tau)x_i = y_i$.
Program of the individual i :

$$\begin{cases} \text{Max } U(c,x) = c_i^{1-\alpha} \cdot x_i^{\alpha} \cdot X^{-\lambda} \\ \\ \text{s.t } c_i + (1+\tau)x_i = y_i \end{cases}$$

FOC :

$$\begin{cases} x_i^\star = \frac{\alpha}{(1+\tau)(1-\alpha)} c_i^\star \\ c_i + (1+\tau)x_i = y_i \end{cases} \Rightarrow c_i^\star = (1-\alpha)y_i \text{ and } x_i^\star = \frac{\alpha}{1+\tau} y_i$$

7) Compute the tax rate $\tau^\star$ that allows to obtain the socially optimal value of consumption of the energy good. (0.5 point).
Answer. $\tau^\star$ is such that $X^\star = \int_0^1 x_i^\star di$, i.e. such that $\frac{\alpha}{1+t^\star} = \frac{\alpha-\lambda}{1-\lambda}$ i.e.

$$t^\star = \frac{\lambda(1-\alpha)}{\alpha-\lambda}$$

Part B : Modified golden rule (2 points)

In the theoretical framework, the economic costs of global warming depend on λ but also on the modified golden rule : $r^\star = \delta + \gamma \cdot g$.

8) What are the parameters δ, γ and g ? (1 point)
Answer : $r^\star$ is the social discount rate, i.e, the rate at which an ideal social planner should discount the future.

– g is the growth rate of the economy.

– δ is the pure social rate of time preference
– γ is the concavity of social welfare function

9) What is the implication of a higher economic growth rate ? (0,5 point)
Answer : When g increases, $r^\star$ increases because future generations are going to be richer and will be more able to clean up our pollution. Therefore, we need to accumulate less capital for the future generations, i.e, we need to sacrifice less welfare today in order to avoid global warming.

10) What is the implication of a higher γ ? (0,5 point)
Answer : γ measures the speed at which the marginal social utility of consumption goes to zero. When γ is very large, it means that extra consumption not so useful for future generations, because they will be very rich anyway. Therefore, we need to accumulate less capital for the future generations, i.e, we need to sacrifice less welfare today in order to avoid global warming.

Exercise 2 : Optimal capital taxation (8 points)

Consider a discrete-time closed economy economy with a continuum $[0;1]$ of infinite-horizon dynasties.

For simplicity, assume a two-points distribution of wealth : dynasties can either own a large capital stock k_t^A or a zero capital stock $k_t^B = 0$. The proportion of high-capital dynasties is exogenous and equal to λ (and the proportion of zero-capital dynasties equal to $1 - \lambda$).

Zero-wealth dynasties have only labor income, which they consume entirely (zero savings). High-wealth dynasties are the only dynasties to own wealth and to save. Assume they maximize a standard dynastic utility function :

$$U_t = \sum_{t \geqslant 0} \frac{U(c_t)}{(1+\theta)^t}$$

All dynasties supply exactly one unit of (homogeneous) labor each period. Output per labor unit is given by a standard production function $f(k_t)$ $(f'(k) > 0, f''(k) < 0)$, where k_t is the average capital stock per capita of the economy at period t. r_t and w_t are respectively the interest rate and wage rate. Markets for labor and capital are assumed to be fully competitive.

1) Compute the marginal products of capital and labor in function of r_t and w_t. (0.5 point)

Answer :

$$\text{Max } \pi_t = f(k_t) - w_t - r_t \cdot k_t$$
$$\text{FOC : } f'(k_t) = r_t \text{ and } f(k_t) - r_t \cdot k_t = w_t$$

2) Explain why interest rate $r^\star$ should be equal to θ at the steady-state ? (No computation is required.) (1 point)

Answer : $r^* = \theta$: this is a direct consequence of the first-order condition of each dynasty's maximization problem (Euler equation). Euler equation $\frac{U'(c_t)}{U'(c_{t+1})} = \frac{1+r_t}{1+\theta}$ if the interest rate r_t is above the rate of time preference θ, then agents choose to accumulate capital and to postpone their consumption indefinitely and this cannot be a steady-state. Conversely, if the interest rate r_t is below the rate of time preference θ, agents choose to desaccumulate capital (i.e. to borrow) indefinitely and to consume more today . This cannot be a steady-state either.

The government wants to introduce a linear redistributive capital taxation into this model. That is, capital income $r_t \cdot k_t$ of the capitalists is taxed at tax rate τ, and the tax revenues are used to finance a wage subsidy s_t to all workers.

3) What does the Golden rule of capital accumulation imply at the steady-state ? Explain the effect of the introduction of the capital taxation on the wealth of capitalist dynasties. (1 point)

Answer : $(1-\tau)f'(k_t^\star) = (1-\tau)r_t^\star = \theta$

The capitalists choose to disaccumulate capital until the point where the net interest rate is back to its initial level (i.e. the rate of time preference).

4) What is the long run income of the workers y_t equal to ? (0.5 point)

Answer : $y_t^\star = w_t^\star + s_t^\star$

with : $w_t^\star = f(k_t^\star) - r_t^\star \cdot k_t^\star$ and $s_t^\star = \tau \cdot r_t^\star \cdot k_t^\star$

That is : $y_t^\star = f(k_t^\star) - (1-\tau)r_t^\star \cdot k_t^\star = f(k_t^\star) - \theta \cdot k_t^\star$

5) What is the capital tax rate τ maximizing workers'income $y_t^\star$? Explain the result.

Answer : $f'(k_t^\star) = \theta \Rightarrow (1-\tau)r_t^\star = \theta \Rightarrow \tau = 0$
The introduction of the capital taxation reduces the capital of the capitalists, which reduces the production and therefore the labor income. The capital tax is shifted on labor in the very long run.

6) How does the elasticity of capital supply play on this result ? (1 point)
Answer : In the infinite-horizon model, the elasticity of capital supply is infinite : any infinitesimal change in the net interest rate generates a savings response that is unsustainable in the long run, unless the net interest rate returns to its initial level. The zero capital tax result breaks down whenever the long run elasticity of capital supply is finite.

7) Why is the optimal capital tax rate equal to 0 in the Atkinson-Stiglitz model ? (Explicit the hypothesis made by the authors) (1.5 points)
Answer : According to Atkinson-Stiglitz (1976), linear optimal capital tax should be equal to 0% if there are perfect capital markets, separable preferences, no inheritance and nonlinear labor income tax. Under separable preferences, there is no point taxing capital income ; it is more efficient to redistribute income by using solely a labor income tax. If inequality entirely comes from labor income inequality, then it is useless to tax capital ; one should rely entirely on the redistributive taxation of labor income

8) According to you, what are the rationales for capital taxation ? (1.5 points)
Answer :

- « Fuzzy frontier » : if the frontier btw labor and capital income flows not so clear (e.g. for self-employed), then it is better to tax both income flows at rates that are not too different
- « Fiscal capacity » : for top wealth holders, wealth is a better indicator of the capacity to contribute than income
- « Meritocracy » : individuals are not responsible for their inherited wealth, so maybe this should be taxed more than their labor income ; but incentives and imperfect k markets imply that part of the ideal inheritance tax tax should be shifted to lifetime k tax

Exercise 2 : Optimal linear taxation (5 points)

We consider an economy made up of individuals who have identical preferences defined over consumption c and labor l, but different wage rates w_i. Assume that each agent i has utility function :

$$u(c, l) = c - \frac{l^{1+\mu}}{\mu + 1}$$

where $\mu > 0$ is a given fixed parameter.

An individual with wage rate w supplying labor l, earns $z = w \cdot l$ (pre-tax earnings) and consumes $c = z - T(z)$ where $T(.)$ is the (possibly nonlinear) income tax.

1) How has the top marginal income tax rate evolved in the U.S. since 1900 ? (1 point)

Answer : In 1900,top marginal tax rate was equal to 0%. It increased from 0% in 1900 to 90% in 1945. It stayed at that level until 1963, and remained at or above 70% until 1981. It declined to 35%-40% since 1990.

2) Consider a linear income tax system $T(z) = -R + \tau \cdot z$ where $R > 0$ is the lump-sum income transfer and τ is a flat tax rate. Compute the optimal level of labor supply that agent i makes (1.5 point).

Answer : Program of the individual i :

$$\text{Max } U(c, l) = R + w \cdot l(1 - \tau) - \frac{l^{1+\mu}}{\mu + 1}$$

$$\text{FOC : } l^{\star} = [w(1 - \tau)]^{1/\mu}$$

3) Show that the tax rate maximizing total tax revenue is equal to $\tau^{\star} = \dfrac{1}{1 + 1/\mu}$ (taking R as given). (1 point)

Answer : Program of the government :

$$\text{Max T} = -R + \sum_i \tau \cdot w_i^{\frac{1}{\mu}+1} \cdot (1 - \tau)^{1/\mu}$$

$$\text{FOC : } w_i^{\frac{1}{\mu}+1}[(1 - \tau)^{1/\mu} - \frac{1}{\mu} \cdot \tau \cdot (1 - \tau)^{1/\mu - 1}] = 0$$

$$\tau^\star = \frac{1}{1+\frac{1}{\mu}}$$

4) What is the parameter $\dfrac{1}{\mu}$? Interpret the formula. (1 point)

Answer : $e = \frac{1}{\mu}$ is the elasticity of income with respect to the net-of-tax rate $1-\tau$. It measures real economic responses to the net-of-tax rate, i.e. labor supply effects, broadly defined (more hours of work, more intense effort per hour worked, occupational choices, etc.). if the elasticity e is higher, then the optimal tax $\tau^\star$ is lower.(basic principle of optimal taxation theory : other things equal, don't tax what's elastic)

5) In Piketty-Saez-Stantcheva, « Optimal Taxation of Top Labor Incomes : A Tale of Three Elasticities »(AEJ 2013), the authors compute an « augmented »optimal tax formula such as :

$$\tau = \frac{1 + t \cdot a \cdot e_2 + a \cdot e_3}{1 + a \cdot e}$$

What is the meaning of e_2 and e_3 ? (1 point)

Answer : e_2 is the tax avoidance elasticity : when tax rates are high, top incomes earners find ways to exploit loopholes and report less of their taxable income. e_3 is the compensation bargaining elasticity : lower top tax rates induces top earners to bargain more aggressively for higher pay.