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ABSTRACT

Large savings and current account surpluses by China and other countries are said to be a contributor to the global current account imbalances and possibly to the recent global financial crisis. This paper proposes a theory of excess savings based on a major, albeit insufficiently recognized by macroeconomists, transformation in many of these societies, namely, a steady increase in the surplus of men relative to women. We construct an OLG model with two sexes and a desire to marry. We show conditions under which an intensified competition in the marriage market can induce men to raise their savings rate, and produce a rise in the aggregate savings and current account surplus. This effect is economically significant if the biological desire to have a partner of the opposite sex is strong. A calibration of the model suggests that this factor could generate economically significant current account responses, or more than 1/2 of the actual current account imbalances observed in the data.

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1 Introduction

High savings rates in excess of domestic investment rates in many countries in East and Southeast Asia have produced a massive current account surplus as a share of GDP, and are said to be a major contributor to the global current account imbalances, to the unusually low long-term interest rates, and possibly to the onset of the 2008-2009 global financial crisis. As to theories of savings behavior, the existing literature has highlighted the roles of life-cycle considerations (Modigliani, 1970), precautionary savings (Kimball, 1990), habit formation (Carroll, Overland, and Weil, 2008), culture (Belton and Uwaifo Oyelere, 2008), and financial under-development (Caballero, Farhi, and Gourinchas, 2008; Ju and Wei, 2006, 2008 and forthcoming; Mendoza, Quadrini and Rios-Rull 2007; and Song, Storesletten, and Zilibotti, forthcoming). The aim of the current paper is to propose an alternative theory that gives prominence to a major, albeit insufficiently recognized by macroeconomists, social transformation in many economies, namely an increasing relative surplus of men in the marriage market. The basic thesis is that as competition intensifies in the marriage market, men or parents with sons raise their savings rates with the hope of improving their relative standing in the marriage market. Because the biological desire to have a partner of the opposite sex is strong, this effect is quantitatively important enough to reveal itself in the aggregate savings rate and the current account balance.

The direct inspiration for the theory is an empirical paper by Wei and Zhang (2009), which studies household savings behavior in China. They provide both cross-regional and cross-household evidence that is consistent with the notion that a worsening prospect for men in the marriage market has motivated them and their parents to raise their savings rates substantially. They call this the "competitive saving motive." Chinese household savings as a share of disposable income rose from 16% in 1990 to 30% in 2007. Wei and Zhang suggest that the rise in the sex ratio imbalance could account for half the total increase in the savings rate. Because their paper does not have a formal theory, there is a need to construct a model to see if the hypothesis can work in a general equilibrium, and whether a calibration of the model can produce an effect whose magnitude is economically significant. Furthermore, one wonders if the hypothesis is applicable to countries beyond China.

In this paper, we aim to fill these important voids. To check if the sex ratio effect goes beyond China, we first provide some international evidence that a country's savings rate is systematically linked to its sex ratio. After controlling for the effects of income, the share of working age people in the population (i.e., a proxy for the life cycle theory), the ratio of credit to GDP (a proxy for financial development), and social security expenditure as a share of GDP (a proxy for the precautionary savings need), we find that a rise in the sex ratio from a balanced level to 1.15 (the sex ratio in China) is associated with a higher current account by over 10% of GDP.

The core part of the paper is to analyze theoretically how a sex ratio imbalance influences the economy-wide savings rate and the current account. We construct a simple overlapping generations (OLG) model with two sexes and a desire to marry. To focus on the macroeconomic implications of sex ratio imbalances, we intentionally shut down channels such as the precautionary savings motive, habit formation, culture, and financial development. Because it is an OLG model, there are still life-cycle considerations, which, however, do not lead to current account imbalances on their own.

Under reasonable conditions, we show that men respond to a rise in the sex ratio by raising their savings rates. Moreover, the increment in their savings is always enough to offset any decrease in women's savings if it happens. As a result, the aggregate savings rises with the sex ratio. To check if the model can deliver an effect that is economically significant, we go to quantitative calibrations. In the benchmark case, for a small open economy, as sex ratio rises from 1 to 1.15, the economy-wide savings rate and the current account will rise by more than 10%. We also consider the case of two large economies, whose relative sizes and income levels are calibrated to mimic China and the United States. The synthetic United States is assumed to always have a balanced sex ratio, while the synthetic China experiences a rise in the sex ratio from 1 (balanced) to 1.5 (very unbalanced). The rise in China's sex ratio produces a rise in its current account surplus, and a corresponding rise in the current account deficit for the United States. The magnitudes of the current account imbalances in the simulations (about 7.7% of GDP for China and -2.6% of GDP for the United States) are such that they are more than one-half of the actual current account imbalances observed in the data. While the sex ratio imbalance is not the sole reason for the global current account imbalances in recent years, it could be one of the significant, and yet thus far unrecognized, factors.

A desire to enhance one's prospect in the marriage market through a higher level of wealth could be a motive for savings even in countries with a balanced sex ratio. But such a motive is not as easy to detect when the competition is modest. When the sex ratio gets out of balance, obtaining a marriage partner becomes much less assured. A host of behaviors that are motivated by a desire to succeed in the marriage market may become magnified. But sex ratio imbalances so far have not been investigated by macroeconomists. This may be a serious omission. A sex ratio imbalance at birth and in the marriage age cohort is a common demographic feature in many economies, especially in East, South, and Southeast Asia, such as Korea, India, Vietnam, Singapore, Taiwan and Hong Kong, in addition to China. In many economies, parents have a preference for a son over a daughter. This used to lead to large families, not necessarily an unbalanced sex ratio. However, in the last three decades, as the technology to detect the gender of a fetus (Ultrasound B) has become less expensive and more widely available, many more parents engage in selective abortions in favor of a son, resulting in an increasing relative surplus of men. The strict family planning policy in China, introduced in the early 1980s, has induced Chinese parents to engage in sex-selective abortions more aggressively than their counterparts in other countries. The sex ratio at birth in China rose from 106 boys per hundred girls in 1980 to 122 boys per hundred girls in 2007 (see Wei and Zhang, 2009, for more detail). It may not be a coincidence that the Chinese current account surplus started to garner international attention around 2003 just when the first cohort born after the implementation of the strict family planning policy was entering the marriage market.

Throughout the model, we assume an exogenous sex ratio. Given the large size of the population in any marriage market, it is reasonable to assume that individual families take the sex ratio as given (and ignore the impact of their individual sex selective abortions on the aggregate sex ratio).

There are three literatures related to the current paper. First, the literature on status goods, positional goods, and social norms (e.g., Cole, Mailath and Postlewaite, 1992, Corneo and Jeanne, 1999, Hopkins and Kornienko, 2004 and 2009) has offered many useful insights. One key point is that when wealth can improve one's social status (including improving one's standing in the marriage market), in addition to affording a greater amount of consumption goods, there is an extra incentive to save. This element is in our model as well. However, all existing theories on status feature a balanced sex ratio. Yet, an unbalanced sex ratio presents some non-trivial challenges. In particular, while a rise in the sex ratio is an unfavorable shock to men (or parents with sons), it is a favorable shock to women (or parents with daughters). Could the latter group strategically reduce their savings so as to completely offset whatever increments in savings by men or parents with sons? In other words, the impact on aggregate savings from a rise in the sex ratio appears ambiguous. Our model will address this question. In any case, this literature has no discernible impact in policy discussions on current account imbalances. For example, while there are voluminous documents produced by the International Monetary Fund and speeches by U.S. officials on China's high savings rate and large current account surplus, no single paper or speech thus far has pointed to a possible connection with its high sex ratio imbalance.

A second related literature is the economics of family, which is too vast to be summarized here comprehensively. One interesting insight from this literature is that a married couple's consumption has a partial public goods feature (Browning, Bourguignon and Chiappori, 1994; Donni, 2006). We make use of this feature in our model as well. None of the papers in this literature explores the general equilibrium implications for aggregate savings and current account from a change in the sex ratio.

The third literature examines causes of a rise in the sex ratio. The key insight is that the proximate cause responsible for a majority of the recent rise in the sex ratio imbalance is sex-selective abortions. The sex selections have been made increasingly possible by the spread of Ultrasound B machines, which explains why the dramatic rise in the sex ratio in so many countries is a relatively recent phenomenon. There are two deeper causes for the parental willingness to disproportionately abort female fetuses. The first is the parental preference for sons, which in part has to do with the relatively inferior economic status of women. When the economic status of women improves, sex-selective abortions appear to decline (Qian, 2008). The second is either something that leads parents to voluntarily choose to have fewer children than the earlier generations, or a government policy that limits the number of children a couple can have. In regions of China where the family planning policy is less strictly enforced, there is also less sex ratio imbalance (Wei and Zhang, 2009). Bhaskar (2009) examines parental sex selections and their welfare consequences.

The rest of the paper is organized as follows: in Section 2, we provide some international evidence that sex ratio may have significant impact on a country's current account. In Section 3, we present a benchmark model with no intra-household bargaining. In Section 4, we consider an extension of the model that allows for intra-household bargaining, and show that the key propositions still hold.

In Section 5, we calibrate the model to see if the sex ratio imbalance can produce changes in the aggregate savings rate and current account whose magnitudes are economically significant. Section 6 offers concluding remarks and discusses possible future research.

2 Some International Data Patterns

To motivate the subsequent theoretical model, we provide some cross-country evidence on the relationship between a country's sex ratio and its (non-government part of the) current account. We define a country's non-governmental part of the current account as its current account balance minus its government savings (or government revenue minus expenditure), divided by its GDP. We exclude government savings because our theory is about private sector savings. (Because the ratio of government savings to GDP has a large variance across countries, it adds noise to the estimated relationship between the non-governmental part of the current account and the sex ratio.)

In Figure 1a, we plot the ratio of the non-governmental part of the current account to GDP in 2006 against the sex ratio for cohort of age 0-15 for all countries for which we can obtain relevant data. The year 2006 is chosen because it is relatively recent (when the global current account imbalances had become a policy issue but the world had not entered a global financial crisis), and the data are available for a large number of countries. The age group 0-15 is chosen for the sex ratio to maximize country coverage. Current account and GDP are from the World Bank's WDI database. The sex ratio data is obtained from the World Factbook.

There is a visible positive correlation between the two variables. The slope coefficient of the fitted line is statistically significant at the one percent level, with a point estimate of 125.5 and a standard error of 44.9. In other words, those countries with a higher sex ratio also tend to have a higher private-sector current account to GDP ratio.

Economic theories suggest that other factors can affect a country's current account. We run a multivariate regression of the ratio of non-governmental current account to GDP on sex ratio and other control variables. To be precise, the specification equation is the following:

$$cagdp_i = \beta_0 + \beta_1 \cdot \text{sex ratio}_i + \beta_2 Z_i + \varepsilon$$

where $cagdp_i$ is the ratio of a country's current account minus government saving to its GDP. Our choice of the control variables is guided by the life-cycle theory, precautionary saving theory, and financial development theory. We therefore include variables in Z_i log per capita GDP, the share of working age people in the population (a proxy for life-cycle theory), social security expenditure as a share of GDP (a proxy for the precautionary saving theory), private credit to GDP ratio (a commonly used proxy for financial development), and continental dummies (a proxy for possible cultural factors).

The share of working age in the population and private credit to GDP ratio are obtained from the World Bank's WDI database. Social security expenditure as a share of GDP data is obtained from the International Labor Organization (ILO) database. A series of regressions are reported in Table 1a, where the set of control variables is progressively enlarged. In each regression, we have a positive and statistically significant coefficient on the sex ratio: as sex ratio becomes more unbalanced, the current account balance tends to go up. This result still holds after we exclude potential outliers (in particular, Kuwait, which has a very large current account surplus in 2006). From Table 1a, as sex ratio rises from 1.03 (normal biological level without sex selection) to 1.15 (China's sex ratio), based on the last column, current account will rise by around 15 percent. Since China is close to be on the regression line, it does not have a significant effect on the magnitude of the slope coefficient.

In Table 1a, we find that the financial development index has a significantly negative coefficient as the theory predicts. This means that a country with an under-developed financial market tends to have a current account surplus. The age profile of populations and the social security expenditure index produce some puzzling patterns. In contrast to the life-cycle theory and precautionary saving theory, the share of working age population has a negative coefficient and social security expenditure as a share of GDP has a positive coefficient. The coefficients mean that old-age households and households with children save more than do households in between, and people save more even though they may get more social security benefits. However, those coefficients are not significant in several regressions which means that life-cycle theory and precautionary saving theory are not capable of explaining the global current account imbalances.

The intertemporal theory predicts that a country's current account should be very sensitive to temporary shocks. To minimize the influence of year-to-year fluctuations in the current account due to temporary shocks, we also conduct a robustness check whereby the dependent variable is the average ratio of non-governmental current account to GDP over a five-year period (2004-2008). The bilateral relationship between the average current account/GDP ratio and the sex ratio is presented in Figure 1b. A strong positive association is still clearly visible. In Table 1b, we report a similar set of regressions where the set of controls is expanded progressively. Again, the positive relationship between the local sex ratio and the local non-governmental current account is robust and statistically significant.

We also study the relationship between a country's sex ratio and its savings rate (% of GDP). In Figures 2a and 2b, we plot a country's aggregate savings rate against its sex ratio, and find a strong positive correlation between the two variables. Tables 2a and 2b provide the regression results. The coefficients on the sex ratio in all the regressions are always positive and significant.

There are many caveats with the empirical patterns. First, in spite of our best effort, there may still be potential control variables that are missing from our list. Second, because the sex ratio data are not available for most countries in earlier years, we are not able to conduct a panel regression. In any case, the sex ratio is likely to be strongly serially correlated, which would have required a long time series to successfully identify the parameters in a panel regression with fixed effects. Third, the sex ratio can be endogenous and/or measured with errors. This would normally call for an instrumental variable approach. At this point, we are not able to come up with convincing instrumental variables in a cross-country context. We note, however, that Wei and Zhang (2009) realize the possibility of endogeneity and measurement errors in their context and address it with a two-stage least squared approach. In particular, they use regional variations in financial penalties for violating birth quotas and variations in the proportion of local population that is legally exempted from the family planning policy as instruments for local sex ratio. With the instrumental variable approach, they find that the positive association between the local savings rate and the local sex ratio continues to hold.

Because of these caveats, we do not claim to have proved a causal relationship from the sex ratio to the current account across countries. The modest aim of this section is to point out the existence of a positive association across countries between a country's sex ratio and its current account, a pattern not previously recognized by the literature. The primary objective of this paper, however, is to construct a theoretical model to investigate how a change in the sex ratio may affect aggregate savings behavior and the current account balance.

3 The Benchmark Model

We construct an overlapping generations model with two sexes. Both men and women live two periods: young and old. An individual (of either sex) receives an exogenous endowment in the first period and nothing in the second period. She or he consumes a part of the endowment in the first period and saves the rest for the second period. Each person can only get married at the beginning of the second period; each person can decide either to be single for life or to enter the marriage market.

A marriage can take place only at the beginning of the second period, and only between a man and a women from the same generation. Once married, the husband and the wife pool their first-period savings together and consume an identical amount in the second period. The second period consumption within a marriage has a partial public good feature. In other words, the husband and the wife can each consume more than half of their combined second period income - the exact proportion is an exogenous parameter to be explained below. In addition, everyone is endowed with an ability to give his/her spouse some emotional utility (or "love" or "happiness"). This emotional utility is a random variable in the first period with a common and known distribution across all members of the same sex, and its value is realized and becomes public information when the individual enters the marriage market.

Each generation is characterized by an exogenous ratio of men to women $\phi (>= 1)$. All men are identical *ex ante*, and all women are identical *ex ante*. (They are different *ex post,* because their realized values of the emotional utility may be different.) Men and women are symmetric in all aspects except that the sex ratio may be unbalanced.

We proceed in six steps. First, we start with a representative woman's optimization, followed by a representative man's optimization problem. Second, we describe how the marriage market works. Third, we perform comparative statics, in particular, on how the savings rates change in response to a rise in the sex ratio. Fourth, we consider a small open economy with production and discuss the current account response to a change in the sex ratio. Fifth, we solve for a two-country model in which the global interest rate is endogenous. Sixth, we use numerical calibrations to see if the model can deliver current account responses that are economically significant.

3.1 A Representative Woman's Optimization Problem

A representative woman makes her consumption/saving decisions in her first period, taking into account the choices by men and all other women, and the likelihood that she will be married. If she remains single, then her second-period consumption is

$$c_{2w,n} = Rs^w y^w$$

where R, y^w and s^w are the gross interest rate, her endowment, and savings rate, respectively. Let V_n^w denote her value function if she chooses to be single for life, which is given by

$$V_n^w = \max_{s_n^w} u(c_{1w,n}) + \beta u(c_{2w,n})$$

If she is married at the beginning of the second period, her second-period consumption is

$$c_{2w} = \kappa \left(Rs^w y^w + Rs^m y^m\right)$$

where y^m and s^m are her husband's first period endowment and savings rate, respectively. κ ($\frac{1}{2} =< \kappa =< 1$) represents the notion that consumption within a marriage is a public good with congestion. As an example, if a couple buys one car, both spouses can use it. If they are both singles, they may need to buy two cars. When $\kappa = \frac{1}{2}$, then the husband and the wife only consume private goods. When $\kappa = 1$, then all the consumption is a public good with no congestion[1].

If she decides to enter the marriage market, her optimal savings rate is chosen to maximize the following objective function:

$$V^w = \max_{s^w} u(c_{1w}) + \beta E\left[u(c_{2w}) + \eta^m\right]$$

[1] By assuming the same κ for the wife and the husband, we abstract from a discussion of bargaining within a household. In an extension later in the paper, we allow κ to be gender specifc, and be a function of the sex ratio and the relative wealth levels of the two spouses, along the line in Chiappori (1988 and 1992) and Browning and Chiappori (1998). This tends to make the response of the aggregate savings stronger to a given rise in the sex ratio.

subject to the budget constraints that

$$\begin{aligned} c_{1w} &= (1-s^w)y^w & (3.1) \\ c_{2w} &= \begin{cases} \kappa\left(Rs^w y^w + Rs^m y^m\right) & \text{if married} \\ Rs^w y^w & \text{otherwise} \end{cases} & (3.2) \end{aligned}$$

where E is the expectation operator. η^m is the emotional utility (or "love") she obtains from her husband, which is a random variable with a distribution function F^m. Bhaskar (2009) also introduces a similar "love" variable.

We allow for a mixed strategy: the representative woman chooses the probability of entering the marriage market ρ^w, a savings rate if she decides to enter, and a separate savings rate if decides to abstain from the marriage market. The optimization problem is

$$\max_{\rho^w, s^w, s_n^w} \rho^w V^w + (1-\rho^w)V_n^w$$

Obviously, she will choose $\rho^w = 1$ if and only if $V^w > V_n^w$.

3.2 A Representative Man's Optimization Problem

A man's problem is symmetric to a women's problem. In particular, if he is single throughout his life, his second period consumption is

$$c_{2m,n} = Rs^m y^m$$

The value function for a man who chooses to be a life-time bachelor is

$$V_n^m = \max_{s_n^m} u(c_{1m,n}) + \beta u(c_{2m,n})$$

If he is married, his second period consumption is

$$c_{2m} = \kappa\left(Rs^w y^w + Rs^m y^m\right)$$

If a man decides to enter the marriage market, he chooses his savings rate to maximize the following value function

$$V^m = \max_{s^m} u(c_{1m}) + \beta E\left[u(c_{2m}) + \eta^w\right]$$

subject to the budget constraints that

$$c_{1m} = (1 - s^m) y^m \tag{3.3}$$

$$c_{2m} = \begin{cases} \kappa \left(R s^w y^w + R s^m y^m \right) & \text{if married} \\ R s^m y^m & \text{otherwise} \end{cases} \tag{3.4}$$

where V^m is his value function when he chooses to enter the marriage market. η^w is the emotional utility he obtains from his wife, which is drawn from a distribution function F^w.

Similar to the representative woman, the representative man chooses the probability of entering the marriage market ρ^m as well as two potentially separate savings rates. The optimization problem is

$$\max_{\rho^m, s^m, s_n^m} \rho^m V^m + (1 - \rho^m) V_n^m$$

He decides to enter the marriage market with probability one if and only if the expected utility of doing so is greater than otherwise, or $V^m > V_n^m$.

3.3 The Marriage Market[2]

In the marriage market, everyone ranks the members of the opposite sex by a combination of two criteria: (1) the level of wealth (which is determined solely by the first-period savings if the endowment is identical across all individuals), and (2) the size of "love" he/she can obtain from his/her spouse. More precisely, a woman i prefers a higher ranked man to a lower ranked one, where the rank on man j is given by $u(c_{2w,i,j}) + \eta_j^m$. Symmetrically, man j assigns a rank to woman i based on the utility he can obtain from her $u(c_{2m,j,i}) + \eta_i^w$. (To ensure that the preference is strict for both men and women, when there is a tie in terms of the above criterion, we break the tie by assuming that a woman prefers j if $j < j'$ and a man does the same.)

The exact size of "love" or emotional utility is only revealed in the second period. Because all men (and women) are the same *ex ante*, all men make the same savings decisions (and all women also make the same decisions).

The marriage market is assumed to follow the Gale-Shapley algorithm, which specifies the following: (1) Each man proposes in the first round to his most preferred woman. Each woman holds the proposal from her most preferred suitor and rejects the rest. (2) Any man who is rejected in round k-1 makes a new proposal in round k to his most preferred woman among those who are still available. Each available women in round k holds the proposal from her most preferred man and rejects the rest. (3) The procedure repeats until no further proposals are made. With our setup, the algorithm produces a unique and stable equilibrium of matching between men and women (Gale and Shapley, 1962; and Roth and Sotomayor, 1990) [3].

[2] We use the word" market" informally here. The pairing of husbands and wives is not done through prices.

[3] If only women can propose and men respond with defered acceptance, the same matching outcomes will emerge in

For woman i, let π_i^w denote the mapping from J to N:

$$\pi_i^w : J \to N$$

where $\pi_i^w(j) \in N$ is the ranking of man j by woman i, J is the set of index on men and N denotes the set of natural numbers. Given all her rivals' (s^w, η^w) and all men's (s^m, η^m), her probability density function to be matched with man j is denoted by $g^i\left(\pi_i^w(j)\right| s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\big)$. Her second period expected utility is

$$\begin{aligned}
& \int \max\Big[\ u\left(c_{2w,i,j}\right) + \eta_j^m, \quad u\left(Rs_i^w y_i^w\right)\ \Big] g^i\left(\pi_i^w(j) | \, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right) d\pi_i^w(j) \\
= \quad & \int^{\bar{\pi}_i^w} \left[u\left(c_{2w,i,j}\right) + \eta_j^m\right] g^i\left(\pi_i^w(j) | \, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right) d\pi_i^w(j) \\
& + \int_{\bar{\pi}_i^w} u\left(Rs_i^w y_i^w\right) g^i\left(\pi_i^w(j) | \, s_{-i}^w, \eta_{-i}^w, s^m, \eta^m\right) d\pi_i^w(j)
\end{aligned}$$

where $\bar{\pi}_i^w$ is her threshold ranking on men. Any lower-ranked man, or any man with $\pi_i^w > \bar{\pi}_i^w$, won't be chosen by her.

Let I denote the index on women. Since there are (weakly) fewer women in the economy than men by assumption, we expand the set I to $\widetilde{I}$ such that $\widetilde{I}$ and J have the same number of elements. For man j, we define π_j^m the mapping from $\widetilde{I}$ to N.

$$\pi_j^m : \widetilde{I} \to N$$

where $\pi_j^m(i) \in N$ is man j's ranking of woman i, $\pi_j^m(k) = k$, $k > i^{\max}$ where $i^{\max}$ is the last woman in set I. Given all his rivals (s^m, η^m) and all women's (s^w, η^w), man j's probability density function to be matched with woman i is denoted by $g^j\left(\pi_j^m \middle| \, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right)$. For all $k > i^{\max}$, we integrate $g^j\left(\pi_j^m \middle| \, h_{-j}^m, \eta_{-j}^m, h^w, \eta^w\right)$ and obtain the probability that man j is matched with any woman. His second-period expected utility is

$$\begin{aligned}
& \int \max\Big[\ u\left(c_{2m,j,i}\right) + \eta_i^w, \quad u\left(Rs_j^m y_j^m\right)\ \Big] g^j\left(\pi_j^m \middle| \, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right) d\pi_j^m \\
= \quad & \int^{\bar{\pi}_j^m} \left[u\left(c_{2m,j,i}\right) + \eta_i^w\right] g^j\left(\pi_j^m \middle| \, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right) d\pi_j^m \\
& + \int_{\bar{\pi}_j^m} u\left(Rs_j^m y_j^m\right) g^j\left(\pi_j^m \middle| \, s_{-j}^m, \eta_{-j}^m, s^w, \eta^w\right) d\pi_j^m
\end{aligned}$$

where $\bar{\pi}_j^m$ is his threshold ranking on all women. Any woman with a poorer rank, $\pi_j^m > \bar{\pi}_j^m$, will not be chosen by man j.

We assume that the density functions of η^m and η^w are continuously differentiable. Since all men

our simple setting. What we have to rule out is that both men and women can propose, in which case, we cannot prove that the matching is unique.

(women) in the marriage market have identical problems, they make the same savings decisions. In equilibrium, a positive assortative matching emerges for those men and women who are matched. In other words, there exists a mapping M from η^w to η^m such that

$$\begin{aligned} 1 - F^w(\eta^w) &= \phi\left(1 - F^m\left(M(\eta^w)\right)\right) \\ &\Leftrightarrow \\ M(\eta^w) &= (F^m)^{-1}\left(\frac{F^w(\eta^w)}{\phi} + \frac{\phi - 1}{\phi}\right) \end{aligned}$$

For simplicity, we assume that η^w and η^m are drawn from the same distribution, $F^w = F^m = F$. Furthermore, the lowest possible value of the emotional utility $\eta^{\min}$ is assumed to be sufficiently large that everyone desires to be married. We also assume that there exists a small and exogenous possibility p that a woman may not find a marriage partner due to frictions in the marriage market. The last assumption plays no role in the analytical part of the model but will simplify the quantitative calibrations later. In equilibrium, given all her rivals' saving decisions and η^w, woman i's second period utility is

$$(1-p)\left[u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) + \int_{\eta^{\min}}^{\eta^{\max}} M(\tilde{\eta}_i^w) d\tilde{F}(\tilde{\eta}_i^w)\right] + pu(Rs^w y^w)$$

where $\tilde{\eta}_i^w = u(\kappa\left(Rs_i^w y^w + Rs^m y^m\right)) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta^w$.

Due to the symmetry, we drop the sub-index i. For a woman who chooses to enter the marriage market, given men's savings decisions, her first order condition is

$$-u'_{1w} y^w + \beta(1-p)\left[u'_{2w}\frac{\partial c_{2w}}{\partial s^w} + \frac{\partial \int M\left(\tilde{\eta}^w\right) d\tilde{F}^w(\tilde{\eta}^w)}{\partial s^w}\right] + pu'_{2w,n} y^w = 0 \qquad (3.5)$$

where

$$\frac{\partial \int M\left(\tilde{\eta}^w\right) d\tilde{F}(\tilde{\eta}^w)}{\partial s^w} = \kappa u'_{2w} R y^w \left[\int M'(\eta^w) dF(\eta^w) + M(\eta^{\min}) f(\eta^{\min})\right]$$

Similarly, for a man who decides to enter the marriage market, his second-period utility is

$$(1-p)\left[\tilde{\delta}_j^m u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) + \int_{M(\eta^{\min})}^{\eta^{\max}} M^{-1}\left(\tilde{\eta}_j^m\right) d\tilde{F}(\tilde{\eta}_j^m)\right] + \left[(1-p)(1-\tilde{\delta}_j^m) + p\right] u(Rs_j^m y^m)$$

where $\tilde{\eta}_j^m = u\left(\kappa\left(Rs^w y^w + Rs_j^m y^m\right)\right) - u(\kappa\left(Rs^w y^w + Rs^m y^m\right)) + \eta_j^m$ and $\tilde{\delta}_j^m$ is the probability he gets married

$$\begin{aligned} \tilde{\delta}_j^m &= \Pr\left(u(c_{2w}(j)) - u(c_{2w}) + \eta_j^m \geq M(\eta^{\min}) \middle| Rs^w y^w, Rs^m y^m\right) \\ &= 1 - F\left(M(\eta^{\min}) - u(c_{2w}(j)) + u(c_{2w})\right) \qquad (3.6) \end{aligned}$$

If he chooses to enter the marriage market, the first order condition for him is

$$-u'_{1m}y^m+\beta(1-p)\left[\begin{array}{c}\delta^m u'_{2m}\frac{\partial c_{2m}}{\partial s^m}+\int_{M(\eta^{\min})}\frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s^m}d\tilde{F}^m(\tilde{\eta}^m)\\ +f\left(M(\eta^{\min})\right)u'_{2w}\frac{\partial c_{2w}}{\partial s^m}\left(u_{2m}-u_{2m,n}+\eta^{\min}\right)\end{array}\right]+\left[\left(1-\delta^m\right)(1-p)+p\right]u'_{2m,n}y^m=0 \tag{3.7}$$

where

$$\int_{M(\eta^{\min})}\frac{\partial M^{-1}(\tilde{\eta}^m)}{\partial s^m}d\tilde{F}^m(\tilde{\eta}^m)=\kappa u'_{2w}Ry\int_{M(\eta^{\min})}\left(M^{-1}\right)'(\eta^m)\,dF(\eta^m)$$

3.4 Equilibrium Savings Rates

We assume that population growth rate is zero, and women and men receive the same first period income ($y^w = y^m = y$). Before period t, the economy has a balanced sex ratio. In period t, the sex ratio for the young cohort becomes ϕ. We first show that if the sex ratio is balanced ($\phi = 1$), and the mean value of emotional utility is large enough, the dominant strategy for both women and men is to enter the marriage market. In this case, $s^w = s^m = s$, and s can be obtained from solving the set of first order conditions (3.5) and (3.7), both of which can be simplified to be:

$$-u'_1 y+\beta(1-p)\left[u'_2\frac{\partial c_2}{\partial s}+\frac{\partial E\left[\eta\right]}{\partial s}\right]+pyu'_2=0$$

where we use the fact that at $\phi = 1$, $M(\eta^{\min}) = \eta^{\min}$.

We rewrite the first order condition as

$$-u'_1+(1-p)\kappa u'_2\left(2+M(\eta^{\min})f(\eta^{\min})\right)+pu'_{2n}=0 \tag{3.8}$$

and can solve for the optimal savings rates.

The first key proposition is about the effect of a rise in the sex ratio on the aggregate savings rate. The thought experiment assumes that people in the old age cohort have made their savings decision when the sex ratio is balanced. When the sex ratio rises, any change in the aggregate savings is driven by a change in the savings by the young cohort. This simplifying assumption is motivated by the reality: A rise in the sex ratio in almost all economies is a recent phenomenon. More precisely, while the diagnostic sonography used for prenatal checkups was available since the 1960s, the procedure became gradually more affordable to ordinary people in countries that currently have a high sex ratio only since the 1980s. (The strict version of the Chinese family planning policy, another contributor to the spread of sex-selective abortions, was also put in place in the early 1980s.) For this reason, the savings pattern for the currently old was largely decided when there was no severe sex ratio imbalance.

In what follows, whenever we say a man (or a woman), we mean a young man, or a young woman, unless otherwise specified. We first state the proposition formally, and then explain the intuition behind the key parts of the proposition. A detailed formal proof is provided in Appendix A.

Proposition 1 *There exist two threshold values, ϕ_1 and ϕ_2, where $\phi_1 < \phi_2$, that satisfy $V^m = V^m_n$ and $\frac{dU^w}{d\phi} = 0$ respectively. For $\phi < \phi_2$, both women and men choose to enter the marriage market with probability one. In addition, (i) for any $\phi < \phi_1$, as the sex ratio rises, a representative man (weakly) increases his savings rate while a representative woman (weakly) reduces her savings rate. However, the economy-wide savings rate increases unambiguously; (ii) For $\phi_1 \leq \phi < \phi_2$, as the sex ratio rises, a representative man keeps his (high) savings rate constant but a representative woman raises her savings rate. The aggregate savings rate increases.*

Proof. See Appendix A. ■

A few remarks are in order. First, if the expected emotional utility from a marriage is sufficiently large, the dominant strategy for both men and women is to enter the marriage market for a broad range of the sex ratio (or more precisely, for $\phi < \phi_2$). We can see the intuition in the following way. When the sex ratio is balanced, both men and women strictly prefer to be married than to be single. If both the utility function and the density function of η are continuously differentiable (which we assume), in the neighborhood of $\phi = 1$, entering the marriage market must also be a dominant strategy for both women and men.

Second, the proposition says that when the sex ratio rises, up to a point, a representative man raises her savings rate, while a representative woman reduces her savings rate, but the aggregate savings rate goes up unambiguously. The representative man raises his savings rate partly because the need to make himself more competitive in the marriage market becomes greater as the sex ratio (or the unconditional probability of not getting a wife) increases. Why does the representative woman reduce her savings rate? Because she understands that a higher sex ratio makes her future husband save more and therefore she doesn't need to sacrifice her first-period consumption as much as she used to. Why is the increment in men's savings greater than the decline in women's savings? Intuitively, a representative man raises his savings rate for two reasons: in addition to improving his relative standing in the marriage market, he also raises his savings rate to make up for the lower savings rate of his future wife. The more he anticipates his future wife to cut down her savings, the more he would have to raise his savings rate to compensate. This ensures that he raises his savings by an amount greater than any reduction in her future wife's savings. In addition, when the sex ratio is above one, a representative man has to save more than a representative woman. A rise in the sex ratio also implies a rise in the proportion of population that has a higher savings rate. Both channels contribute to a rise in the aggregate savings rate, but the first channel (the incremental competitive savings by any given man) is more important than the second effect (a change in the composition of the population with different saving propensities).

Third, ϕ_1 is the threshold at which a man is indifferent between increasing his savings rate further in order to improve his chance in the marriage market and switching to a mixed strategy that assigns some probability of not entering the marriage market. In other words, ϕ_1 is the level of sex ratio at which the representative man is indifferent between entering the marriage market or not, $V^m = V^m_n$.

The value of the threshold can be very high if a man's biological desire to be matched with a female is strong enough. In the calibrations that we will report later, the threshold is always higher than the highest sex ratio observed in the data. In other words, the sex ratios in the real world appear to be always below this threshold.

Finally, beyond the first threshold, ϕ_1, if the sex ratio continues to rise, up to another threshold (ϕ_2), men would switch to a mixed strategy (that assigns some probability of not entering the marriage market) if women do nothing. Due to the benefits associated with marriage, women would not find it optimal to do nothing and let some good men to skip the marriage market. By raising their own savings rate, women can succeed in enticing men to come back to the marriage market. However, beyond the second threshold ϕ_2, if the sex ratio continues to rise, the required sacrifice for women in terms of reduced first-period consumption becomes too large for them to continue to raise their savings rate. At that point, women would keep their (high) savings rate constant, and let men play their mixed strategy. More formally, ϕ_2 is the level of sex ratio at which $\frac{dU^w}{d\phi} = 0$. In other words, ϕ_2 can be solved by equation

$$y\left(-u'_{1w} + 2(1-p)\kappa u'_{2w} + pu'_{2w,n}\right)\frac{ds^w}{d\phi} + \frac{\beta}{\phi^2}E\left[\eta\right] = 0$$

Since the sex ratios in the real world are unlikely to reach the first threshold ϕ_1, it is even less likely to reach the second threshold ϕ_2.

3.5 A Production Economy

To analyze how the sex ratio imbalance affects a country's current account imbalance, we need to compare economy-wide savings with investments. In this subsection, we introduce a production sector. We assume that both the final good market and the factor markets are perfectly competitive. The production function is Cobb-Douglas:

$$Q_t = \zeta K_t^{\alpha} L_t^{1-\alpha} \tag{3.9}$$

where K_t is the capital stock and L_t is the labor input. α is the share of capital input to total output and ξ is the total factor productivity (TFP). Everyone in the economy inelastically supplies one unit of labor and earns the same income[4].

A representative firm maximizes the profit

$$\max_{K_t, L_t} Q_t - R_t K_t - W_t L_t$$

[4] Allowing men and women to earn different wages (with a fixed proportional gap) would not change our results.

The capital return and the wage rate are determined by

$$R_t = \frac{\partial Q_t}{\partial K_t} = \alpha\zeta\left(\frac{1}{K_t}\right)^{1-\alpha} \quad (3.10)$$

$$W_t = \frac{\partial Q_t}{\partial L_t} = (1-\alpha)\,\zeta K_t^{\alpha} \quad (3.11)$$

where we normalize the aggregate labor supply in the economy to be 1, i.e., $L_t = 1$.

For simplicity, we assume no tax or government expenditure; then $y_t = W_t$ where y_t is the corresponding first period income in the endowment economy. We also assume complete depreciation in each period. The aggregate capital supply in period $t+1$ is predetermined by the aggregate savings in period t

$$K_{t+1}^s = \frac{\phi}{1+\phi}s_t^m W_t + \frac{1}{1+\phi}s_t^w W_t \quad (3.12)$$

3.6 Current Account in a Small Open Economy

In a small open economy, we assume that capital can flow freely among countries and the gross interest rate R is exogenously determined by the rest of the world. By (3.10) and (3.11), the wage rate is also a constant, and the aggregate investment in the economy is

$$K_t^d = \frac{\alpha W_t}{(1-\alpha)R_t} \quad (3.13)$$

Substituting (3.10) and (3.13) into the production function, we have

$$Q_t = \frac{W_t}{1-\alpha}$$

The current account in period t equals the increase in the country's net foreign assets,

$$\Delta NFA_t = Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t} - K_{t+1}^d$$

where $(R-1)\cdot NFA_{t-1}$ is the factor income from abroad. C_{1t} and C_{2t} represent the aggregate consumptions by the young and the old cohorts, respectively. Then

$$\Delta NFA_t = \frac{\phi}{1+\phi}s_t^m W_t + \frac{1}{1+\phi}s_t^w W_t - NFA_{t-1} - K_{t+1}^d$$

The economy-wide savings rate is

$$s_t = \frac{Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t}}{Q_t} \quad (3.14)$$

We assume that the country has a balanced sex ratio in period $t-1$, and the sex ratio in the young cohort in period t, rises from one to $\phi(>1)$. Then the ratio of the current account to GDP is

$$\begin{aligned} ca_t &= \frac{Q_t + (R-1)\cdot NFA_{t-1} - C_{1t} - C_{2t} - K^d_{t+1}}{Q_t} \\ &= (1-\alpha)\left(\frac{\phi}{1+\phi}s^m_t + \frac{1}{1+\phi}s^w_t - s_{t-1}\right) \end{aligned} \tag{3.15}$$

where the second equality holds because[5]

$$NFA_{t-1} = s_{t-1}W_{t-1} - K^d_t$$

where s_{t-1} is the savings rate by the cohort born in period $t-1$. Since the sex ratio is balanced at that time, both the women and the men will have the same savings rate.

Since the wage rate is constant in the small open economy, we can show that a country's current account rises as its sex ratio rises (up to a point).

Proposition 2 *If $\phi \le \phi_2$, the economy-wide savings rate and the current account in period t are increasing in ϕ, where ϕ_2 is as defined in Proposition 1.*[6]

Proof. See Appendix B. ■

The assumption of an exogenous interest rate holds only for a small open economy. But some of the countries that motivate this study are large. An increase in the savings rate in such economies could lower the world interest rate, which could alter investment and savings decisions in all countries. We examine the large country case in the next subsection.

3.7 Two Large Countries

Consider a world consisting of only two countries. They are identical in every respect except for their sex ratios in period t (they both have balanced sex ratios in period $t-1$). Country 1's sex ratio ϕ^1 is smaller than Country 2's sex ratio ϕ^2. There are no barriers to either goods trade or capital flows (although labor is not mobile internationally). We can show the following result:

Proposition 3 *Country 1 runs a current account deficit while country 2 runs a current account surplus in period t as long as $1<=\phi^1 < \phi^2 < \phi_2$.*

Proof. See Appendix C. ■

[5] In overlapping generations models, net foreign asset is equal to the difference between the savings by the young cohort and the domestic investment demand.

[6] If we introduce tax and government expenditure, then the non-governmental part of the current account will rise as the sex ratio becomes more unbalanced.

To see the intuition, let us fix $\phi^1 = 1$ (i.e., Country 1 has a balanced sex ratio). If Country 2 were to have a balanced sex ratio, the current account must be zero for both countries since they are identical in every respect. In other words, within each country, the investment must be equal to the aggregate savings. However, the sex ratio imbalance in Country 2 causes it to have a higher aggregate savings for a given world interest rate. This depresses the world interest rate. The lower interest rate raises the investment level in both economies (and reduces the savings rate a little bit). This must imply that the desired investment level in Country 1 is now greater than its desired savings rate. As a result, capital flows from Country 2 to Country 1. That is, Country 1 runs a current account deficit, and Country 2 a surplus.

3.8 Welfare

We now consider what a hypothetical welfare-maximizing central planner would do. There are two sources of market failures that the central planner would avoid: (a) men save competitively to improve their relative standing in the marriage market; and (b) both men and women may under-save as they do not take into account the benefits of their own savings for the well-being of their future spouses. The central planner assigns the marriage market matching outcome and optimally chooses women's and men's savings rates to maximize the social welfare function,

$$\max\ U = \frac{1}{1+\phi}U^w + \frac{\phi}{1+\phi}U^m$$

The first order conditions are

$$-u'_{1w} + 2(1-p)\kappa u'_{2w} + pu'_{2w,n} = 0 \tag{3.16}$$

$$-u'_{1m} + \frac{2(1-p)}{\phi}\kappa u'_{2w} + \left(1 - \frac{1}{\phi}(1-p)\right) u'_{2w,n} = 0 \tag{3.17}$$

Comparing (3.16), (3.17) to (3.5) and (3.7), in general, it is not obvious whether women or men will save at a higher rate in a decentralized equilibrium than that under central planning. However, when $\phi = 1$, since women and men have the same optimization problem, by (3.8), if $f(\eta^{\min})M(\eta^{\min}) > 0$, then women and men will save more in the competitive equilibrium. If $f(\eta^{\min})M(\eta^{\min})$ is sufficiently small, the competitive equilibrium is very close to the central planner's economy.

There are two opposing effects. On one hand, a part of the savings in the competitive equilibrium is motivated by a desire to out-save one's competitors in the marriage market. The increment in the savings, while individually rational, is not useful in the aggregate, since when everyone raises the savings rate by the same amount, the ultimate marriage market outcome is not affected by the increase in the savings. In this sense, the competitive equilibrium produces too much savings. On the other hand, because the savings contribute to a pubic good in a marriage (an individual's savings raises the utility of his/her partner), but the individual in the first period does not take this into account and may under-save relative to the social optimum. These two effects offset each other. Therefore, when

$\phi = 1$, the final savings rate in the decentralized equilibrium could be close to the social optimum.

In calibrations with a log utility function, we will show that men's welfare under a decentralized equilibrium relative to the central planner's economy declines as the sex ratio increases. In comparison, women's relative welfare increases as the sex ratio goes up. The social welfare (a weighted average of men's and women's welfare) goes down as the sex ratio rises.[7]

As a thought experiment, one may also consider what the central planner would do if she can choose the sex ratio (in addition to the savings rates) to maximize the social welfare, the new first order condition with respect to ϕ is

$$U^m - U^w = 0 \tag{3.18}$$

The only sex ratio that satisfies (3.18) is $\phi = 1$. In other words, the central planner would have chosen a balanced sex ratio. Deviations from a balanced sex ratio represent welfare losses.

4 Extension: Intra-household Bargaining

In the benchmark model, we abstract from bargaining between a husband and a wife within a marriage. Chiappori (1988 and 1992) and Browning and Chiappori (1998), among others in the literature on the economics of family, have considered intra-household bargaining, and emphasized that the relative income between the husband and the wife matters. In this section, we extend the benchmark model by allowing the relative bargaining power to be a function of the sex ratio and relative wealth. We assume that everyone consumes two goods in the second period, a public good (e.g., a house) and a private good. The aggregate second period consumption index is

$$c_{2i} = \frac{z_i^{\gamma} h^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}} \quad i = w, m$$

where z_w and z_m are private goods consumption by women and men respectively. h is the public good consumption.

A representative household maximizes the weighted sum of the utilities of the husband and the wife. Let μ denote the weight on the wife's utility, which represents her bargaining power in the family. Then the household's optimization problem is

$$\max_{h, z_w, z_m} \mu u(c_{2w}) + (1-\mu) u(c_{2m})$$

with the resource constraint

$$z_w + z_m + h = Rs^w y^w + Rs^m y^m \tag{4.1}$$

[7] The results are similar if we change the utility function to a CRRA form.

If we assume $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ $(\theta \geq 1)$, solving the household's maximization problem, we have

$$
\begin{aligned}
c_{2w} &= \frac{Rs^w y^w + Rs^m y^m}{\left[1 + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\gamma+\theta\gamma}}\right]^{\gamma}} \\
c_{2m} &= \frac{Rs^w y^w + Rs^m y^m}{\left[1 + \left(\frac{\mu}{1-\mu}\right)^{\frac{1}{1-\gamma+\theta\gamma}}\right]^{\gamma}}
\end{aligned}
$$

If $\mu = \frac{1}{2}$, this is the case in our benchmark model and $\frac{1}{2} < \kappa = 2^{-\gamma} < 1$.

More generally, similar to Browning et al (1994), we assume μ to be an increasing function in the sex ratio ϕ, the relative wealth, and η^w, a decreasing function in η^m, then

$$
\begin{aligned}
c_{2w} &= \kappa^w \left(Rs^w y^w + Rs^m y^m\right) \\
c_{2m} &= \kappa^m \left(Rs^w y^w + Rs^m y^m\right)
\end{aligned}
$$

where

$$
\kappa^w = \left[1 + \left(\frac{1-\mu}{\mu}\right)^{\frac{1}{1-\gamma+\theta\gamma}}\right]^{-\gamma} \tag{4.2}
$$

$$
\kappa^m = \left[1 + \left(\frac{\mu}{1-\mu}\right)^{\frac{1}{1-\gamma+\theta\gamma}}\right]^{-\gamma} \tag{4.3}
$$

We denote the derivatives we will use as following:

$$
\begin{aligned}
\kappa_1^w &= \frac{\partial \kappa^w}{\partial \phi^{-1}} \leq 0,\ \kappa_1^m = \frac{\partial \kappa^m}{\partial \phi} \leq 0 \\
\kappa_2^w &= \frac{\partial \kappa^w}{\partial \left(\frac{Rs^w y^w}{Rs^m y^m}\right)} \geq 0,\ \kappa_2^m = \frac{\partial \kappa^m}{\partial \left(\frac{Rs^m y^m}{Rs^w y^w}\right)} \geq 0 \\
\kappa_3^w &= \frac{\partial \kappa^w}{\partial \eta^w} \geq 0,\ \kappa_3^m = \frac{\partial \kappa^m}{\partial \eta^m} \geq 0 \\
\kappa_4^w &= \frac{\partial \kappa^w}{\partial \eta^m} \leq 0,\ \kappa_4^m = \frac{\partial \kappa^m}{\partial \eta^w} \leq 0
\end{aligned}
$$

For simplicity, we assume that $\kappa^w = \kappa^m$, if $\phi = 1$, $Rs^w y^w = Rs^m y^m$ and $\eta^w = \eta^m$.

We assume $y^w = y^m = y$. To simplify the discussions, we make several additional assumptions.

Assumption E1:

$$
-\frac{\partial \kappa^i}{\partial \eta^j} \frac{\eta^j}{\kappa^i} \leq \frac{\eta^j}{u'_{2i} c_{2i}}
$$

This means that the elasticity of the bargaining power for household member i is small enough given the relative wealth between women and men. This assumption ensures that $u(\kappa^i(Rs^w y^w + Rs^m y^m)) + \eta^j$ is increasing in η^j, i.e., everybody wants her/his spouse to have a higher η. Therefore, in equilibrium, the mapping that results in an assortative matching M still holds.

Assumption E2:

$$\frac{\partial \kappa^i}{\partial \left(\frac{Rs^i y^i}{Rs^j y^j}\right)} \frac{\frac{Rs^i y^i}{Rs^j y^j}}{\kappa^i} \leq \frac{Rs^j y^j}{Rs^i y^i + Rs^j y^j}$$

This ensures that each person prefers a wealthy spouse even though this may reduce his/her bargaining power within the household.

Assumption E3: In the second period, for the man who enters the marriage market with type $M(\eta^{\min})$, $u(c_{2m}) + \eta^{\min} = u(c_{2m,n})$.

This implies that the marginal married man is indifferent between getting married and staying single. This assumption is reasonable because *ex ante* men do not know their type and make the same savings decision. In the second period, the woman matched to the man of type $M(\eta^{\min})$ makes an intra-household allocation offer that would make the man slightly happier than being single. If he does not accept the offer, the next ranked man may accept the offer, which would have resulted in a welfare loss to the man of type $M(\eta^{\min})$. Hence, he accepts the offer.

Assumption E4: κ^i_{12} is either positive or small in its absolute value, $i = w, m$.

Given these assumptions, and as assumed in the benchmark that the sex ratio in the old cohort is balanced, we can show the following proposition.

Proposition 4 *(i) When the sex ratio is in the neighborhood of $\phi = 1$, both men and women enter the marriage market with probability one. As the sex ratio rises, a representative man's savings rate rises, but the response by a woman's savings rate is ambiguous. However, the aggregate savings rate rises unambiguously. (ii) For $\phi < \phi_2$,[8] women and men enter the marriage market with probability one. For sufficiently large κ^i_2 ($i = w, m$), as the sex ratio rises, up to ϕ_1, a representative man's savings rate rises while a representative woman's savings rate declines. The aggregate savings rate increases unambiguously. As the sex ratio continues to rise, beyond ϕ_1 but up to ϕ_2, a representative woman's savings rate rises while a representative man keeps his (high) savings rate constant. The aggregate savings rate increases unambiguously.*

Proof. (See Appendix D). ■

In Part (i) of Proposition 4, the ambiguity in a woman's response to a rise in the sex ratio comes from two opposing forces. On the one hand, she wishes to free ride on her future husband's higher

[8] ϕ_1 and ϕ_2 are defined in Proposition 1.

savings rate by reducing her own savings rate. On the other hand, she does not wish to see an erosion in her intra-household bargaining power, and one way to protect her bargaining power is to raise her own savings rate. This ambiguity on the part of the women's savings does not affect the comparative statics on the aggregate savings. Even if the women do not raise their savings rate to preserve intra-household bargaining power, we already know from Proposition 1 that in the aggregate, the increment in men's savings is always more than enough to offset any decline in women's savings. Any increase in women's savings out of concern for intra-household bargaining power can only further increase the response of the aggregate savings to a rise in the sex ratio.

In Part (ii), when κ_2^i is sufficiently large, men's incentive to raise savings is strong. Given our other assumptions, even if a woman raises her savings, the contribution to her bargaining power within the family is relatively small. In this case, she may as well consume more in the first period and free ride on her husband's savings in the second period.

As for the aggregate savings rate, the qualitative results are similar to Proposition 1. As a result, it is easy to verify that all open-economy conclusions still hold with this extension.

5 Calibrations

The analytical model presents conditions under which a rise in the sex ratio leads to an increase in a country's current account surplus. Are the actual sex ratios observed in the data capable of generating a current account response whose magnitude is economically significant? We answer this question in this section by quantitative calibrations of the model. We start with a small open economy, and move on to two cases of a large economy.

5.1 The Small Open Economy

Assume that the utility function is of the log form

$$u(c) = \ln(c)$$

In the calibrations for a small open economy, we fix $R = \beta^{-1}$. (In the large country case, the interest rate is endogenously determined.)

The emotional utility η needs to follow a continuously differential distribution. In the benchmark calibration, we assume a truncated normal distribution which might be more realistic than a uniform distribution used in the analytical model. We choose a standard deviation that is relatively tight, $\sigma = 0.01$. This limits the extent of heterogeneity among women (or men) in the eye of the opposite sex. We truncate the distribution at the 1% in the left tail and at the 99% in the right tail.

We choose the mean value of the emotional utility/love to match the existing empirical estimates of the value of marriage. Holding other factors constant, the income equivalent of a marriage can be defined as the extra monetary compensation to a life-time bachelor that would make him indifferent between being married and single.

$$u\left(\frac{1}{1+\beta}(1+m)y\right)=u\left(\frac{1}{1+\beta}y\right)+E\left(\eta\right)$$

where m is additional income multiplier paid to a life-time bachelor for being single, and $\frac{1}{1+\beta}(1+m)y$ is his second period consumption. Blanchflower and Oswald (2004) estimated econometrically a relationship between subjective well-being and income and marital status (and other determinants of happiness) in the United States during 1972-1998. From that estimation, they infer that, on average, a lasting marriage is equivalent to augmenting one's income by $100,000 (in 1990 dollars) per year every year. Since the average income per working person was about $48,000 during that period, a sustained marriage is worth 2 times the average income. We therefore choose $m = 2$ as the benchmark. This implies that the mean value of the emotional utility/love is:

$$E\left(\eta\right)=u\left(\frac{3y}{1+\beta}\right)-u\left(\frac{y}{1+\beta}\right)$$

We will vary the value of m in the robustness checks.

For other parameters, whenever possible, we assign values that are consistent with the standard literature.

Choice of Parameter Values

Parameters	Benchmark	Source and robustness checks
Discount factor	$\beta = 0.45$	Prescott (1986), discount factor takes value 0.96 based on annual frequency. We take 20 years as one period, then $\beta = 0.96^{20} \simeq 0.45$
Share of capital input	$\alpha = 0.35$	Bernanke, Gertler and Gilchrist (1999)
Congestion index	$\kappa = 0.8$	$\kappa = 0.7, 0.9$ in the robustness checks.
Marriage market friction[9]	$p = 0.02$	$p = 0.05$ in the robustness checks
Love, standard deviation	$\sigma = 0.01$	$\sigma = 0.05$ in the robustness checks
Love, mean	$m = 2$	$m = 0.5$ in the robustness checks

In Figure 3, we plot the aggregate savings rate as a function of the sex ratio (which changes from 1 to 1.5). In the benchmark calibration, when the sex ratio goes up from 1 to 1.15, the savings rate would go up by 10.3 percentage points. For comparison, according to the last regression reported in Table 1b, the same increase in the sex ratio is associated with an increase in the savings rate by 16.1

[9] p is the exogenous possibility that any individual (a women or a man) entering the marriage market is bumped off the market independent of the sex ratio.

percentage points. This suggests that the benchmark model, while producing quantitatively significant current account responses, may still under-predict the true magnitude of the responses in the data.

We conduct a series of robustness checks by considering different combinations of the parameter values. They include varying $\kappa = 0.7, 0.8$ and 0.9, $m = 2$ and 0.5, $\sigma = 0.01$ and 0.05, and $p = 0.02$ and 0.05. We tabulate these results in the working paper version, but do not include the large number of tables due to the space constraint. The three panels in Figure 3 trace out the savings response under some selected parameter combinations.

There are a few noteworthy patterns. First, generally speaking, the economy-wide savings rate always rises in response to a rise in the sex ratio. With all parameter combinations, as we raise the sex ratio from 1 to 1.5, we do not see a turning point in the savings response. In other words, the theoretical values of ϕ_1 and ϕ_2 in Proposition 1 must be greater than 1.5. Since no economy in the real world has a sex ratio that exceeds 1.5, all real economies appear to satisfy the conditions under which the savings rate (and the current account) rises as the sex ratio rises.

Second, as κ becomes larger, the economy-wide savings rate and the current account respond more strongly to a rise in the sex ratio. Intuitively, as κ becomes larger, consumption within a marriage acquires more public goods feature. Consequently, the desire to marry (and the need to compete in the marriage market) also increase. However, the response of the aggregate savings is not very sensitive to small perturbations of this parameter.

Third, when the mean value of the love factor is higher (e.g., when $m = 2$ compared to $m = 0.5$), the economy-wide savings rate and the current account respond more strongly to a given rise in the sex ratio. This is intuitive since men have a stronger desire to compete for a marriage partner.

Fourth, when there are more frictions in the marriage market (unrelated to the sex ratio) (e.g., when $p = 0.05$ rather than 0.02), a women's savings response to a change in the sex ratio becomes smaller. Intuitively, with an increased probability that a woman could stay single in her life (and therefore may not free ride on her husband's savings), she would not reduce her savings as much in response to a given rise in the sex ratio as she otherwise would. (However, the effect on the aggregate savings rate is not very sensitive to p. If women are induced to save more due to a fear of singlehood, men may save less as there is less need to compensate for the lower savings of his future wife. As a result, the net effect on the aggregate savings is smaller than otherwise.)

Fifth, as the dispersion for emotional utility becomes smaller, the economy-wide savings rate and the current account respond more strongly to a rise in the sex ratio. This is because, since all men are more similar in terms of the amount of "love" they can offer to women, the need to compete on the basis of wealth also rises.

5.2 Two Large Countries

We consider two cases of calibrations. In the first case, we assume that the two countries are identical in every respect except for their sex ratios. While Country 1 always has a balanced sex ratio ($\phi_1 = 1$), we vary the sex ratio in country 2 from 1 to 1.5. The interest rate R and the wage rate W are now endogenously determined. All other parameter values are the same as in the small open economy case. Figure 4 traces out the current account responses in both countries as Country 2's sex ratio increases. The most important result is that a rise in Country 2's sex ratio triggers a rise in its current account surplus and a rise in Country 1's current account deficit. Note that Country 1's current account exhibits a bigger change than Country 2. This is because as Country 2's sex ratio rises, its economy-wide savings rate rises. As a result, it becomes a progressively larger country than Country 1. We have also done robustness checks by varying the values of κ, m, p and σ (not reported to save space). Based on the same reasoning as in the small open economy case, for larger κ, m, p or smaller σ, a given increase in Country 2's sex ratio results in greater current account imbalances in the two countries.

In the second case, we attempt to let Countries 1 and 2 mimic the United States and China, respectively. In particular, we assume that $L_1 = 1/5 \cdot L_2$ to match the fact that the U.S. population is around 1/5 of that of China. In addition, we choose the TFP parameter in Country 1, ζ_1, to match the fact that the U.S. per capita GDP was about 15 times the Chinese level around 1990 when the sex ratio in China for the marriage age cohort was not yet seriously out of balance. The remaining parameters are set to be the same as before. We let the sex ratio in the United States be always balanced, and vary the Chinese sex ratio from 1 to 1.5.

Figure 5 plots the calibration results. Qualitatively, they look similar to the first large-country experiment. Quantitatively, Country 2's (China) current account response (as a share of GDP) becomes stronger. With China's sex ratio at 1.15 (and $\kappa = 0.8$), the United States runs a current account deficit of 2.6% of GDP, and China runs a surplus of 7.7% of GDP. This resembles the real world pattern in which the U.S. deficit is about 4-6% of GDP, whereas the Chinese surplus is on the order of 7-10% of GDP in recent years. In other words, the rise in the Chinese sex ratio could potentially generate an economically significant current account imbalance, or more than half of what is observed in the data.

To summarize, the calibrations suggest that a rise in the sex ratio could produce an economically significant increase in the aggregate savings rate that results in a current account surplus. If the country is large enough, this could induce other countries to run a current account deficit even if they have a balanced sex ratio.

5.3 Welfare

In the welfare calibrations, we compare the welfare under a decentralized equilibrium relative to the central planner's economy. Figures 6, 6a and 6b trace out, for the case of a small open economy, the savings rates for men (the upper left panel), women (the upper right panel), the economy as a whole (the lower left panel) and the welfare (the lower right panel), respectively. With a log-utility function, the optimal savings rates for men and women chosen by the planner do not depend on the sex ratio and intra-household bargaining powers.[10] When the sex ratio is balanced, the savings rates by women, men and the economy as a whole are almost the same as those under the planner's economy. With unbalanced sex ratios, men's (decentralized) savings rates overshoot the socially optimally level, and the extent of excessive savings rises with the sex ratio. Women's savings rates follow an opposite pattern. The economy-wide savings rate follows a pattern that is qualitatively similar to the men's savings rate. In particular, the economy in a decentralized equilibrium tends to save too much relative to the social optimum, and the excess savings rises with the sex ratio. In the lower right panel, we can see that the welfare levels for both men and the economy as a whole decline as the sex ratio increases, while the welfare for women rises with the sex ratio.

5.4 Endogenous Intra-household Bargaining

One problem in the benchmark calibration is that, as the sex ratio rises, women's savings rate declines very quickly. As we have noted earlier, if the analytical model allows for intra-household bargaining, then the effect of a rise in the sex ratio on the change in women's savings rates becomes ambiguous. As men start to save more, women do not wish to reduce their bargaining power in a marriage if the relative size of the savings rates is a determinant of the bargaining power. We use calibrations to demonstrate this effect. For simplicity, we assume that the intra-household bargaining power depends only on the relative wealth of household members. In particular, we assume that the wife's bargaining power within a family is

$$\mu = \frac{(s^w)^\varepsilon}{(s^w)^\varepsilon + (s^m)^\varepsilon}$$

and the husband's bargaining power is $1-\mu$. $\varepsilon \in (0,1)$ is the parameter that governs the sensitivity of bargaining power to relative wealth. A larger ε means that household bargaining power depends more strongly on the relative wealth.

The log-utility function implies $\theta = 1$ in (4.2) and (4.3), then

$$\kappa^w = \mu^\gamma \text{ and } \kappa^m = (1-\mu)^\gamma$$

where γ is the share of private expenditure in the second period consumption index. Other parameters take the same values as in the benchmark. Figures 7a and 7b plot the saving rates from the calibrations. Relative to the case of no intra-household bargaining, women now change their savings rates much more slowly as the sex ratio rises. Since there is no big change in men's response to the rise in the sex ratio, the economy-wide savings rate responds more strongly to a rise in the sex ratio than the

[10] This feature does not hold when we use CRRA utility function.

benchmark case. For $\varepsilon = 0.5$ and $\gamma = 0.5$, as the sex ratio rises from 1 to 1.15, the current account to GDP ratio rises by 14%. (Recall from the last regression in Table 1b, the same increase in the sex ratio is associated with an increase in the current account by 16%). Thus, the calibration result with intra-household bargaining generates a current account response whose magnitude is very close to what is observed in the data. (As reported in Wei and Zhang (2009), the estimated elasticity of savings by rural households with a daughter in China to a rise in the sex ratio is close to be zero. This suggests that the model with endogenous intra-household bargaining may be more realistic than the benchmark model without intra-household bargaining.)

We re-calibrate the case of two otherwise identical countries except for the sex ratio (in Figure 8), and also the case of the United States versus China (in Figure 9). In both cases, the only difference relative to the benchmark model is the allowance for the endogenous bargaining power within a family. The qualitative results on the aggregate savings and the current account are similar to before. However, the savings response by women becomes more realistic. For $\varepsilon = 0.5$ and $\gamma = 0.5$, we see that as the sex ratio in China rises from 1 to 1.15, this can generate a 10.1% current account surplus (as a share of GDP) in China and 3.4% deficit in the U.S. The calibration result can account for almost all the current account surplus in China and more than half of the deficit in the United States.

In the right panel in both Figures 8 and 9, we trace out the economy-wide welfare in a decentralized equilibrium for a given sex ratio relative to the welfare in a decentralized equilibrium with a balanced sex ratio. The country that experiences a rise in the sex ratio (e.g., China) clearly suffers from an ever-deteriorating welfare due to a rise in socially inefficient competitive savings. Interestingly, the country with a balanced sex ratio (e.g., the United States) could enjoy a small welfare gain initially as China's sex ratio starts to be out of balance. Intuitively, a rise in the Chinese sex ratio depresses the global interest rate, but this produces two effects with opposite signs for the United States. On one hand, the lower cost of capital boosts the real wage in the United States, which is positive for the Americans. On the other hand, the lower interest rate also implies a lower interest income for a given amount of savings, which is negative for the Americans. For a moderately unbalanced sex ratio in China, the positive effect for the Americans dominates. As the Chinese sex ratio becomes seriously out of balance, the welfare levels in both countries can go down.

We note, however, that the quantitative effect of a rise in the Chinese sex ratio on the U.S. welfare is small. The Chinese are the biggest victims from a rise in their sex ratio. As an illustration, based on the right panel of Figure 9, if the Chinese sex ratio reaches 1.15, the U.S. suffers a loss of utility that is equivalent to a decrease in consumption by 0.3% (relative to the level of consumption with a balanced Chinese sex ratio). In contrast, China suffers a significant welfare loss that is equivalent to a decline in consumption by 40.1%.

6 Concluding Remarks and Future Research

This paper builds a theoretical model to analyze whether and how a rise in the sex ratio may trigger a competitive race in the savings by men (or households with sons). Generally speaking, men raise their savings rate in order to improve their relative standing in the marriage market. If we don't consider intra-household bargaining, women may respond to a rise in the sex ratio by reducing their savings rate because they may free ride on the increased savings from their husbands. If we consider intra-household bargaining, then women's response becomes ambiguous because they also have an incentive to raise their savings rate in order to protect their bargaining power within a family. In any case, the aggregate savings always rises unambiguously in response to a rise in the sex ratio, as long as the sex ratio is below some threshold. We argue conceptually and through calibrations that the sex ratios in real economies are unlikely to exceed the threshold. The increase in the aggregate savings is socially inefficient since the number of unmarried men in the aggregate is not altered by the savings race.

When the country with an unbalanced sex ratio is large, this could have global impact. In particular, when the sex ratio rises, the world interest rate becomes lower. Other countries with a balanced sex ratio could be induced to run a current account deficit. Calibration results suggest that the sex ratio effect could potentially explain more than 1/2 of China's current account surplus and the U.S. current account deficit. In other words, the effect is economically significant.

In our model, men and women are symmetric except for the sex ratio. This implies that if the sex ratio imbalance increases in the opposite direction (i.e., more women than men), we would obtain the same qualitative results. That is, an increase in the relative surplus of women would also trigger an increase in the aggregate savings. The quantitative elasticity of the savings rate to a given rise in the sex ratio may not be same, because the amount of emotional utility that women derive from their spouses may not be the same as the other way around. Wars and famines often result in a relative shortage of young men. While the competitive savings channel outlined in our model may very well be operating in that context, there are other confounding factors. For example, instability generally deters savings and investment for reasons unrelated to sex ratio imbalance. For these reasons, such episodes may not be suitable to test our theory. In the years immediately after a war, peace has resumed yet a shortage of young men often persists for a while. For example, in the first decade after World War II, many countries including Japan and Germany simultaneously exhibited a sex ratio imbalance (more young women than young men) and a high savings rate. Chinese immigrants to Southeast Asia in the 19th century and European immigrants to the United States in the previous centuries often had a skewed sex ratio. An informal reading of historical accounts suggests that these immigrants also worked very hard and saved a lot. These experiences are consistent with our theory. However, post-war periods and immigration could be special for other reasons, which need to be taken into account when one performs a formal test.

The theory can be extended in a number of directions. First, the sex ratio could endogenously respond to the economic burden of raising a son (as in Bhaskar, 2009). As a result, there may be forces that will eventually induce a correction in the trajectory of a country's sex ratio. It will be a useful extension to endogenize the sex ratio in an extension of the model. This will help us understand better the future trajectories of the global current account imbalances. Second, while the model focuses on the responses of the savings and current account to a rise in the sex ratio, one may extend it to study entrepreneurship and growth effects. Third, while we have provided some cross-country evidence on the connection between a country's sex ratio and its current account, the evidence is based on OLS regressions. It will be useful to find instrumental variables to prove the causality. These will be useful topics for future research.

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Figures and Tables

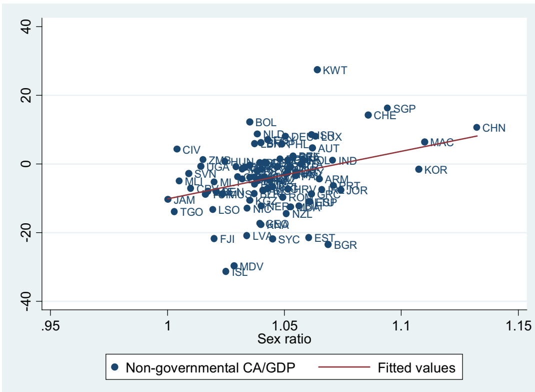

Figure 1a: (CA-Gov Saving)/GDP vs Sex ratio, year 2006

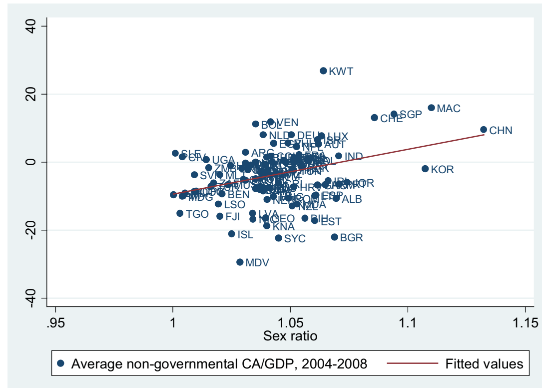

Figure 1b: (CA-Gov Saving)/GDP vs Sex ratio, average 2004-2008

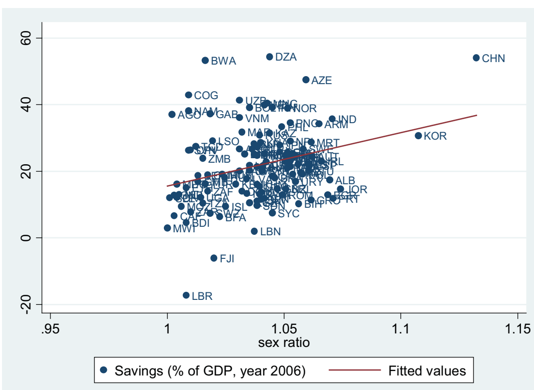

Figure 2a: Savings (% of GDP) vs Sex ratio, year 2006

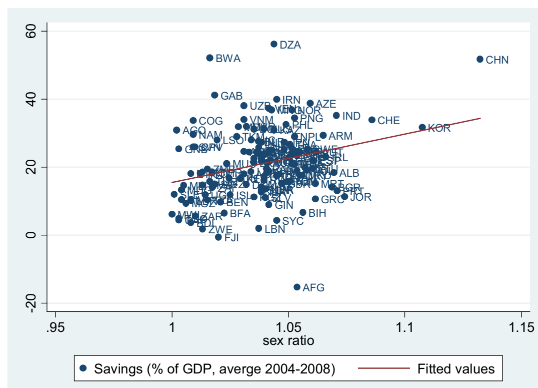

Figure 2b: Savings (% of GDP) vs Sex ratio, average 2004-2008

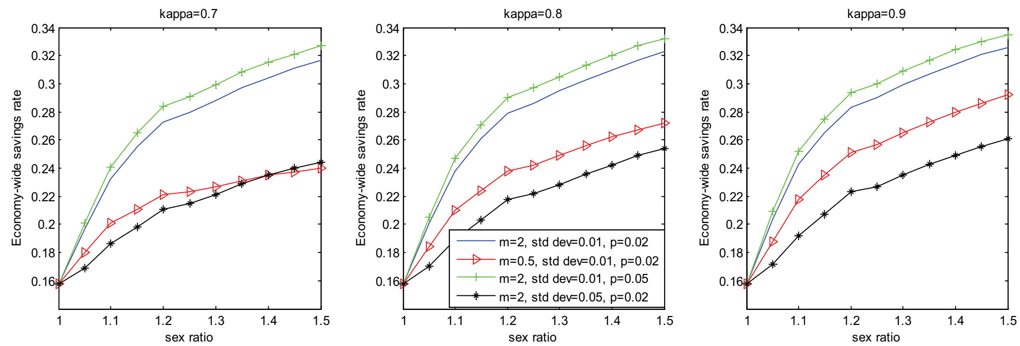

Figure 3: Economy-wide savings rate vs sex ratio

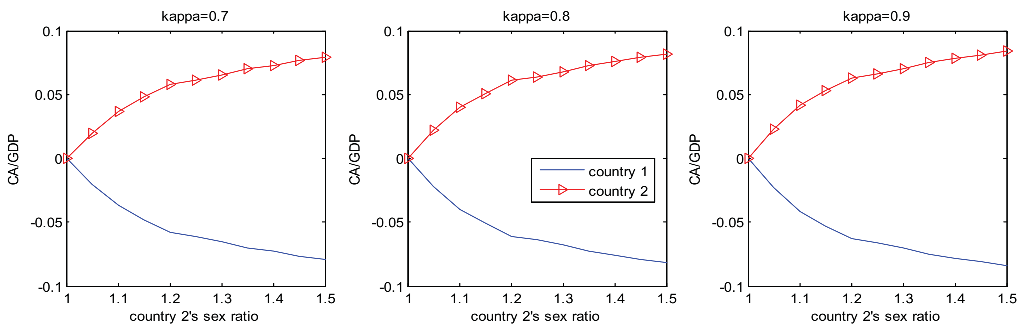

Figure 4: Two large countries, differing only in sex ratio

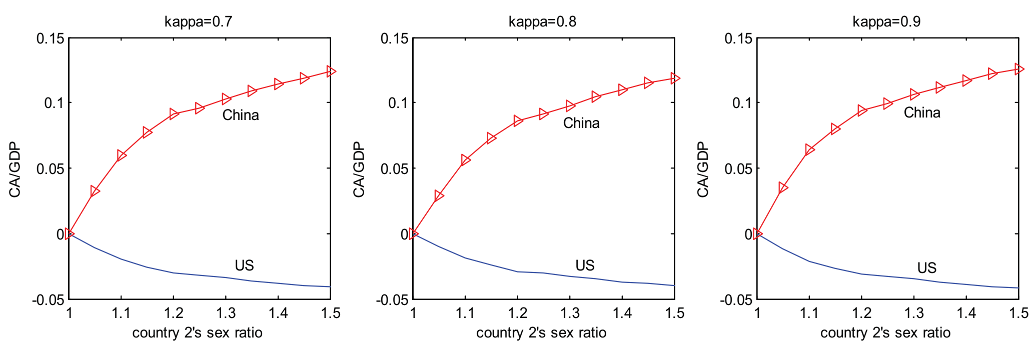

Figure 5: China and the United States

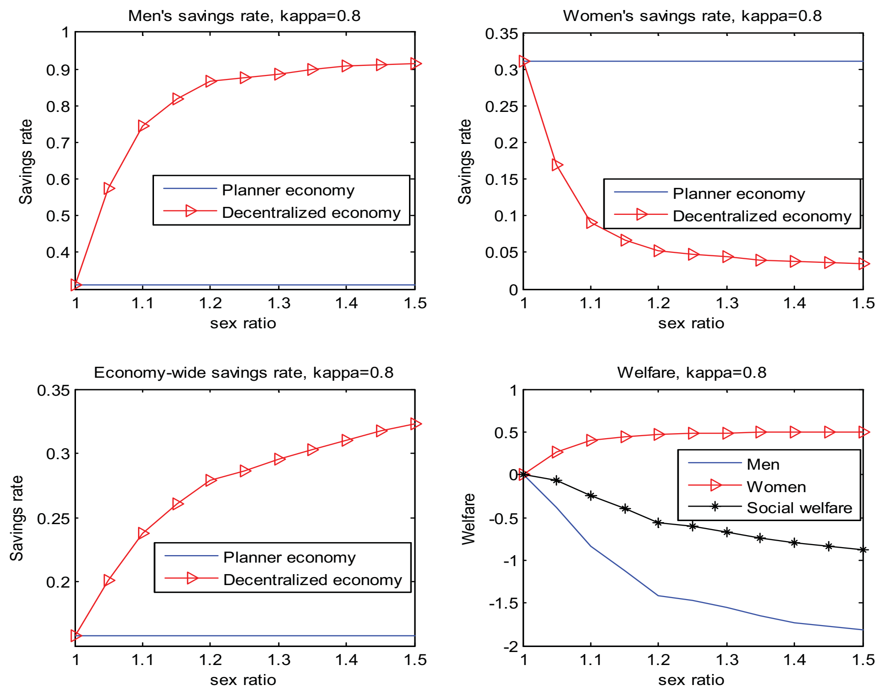

Figure 6: Planner's economy vs decentralized economy, ĸ=0.8

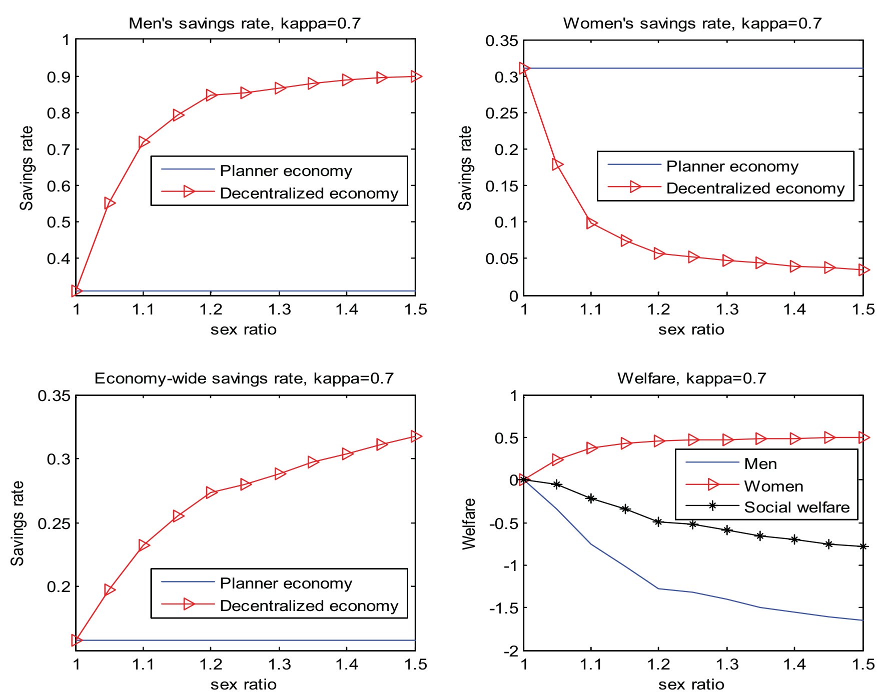

Figure 6a: Planner's economy vs decentralized economy, ĸ=0.7

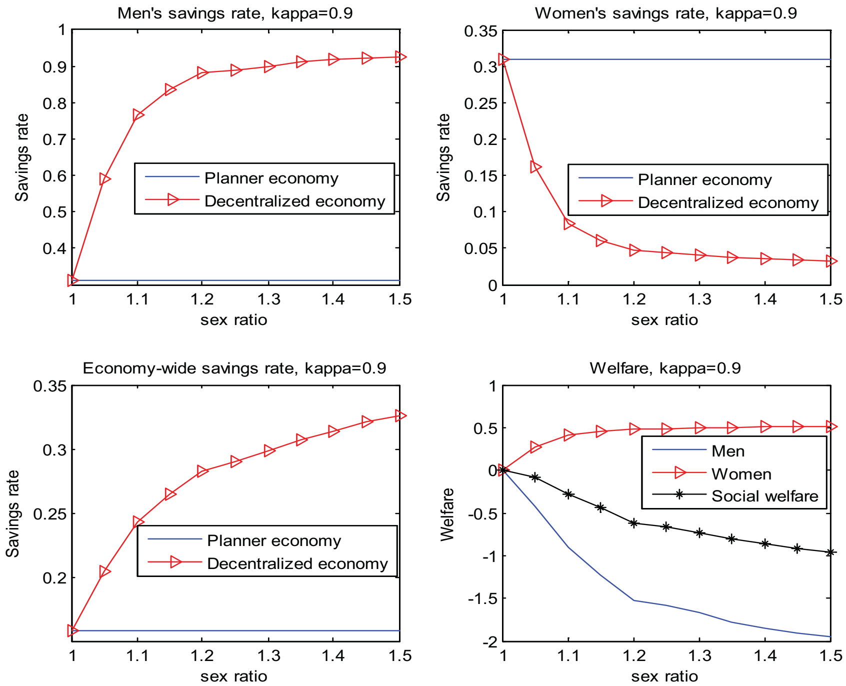

Figure 6b: Planner's economy vs decentralized economy, κ=0.9

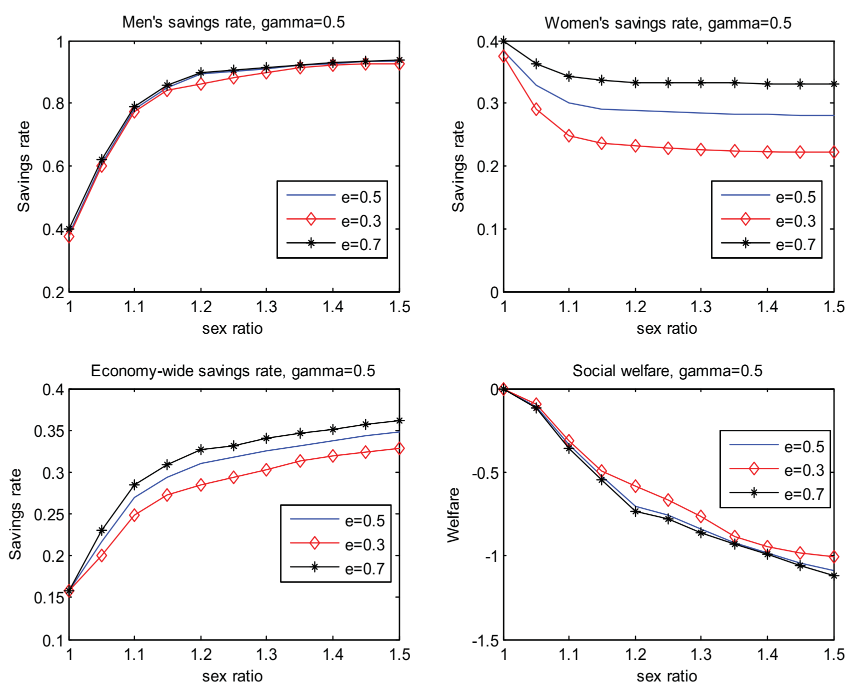

Figure 7a: Savings rate vs sex ratio, endogenous intra-household bargaining, γ=0.5

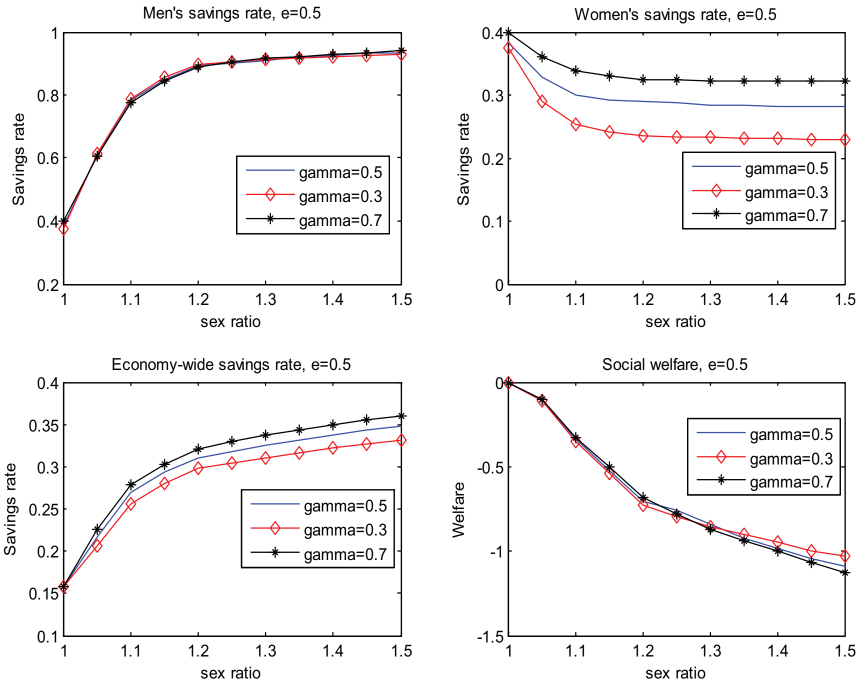

Figure 7b: Savings rate vs sex ratio, endogenous intra-household bargaining, ε=0.5

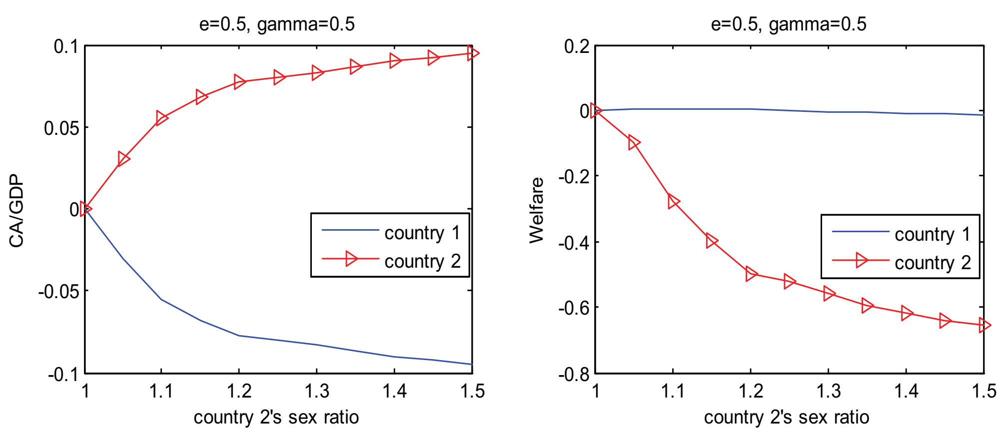

Figure 8: Two large countries, differing in sex ratio, endogenous bargaining power, welfare loss in units of consumption goods relative to the case of Φ=1

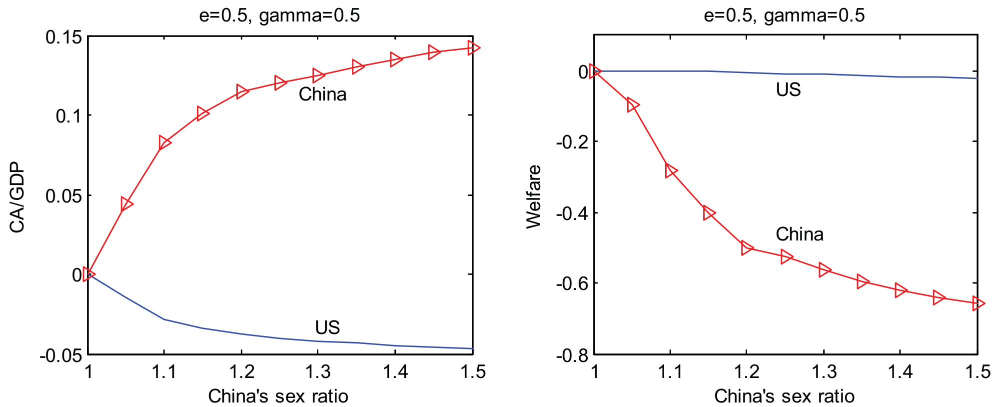

Figure 9: China and the United States, endogenous bargaining power, welfare loss in units of consumption goods relative to the case of a balanced sex ratio

Table 1a: Current account vs sex ratio, year 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	125.5***	126.6***	134.3**	166.3***	171.6***	131.1**	125.1**
	(44.94)	(44.31)	(54.60)	(55.71)	(55.14)	(55.41)	(53.27)
ln(real GDP per capita)	0.554	-12.40*	0.269	0.603	2.307	3.280**	-20.58*
	(0.664)	(6.846)	(1.088)	(1.075)	(1.418)	(1.437)	(10.590)
social security expenditure/GDP			0.044	0.122	0.109	0.479**	0.434*
			(0.179)	(0.179)	(0.177)	(0.229)	(0.221)
working age population				-0.545*	-0.596**	-0.405	-0.103
				(0.277)	(0.276)	(0.273)	(0.294)
private credit/GDP					-0.0478*	-0.0639**	-0.0836***
					(0.026)	(0.025)	(0.026)
Africa						17.89***	17.32***
						(6.121)	(5.881)
Asia						16.98***	15.58***
						(5.484)	(5.301)
Europe						5.546	4.645
						(5.276)	(5.081)
North America						11.03*	11.48**
						(5.819)	(5.591)
Oceania						16.74**	18.09***
						(6.565)	(6.330)
ln(real GDP per capita) square		0.793*					1.444**
		(0.417)					(0.636)
Observations	93	93	62	62	61	60	60
R-squared	0.12	0.15	0.11	0.17	0.21	0.41	0.47

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 1b: Current account vs sex ratio, average 2004-2008

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	110.0***	112.0***	107.2**	142.3***	144.8***	109.4**	107.5**
	(38.73)	(37.82)	(44.37)	(45.07)	(45.35)	(46.11)	(43.94)
ln(real GDP per capita)	0.817	-12.59**	0.507	0.955	1.85	2.667**	-17.66**
	(0.558)	(5.514)	(0.891)	(0.876)	(1.170)	(1.203)	(8.135)
social security expenditure/GDP			-0.010	0.071	0.064	0.355*	0.320*
			(0.151)	(0.149)	(0.150)	(0.192)	(0.183)
working age population				-0.542**	-0.576**	-0.454**	-0.144
				(0.223)	(0.227)	(0.226)	(0.247)
private credit/GDP					-0.026	-0.0404*	-0.0594***
					(0.022)	(0.021)	(0.021)
Africa						13.71***	13.02***
						(5.084)	(4.848)
Asia						14.55***	13.21***
						(4.631)	(4.440)
Europe						5.076	4.161
						(4.452)	(4.254)
North America						8.337*	8.925*
						(4.941)	(4.710)
Oceania						14.00**	15.35***
						(5.566)	(5.326)
ln(real GDP per capita) square		0.832**					1.241**
		(0.341)					(0.492)
Observations	104	104	65	65	64	63	63
R-squared	0.14	0.19	0.12	0.20	0.21	0.39	0.46

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 2a: Savings (% of GDP) vs sex ratio, year 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	144.1***	102.9**	182.7***	169.5***	165.9***	113.5*	121.8**
	(50.57)	(50.22)	(54.72)	(57.69)	(56.31)	(56.63)	(57.41)
ln(real GDP per capita)	0.551	19.67***	-0.13	-0.303	-0.839	-0.56	-11.51
	(0.688)	(5.850)	(1.085)	(1.113)	(1.590)	(1.634)	(11.920)
social security expenditure/GDP			-0.0855	-0.111	-0.103	0.221	0.153
			(0.190)	(0.193)	(0.193)	(0.259)	(0.270)
working age population				0.209	0.281	0.443	0.641*
				(0.278)	(0.278)	(0.284)	(0.356)
private credit/GDP					0.00303	-0.000909	-0.0133
					(0.028)	(0.027)	(0.030)
Africa						12.00*	11.75*
						(6.063)	(6.078)
Asia						16.05***	15.10***
						(5.326)	(5.431)
Europe						6.424	6.243
						(5.217)	(5.228)
North America						6.944	6.259
						(6.267)	(6.320)
Oceania						18.17***	18.88***
						(6.316)	(6.372)
ln(real GDP per capita) square		-1.219***					0.683
		(0.371)					(0.736)
Observations	124	124	60	60	59	58	58
R-squared	0.10	0.18	0.17	0.18	0.21	0.39	0.40

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 2b: Savings (% of GDP) vs sex ratio, average 2004-2008

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
sex ratio	127.8***	102.5**	180.5***	159.3***	157.5***	118.6**	124.6**
	(43.18)	(42.94)	(47.47)	(49.21)	(48.07)	(49.74)	(50.25)
ln(real GDP per capita)	0.713	14.72***	0.12	-0.111	-0.63	0.248	-8.574
	(0.589)	(4.898)	(0.939)	(0.944)	(1.338)	(1.439)	(9.788)
social security expenditure/GDP			-0.104	-0.16	-0.153	0.0083	-0.0419
			(0.163)	(0.166)	(0.165)	(0.218)	(0.225)
working age population				0.335	0.390*	0.412*	0.553*
				(0.229)	(0.227)	(0.235)	(0.281)
private credit/GDP					0.00469	-0.0051	-0.0163
					(0.024)	(0.024)	(0.027)
Africa						10.07*	9.926*
						(5.253)	(5.264)
Asia						13.42***	12.65**
						(4.827)	(4.908)
Europe						6.535	6.365
						(4.624)	(4.635)
North America						6.154	6.224
						(5.291)	(5.300)
Oceania						12.73**	13.19**
						(5.709)	(5.741)
ln(real GDP per capita) square		-0.895***					0.55
		(0.311)					(0.603)
Observations	134	134	63	63	62	61	61
R-squared	0.12	0.17	0.21	0.24	0.26	0.39	0.40

Standard errors in parentheses, *** p<0.01, ** p<0.05, * p<0.1

Table 3a: Saving rate vs sex ratio, small country (not for publication)

K=0.8	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000
1.050	0.573	0.170	0.201	0.043	0.481	0.214	0.184	0.026	0.576	0.180	0.205	0.047	0.399	0.255	0.170	0.012
1.100	0.745	0.090	0.238	0.080	0.608	0.151	0.210	0.052	0.750	0.115	0.247	0.089	0.509	0.193	0.189	0.031
1.150	0.818	0.066	0.261	0.103	0.668	0.118	0.224	0.066	0.822	0.095	0.271	0.113	0.576	0.153	0.203	0.045
1.200	0.867	0.052	0.279	0.121	0.717	0.094	0.238	0.080	0.871	0.085	0.290	0.132	0.637	0.119	0.218	0.060
1.250	0.875	0.048	0.286	0.128	0.723	0.086	0.242	0.084	0.878	0.080	0.297	0.139	0.647	0.110	0.222	0.064
1.300	0.887	0.044	0.295	0.137	0.736	0.078	0.249	0.091	0.891	0.074	0.305	0.147	0.666	0.097	0.228	0.070
1.350	0.899	0.040	0.303	0.145	0.750	0.070	0.256	0.098	0.902	0.071	0.313	0.155	0.685	0.085	0.236	0.078
1.400	0.907	0.037	0.310	0.152	0.759	0.064	0.262	0.104	0.910	0.068	0.320	0.162	0.699	0.077	0.242	0.084
1.450	0.912	0.036	0.317	0.159	0.766	0.060	0.267	0.109	0.916	0.068	0.327	0.169	0.710	0.073	0.249	0.091
1.500	0.916	0.035	0.323	0.165	0.771	0.056	0.272	0.114	0.919	0.064	0.332	0.174	0.717	0.069	0.254	0.096
1.550	0.919	0.033	0.328	0.170	0.774	0.053	0.276	0.118	0.922	0.063	0.337	0.179	0.723	0.063	0.258	0.100
1.600	0.922	0.032	0.333	0.175	0.777	0.051	0.280	0.122	0.925	0.061	0.342	0.184	0.728	0.061	0.263	0.105
1.650	0.924	0.031	0.338	0.180	0.779	0.049	0.284	0.126	0.927	0.061	0.346	0.188	0.731	0.058	0.267	0.109
1.700	0.925	0.030	0.342	0.184	0.780	0.047	0.287	0.129	0.927	0.594	0.479	0.321	0.734	0.055	0.270	0.112
1.750	0.926	0.029	0.347	0.189	0.781	0.045	0.290	0.132	0.929	0.058	0.354	0.196	0.736	0.053	0.274	0.116
1.800	0.927	0.029	0.351	0.193	0.782	0.044	0.293	0.135	0.930	0.058	0.358	0.200	0.737	0.051	0.276	0.118
1.850	0.928	0.028	0.354	0.196	0.782	0.043	0.296	0.138	0.930	0.058	0.362	0.204	0.738	0.049	0.279	0.121
1.900	0.928	0.028	0.358	0.200	0.782	0.042	0.299	0.141	0.931	0.056	0.365	0.207	0.739	0.048	0.282	0.124
1.950	0.928	0.028	0.361	0.203	0.781	0.041	0.301	0.143	0.931	0.056	0.369	0.211	0.739	0.046	0.284	0.126
2.000	0.928	0.027	0.365	0.207	0.780	0.040	0.303	0.145	0.931	0.055	0.372	0.214	0.738	0.046	0.286	0.128

sm—men's saving rate, sw—women's saving rate, st—economy-wide saving rate, ca—current account to GDP ratio

Table 3b: Saving rate vs sex ratio, small country (not for publication)

K=0.7	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000
1.050	0.552	0.179	0.197	0.039	0.455	0.227	0.180	0.022	0.556	0.189	0.201	0.043	0.391	0.260	0.169	0.011
1.100	0.720	0.099	0.232	0.074	0.561	0.173	0.201	0.043	0.724	0.122	0.241	0.083	0.491	0.201	0.186	0.028
1.150	0.793	0.075	0.255	0.097	0.608	0.144	0.211	0.053	0.798	0.103	0.265	0.107	0.553	0.164	0.198	0.040
1.200	0.846	0.057	0.273	0.115	0.645	0.122	0.221	0.063	0.851	0.088	0.284	0.126	0.610	0.131	0.211	0.053
1.250	0.854	0.052	0.280	0.122	0.648	0.115	0.223	0.065	0.859	0.083	0.291	0.133	0.619	0.121	0.215	0.057
1.300	0.867	0.047	0.288	0.130	0.656	0.106	0.227	0.069	0.872	0.079	0.299	0.141	0.638	0.108	0.221	0.063
1.350	0.881	0.044	0.297	0.139	0.664	0.098	0.231	0.073	0.885	0.076	0.308	0.150	0.656	0.101	0.229	0.071
1.400	0.889	0.040	0.304	0.146	0.668	0.092	0.235	0.077	0.894	0.072	0.315	0.157	0.669	0.092	0.235	0.077
1.450	0.896	0.037	0.311	0.153	0.670	0.087	0.237	0.079	0.900	0.069	0.321	0.163	0.679	0.084	0.240	0.082
1.500	0.900	0.035	0.317	0.159	0.671	0.084	0.240	0.082	0.904	0.067	0.327	0.169	0.685	0.079	0.244	0.086
1.550	0.904	0.035	0.322	0.164	0.671	0.080	0.242	0.084	0.908	0.065	0.332	0.174	0.691	0.074	0.248	0.090
1.600	0.906	0.034	0.328	0.170	0.670	0.077	0.244	0.086	0.910	0.064	0.337	0.179	0.695	0.070	0.252	0.094
1.650	0.909	0.032	0.332	0.174	0.669	0.075	0.246	0.088	0.913	0.063	0.341	0.183	0.698	0.066	0.255	0.097
1.700	0.910	0.032	0.337	0.179	0.668	0.073	0.247	0.089	0.914	0.061	0.345	0.187	0.701	0.063	0.258	0.100
1.750	0.911	0.031	0.341	0.183	0.666	0.071	0.249	0.091	0.915	0.060	0.349	0.191	0.703	0.061	0.262	0.104
1.800	0.912	0.030	0.345	0.187	0.665	0.069	0.250	0.092	0.916	0.060	0.353	0.195	0.704	0.058	0.264	0.106
1.850	0.913	0.030	0.348	0.190	0.663	0.068	0.252	0.094	0.917	0.059	0.357	0.199	0.704	0.056	0.266	0.108
1.900	0.913	0.029	0.352	0.194	0.660	0.066	0.252	0.094	0.917	0.058	0.360	0.202	0.704	0.054	0.269	0.111
1.950	0.913	0.028	0.355	0.197	0.658	0.065	0.254	0.096	0.917	0.056	0.363	0.205	0.704	0.053	0.271	0.113
2.000	0.913	0.028	0.358	0.200	0.656	0.064	0.255	0.097	0.917	0.057	0.366	0.208	0.704	0.053	0.273	0.115

sm—men's saving rate, sw—women's saving rate, st—economy-wide saving rate, ca—current account to GDP ratio

Table 3c: Saving rate vs sex ratio, small country (not for publication)

K=0.9	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000	0.310	0.310	0.158	0.000
1.050	0.590	0.162	0.204	0.046	0.503	0.203	0.188	0.030	0.592	0.174	0.209	0.051	0.406	0.252	0.172	0.014
1.100	0.765	0.084	0.243	0.085	0.646	0.135	0.218	0.060	0.769	0.110	0.252	0.094	0.524	0.185	0.192	0.034
1.150	0.835	0.060	0.265	0.107	0.713	0.100	0.235	0.077	0.839	0.091	0.275	0.117	0.595	0.145	0.207	0.049
1.200	0.882	0.048	0.283	0.125	0.767	0.077	0.251	0.093	0.885	0.080	0.294	0.136	0.659	0.110	0.223	0.065
1.250	0.889	0.044	0.290	0.132	0.775	0.071	0.257	0.099	0.892	0.076	0.300	0.142	0.670	0.101	0.227	0.069
1.300	0.900	0.041	0.299	0.141	0.790	0.063	0.265	0.107	0.903	0.073	0.309	0.151	0.689	0.089	0.235	0.077
1.350	0.911	0.038	0.307	0.149	0.806	0.056	0.273	0.115	0.914	0.069	0.317	0.159	0.709	0.078	0.243	0.085
1.400	0.918	0.036	0.314	0.156	0.816	0.052	0.280	0.122	0.921	0.069	0.324	0.166	0.722	0.070	0.249	0.091
1.450	0.923	0.034	0.321	0.163	0.824	0.048	0.286	0.128	0.926	0.065	0.330	0.172	0.733	0.064	0.255	0.097
1.500	0.927	0.033	0.326	0.168	0.829	0.045	0.292	0.134	0.929	0.063	0.335	0.177	0.741	0.059	0.261	0.103
1.550	0.930	0.032	0.332	0.174	0.834	0.043	0.297	0.139	0.932	0.062	0.340	0.182	0.746	0.056	0.266	0.108
1.600	0.932	0.031	0.337	0.179	0.837	0.041	0.302	0.144	0.934	0.063	0.346	0.188	0.752	0.052	0.270	0.112
1.650	0.934	0.030	0.342	0.184	0.840	0.040	0.306	0.148	0.936	0.059	0.350	0.192	0.755	0.050	0.274	0.116
1.700	0.935	0.030	0.346	0.188	0.842	0.038	0.310	0.152	0.936	0.058	0.354	0.196	0.758	0.051	0.279	0.121
1.750	0.936	0.029	0.350	0.192	0.844	0.037	0.314	0.156	0.938	0.057	0.358	0.200	0.760	0.046	0.282	0.124
1.800	0.937	0.029	0.354	0.196	0.845	0.036	0.318	0.160	0.939	0.057	0.362	0.204	0.762	0.044	0.285	0.127
1.850	0.937	0.028	0.358	0.200	0.845	0.035	0.321	0.163	0.939	0.561	0.481	0.323	0.763	0.045	0.288	0.130
1.900	0.937	0.027	0.362	0.204	0.845	0.035	0.324	0.166	0.939	0.055	0.369	0.211	0.763	0.044	0.291	0.133
1.950	0.937	0.027	0.365	0.207	0.845	0.034	0.327	0.169	0.939	0.055	0.372	0.214	0.763	0.042	0.294	0.136
2.000	0.938	0.026	0.369	0.211	0.845	0.033	0.330	0.172	0.940	0.055	0.375	0.217	0.763	0.040	0.296	0.138

sm—men's saving rate, sw—women's saving rate, st—economy-wide saving rate, ca—current account to GDP ratio

Table 4: CA/GDP vs country 2's sex ratio, two large countries, differing in sex ratio (not for publication)

K=0.8	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000
1.050	0.202	0.245	-0.022	0.022	0.202	0.228	-0.013	0.013	0.202	0.249	-0.024	0.024	0.202	0.214	-0.006	0.006
1.100	0.202	0.282	-0.040	0.040	0.202	0.254	-0.026	0.026	0.202	0.291	-0.045	0.045	0.202	0.233	-0.016	0.016
1.150	0.202	0.304	-0.051	0.051	0.202	0.268	-0.033	0.033	0.202	0.315	-0.057	0.057	0.202	0.247	-0.023	0.023
1.200	0.202	0.323	-0.061	0.061	0.202	0.282	-0.040	0.040	0.202	0.334	-0.066	0.066	0.202	0.261	-0.030	0.030
1.250	0.202	0.330	-0.064	0.064	0.202	0.286	-0.042	0.042	0.202	0.340	-0.069	0.069	0.202	0.265	-0.032	0.032
1.300	0.202	0.338	-0.068	0.068	0.202	0.292	-0.045	0.045	0.202	0.348	-0.073	0.073	0.202	0.272	-0.035	0.035
1.350	0.202	0.347	-0.073	0.073	0.202	0.299	-0.049	0.049	0.202	0.356	-0.077	0.077	0.202	0.279	-0.039	0.039
1.400	0.202	0.354	-0.076	0.076	0.202	0.305	-0.052	0.052	0.202	0.363	-0.081	0.081	0.202	0.286	-0.042	0.042
1.450	0.202	0.360	-0.079	0.079	0.202	0.311	-0.055	0.055	0.202	0.370	-0.084	0.084	0.202	0.293	-0.046	0.046
1.500	0.202	0.366	-0.082	0.082	0.202	0.315	-0.057	0.057	0.202	0.375	-0.087	0.087	0.202	0.298	-0.048	0.048
1.550	0.202	0.372	-0.085	0.085	0.202	0.319	-0.059	0.059	0.202	0.380	-0.089	0.089	0.202	0.302	-0.050	0.050
1.600	0.202	0.377	-0.088	0.088	0.202	0.324	-0.061	0.061	0.202	0.385	-0.092	0.092	0.202	0.306	-0.052	0.052
1.650	0.202	0.382	-0.090	0.090	0.202	0.327	-0.063	0.063	0.202	0.390	-0.094	0.094	0.202	0.310	-0.054	0.054
1.700	0.202	0.386	-0.092	0.092	0.202	0.331	-0.065	0.065	0.202	0.522	-0.160	0.160	0.202	0.314	-0.056	0.056
1.750	0.202	0.390	-0.094	0.094	0.202	0.334	-0.066	0.066	0.202	0.398	-0.098	0.098	0.202	0.317	-0.058	0.058
1.800	0.202	0.394	-0.096	0.096	0.202	0.337	-0.068	0.068	0.202	0.402	-0.100	0.100	0.202	0.320	-0.059	0.059
1.850	0.202	0.398	-0.098	0.098	0.202	0.340	-0.069	0.069	0.202	0.406	-0.102	0.102	0.202	0.323	-0.061	0.061
1.900	0.202	0.401	-0.100	0.100	0.202	0.342	-0.070	0.070	0.202	0.409	-0.104	0.104	0.202	0.325	-0.062	0.062
1.950	0.202	0.405	-0.102	0.102	0.202	0.345	-0.072	0.072	0.202	0.412	-0.105	0.105	0.202	0.328	-0.063	0.063
2.000	0.202	0.408	-0.103	0.103	0.202	0.347	-0.073	0.073	0.202	0.415	-0.107	0.107	0.202	0.330	-0.064	0.064

s1—country 1's economy-wide saving rate, s2—country 2's economy-wide saving rate,

ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Table 5a: CA/GDP vs country 2's sex ratio, two large countries: US and China (not for publication)

K=0.8	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000
1.050	0.202	0.245	-0.011	0.032	0.202	0.228	-0.007	0.020	0.202	0.249	-0.012	0.036	0.202	0.214	-0.003	0.009
1.100	0.202	0.282	-0.020	0.060	0.202	0.254	-0.013	0.039	0.202	0.291	-0.022	0.067	0.202	0.233	-0.008	0.024
1.150	0.202	0.304	-0.026	0.077	0.202	0.268	-0.017	0.050	0.202	0.315	-0.028	0.085	0.202	0.247	-0.011	0.034
1.200	0.202	0.323	-0.030	0.091	0.202	0.282	-0.020	0.060	0.202	0.334	-0.033	0.099	0.202	0.261	-0.015	0.045
1.250	0.202	0.330	-0.032	0.096	0.202	0.286	-0.021	0.063	0.202	0.340	-0.035	0.104	0.202	0.265	-0.016	0.048
1.300	0.202	0.338	-0.034	0.103	0.202	0.292	-0.023	0.068	0.202	0.348	-0.037	0.110	0.202	0.272	-0.018	0.053
1.350	0.202	0.347	-0.036	0.109	0.202	0.299	-0.024	0.073	0.202	0.356	-0.039	0.116	0.202	0.279	-0.019	0.058
1.400	0.202	0.354	-0.038	0.114	0.202	0.305	-0.026	0.078	0.202	0.363	-0.040	0.121	0.202	0.286	-0.021	0.063
1.450	0.202	0.360	-0.040	0.119	0.202	0.311	-0.027	0.082	0.202	0.370	-0.042	0.127	0.202	0.293	-0.023	0.068
1.500	0.202	0.366	-0.041	0.124	0.202	0.315	-0.028	0.085	0.202	0.375	-0.043	0.130	0.202	0.298	-0.024	0.072
1.550	0.202	0.372	-0.043	0.128	0.202	0.319	-0.029	0.088	0.202	0.380	-0.045	0.134	0.202	0.302	-0.025	0.075
1.600	0.202	0.377	-0.044	0.131	0.202	0.324	-0.031	0.092	0.202	0.385	-0.046	0.138	0.202	0.306	-0.026	0.079
1.650	0.202	0.382	-0.045	0.135	0.202	0.327	-0.031	0.094	0.202	0.390	-0.047	0.141	0.202	0.310	-0.027	0.081
1.700	0.202	0.386	-0.046	0.138	0.202	0.331	-0.032	0.097	0.202	0.522	-0.080	0.241	0.202	0.314	-0.028	0.084
1.750	0.202	0.390	-0.047	0.141	0.202	0.334	-0.033	0.099	0.202	0.398	-0.049	0.147	0.202	0.317	-0.029	0.087
1.800	0.202	0.394	-0.048	0.144	0.202	0.337	-0.034	0.102	0.202	0.402	-0.050	0.150	0.202	0.320	-0.030	0.089
1.850	0.202	0.398	-0.049	0.147	0.202	0.340	-0.035	0.104	0.202	0.406	-0.051	0.153	0.202	0.323	-0.030	0.091
1.900	0.202	0.401	-0.050	0.150	0.202	0.342	-0.035	0.106	0.202	0.409	-0.052	0.156	0.202	0.325	-0.031	0.093
1.950	0.202	0.405	-0.051	0.153	0.202	0.345	-0.036	0.107	0.202	0.412	-0.053	0.158	0.202	0.328	-0.032	0.095
2.000	0.202	0.408	-0.052	0.155	0.202	0.347	-0.036	0.109	0.202	0.415	-0.053	0.160	0.202	0.330	-0.032	0.096

s1—country 1's economy-wide saving rate, s2—country 2's economy-wide saving rate,

ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Table 5b: CA/GDP vs country 2's sex ratio, two large countries: US and China (not for publication)

K=0.7	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000
1.050	0.202	0.241	-0.010	0.029	0.202	0.223	-0.005	0.016	0.202	0.245	-0.011	0.033	0.202	0.213	-0.003	0.008
1.100	0.202	0.276	-0.019	0.056	0.202	0.245	-0.011	0.032	0.202	0.284	-0.021	0.062	0.202	0.229	-0.007	0.021
1.150	0.202	0.298	-0.024	0.073	0.202	0.255	-0.013	0.040	0.202	0.309	-0.027	0.080	0.202	0.242	-0.010	0.030
1.200	0.202	0.317	-0.029	0.086	0.202	0.265	-0.016	0.047	0.202	0.328	-0.032	0.095	0.202	0.255	-0.013	0.040
1.250	0.202	0.323	-0.030	0.091	0.202	0.267	-0.016	0.049	0.202	0.334	-0.033	0.100	0.202	0.258	-0.014	0.043
1.300	0.202	0.332	-0.033	0.098	0.202	0.271	-0.017	0.052	0.202	0.343	-0.035	0.106	0.202	0.265	-0.016	0.048
1.350	0.202	0.341	-0.035	0.105	0.202	0.275	-0.018	0.055	0.202	0.351	-0.037	0.112	0.202	0.273	-0.018	0.054
1.400	0.202	0.348	-0.037	0.110	0.202	0.278	-0.019	0.058	0.202	0.358	-0.039	0.118	0.202	0.279	-0.019	0.058
1.450	0.202	0.355	-0.038	0.115	0.202	0.281	-0.020	0.059	0.202	0.365	-0.041	0.122	0.202	0.283	-0.020	0.061
1.500	0.202	0.360	-0.040	0.119	0.202	0.284	-0.021	0.062	0.202	0.370	-0.042	0.126	0.202	0.288	-0.022	0.065

K=0.9	σ=0.01, m=2, p=0.02				σ =0.01, m=0.5, p=0.02				σ =0.01, m=2, p=0.05				σ =0.05, m=2, p=0.02			
	(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)				(s1, s2, ca1, ca2)			
1.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000	0.202	0.202	0.000	0.000
1.050	0.202	0.248	-0.012	0.035	0.202	0.232	-0.008	0.023	0.202	0.252	-0.013	0.038	0.202	0.215	-0.003	0.010
1.100	0.202	0.286	-0.021	0.064	0.202	0.262	-0.015	0.045	0.202	0.296	-0.024	0.071	0.202	0.236	-0.009	0.026
1.150	0.202	0.308	-0.027	0.080	0.202	0.278	-0.019	0.057	0.202	0.319	-0.029	0.088	0.202	0.251	-0.012	0.037
1.200	0.202	0.327	-0.031	0.094	0.202	0.295	-0.023	0.070	0.202	0.337	-0.034	0.102	0.202	0.266	-0.016	0.048
1.250	0.202	0.334	-0.033	0.099	0.202	0.300	-0.025	0.074	0.202	0.344	-0.036	0.107	0.202	0.271	-0.017	0.052
1.300	0.202	0.342	-0.035	0.106	0.202	0.308	-0.027	0.080	0.202	0.352	-0.038	0.113	0.202	0.278	-0.019	0.058
1.350	0.202	0.351	-0.037	0.112	0.202	0.316	-0.029	0.086	0.202	0.360	-0.040	0.119	0.202	0.286	-0.021	0.064
1.400	0.202	0.358	-0.039	0.117	0.202	0.323	-0.030	0.091	0.202	0.368	-0.042	0.125	0.202	0.293	-0.023	0.068
1.450	0.202	0.364	-0.041	0.122	0.202	0.330	-0.032	0.096	0.202	0.373	-0.043	0.129	0.202	0.299	-0.024	0.073
1.500	0.202	0.370	-0.042	0.126	0.202	0.335	-0.033	0.100	0.202	0.379	-0.044	0.133	0.202	0.304	-0.026	0.077

s1—country 1's economy-wide saving rate, s2—country 2's economy-wide saving rate,
ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio

Table 6: Planner's economy vs competitive equilibrium, small country (not for publication)

	planner's economy			κ =0.7						κ =0.8						κ =0.9					
	(sm, sw, st)			(sm, Δw(m), sw, Δw(w), st, Δw)						(sm, Δw(m), sw, Δw(w), st, Δw)						(sm, Δw(m), sw, Δw(w), st, Δw)					
1.000	0.310	0.310	0.158	0.310	0.000	0.310	0.000	0.158	0.000	0.310	0.000	0.310	0.000	0.158	0.000	0.310	0.000	0.310	0.000	0.158	0.000
1.050	0.310	0.310	0.158	0.552	-0.345	0.179	0.242	0.197	-0.059	0.573	-0.385	0.170	0.259	0.201	-0.071	0.590	-0.420	0.162	0.274	0.204	-0.082
1.100	0.310	0.310	0.158	0.720	-0.749	0.099	0.379	0.232	-0.212	0.745	-0.833	0.090	0.397	0.238	-0.247	0.765	-0.907	0.084	0.410	0.243	-0.280
1.150	0.310	0.310	0.158	0.793	-1.012	0.075	0.429	0.255	-0.342	0.818	-1.132	0.066	0.445	0.261	-0.398	0.835	-1.224	0.060	0.456	0.265	-0.442
1.200	0.310	0.310	0.158	0.846	-1.279	0.057	0.463	0.273	-0.487	0.867	-1.417	0.052	0.475	0.279	-0.557	0.882	-1.531	0.048	0.484	0.283	-0.615
1.250	0.310	0.310	0.158	0.854	-1.321	0.052	0.469	0.280	-0.525	0.875	-1.467	0.048	0.481	0.286	-0.601	0.889	-1.581	0.044	0.489	0.290	-0.661
1.300	0.310	0.310	0.158	0.867	-1.401	0.047	0.477	0.288	-0.584	0.887	-1.555	0.044	0.488	0.295	-0.667	0.900	-1.672	0.041	0.495	0.299	-0.730
1.350	0.310	0.310	0.158	0.881	-1.498	0.044	0.485	0.297	-0.654	0.899	-1.655	0.040	0.495	0.303	-0.740	0.911	-1.776	0.038	0.501	0.307	-0.807
1.400	0.310	0.310	0.158	0.889	-1.558	0.040	0.490	0.304	-0.704	0.907	-1.727	0.037	0.500	0.310	-0.799	0.918	-1.848	0.036	0.505	0.314	-0.867
1.450	0.310	0.310	0.158	0.896	-1.613	0.037	0.494	0.311	-0.753	0.912	-1.773	0.036	0.502	0.317	-0.844	0.923	-1.902	0.034	0.508	0.321	-0.918
1.500	0.310	0.310	0.158	0.900	-1.644	0.035	0.497	0.317	-0.788	0.916	-1.811	0.035	0.504	0.323	-0.885	0.927	-1.947	0.033	0.510	0.326	-0.964
1.550	0.310	0.310	0.158	0.904	-1.677	0.035	0.499	0.322	-0.824	0.919	-1.840	0.033	0.506	0.328	-0.920	0.930	-1.981	0.032	0.512	0.332	-1.004
1.600	0.310	0.310	0.158	0.906	-1.692	0.034	0.500	0.328	-0.849	0.922	-1.871	0.032	0.508	0.333	-0.956	0.932	-2.004	0.031	0.513	0.337	-1.036
1.650	0.310	0.310	0.158	0.909	-1.718	0.032	0.502	0.332	-0.880	0.924	-1.891	0.031	0.509	0.338	-0.985	0.934	-2.028	0.030	0.514	0.342	-1.068
1.700	0.310	0.310	0.158	0.910	-1.723	0.032	0.503	0.337	-0.899	0.925	-1.899	0.030	0.510	0.342	-1.007	0.935	-2.037	0.030	0.515	0.346	-1.092
1.750	0.310	0.310	0.158	0.911	-1.729	0.031	0.503	0.341	-0.917	0.926	-1.907	0.029	0.511	0.347	-1.028	0.936	-2.048	0.029	0.515	0.350	-1.116
1.800	0.310	0.310	0.158	0.912	-1.736	0.030	0.504	0.345	-0.936	0.927	-1.916	0.029	0.511	0.351	-1.049	0.937	-2.058	0.029	0.516	0.354	-1.139
1.850	0.310	0.310	0.158	0.913	-1.742	0.030	0.505	0.348	-0.954	0.928	-1.925	0.028	0.512	0.354	-1.070	0.937	-2.054	0.028	0.516	0.358	-1.152
1.900	0.310	0.310	0.158	0.913	-1.738	0.029	0.505	0.352	-0.965	0.928	-1.921	0.028	0.512	0.358	-1.082	0.937	-2.050	0.027	0.516	0.362	-1.165
1.950	0.310	0.310	0.158	0.913	-1.735	0.028	0.505	0.355	-0.975	0.928	-1.917	0.028	0.512	0.361	-1.093	0.937	-2.046	0.027	0.516	0.365	-1.177
2.000	0.310	0.310	0.158	0.913	-1.731	0.028	0.505	0.358	-0.986	0.928	-1.913	0.027	0.512	0.365	-1.105	0.938	-2.058	0.026	0.517	0.369	-1.200

sm—men's saving rate, sw—women's saving rate, st—economy-wide saving rate, Δw(m)—men's welfare gain under the decentralized economy compared to that under the planner's economy, Δw(w)—women's welfare gain under the decentralized economy compared to that under the planner's economy, Δw—social welfare gain under the decentralized economy compared to that under the planner's economy

Table 7: Saving rate vs sex ratio, endogenous intra-household bargaining, small country (not for publication)

γ=0.5	ε=0.5				ε =0.3				ε =0.7			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.000	0.382	0.382	0.158	0.000	0.375	0.375	0.158	0.000	0.399	0.399	0.158	0.000
1.050	0.610	0.329	0.217	0.059	0.598	0.290	0.201	0.043	0.621	0.363	0.231	0.073
1.100	0.782	0.300	0.269	0.111	0.771	0.249	0.249	0.091	0.790	0.343	0.285	0.127
1.150	0.850	0.291	0.293	0.135	0.841	0.236	0.273	0.115	0.856	0.337	0.309	0.151
1.200	0.893	0.289	0.311	0.153	0.863	0.232	0.284	0.126	0.898	0.333	0.327	0.169
1.250	0.900	0.287	0.318	0.160	0.880	0.229	0.294	0.136	0.904	0.333	0.332	0.174
1.300	0.911	0.284	0.326	0.168	0.897	0.226	0.303	0.145	0.915	0.332	0.340	0.182
1.350	0.921	0.283	0.332	0.174	0.915	0.224	0.313	0.155	0.922	0.332	0.346	0.188
1.400	0.927	0.282	0.338	0.180	0.922	0.223	0.320	0.162	0.928	0.330	0.351	0.193
1.450	0.932	0.281	0.343	0.185	0.924	0.222	0.324	0.166	0.934	0.330	0.357	0.199
1.500	0.935	0.281	0.348	0.190	0.925	0.222	0.328	0.170	0.939	0.330	0.362	0.204

ε =0.5	γ =0.5				γ =0.3				γ =0.7			
	(sm, sw, st, ca)				(sm, sw, st, ca)				(sm, sw, st, ca)			
1.000	0.382	0.382	0.158	0.000	0.374	0.374	0.158	0.000	0.399	0.399	0.158	0.000
1.050	0.610	0.329	0.217	0.059	0.614	0.290	0.206	0.048	0.607	0.361	0.226	0.068
1.100	0.782	0.300	0.269	0.111	0.788	0.253	0.256	0.098	0.775	0.338	0.278	0.120
1.150	0.850	0.291	0.293	0.135	0.855	0.242	0.280	0.122	0.843	0.330	0.303	0.145
1.200	0.893	0.289	0.311	0.153	0.898	0.235	0.298	0.140	0.888	0.325	0.321	0.163
1.250	0.900	0.287	0.318	0.160	0.907	0.234	0.305	0.147	0.904	0.324	0.330	0.172
1.300	0.911	0.284	0.326	0.168	0.914	0.233	0.311	0.153	0.916	0.323	0.338	0.180
1.350	0.921	0.283	0.332	0.174	0.918	0.232	0.317	0.159	0.923	0.323	0.344	0.186
1.400	0.927	0.282	0.338	0.180	0.922	0.231	0.322	0.164	0.929	0.322	0.349	0.191
1.450	0.932	0.281	0.343	0.185	0.926	0.230	0.327	0.169	0.935	0.322	0.355	0.197
1.500	0.935	0.281	0.348	0.190	0.928	0.229	0.331	0.173	0.940	0.322	0.360	0.202

sm—men's saving rate, sw—women's saving rate, st—economy-wide saving rate, ca—current account to GDP ratio

Table 8: CA/GDP vs country 2's sex ratio, endogenous intra-household bargaining, two large countries, differing in sex ratio (not for publication)

γ=0.5	ε=0.5						ε =0.3						ε =0.7					
	(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)					
1.000	0.248	0.248	0.000	0.000	0.000	0.000	0.244	0.244	0.000	0.000	0.000	0.000	0.259	0.259	0.000	0.000	0.000	0.000
1.050	0.248	0.307	-0.030	0.030	0.003	-0.095	0.244	0.291	-0.024	0.024	0.002	-0.081	0.259	0.322	-0.031	0.031	0.003	-0.106
1.100	0.248	0.359	-0.055	0.055	0.004	-0.277	0.244	0.340	-0.048	0.048	0.003	-0.254	0.259	0.375	-0.058	0.058	0.004	-0.294
1.150	0.248	0.384	-0.068	0.068	0.003	-0.397	0.244	0.364	-0.060	0.060	0.002	-0.372	0.259	0.399	-0.070	0.070	0.003	-0.414
1.200	0.248	0.402	-0.077	0.077	0.002	-0.497	0.244	0.375	-0.065	0.065	0.000	-0.424	0.259	0.417	-0.079	0.079	0.001	-0.514
1.250	0.248	0.408	-0.080	0.080	-0.001	-0.523	0.244	0.384	-0.070	0.070	-0.002	-0.469	0.259	0.423	-0.082	0.082	-0.001	-0.537
1.300	0.248	0.415	-0.083	0.083	-0.003	-0.559	0.244	0.393	-0.075	0.075	-0.005	-0.517	0.259	0.430	-0.085	0.085	-0.004	-0.574
1.350	0.248	0.422	-0.087	0.087	-0.005	-0.593	0.244	0.404	-0.080	0.080	-0.006	-0.571	0.259	0.436	-0.088	0.088	-0.006	-0.600
1.400	0.248	0.428	-0.090	0.090	-0.008	-0.618	0.244	0.410	-0.083	0.083	-0.009	-0.598	0.259	0.441	-0.091	0.091	-0.008	-0.624
1.450	0.248	0.433	-0.092	0.092	-0.010	-0.639	0.244	0.414	-0.085	0.085	-0.011	-0.611	0.259	0.447	-0.094	0.094	-0.010	-0.649
1.500	0.248	0.438	-0.095	0.095	-0.012	-0.655	0.244	0.418	-0.087	0.087	-0.013	-0.621	0.259	0.452	-0.096	0.096	-0.012	-0.670

ε=0.5	γ=0.5						γ =0.3						γ =0.7					
	(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)					
1.000	0.248	0.248	0.000	0.000	0.000	0.000	0.243	0.243	0.000	0.000	0.000	0.000	0.259	0.259	0.000	0.000	0.000	0.000
1.050	0.248	0.307	-0.030	0.030	0.003	-0.095	0.243	0.296	-0.027	0.027	0.001	-0.095	0.259	0.317	-0.029	0.029	0.004	-0.093
1.100	0.248	0.359	-0.055	0.055	0.004	-0.277	0.243	0.347	-0.052	0.052	0.001	-0.282	0.259	0.368	-0.055	0.055	0.007	-0.268
1.150	0.248	0.384	-0.068	0.068	0.003	-0.397	0.243	0.370	-0.064	0.064	-0.002	-0.403	0.259	0.393	-0.067	0.067	0.007	-0.384
1.200	0.248	0.402	-0.077	0.077	0.002	-0.497	0.243	0.388	-0.072	0.072	-0.004	-0.506	0.259	0.411	-0.076	0.076	0.006	-0.485
1.250	0.248	0.408	-0.080	0.080	-0.001	-0.523	0.243	0.395	-0.076	0.076	-0.008	-0.538	0.259	0.420	-0.080	0.080	0.005	-0.531
1.300	0.248	0.415	-0.083	0.083	-0.003	-0.559	0.243	0.402	-0.079	0.079	-0.012	-0.566	0.259	0.428	-0.084	0.084	0.004	-0.571
1.350	0.248	0.422	-0.087	0.087	-0.005	-0.593	0.243	0.407	-0.082	0.082	-0.015	-0.585	0.259	0.434	-0.087	0.087	0.003	-0.597
1.400	0.248	0.428	-0.090	0.090	-0.008	-0.618	0.243	0.412	-0.085	0.085	-0.018	-0.604	0.259	0.439	-0.090	0.090	0.002	-0.622
1.450	0.248	0.433	-0.092	0.092	-0.010	-0.639	0.243	0.417	-0.087	0.087	-0.021	-0.622	0.259	0.445	-0.093	0.093	0.001	-0.646
1.500	0.248	0.438	-0.095	0.095	-0.012	-0.655	0.243	0.421	-0.089	0.089	-0.025	-0.635	0.259	0.450	-0.095	0.095	0.000	-0.668

s1—country 1's economy-wide saving rate, s2—country 2's economy-wide saving rate, ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio, dw1—welfare change in country 1 in units of consumption goods relative to the case Φ=1, dw1—welfare change in country 2 in units of consumption goods relative to the case Φ=1

Table 9: CA/GDP vs country 2's sex ratio, endogenous intra-household bargaining, two large countries, US vs China (not for publication)

γ=0.5	ε=0.5						ε =0.3						ε =0.7					
	(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)					
1.000	0.248	0.248	0.000	0.000	0.000	0.000	0.244	0.244	0.000	0.000	0.000	0.000	0.259	0.259	0.000	0.000	0.000	0.000
1.050	0.248	0.307	-0.015	0.044	0.000	-0.098	0.244	0.291	-0.012	0.035	-0.001	-0.084	0.259	0.322	-0.016	0.047	0.000	-0.109
1.100	0.248	0.359	-0.028	0.083	-0.001	-0.281	0.244	0.340	-0.024	0.072	-0.002	-0.257	0.259	0.375	-0.029	0.087	-0.001	-0.298
1.150	0.248	0.384	-0.034	0.101	-0.003	-0.401	0.244	0.364	-0.030	0.090	-0.004	-0.376	0.259	0.399	-0.035	0.105	-0.003	-0.418
1.200	0.248	0.402	-0.038	0.115	-0.006	-0.501	0.244	0.375	-0.033	0.098	-0.007	-0.427	0.259	0.417	-0.039	0.118	-0.006	-0.517
1.250	0.248	0.408	-0.040	0.120	-0.008	-0.526	0.244	0.384	-0.035	0.105	-0.009	-0.472	0.259	0.423	-0.041	0.122	-0.009	-0.541
1.300	0.248	0.415	-0.042	0.125	-0.011	-0.562	0.244	0.393	-0.037	0.112	-0.012	-0.520	0.259	0.430	-0.043	0.128	-0.011	-0.577
1.350	0.248	0.422	-0.043	0.130	-0.013	-0.597	0.244	0.404	-0.040	0.120	-0.014	-0.574	0.259	0.436	-0.044	0.133	-0.014	-0.604
1.400	0.248	0.428	-0.045	0.135	-0.016	-0.621	0.244	0.410	-0.042	0.125	-0.016	-0.601	0.259	0.441	-0.045	0.136	-0.016	-0.627
1.450	0.248	0.433	-0.046	0.139	-0.018	-0.642	0.244	0.414	-0.043	0.128	-0.019	-0.614	0.259	0.447	-0.047	0.141	-0.018	-0.652
1.500	0.248	0.438	-0.047	0.142	-0.020	-0.658	0.244	0.418	-0.044	0.131	-0.021	-0.624	0.259	0.452	-0.048	0.144	-0.021	-0.673

ε=0.5	γ=0.5						γ =0.3						γ =0.7					
	(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)						(s1, s2, ca1, ca2, dw1, dw2)					
1.000	0.248	0.248	0.000	0.000	0.000	0.000	0.243	0.243	0.000	0.000	0.000	0.000	0.259	0.259	0.000	0.000	0.000	0.000
1.050	0.248	0.307	-0.015	0.044	0.000	-0.098	0.243	0.296	-0.013	0.040	-0.002	-0.098	0.259	0.317	-0.014	0.043	0.001	-0.095
1.100	0.248	0.359	-0.028	0.083	-0.001	-0.281	0.243	0.347	-0.026	0.078	-0.004	-0.285	0.259	0.368	-0.027	0.082	0.001	-0.272
1.150	0.248	0.384	-0.034	0.101	-0.003	-0.401	0.243	0.370	-0.032	0.095	-0.008	-0.407	0.259	0.393	-0.033	0.100	0.001	-0.388
1.200	0.248	0.402	-0.038	0.115	-0.006	-0.501	0.243	0.388	-0.036	0.109	-0.011	-0.510	0.259	0.411	-0.038	0.114	0.000	-0.488
1.250	0.248	0.408	-0.040	0.120	-0.008	-0.526	0.243	0.395	-0.038	0.114	-0.015	-0.542	0.259	0.420	-0.040	0.121	-0.002	-0.535
1.300	0.248	0.415	-0.042	0.125	-0.011	-0.562	0.243	0.402	-0.040	0.119	-0.019	-0.569	0.259	0.428	-0.042	0.126	-0.003	-0.574
1.350	0.248	0.422	-0.043	0.130	-0.013	-0.597	0.243	0.407	-0.041	0.123	-0.023	-0.588	0.259	0.434	-0.044	0.131	-0.005	-0.601
1.400	0.248	0.428	-0.045	0.135	-0.016	-0.621	0.243	0.412	-0.042	0.127	-0.026	-0.607	0.259	0.439	-0.045	0.135	-0.006	-0.625
1.450	0.248	0.433	-0.046	0.139	-0.018	-0.642	0.243	0.417	-0.044	0.131	-0.029	-0.626	0.259	0.445	-0.046	0.139	-0.007	-0.649
1.500	0.248	0.438	-0.047	0.142	-0.020	-0.658	0.243	0.421	-0.045	0.134	-0.033	-0.638	0.259	0.450	-0.048	0.143	-0.009	-0.671

s1—country 1's economy-wide saving rate, s2—country 2's economy-wide saving rate, ca1—country 1's current account to GDP ratio, ca2—country 2's current account to GDP ratio, dw1—welfare change in country 1 in units of consumption goods relative to the case Φ=1, dw1—welfare change in country 2 in units of consumption goods relative to the case Φ=1

A Proof of Proposition 1 (not for publication)

Proof. We have shown that when $\phi = 1$, entering the marriage market is a dominant strategy for both women and men. Then if ϕ is close to one, we still have $V^i > V_n^i$ $(i = w, m)$, all women and men choose to enter the marriage market in the second period. Since $\frac{1}{2} \leq \kappa \leq 1$,

$$\kappa(Rs^m y + Rs^w y) > \max(Rs^w y,\, Rs^m y)$$

which means that within the neighbourhood of $\phi = 1$, we have $\kappa u'_{2m} < u'_{2m,n}$.

1. If for all $\phi \in [1, \phi_1)$, we have $\kappa u'_{2m} < u'_{2m,n}$

The first order conditions for a woman and a man, respectively, are:

$$-u'_{1w} + (1-p)\kappa u'_{2m}\left[1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right] + pu'_{2w,n} = 0 \quad \text{(A.1)}$$

$$-u'_{1m} + (1-p)\left[\kappa\delta^m u'_{2m} + \kappa u'_{2w}\left(\frac{u_{2m} + \eta^{\max} - u_{2m,n}}{\eta^{\max} - \eta^{\min}}\right)\right] + \left[(1-\delta^m)(1-p) + p\right]u'_{2m,n} = 0 \quad \text{(A.2)}$$

Totally differentiating the system, we obtain

$$\Omega \cdot \mathbf{ds} = \mathbf{dz} \quad \text{(A.3)}$$

where

$$\Omega = \begin{pmatrix} A & (1-p)\kappa^2 Ryu''_{2w}\left[1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right] \\ D & B \end{pmatrix}, \ \mathbf{ds} = \begin{pmatrix} ds^w \\ ds^m \end{pmatrix} \text{ and } \mathbf{d\boldsymbol{\phi}} = \begin{pmatrix} 0 \\ E \end{pmatrix}$$

where

$$\begin{aligned}
A &= y\left[u''_{1w} + (1-p)\kappa^2 Ru''_{2w}\left(1 + \frac{1}{\phi} + M(\eta^{\min})f(\eta^{\min})\right) + pRu''_{2w,n}\right] < 0 \\
B &= y\left[\begin{array}{c} u''_{1m} + \kappa^2 R\delta^m u''_{2m} + \kappa^2 u''_{2m}R \\ +\kappa^2 Ru''_{2w}\left(\frac{u_{2m} - u_{2m,n} + \eta^{\min}}{\eta^{\max} - \eta^{\min}}\right) + R\kappa u'_{2w}\left(\frac{\kappa u'_{2m} - u'_{2m,n}}{\eta^{\max} - \eta^{\min}}\right) \end{array}\right] + \left[p + (1-p)(1-\delta^m)\right] Ryu''_{2m,n} < 0 \\
D &= (1-p)\kappa^2\left[\left((\delta^m + 1) + \left(\frac{u_{2m} - u_{2m,n} + \eta^{\min}}{\eta^{\max} - \eta^{\min}}\right)\right) yu''_{2m}R + \frac{u'_{2w}u'_{2m}}{\eta^{\max} - \eta^{\min}}\right] \\
E &= (1-p)\frac{\partial \delta^m}{\partial \phi}\left(u'_{2m}(n) - \kappa u'_{2m}\right) < 0
\end{aligned}$$

It is easy to show that

$$\det(\Omega) > 0$$

and

$$
\begin{aligned}
\frac{ds^w}{d\phi} &= -\frac{\kappa^2 Ryu''_{2w}\left[1+\frac{1}{\phi}+M(\eta^{\min})f(\eta^{\min})\right]E}{\det(\Omega)} < 0 \\
\frac{ds^m}{d\phi} &= \frac{AE}{\det(\Omega)} > 0
\end{aligned}
$$

The aggregate savings rate of the young cohort is

$$
s^y = \frac{\phi}{1+\phi}s^m + \frac{1}{1+\phi}s^w
$$

and we can show that

$$
\begin{aligned}
\frac{ds^y}{d\phi} &= \frac{s^m - s^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds^m}{d\phi}+\frac{ds^w}{d\phi}\right) \\
&= \frac{s^m - s^w}{(1+\phi)^2} + \frac{\phi-1}{1+\phi}\frac{ds^m}{d\phi} + \frac{1}{1+\phi}\frac{yu''_{1w}E}{\det(\Omega)}
\end{aligned}
$$

Obviously, the second and the third term on the right hand side are postive. When $\phi = 1$, women and men have the same savings rates. As the sex ratio rises, we have showed that men raise their savings while women reduce their savings. Then $s^m > s^w$, which implies that the first term on the right hand side is positive. Therefore, the aggregate savings rate of the young cohort increases as the sex ratio rises. Since the (dis-)savings rate of the old cohort is fixed (their sex ratio is balanced), an increase in the savings rate by the young cohort translates into an increase in the economy-wide savings rate.

The impact of a rise in the sex ratio on the social welfare is

$$
\begin{aligned}
\frac{\partial U^w}{\partial \phi} &= y\left(-u'_{1w}+(1-p)\kappa u'_{2w}+pu'_{2w,n}\right)\frac{ds^w}{d\phi} + (1-p)\left(y\kappa u'_{2w}\frac{ds^m}{d\phi}+\frac{\beta}{\phi^2}E\left[\eta\right]\right) \qquad \text{(A.4)} \\
&> (1-p)\left(y\kappa u'_{2w}\left(\frac{ds^m}{d\phi}+\frac{ds^w}{d\phi}\right)+\frac{\beta}{\phi^2}E\left[\eta\right]\right) > 0 \\
\frac{\partial U^m}{\partial \phi} &= y\left(-u'_{1m}+\kappa\delta(1-p)u'_{2m}+\left[(1-p)\delta+(1-\delta)\right]u'_{2m,n}\right)\frac{ds^m}{d\phi} + (1-p)y\kappa\delta u'_{2w}\frac{ds^w}{d\phi} \\
&\quad +(1-p)\left[\beta\frac{\partial\delta}{\partial\phi}\left(u_{2m}-u_{2m,n}+\eta^{\min}\right)+\beta\frac{\partial\left(\int_{M(\eta^{\min})}M^{-1}(\eta^m)dF(\eta^m)\right)}{\partial\phi}\right] \qquad \text{(A.5)} \\
&< (1-p)\left(y\kappa\delta u'_{2w}\frac{ds^w}{d\phi}-\frac{\beta}{\phi^2}\left(u_{2m}-u_{2m,n}+\eta^{\min}\right)-\frac{\beta}{\phi^2}E\left[\eta\right]\right) < 0
\end{aligned}
$$

where the first inequality in (A.4) holds because

$$
-u'_{1w}+(1-p)\kappa u'_{2w}+pu'_{2w,n} = -(1-p)\kappa u'_{2m}\left[\frac{1}{\phi}+M(\eta^{\min})f(\eta^{\min})\right] < 0
$$

and the first inequality in (A.5) holds because

$$-u'_{1m} + \kappa(1-p)(1+\delta)u'_{2m} + p(1-\delta)u'_{2m,n} = -(1-p)\kappa u'_{2w}\left(\frac{u_{2m} + \eta^{\min} - u_{2m,n}}{\eta^{\max} - \eta^{\min}}\right) < 0$$

As the sex ratio rises further, the men lose while the women gain. However, as ϕ approaches infinity, the expected emotional utility gain from a marriage is close to zero, which means that a representative man can achieve a higher welfare level by staying single. Therefore, there exists a threshold value $\phi_1 < \infty$ such that the man is indifferent between entering the marriage market and being a life-time bachelor. For any $\phi < \phi_1$, entering the marriage market is still a dominant strategy for both women and men.

Lemma 1 *For ϕ slightly above ϕ_1, the women increase their savings rate.*

Suppose the sex ratio rises just above ϕ_1, if women do not raise their savings rates, men would choose a mixed strategy: with probability $\frac{\phi_1}{\phi}$, they enter the marriage market and with probability $1 - \frac{\phi_1}{\phi}$ they opt to be out of the marriage market. However, women would not choose to do nothing, because they strictly prefer to be married and have the men with the highest values of the emotional utility staying in the marriage pool. By raising her savings rate by a small amount, she won't experience a first-order welfare loss. At the same time, by raising her second-period wealth, men will be induced to go back to always entering the marriage market. Woman i will have a welfare gain because she will have a higher probability to marry a man with a higher η^m.

Therefore, if ϕ rises beyond ϕ_1, women will increase their savings rates to induce all men to enter the marriage market. A woman's savings decision must satisfy the following equation:

$$\max_{s^m} u(c_{1m}) + \beta\left[\begin{array}{c}(1-p)\delta^m u(c_{2m}) + (1-p)E(\eta^w \,|\, s^m, s^w, \phi) \\ +\left(p + (1-p)(1-\delta^m)\right) u(c_{2m}(n))\end{array}\right] = (1+\beta)u\left(\frac{R}{1+R}y\right) \qquad \text{(A.6)}$$

where we have used the fact that the optimal savings rate of a life-time bachelor equals $\frac{1}{1+R}$.

Totally differentiating (A.6) and (A.2), we obtain

$$\Omega \cdot \mathbf{ds} = \mathbf{d}\boldsymbol{\phi}$$

where

$$\Omega = \begin{pmatrix} \kappa(1-p)\delta u'_{2m}y & 0 \\ D & B \end{pmatrix},\ \mathbf{ds} = \begin{pmatrix} ds^w \\ ds^m \end{pmatrix} \text{ and } \mathbf{d}\boldsymbol{\phi} = \begin{pmatrix} I \\ E \end{pmatrix}$$

where

$$I = \frac{\beta(1-p)}{\phi^2}\left[u_{2m} - u_{2m,n} + \eta^{\min} + E\left(\eta^w\right)\right] > 0$$

Then

$$\det(\Omega) = (1-p)\kappa\delta u'_{2m} y \cdot B < 0$$

and

$$\frac{ds^w}{d\phi} = \frac{I \cdot B}{\det(\Omega)} > 0$$

Since (A.6) holds, we have

$$\frac{\partial U^m}{\partial \phi} = 0$$

and then

$$\begin{aligned}
\frac{ds^m}{d\phi} &= -(1-p)\frac{\kappa\delta u'_{2m} y \frac{ds^w}{d\phi} - \frac{\beta}{\phi^2}\left(u_{2m} - u_{2m,n} + \eta^{\min}\right) - \frac{\beta}{\phi^2}E\left[\eta\right]}{y\left(-u'_{1m} + \kappa\left(1+\delta\right)u'_{2m} y + (1-\delta)u'_{2m,n}\right)} \\
&= -(1-p)\frac{I - \frac{\beta}{\phi^2}\left(u_{2m} - u_{2m,n} + \eta^{\min}\right) - \frac{\beta}{\phi^2}E\left[\eta\right]}{y\left(-u'_{1m} + \kappa\left(1+\delta\right)u'_{2m} y + (1-\delta)u'_{2m,n}\right)} \\
&= 0
\end{aligned}$$

Therefore, if $\phi \geq \phi_1$, as the sex ratio rises, a representative man's savings rate stays constant and a representative woman's savings rate increases. However, by (A.4), as the sex ratio becomes sufficiently high (over another threshold), raising the savings rate further by the representative woman may lead to a welfare loss for her. Therefore, at a certain point, the representative woman stops responding to any additional increase in the sex ratio by raising her savings rate. We use ϕ_2 to denote this threshold value, and ϕ_2 satisfies the condition that $\frac{dU^w}{d\phi} = 0$, which means that

$$y\left(-u'_{1w} + 2(1-p)\kappa u'_{2w} + p u'_{2w,n}\right)\frac{ds^w}{d\phi} + \frac{\beta}{\phi^2}E\left[\eta\right] = 0 \tag{A.7}$$

Lastly, we will show that the aggregate savings rate of the young cohort rises as the sex ratio rises. Consider the case $\phi = \phi_2$, substituting the expression of $\frac{ds^w}{d\phi}$ into (A.7), we have

$$-u'_{1w} + (1-p)\kappa u'_{2m} + \frac{(1-p)E\left[\eta\right]}{\phi\left[u_{2m} - u_{2m,n} + \eta^{\min} + E\left(\eta^w\right)\right]}\kappa u'_{2m} + p u'_{2w,n} = 0$$

Since $u_{2m} - u_{2m,n} + \eta^{\min} > 0$, $u'_{2m,n} > \kappa u'_{2m}$, and $u_{2w} = u_{2m}$, then

$$-u'_{1m} + (1-p)\left[\kappa\delta^m u'_{2m} + \kappa u'_{2w}\left(\frac{u_{2m} + \eta^{\max} - u_{2m,n}}{\eta^{\max} - \eta^{\min}}\right)\right] + \left[\left(1-\delta^m\right)(1-p) + p\right]u'_{2m}(n) > 0$$

which means that $s^w < s^m$. Therefore, the aggregate savings rate of the young cohort will rise because (1) the women raise their savings rates; (2) the men do not reduce their savings rates, and (3) the men (with a higher savings rate than women) represent a larger share in the population. The economy-wide savings rate will also rise if we assume the (dis-)savings of the old people are fixed. In any case, entering the marriage market is a dominant strategy for both sexes.

Starting from a high enough sex ratio that exceeds the threshold, $\phi > \phi_2$, any additional increase in the sex ratio would not induce the women to cut down their consumption further, and would induce some men to quit the marriage market. The aggregate savings rate of the young cohort declines since a life-time bachelor's savings rate is now lower than a wife-seeker's savings rate.

2. If there exists a $\phi_3(< \phi_1)$ at which $\kappa u'_{2m} = u'_{2m,n}$

In this case, we still have $V^m > V^m_n$, i.e., no man will choose to be single and at ϕ_3. Substituting the equation $\kappa u'_{2m} = u'_{2m,n}$ into (A.3), we can easily show that

$$\frac{ds^m}{d\phi} = \frac{ds^w}{d\phi} = 0$$

For any ϕ slightly greater than ϕ_3, men and women will not change their saving decisions. Since we have shown that for $\phi \leq \phi_3$, a man has a higher savings rate than a woman. Therefore, as the sex ratio becomes more unbalanced, the aggregate savings rate of the young cohort rises unambiguously since men represent a larger share in the total population.

For any additional increase in the sex ratio beyond $\phi \geq \phi_3$, both women and men will keep their savings rates as constants. And since men save at a higher rate than women, the aggregate savings rate of the young cohort increases as the sex ratio rises. If the (dis-)savings of the old cohort are fixed, the economy-wide savings rate rises unambiguously as the sex ratio becomes more unbalanced.

If the sex ratio keeps rising, similar to the analysis in part 1, there exists a threshold value ϕ_1 at which the representative man becomes indifferent between entering the marriage market and remaining single. The rest of the discussions in the analysis of part 1 is applicable here in terms of how a rise in the sex ratio affects the savings rates.

In this proof, we used the assumption of the uniform distribution of η^i $(i = w, m)$. In fact, there exist many other distributions that can give us the same results. Two sufficient conditions are:

$$\frac{\partial \int f(M(\eta^w))d\eta^w}{\partial \phi} \geq 0$$

and

$$\frac{\partial \left[\frac{1}{\phi} \int \frac{f(\eta^w)}{f(M(\eta^w))} dF(\eta^w) + M(\eta^{\min}) f(\eta^{\min})\right]}{\partial \phi} \leq 0$$

The first sufficient condition is equivalent to

$$\int_{\eta^{\min}}^{\eta^{\max}} \frac{f'(M(\eta^w))}{f(M(\eta^w))} \frac{1 - F(\eta^w)}{\phi^2} d\eta^w \geq 0$$

and the second one is equivalent to

$$\frac{1}{\phi^2}\int\left[\frac{f(\eta^w)}{f(M(\eta^w))}-\frac{f(\eta^{\min})}{f(M(\eta^{\min}))}\right]dF(\eta^w)+\frac{1}{\phi}\int_{\eta^{\min}}^{\eta^{\max}}\frac{f(\eta^w)f'(M(\eta^w))}{f^3(M(\eta^w))}\frac{1-F(\eta^w)}{\phi^2}d\eta^w\geq 0$$

Distributions that are similar to normal distributions (i.e., symmetric around the mean and $f(\eta^{\min})$ is small) may satisfy the two conditions. ■

B Proof of Proposition 2 (not for publication)

Proof. We rewrite the the economy-wide savings rate as following

$$s_t^P=(1-\alpha)\left(\frac{\phi}{1+\phi}s_t^m+\frac{1}{1+\phi}s_t^w-s_{t-1}^y\right)+\frac{\alpha}{R}$$

As we have shown in Proposition 1, even if s_t^m and s_t^w may be weakly increasing in ϕ when $\phi<\phi_2$, $\frac{\phi}{1+\phi}s_t^m+\frac{1}{1+\phi}s_t^w$ strictly increases in ϕ since men will save at a higher rate than women. s^P then is an increasing function of ϕ. By the expression of the current account to GDP ratio, this is also the condition that the current account is an increasing function of the sex ratio. Therefore, the economy-wide savings rate and the current account rise as the sex ratio becomes more unbalanced. ■

C Proof of Proposition 3 (not for publication)

Proof. Since capital can flow freely between countries, the interest rates R are equal in both countries. By (**??**) and (**??**), the wage rates are also equal in the two countries.

Given the same wage rates, the households in the two countries have the same first period endowment. By Proposition 1, country 2 will have a higher savings rate than country 1. On the other hand, in equilibrium, given a constant R, the investments in both countries are the same, and the world capital market always clears. Therefore, country 2 runs a current account surplus and country 1 runs a current account deficit. ■

D Proof of Proposition 4 (not for publication)

Proof. (i) If the sex ratio is balanced, then similar to the previous analysis, it is easy to verify that $s^w=s^m$, and that entering the marriage market is the dominant strategy for both women and men.

For a moderately unbalanced sex ratio, $\phi \geq 1$, all women and men still enter the marriage market with probability one. We can rewrite the first order conditions for women and men, respectively,

$$-u'_{1w} + (1-p)\left[\begin{array}{c} \int u'_{2w}\left(\kappa^w + \frac{s^w+s^m}{s^m}\kappa_2^w\right)dF(\eta^w) \\ +\int u'_{2m}\left(\kappa^m - \frac{(s^w+s^m)s^m}{(s^w)^2}\kappa_2^m\right)dF(\eta^w) \end{array}\right] + pu'_{2w,n} = 0 \tag{D.1}$$

$$-u'_{1m} + (1-p)\left[\begin{array}{c} \int_{M(\eta^{\min})} u'_{2m}\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)dF(\eta^m) \\ +\int_{M(\eta^{\min})} u'_{2w}\left(\kappa^w - \frac{(s^w+s^m)s^w}{(s^m)^2}\kappa_2^w\right)dF(\eta^m) \end{array}\right] + (1-\delta^m(1-p))u'_{2m,n} = 0 \tag{D.2}$$

We totally differentiate the system and denote the outcome by

$$\Omega \cdot \mathbf{ds} = \mathbf{dz}$$

where

$$\Omega = \begin{pmatrix} A' & Q' \\ D' & B' \end{pmatrix},\ d\mathbf{s} = \begin{pmatrix} ds^w \\ ds^m \end{pmatrix} \text{ and } d\boldsymbol{\phi} = \begin{pmatrix} V' \\ E' \end{pmatrix}$$

where

$$A' = Ry(1-p)\left[\begin{array}{c} R^{-1}u''_{1w} + \int u''_{2w}\left(\kappa^w + \frac{s^w+s^m}{s^m}\kappa_2^w\right)^2 dF(\eta^w) + \int u'_{2w}\left(\frac{s^m+s^w}{s^m}\frac{\kappa_{22}^w}{s^m}\right)dF(\eta^w) \\ +\int u''_{2m}\left(\kappa^m - \frac{(s^w+s^m)s^m}{(s^w)^2}\kappa_2^m\right)^2 dF(\eta^w) + \int u'_{2m}\frac{(s^w+s^m)(s^m)^2}{(s^w)^4}\kappa_{22}^m dF(\eta^w) \end{array}\right] + pRyu''_{2w,n}$$

$$B' = R(1-p)y\left[\begin{array}{c} R^{-1}u''_{1m} + \int_{M(\eta^{\min})} u''_{2m}\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)^2 dF(\eta^m) + \int_{M(\eta^{\min})} u'_{2m}\left(\frac{s^w+s^m}{s^w}\frac{\kappa_{22}^m}{s^w}\right)dF(\eta^m) \\ +\int u''_{2w}\left(\kappa^w - \frac{(s^w+s^m)s^w}{(s^m)^2}\kappa_2^w\right)^2 dF(\eta^w) + \int u'_{2w}\frac{(s^w+s^m)(s^w)^2}{(s^m)^4}\kappa_{22}^w dF(\eta^w) \end{array}\right]$$
$$+\left[p + (1-\delta^m)(1-p)\right]u''_{2m}(n)Ryu''_{2m,n}$$

$$D' = R(1-p)y\left[\begin{array}{c} \int_{M(\eta^{\min})} u''_{2m}\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)\left(\kappa^m - \frac{(s^w+s^m)s^m}{(s^w)^2}\kappa_2^m\right)dF(\eta^m) \\ -\int_{M(\eta^{\min})} u'_{2m}\left(\frac{s^w+s^m}{s^w}\frac{s^m}{(s^w)^2}\kappa_{22}^m\right)dF(\eta^m) + \int u''_{2w}\left(\kappa^w - \frac{(s^w+s^m)s^w}{(s^m)^2}\kappa_2^w\right)^2 dF(\eta^w) \\ +\int u'_{2w}\left(-\frac{2s^w}{(s^m)^2}\kappa_2^w + \frac{(s^w+s^m)s^w}{(s^m)^3}\kappa_{22}^w\right)dF(\eta^w) \end{array}\right]$$

$$Q' = R(1-p)y\left[\begin{array}{c} \int u''_{2w}\left(\kappa^w + \frac{s^w+s^m}{s^m}\kappa_2^w\right)\left(\kappa^w - \frac{(s^w+s^m)s^w}{(s^m)^2}\kappa_2^w\right)dF(\eta^w) \\ +\int u'_{2w}\left(\frac{2(s^w)^2}{(s^m)^3}\kappa_2^w + \frac{(s^w+s^m)(s^w)^2}{(s^m)^4}\kappa_{22}^w\right)dF(\eta^w) + \int u'_{2m}\left(\frac{s^w+s^m}{s^w}\frac{s^m}{(s^w)^2}\kappa_{22}^m\right)dF(\eta^w) \\ +\int u''_{2m}\left(\kappa^m - \frac{(s^w+s^m)s^m}{(s^w)^2}\kappa_2^m\right)\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)dF(\eta^w) \end{array}\right]$$

$$
\begin{aligned}
E' &= -(1-p)\int_{M(\eta^{\min})} u'_{2m}\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2\left(\eta^{\max}-\eta^{\min}\right)} + \frac{s^w+s^m}{s^w}\kappa_{12}^m\right) dF(\eta^m) \\
&\quad -(1-p)R(s^m+s^w)y\int_{M(\eta^{\min})} u''_{2m}\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2\left(\eta^{\max}-\eta^{\min}\right)}\right) dF(\eta^m) \\
&\quad -(1-p)R(s^m+s^w)y\int_{M(\eta^{\min})} u''_{2w}\left(\kappa^w - \frac{\left(s^w+s^m\right)s^w}{\left(s^m\right)^2}\kappa_2^w\right)\left(-\frac{\kappa_1^w}{\phi^2} - \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right) dF(\eta^m) \\
&\quad -(1-p)\int_{M(\eta^{\min})} u'_{2w}\left(-\frac{\kappa_1^w}{\phi^2} - \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}} - \frac{\left(s^w+s^m\right)s^w}{\left(s^m\right)^2}\frac{\kappa_{12}^w}{\phi^2}\right) dF(\eta^m)
\end{aligned}
$$

$$
\begin{aligned}
V' &= -(1-p)\int u'_{2w}\left(-\frac{\kappa_1^w}{\phi^2} - \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}} - \frac{s^w+s^m}{s^m}\frac{\kappa_{12}^w}{\phi^2}\right) dF(\eta^w) \\
&\quad -(1-p)R(s^m+s^w)y\int u''_{2w}\left(\kappa^w + \frac{s^w+s^m}{s^m}\kappa_2^w\right)\left(-\frac{\kappa_1^w}{\phi^2} - \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right) dF(\eta^w) \\
&\quad -(1-p)\int u'_{2m}\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2\left(\eta^{\max}-\eta^{\min}\right)} - \frac{\left(s^w+s^m\right)s^m}{\left(s^w\right)^2}\kappa_{12}^m\right) dF(\eta^w) \\
&\quad -(1-p)R(s^m+s^w)y\int u''_{2m}\left(\kappa^m - \frac{\left(s^w+s^m\right)s^m}{\left(s^w\right)^2}\kappa_2^m\right)\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2\left(\eta^{\max}-\eta^{\min}\right)}\right) dF(\eta^w)
\end{aligned}
$$

Under Assumptions E1 to E3, we can show that $\det(\Omega) > 0$ and

$$
A' < 0,\ B' < 0,\ D' < 0,\ Q' < 0
$$

At $\phi = 1$, both men and women have the symmetric problem, $\kappa_{12}^m = \kappa_{12}^w$, then

$$
\begin{aligned}
E' &= -2R(1-p)(s^m+s^w)y\int_{M(\eta^{\min})} u''_{2m}\left(\frac{s^w+s^m}{s^w}\kappa_2^m\right)\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2\left(\eta^{\max}-\eta^{\min}\right)}\right) dF(\eta^m) < 0 \\
V' &= 2(1-p)\int u'_{2w}\frac{s^w+s^m}{s^m}\frac{\kappa_{12}^w}{\phi^2} dF(\eta^w) + 2R(1-p)(s^m+s^w)y\int u''_{2w}\frac{s^w+s^m}{s^m}\kappa_2^w\left(\frac{\kappa_1^w}{\phi^2} + \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right) dF(\eta^w)
\end{aligned}
$$

The sign of V' is ambiguous:

(1) If $\kappa_{12}^i > 0$, then $V' > 0$.

At $\phi = 1$

$$
\frac{ds^m}{d\phi} = \frac{A'E' - D'V'}{\det(\Omega)} > 0
$$

and

$$
\frac{ds^w}{d\phi} = \frac{B'V' - D'E'}{\det(\Omega)} < 0
$$

The aggregate savings rate of the young cohort is s^y,

$$\frac{ds^y}{d\phi} = \frac{s^m - s^w}{(1+\phi)^2} + \frac{\phi - 1}{1+\phi}\frac{ds^m}{d\phi} + \frac{1}{1+\phi}\left(\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi}\right)$$

We have shown in this case that men increase their savings rate as the sex ratio rises while women reduce their savings. Then $s^m > s^w$. Plugging the expressions of A', B', D', E' and V', we can also show that $\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} > 0$. Therefore, $\frac{ds^y}{d\phi} > 0$. If the (dis-)savings of the old cohort are fixed, the economy-wide savings rate rises as the sex ratio becomes more unbalanced.

(2) If $\kappa_{12}^i < 0$, the sign of V' is ambiguous. However, if we assume that κ_{12}^i is small enough in its absolute value, then we still have $\frac{ds^m}{d\phi} > 0$ by substituting the expressions of A' and D'. The sign of $\frac{ds^w}{d\phi}$ is ambiguous.

If $\frac{ds^w}{d\phi} > 0$, then the aggregate savings rate of the young cohort and the economy-wide savings rate rise as the sex ratio becomes more unbalanced. Even if $\frac{ds^w}{d\phi} \leq 0$, we still can show that

$$\frac{ds^m}{d\phi} + \frac{ds^w}{d\phi} = \frac{(A' - Q')\,E' + (B' - D')\,V'}{\det(\Omega)} > 0$$

By the same logic in (1), the aggregate savings rate of the young cohort and the economy-wide savings rate rise as the sex ratio becomes more unbalanced.

Since all functions in the model are continuously differentiable, then for moderately unbalanced sex ratios (i.e., within the neighbourhood of $\phi = 1$), we have similar results: as the sex ratio rises, a representative man raises his savings rate, but the response by a representative woman is ambiguous. Nonetheless, the economy-wide savings rate rises unambiguously.

(ii) We can rewrite E' and V' as following:

$$\begin{aligned}
E' \;&\simeq\; -(\theta-1)(1-p)\int_{M(\eta^{\min})}\left[u'_{2m}\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2(\eta^{\max}-\eta^{\min})}\right) - u'_{2w}\left(\frac{\kappa_1^w}{\phi^2} + \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right)\right] dF(\eta^m) \\
&\quad + \frac{\theta(s^w + s^m)}{s^w}(1-p)\int_{M(\eta^{\min})}\left[\begin{array}{c} u'_{2w}\kappa_2^w\left(\frac{s^w}{s^m}\right)^2\left(\frac{\kappa_1^w}{\phi^2} + \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right) \\ +u'_{2m}\kappa_2^m\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2(\eta^{\max}-\eta^{\min})}\right)\end{array}\right] dF(\eta^m) \\
V' \;&\simeq\; (\theta-1)(1-p)\int_{M(\eta^{\min})}\left[u'_{2m}\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2(\eta^{\max}-\eta^{\min})}\right) - u'_{2w}\left(\frac{\kappa_1^w}{\phi^2} + \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right)\right] dF(\eta^m) \\
&\quad - \frac{\theta(s^w + s^m)}{s^w}(1-p)\int_{M(\eta^{\min})}\left[\begin{array}{c} u'_{2w}\kappa_2^w\left(\frac{s^w}{s^m}\right)^2\left(\frac{\kappa_1^w}{\phi^2} + \frac{\kappa_4^w}{\eta^{\max}-\eta^{\min}}\right) \\ +u'_{2m}\kappa_2^m\left(\kappa_1^m + \frac{\kappa_4^m}{\phi^2(\eta^{\max}-\eta^{\min})}\right)\end{array}\right] dF(\eta^m)
\end{aligned}$$

If κ_2^i is large enough, $E' < 0$ and $V' > 0$.

Then it is easy to show that

$$\frac{ds^m}{d\phi} > 0,\ \frac{ds^w}{d\phi} < 0, \text{ and } \frac{ds^y}{d\phi} > 0$$

As the sex ratio rises, a representative man's savings rate rises while a representative woman's savings rate declines. The economy-wide savings rate rises.

The impacts of a rise in the sex ratio on women and men's welfare are, respectively:

$$\begin{aligned}
\frac{\partial U^w}{\partial \phi} &= y\left(-u_{1w}' + (1-p)\int u_{2w}'\left(\kappa^w + \frac{s^w+s^m}{s^m}\kappa_2^w\right)dF(\eta^w) + pu_{2w,n}'\right)\frac{ds^w}{d\phi} \\
&\quad +(1-p)\left[y\int u_{2w}'\left(\kappa^w - \frac{(s^w+s^m)\,s^w}{(s^m)^2}\kappa_2^w\right)dF(\eta^w)\frac{ds^m}{d\phi} + \frac{\beta}{\phi^2}E\left[\eta\right]\right] > 0 \\
\frac{\partial U^m}{\partial \phi} &= y\left(\begin{array}{c} -u_{1m}' + (1-p)\int_{M(\eta^{\min})} u_{2m}'\left(\kappa^m + \frac{s^w+s^m}{s^w}\kappa_2^m\right)dF(\eta^m) \\ +\left((1-\delta^m)(1-p)+p\right)u_{2m,n}' \end{array}\right)\frac{ds^m}{d\phi} \\
&\quad +(1-p)\left[\begin{array}{c} y\int_{M(\eta^{\min})} u_{2m}'\left(\kappa^m - \frac{(s^w+s^m)s^m}{(s^w)^2}\kappa_2^m\right)dF(\eta^m)\frac{ds^w}{d\phi} \\ +\beta\frac{\partial\left(\int_{M(\eta^{\min})} M^{-1}(\eta^m)dF(\eta^m)\right)}{\partial\phi} \end{array}\right] \\
&< (1-p)\left[y\int_{M(\eta^{\min})} u_{2m}'\left(\kappa^m - \frac{(s^w+s^m)\,s^m}{(s^w)^2}\kappa_2^m\right)dF(\eta^m)\frac{ds^w}{d\phi} - \frac{\beta}{\phi^2}E\left[\eta\right]\right] < 0
\end{aligned}$$

Similar to the analysis in Proposition 1, it is possible that as the sex ratio becomes very unbalanced, men are indifferent between entering the marriage market and being single. ϕ_1 is the threshold value.

If the sex ratio rises just beyond this threshold, $\phi > \phi_1$, women increase their savings rates in order to induce all men entering the marriage market. The representative woman's savings decision must satisfy the following equation:

$$\max_{s^m} u(c_{1m}) + \beta(1-p)\left[\int u(c_{2m})dF(\eta^m) + E(\eta^w \,|\, s^m, s^w, \phi)\right] + \beta\left[p + (1-p)(1-\delta^m)\right]u(c_{2m}(n)) = (1+\beta)u\left(\frac{R}{1+R}y\right)$$

Total differentiating the entry condition and the first order condition of the representative man, we have

$$\Omega \cdot \mathbf{ds} = \mathbf{d\boldsymbol{\phi}}$$

where

$$\Omega = \left(\begin{array}{cc} \kappa(1-p)y\int u_{2m}'dF(\eta^m) & 0 \\ D' & B' \end{array}\right), \ \mathbf{ds} = \left(\begin{array}{c} ds^w \\ ds^m \end{array}\right) \text{ and } \mathbf{d\boldsymbol{\phi}} = \left(\begin{array}{c} I' \\ E' \end{array}\right)$$

where

$$I' = \frac{(1-p)\beta}{\phi^2}E\left(\eta^w\right) > 0$$

Then

$$\det(\Omega) = \kappa(1-p)y\int u_{2m}'dF(\eta^m) \cdot B' < 0$$

and

$$\frac{ds^w}{d\phi} = \frac{I' \cdot B'}{\det(\Omega)} > 0$$

Since (A.6) holds, we have

$$\frac{\partial U^m}{\partial \phi} = 0$$

and then

$$\begin{aligned}
\frac{ds^m}{d\phi} &= -(1-p)\frac{\kappa y \int u'_{2m} dF(\eta^m) \frac{ds^w}{d\phi} - \frac{\beta}{\phi^2} E[\eta]}{y\left(-u'_{1m} + \kappa y \int u'_{2m} dF(\eta^m) + (1-\delta) u'_{2m,n}\right)} \\
&= -(1-p)\frac{I - \frac{\beta}{\phi^2} E[\eta]}{y\left(-u'_{1m} + \kappa y \int u'_{2m} dF(\eta^m) + (1-\delta) u'_{2m,n}\right)} \\
&= 0
\end{aligned}$$

We obtain the same result as in Proposition 1. ■