Public Economics: Tax & Transfer Policies

Final Exam, February 18, 2016 - 2 hours

The exam is 2 hours long and can be done either in French or English. ***No*** *document whatsoever is allowed.*

1 Welfare theorem and taxation (4.5 points)

1) According to the first welfare theorem, what are the rationales for taxation ? (1.5 points)

Answer : First welfare theorem : Under standard convexity assumptions (no externalities and public goods), all market equilibria are Pareto efficient. Therefore, taxation can generate Pareto improvements in the case of public good provision, externalities and stabilization.

2) According to the second welfare theorem, what should be a non-distortionnary redistributive policy ? Why is it difficult in practice to implement such a policy ? (1.5 points)

Answer : All Pareto optima can be obtained as market equilibria under adequate lump-sum transfers. Second-best Pareto optima due to informational imperfections (moral hazard, adverse selection, etc.). Only distortionnary taxation can redistribute resources : equity/efficiency trade-off

3) How has evolved tax revenues in France and in the UK during the XXth century ? How could we explain the different levels of taxation between these two countries today ? (1.5 points)

Answer : Tax revenues represented less than 10% of national income for both countries in 1900. It is now equal to 40% of national income in the UK against

50% for France. Large variations in tax regimes also correspond to large variations in welfare state regimes (Esping-Andersen (1990) The Three Worlds of Welfare Capitalism) : Bismarck (France) vs Beveridge (UK) welfare state organization. Main difference is due to different pension systems (mainly public in France, large private pensions in the UK)

2 Exercise 1 : Income inequality and redistribution (7.5 points)

Consider an economy made up of a continuum of agents i in $[0, 1]$. The utility function of an agent i is given by :

$$U_i = y_i - C(e_i)$$

$$\text{with } C(e_i) = \frac{e^2}{2a},\ a > 0$$

Each individual i can obtain one of two possible pretax incomes $y_i = y_0$ or y_1 (with $y_1 > y_0 > 0$) depending on his/her ability. We note L the fraction of low-ability individuals , i.e individuals with ability θ_0.

The probability for an individual with low ability making effort e to have labor income y_1 is :

$$P[y_i = y_1 | e_i = e,\ \theta = \theta_0] = \theta_0 e$$

The probability for an individual with high ability making effort e to have labor income y_1 is :

$$P[y_i = y_1 | e_i = e,\ \theta = \theta_1] = \theta_1 e$$

where $0 < \theta_0 < \theta_1$ and $\theta > 0$.

Let's introduce a redistributive tax system : all incomes are taxed at rate $0 < \tau < 1$ and all tax revenues are redistributed in a lump-sum way $\tau \cdot Y$, where Y is aggregate income at the corresponding period.

1) Express the after-tax income of a person with low or high pre-tax income.

(1 point).

Answer :

$$\text{Low after-tax income} : (1-\tau)y_0 + \tau Y$$

$$\text{High after-tax income} : (1-\tau)y_1 + \tau Y$$

2) Express the expected utility of a person with low ability. (1 point).

Answer :

$$U_i = E(Y_i) - C(e_i) = (1-\theta_0 e)(1-\tau)y_0 + \theta_0 e(1-\tau)y_1 + \tau Y - e_i^2/2a \qquad (1)$$

3) Compute the effort level $e^\star$ that maximizes the expected utility of an individual with low ability and interpret the result. (1 point)

Answer : Maximizing (1) with respect to e yields the first order condition :

$$e^\star = a\theta_0(1-\tau)(y_1 - y_0)$$

The higher the tax rate the lower the effort, the effect depending on the ability i.e the elasticity of income with respect to effort θ_0 and the cost of effort a.

4) Compute the optimal effort level for an individual with high ability. (0.5 point).

Answer :

$$e_H = a\theta_1(1-\tau)(y_1 - y_0)$$

Suppose that the government decides to set the tax rate t at the level that maximizes the expected utility of individuals with low ability. Note that the government can not observe directly the effort made by individuals.

5) Express the aggregate income Y as a function of the optimal effort level e^*. (1 point)

Answer

$$Y = L[y_1\theta_0 e^* + y_0(1-\theta_0 e^*)] + (1-L)[y_1\theta_1 e_H + y_0(1-\theta_1)e_H)]$$

$$Y = L[\theta_0 e^*(y_1 - y_0)] + (1-L)[\theta_1 e_H(y_1 - y_0)] + y_0$$

or with $e_H = e^*(\theta_1/\theta_0)$

$$Y = e^*(y_1 - y_0)[L\theta_0 + (1-L)\frac{\theta_1^2}{\theta_0}] + y_0$$

6) Compute the optimal tax rate. (2 points)

Answer We want to maximize equation (1) given that $e^\star = a\theta_0(1-\tau)(y_1-y_0)$ and $Y = e^*(y_1-y_0)[L\theta_0 + (1-L)\frac{\theta_1^2}{\theta_0}] + y_0$. After some algebra, the utility of agents with low ability can be written :

$$U_i = \tau(y_1 - y_0)(1-L)e^\star[\frac{\theta_1^2 - \theta_0^2}{\theta_0}] - \frac{e^{\star 2}}{2a} + \theta_0 e^\star(y_1 - y_0) + y_0$$

The first order condition is :

$$\tau = (1-L)[\frac{\theta_1^2 - \theta_0^2}{\theta_0}](1 - 2\tau)$$

$$\tau = \frac{(1-L)[\theta_1^2 - \theta_0^2]}{\theta_0^2 + 2(1-L)[\theta_1^2 - \theta_0^2]}$$

7) Explain why the optimal tax rate would be equal to zero if $\theta_1 = \theta_0$. What assumption should be changed in order to get a different result ? (1 point)

Answer : If $\theta_1 - \theta_0$, all individuals are similar and there is no point to redistribute and therefore tax labor income. However, this result holds because all individuals are risk-neutral. If individuals were risk-averse, taxation and redistribution would be desirable but would decrease incitives to make effort.

3 Exercise 2 : Optimal capital taxation (8 points)

We consider a simple model of wealth accumulation with one period and a continuum of agents i in $[0,1]$. At the beginning of the period, individuals work to earn exogeneous lifetime earnings y_{Li} and receive an inheritance b_i^r. In this simple economy, there is no growth, no return on capital and no discount factor. There is no taxation on labor, nor taxation on inheritance received. For redistribution purpose, the government decides to introduce a wealth tax on wealth accumulated at the end of the first period in order to finance a lump-sum transfer. Therefore, the lifetime ressources of individuals during the first period is equal to the sum of labor income y_{Li}, bequests received b_i^r and lump-sum transfer g for all individuals. The lump-sum transfer g is entirely financed by linear taxation on end-of-period wealth such as $g = \tau_w \cdot w$, with w the average end-of-period wealth and τ_w, the tax rate on wealth. At the end of the period, individuals split their ressources into consumption c_i and wealth accumulated w_i. They derive utility over consumption and after-tax wealth (partly because of bequest motives and partly because wealth brings utility per se. The utility function is given by :

$$V_i = U_i(c_i) + \phi_i((1-\tau_w)w_i)$$

We take as given the joint distribution $f(y_{Li}, b_i^r, U_i, \phi_i)$ of labor income, bequests received and preferences over consumption and wealth.

1) Define the budget constraint and the the maximisation program of an individual i. (2 points)

Answer : Budget constraint

$$y_{Li} + b_i^r + g = w_i + c_i$$

Answer : Maximisation program

$$\text{Max } V_i = U_i(c_i) + \phi_i((1-\tau_w)w_i)$$
$$\text{st : } y_{Li} + b_i^r + \tau_w \cdot w = w_i + c_i$$

2) What is the first order condition for wealth accumulation w_i ? (2 points)

Answer :

Plotting the budget constraint into the utility function gives :

$$\text{Max } V_i = U_i(y_{Li} + b_i^r + \tau_w \cdot w - w_i) + \phi_i((1-\tau_w)w_i)$$

$$\text{FOC : } U' = (1-\tau_w)\phi'$$

3) The government wants to define wealth tax rate τ_w in order to maximize the welfare of individuals. The average wealth w is a function of $(1-\tau_w)$. We define the aggregate elasticity of wealth to wealth tax such as $e_w = \frac{1-\tau_w}{w}\frac{dw}{d(1-\tau_w)}$. The aggregate elasticity e_w comes from the aggregation of all individual maximisation programmes.

Show that the wealth tax rate maximizing the utility of a given individual i is equal to $\tau_w = (1 - w_i/w)/(1+e_w)$? (2 points)

Answer :

$$\frac{dU_i}{d\tau_w} = 0 = (U'[w - \tau_w \frac{dw}{d(1-\tau_w)}) - w_i\phi'$$

First order condition of individual maximisation problem gives $U' = (1-\tau_w)\phi'$ and $e_w = \frac{1-\tau_w}{w}\frac{dw}{d(1-\tau_w)}$

$$\Rightarrow 0 = U' \cdot w[1 - \frac{\tau_w}{1-\tau_w}e_w] - U'\frac{w_i}{1-\tau_w}$$

$$\Rightarrow 0 = w[1 - \frac{\tau_w}{1-\tau_w}e_w] - \frac{w_i}{1-\tau_w}$$

$$\Rightarrow 0 = 1 - \tau_w - \tau_w \cdot e_w - \frac{w_i}{w}$$

$$\Rightarrow \tau_w = (1 - w_i/w)/(1 + e_w)$$

4)Interpret the formula (2 points)

Answer :

When the government wants to maximize wealth tax revenues, the optimal tax rate is $\tau_w = 1/(1+e_w)$. Don't tax what is elastic! In our case, $\tau_w \neq 1/(1+e_w)$ because individuals care about wealth-holding (bequest motive or utility of wealth per se). The more they want to accumulate as compared to average (the higher w_i/w), the less is the tax rate τ_w.

5) Bonus question : Now assume that the government wants to maximise the sum of utilities of all individuals with zero bequest received, possibly with different weights ω_i in order to normalize for preference heterogeneity. The social welfare function of the government is given by :

$$SWF = \int_{i \in [0,1] s.t b_i^r = 0} \omega_i [U_i(y_{Li} + \tau_w \cdot w - w_i) + \phi_i((1-\tau_w)w_i)]$$

What would be the optimal tax rate? (3 points)

Answer :

$$\frac{dSWF}{d\tau_w} = 0 = \int_i \omega_i [(U_i'[w - \tau_w \frac{dw}{d(1-\tau_w)}) - w_i \phi_i']$$

First order condition of individual maximisation problem gives $U_i' = (1-\tau_w)\phi_i'$ and $e_w = \frac{1-\tau_w}{w} \frac{dw}{d(1-\tau_w)}$

$$\Rightarrow 0 = \int_i \omega_i [U_i' \cdot b[1 - \frac{\tau_w}{1-\tau_w} e_w] - U_i' \frac{w_i}{1-\tau_w}]$$

$$\Rightarrow 0 = w[1 - \frac{\tau_w}{1-\tau_w} e_w] - \frac{1}{1-\tau_w} \frac{\int_i \omega_i U_i' w_i}{\int_i \omega_i U_i'}$$

$$\Rightarrow 0 = 1 - \tau_w - \tau_w \cdot e_w - \frac{\int_i \omega_i U_i' w_i}{\int_i \omega_i U_i' w}$$

$$\Rightarrow \tau_w = \frac{1 - \frac{\int_i \omega_i U_i' w_i}{\int_i \omega_i U_i' w}}{1 + e_w}$$

Note on $\frac{\int_i \omega_i U_i' w_i}{\int_i \omega_i U_i' w}$: In a more simplified setting in which preferences are identical among zero-bequest receivers and where $\omega_i = 0$, those with high earnings would consume more and accumulate more wealth than those with low earnings. Therefore, high earnings individuals would have a lower marginal utility of consumption U_i' and would have a lower importance in the social welfare function than zero-bequest receivers with low earnings. In the Meritocratic-Rawlsian case considered in this model, the government wants to favor all zero-bequest receivers, with an equal weight, irrespective of their earnings. Indeed, the government considers that individuals should be compensated for inequality they are not responsible forŮsuch as bequests received-but not for inequality they are responsible for-such as labor income. Therefore the government should put uniform weight on zero-bequest receivers irrespective of their earnings. To take into account that, the weight ω_i is constructed such as $\omega_i \cdot U_i' = 1$ for all zero bequest receivers, i.e all zero-bequest receivers have a uniform weight in the social welfare function. In this case, $\frac{\int_i \omega_i U_i' w_i}{\int_i \omega_i U_i'}$ is equal to average wealth of the zero-bequest receivers w_p and the optimal tax rate formula becomes $\tau_w = \frac{1 - w_p/w}{1+e_w}$.

More generally, the weight ω_i is also used to ensure that all zero-bequest receivers have the same weight in the social welfare function even if they have different preferences over consumption and wealth, different degree of risk aversion ...