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Public Pensions: To What Extent Do They Account for Swedish Wealth Inequality?[1]

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Sweden's distribution of disposable income is very even, with a Gini coefficient of just 0.31. Yet its wealth distribution is extremely unequal, with a Gini coefficient of 0.79. Moreover, Swedish wealth inequality is, to a large extent, driven by the large fraction of households with zero or negative wealth. In this paper, we investigate to what extent the redistributive public pension scheme is responsible for these features of the data. To address this problem, we study the properties of two overlapping generations economies with uninsurable idiosyncratic risk. The first has a pension system modeled on the actual system; the second has no public pension scheme at all. Our findings support the view that the public pension scheme is, to a large extent, responsible for the features of the data that we focus on. *Journal of Economic Literature* Classification Numbers: E13, D31, H55.
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1. INTRODUCTION

The distribution of wealth tends to be more unequal than the distribution of income. This is especially true of Sweden—indeed spectacularly so. Sweden's distribution of disposable income is second only to Finland in terms of equality among the 15 OECD countries studied by Atkinson (1995). Yet its wealth distribution is more unequal than that of the United States, whose income distribution is the most unequal among the countries in Atkinson's study. Using data from 1992 (see Domeij and Klein (1998)), we find that Sweden's distribution of disposable income by household exhibits a Gini coefficient of just 0.31, but that the distribution of wealth has a Gini coefficient of 0.79 (or even as high as 0.86 when the very rich are oversampled).

For this discrepancy to arise, it has to be that low-income earners save less as a proportion of income than high-income earners, and, of course, they do (see Huggett and Ventura (2000) for a survey of the evidence on U.S. savings rates across households). The question is why. Several recent papers have tried to account for this fact as it applies to the United States. Quadrini (2000) studies the effects on savings of giving agents the opportunity to set up their own business. He finds that those who choose to become entrepreneurs tend to earn more and to save a higher fraction of their income than others.

Hubbard *et al.* (1995) focus on the other end of the income distribution and stress the importance of means-tested social insurance programs in accounting for the low savings rates of low-income households. Huggett and Ventura (2000) stress the demographic structure and the structure of social security payments and find that these features are quantitatively important in accounting for differences in savings rates across households. Huggett (1996) finds that it is possible to replicate many (but not all) of the features of the U.S. wealth distribution by using a life-cycle model featuring uninsurable earnings and longevity risk as well as a simple social security scheme.

Cubbedu and Ríos-Rull (1997) consider an OLG model with shocks to household formation. They focus on the impact of changes in household structure on aggregate savings and find that it is very small. However, they also suggest that the explicit modeling of household arrangements can go a long way toward accounting for wealth inequality. Using the same data as in Domeij and Klein (1998), we find that the wealth/income ratio of married households in 1992 was 26% greater than that of single men and 38% greater than that of single women. A possible reason for this is that married households face more downside risk than unmarried households. The dissolution of a household typically leads to a fall in the ratio of income to the number of "mouths to feed" (and hence a rise in marginal utility); this is especially true for women.

This paper investigates to what extent Sweden's redistributive public pension scheme can account for the big difference between the degree of inequality in its income and its wealth distribution. Our reason for focusing on the public pension scheme is the following. Wealth inequality in Sweden is, to a large extent, driven by the large fraction of households with negative or zero net wealth. This fraction is about 24%.[2] Given that, it makes sense to focus first on mechanisms that reduce incentives for low-income earners to save.

There are good reasons for thinking that a public pension scheme is such a mechanism. In particular, a common benefit payable to each senior citizen (independent of lifetime earnings) will reduce the savings of low-income earners proportionately more than for high-income earners and thus increases the inequality of wealth (provided that claims on future pensions are not included in measured wealth). An upper limit to benefits from an earnings-based pension scheme has the same effect. In Sweden, both these elements are present—indeed very much so. For example, the minimum (cash) benefit level is about a third of GDP per adult and the upper limit is such that earnings above the median among the full-time employed do not generate further entitlements to earnings-based pension benefits.[3]

To assess the impact on the wealth distribution of the public pension scheme, we build and calibrate a life-cycle model for a small open economy.[4] Agents face uninsurable idiosyncratic shocks to earnings and marital status and also an age-dependent probability of surviving into the next period. Earnings depend on age, sex, marital status, and a persistent but transitory shock. We estimate the parameters of the process using panel data, as in Flodén and Lindé (2001). Individuals supply labor inelastically and household saving is chosen optimally.[5] There are no aggregate dynamics. First, we solve the model for the invariant wealth distribution in the presence of a Swedish-style public pension scheme. Then, to assess the impact of public pensions, we re-solve the model without them.

Our main results are the following. In the first place, we are able to replicate the degree of disposable income inequality with a reasonable degree of accuracy. In the baseline economy with a pension scheme present, the Gini coefficient for disposable income is 0.34. Second, the pension system goes

[2]For an account of the data, see Appendix A.

[3]According to Huggett and Ventura (2000), social security in the United States is characterized by a common benefit level, including various health-care benefits, of about an eighth of GDP per capita and an upper limit such that earnings above 2.47 times average earnings do not generate further entitlements to earnings-based pension benefits.

[4]Huggett (1996) and Quadrini and Ríos-Rull (1997) note that OLG economies where agents never experience zero earnings can generate a large fraction of households with zero or negative wealth.

[5]The settlement of disagreements within households is discussed in Section 3.1.

a long way in accounting for wealth inequality. With the pension scheme, the model Gini coefficient for net wealth is 0.66. Thus the model is, to a large extent, able to replicate the gap between income and wealth inequality. In addition to this, we are able to replicate fairly well the fraction of households with zero or negative net wealth. In the presence of the pension scheme, the model generates an economy where 22% of households have non-positive net wealth. By contrast, the model without pensions exhibits a Gini coefficient for disposable income of 0.45 and a Gini coefficient for net wealth of 0.61.

The paper is organized as follows. Section 2 describes our model economies. Section 3 describes the calibration. Section 4 presents the results. Section 5 concludes. In Appendix A, we discuss our data source, especially the measurement of wealth inequality, and in Appendix B, we discuss the numerical solution method that we use.

2. THE MODEL

We study a small open economy with 80 overlapping generations. Individuals enter the economy at the age of 20 and work until the age of 64. They face idiosyncratic shocks to household formation (marriages and separations),[6] longevity, and earnings. There is no aggregate risk. Markets are incomplete in that earnings and marital risk are uninsurable. We consider two economies: one with a public pension scheme modeled on that of Sweden, and another with no public pension scheme at all. Especially in the latter economy, there is a strong life-cycle savings motive in addition to the precautionary savings motive.

We consider only steady states, where the distribution of individuals with respect to age, sex, marital status, earnings, and assets is invariant over time.

The rather detailed modeling of the demographic features is motivated by the following considerations. First, the estimation of an income process for households rather than individuals runs into some conceptual difficulties. In particular, it is not clear how observations featuring the formation and separation of households in the data should be satisfactorily dealt with. Second, it is not clear how to measure mortality rates for households as opposed to individuals. Third, the Swedish pension system is geared toward individuals, not households. It is not obvious how one would translate its provisions to apply to an economy consisting of undifferentiated households. Moreover, household formation and dissolution is an important determinant of the age

[6]By *marriage* we mean two people of the opposite sex moving in together, whether or not they literally marry.

profile of inequality: young people marry, the middle-aged divorce, and the old become widowed; this has significant consequences for the evolution of the within-cohort distribution of income and wealth. More generally, a rich description of demographic features enables the model to shed light upon a larger set of facts and creates further dimensions in which the model can be evaluated.

In what follows, we will be developing the description of the population. We first describe the distribution of demographic characteristics. We then characterize the joint distribution of demographics and productivity. The same extension procedure is then applied, step by step, to the pension claims and partner characteristics, finally enabling us to describe the distribution across all the relevant characteristics.

2.1. Demographics

The demographic dynamics are similar to those of Cubbedu and Ríos-Rull (1997). The economy is inhabited by individuals who differ by sex, age, and marital status. They face mortality risk which depends on age, sex, and marital status. The survival probability of an individual indexed by age $i \in \mathcal{I} = \{20, 21, \ldots, 99\}$, sex $s \in \mathcal{S} = \{\text{male, female}\}$, and marital status $g \in \mathcal{G} = \{\text{unmarried, divorced, widowed, married}\}$ is $\gamma_{i,s,g}$. We sometimes write, abusing the notation somewhat, $g = \text{single}$ when we mean $g \in \{\text{unmarried, divorced, widowed}\}$.

At age 99, the probability of death is 1. We assume that individuals can only marry a partner of the same age and opposite sex. By marriage, we mean that partners share assets and current income and enjoy the same level of consumption. They also have the same current period utility function. Married individuals, by definition, have partners. Variables pertaining to a partner are denoted by an asterisk. In particular, if $s = \text{male}$, then $s^* = \text{female}$ and vice versa.

We assume that the process for marital status is exogenous where the probability (conditional on survival) of an individual of age i, sex s, and marital status g of transiting to marital status g' is $\pi_{i,s}(g'|g)$.[7] Define transition

[7]The probability of meeting a new spouse, as well as the characteristics of this new spouse, is independent of own income or wealth. Thus there is no assortative matching with respect to income or wealth. The reason for this assumption is that there are no data available on this kind of assortative matching. On the other hand, there *is* strong evidence of assortative matching *with respect to education* in Sweden (SCB, 1993a). For example, among the people who do marry in a given year, a college graduate is twice as likely as a non-graduate is to marry a college graduate. However, in order to incorporate assortative matching with respect to education, we would need to extend the state space, which is already quite large.

probabilities $\pi_{i,s}(g'|g)$ so that

$$\sum_{g' \in \mathcal{G}} \pi_{i,s}(g'|g) = \gamma_{i,s,g}.$$

We also define the transition measure via

$$\Pi_{i,s}(G'|g) = \sum_{g' \in G'} \pi_{i,s}(g'|g),$$

where $G' \subset \mathcal{G}$.

Age and sex evolve over time in the obvious way: next-period agents who survive are one year older and have the same sex as today.

A stable population is characterized by constant ratios over time across the different demographic groups. Assume the population grows at rate χ. This implies that the measure of different types $\mu_{i,s,g}$ satisfies the difference equation:

$$\mu_{i+1,s,g'} = \sum_{g \in \mathcal{G}} \frac{\pi_{i,s}(g'|g)}{1+\chi} \mu_{i,s,g}. \tag{1}$$

A consistency requirement is that, for each i, we have

$$\mu_{i,\text{male, married}} = \mu_{i,\text{female, married}}.$$

We normalize so that

$$\sum_{i,s,g} \mu_{i,s,g} = 1.$$

2.2. Preferences

Following Cubbedu and Ríos-Rull (1997), a household member enjoys consumption according to a period utility function of the form $u(c/\eta)$, where η is the number of consumer equivalents in the household. Individuals rank stochastic consumption sequences according to the intertemporal utility function

$$E\left[\sum_{i=20}^{T} \beta^{i-20} u \frac{c_i}{\eta_i}\right], \tag{2}$$

where β is the subjective discount factor, and

$$u(x) = \frac{x^{1-\sigma} - 1}{1-\sigma}, \tag{3}$$

where σ is the reciprocal of the intertemporal elasticity of substitution and T is the (stochastic) final year of life.[8]

[8]Given this specification, marginal utility is $(1/\eta_t)(c_t/\eta_t)^{-\sigma}$. This means that behavior is the net result of two competing forces. On the one hand, as expressed by the first factor,

2.3. Process for Productivity

Each individual under the age of 65 supplies one unit of labor inelastically.[9] Labor earnings are given by $e(i, s, g, z)$. Here z is a persistent stochastic process which is independent of i, s, and g. In particular, $z \in \mathscr{Z} = \{z_1, \ldots, z_9\}$ and the transition probability measure for z is denoted as

$$P[z' \in Z | z = x] = Q(Z, x),$$

where $Z \subset \mathscr{Z}$. The associated stationary probability measure is denoted by θ.

The function $e(i, s, g, z)$ is designed so that $e(i, s, g, z) = 0$ for $i > 64$ and that average idiosyncratic earnings among those below 65 is normalized to 1.

We now introduce the measure m_1 on $\mathscr{I} \times \mathscr{S} \times \mathscr{G} \times \mathscr{Z}$, defined via

$$m_1(A \times Z) = \theta(Z) \sum_{(i, s, g) \in A} \mu_{i, s, g},$$

where $A \subset \mathscr{I} \times \mathscr{S} \times \mathscr{G}$ and $Z \subset \mathscr{Z}$. We can now be precise about what we mean by normalizing earnings to 1. Mathematically, this can be written as

$$\int_{\mathscr{I} \times \mathscr{S} \times \mathscr{G} \times \mathscr{Z}} e(i, s, g, z)\, dm_1 = \int_{\mathscr{I} \times \mathscr{S} \times \mathscr{G} \times \mathscr{Z}} I_{\{i<65\}}\, dm_1.$$

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utility is spread out more thinly in periods with a high value of η_t, making it more attractive to consume in periods with a low value of η_t. On the other hand, the second factor means that the agent wants to smooth consumption per consumption equivalent. An intertemporal elasticity equal to 1 means that the two effects cancel out; hence consumption decisions are independent of η_t. An intertemporal elasticity of substitution less than 1 means that the second effect dominates so that a household tends to consume more in periods with high values of η_t and less in periods with a low value η_t. However, even when the intertemporal elasticity of substitution is less than 1, consumption per consumption equivalent tends to be lower in periods with a high value of η_t.

[9]Although we do not focus on labor supply as such, this assumption may affect savings behavior, which we do focus on. However, the effect is ambiguous in theory and not likely to be large quantitatively. As explained in Flodén (1999), there are two competing considerations. First, shocks to wages, if the substitution effect dominates, tend to increase precautionary savings. This is because a negative shock to wages will be reinforced by a reduction in hours worked. (But for borrowing-constrained households, this effect may be mitigated or even overturned; see Domeij and Flodén (2001).) Second, the impact of other shocks tends to be affected in the opposite way by endogenous labor supply; a negative shock to wealth can be partially offset by an increase in hours worked, thus reducing the need for precautionary savings. Meanwhile, Flodén (2001) shows that the quantitative impact of endogenizing labor supply on wealth inequality is small.

2.4. The Public Pension Scheme

Under the public pension scheme, an individual of age 65 or above is entitled to a pension benefit given by $p(g_i, h_i, \tau^n)$, where g_i is the individual's marital status at age i, h_i is the individual's pension claim at age i, and τ^n is the labor income tax, payable on part of the pension benefit. Pension claims evolve according to $h_i = \widetilde{H}(h_{i-1}, e_{i-1})$, where e_i is the idiosyncratic earnings component at age i. The set of all possible pension claims is denoted by $\mathscr{H} \subset R_+$.

2.5. Distributions

We now introduce the measure $m_2^{i,s}$ on $\mathscr{G} \times \mathscr{Z} \times \mathscr{H}$, describing the joint distribution of marital status, productivity, and pension claims among individuals of age i and sex s. We define it via

$$m_2^{i+1,s}(G' \times Z' \times H') = \int_{\mathscr{G}\times\mathscr{Z}\times\mathscr{H}} \frac{\Pi_{i,s}(G'|g)}{1+\chi} Q(Z', z) I_{\{\widetilde{H}(h, e(i,s,g,z))\in H'\}} dm_2^{i,s},$$

where $H' \subset \mathscr{H}$ is a Borel-measurable set and $I_{\{x\in X\}}$ is a function that takes the value 1 if $x \in X$ and 0 otherwise. We need a further equation in order to pin down m_2 uniquely. We choose to normalize the population so that

$$\sum_{i,s} m_2^{i,s}(\mathscr{G} \times \mathscr{Z} \times \mathscr{H}) = 1.$$

This defines the measure for rectangles; by Hahn's theorem, there is a unique extension to all the measurable sets. This argument will be applied repeatedly below.

As a further preliminary, we need to define the (marginal) distribution of (z, h) among individuals with a given (i, s, g). We define

$$\delta^{i,s,g}(Z \times H) = \frac{m_2^{i,s}(\{g\} \times Z \times H)}{m_2^{i,s}(\{g\} \times \mathscr{Z} \times \mathscr{H})}$$

and we use this to derive the distribution of (z, h) among newlyweds of a certain age and sex. Since the event of getting married is independent of shocks to z, this is just the distribution of (z, h) among those who were single in the previous year. We have

$$\tilde{\delta}^{i+1,s}(Z' \times H') = \int_{\mathscr{Z}\times\mathscr{H}} Q(Z', z) I_{\{\widetilde{H}(h,\, e(i,\, s,\, \text{single},\, z))\in H'\}} d\delta^{i,\, s,\, \text{single}}.$$

Using these definitions, we can define the measures $m_3^{i,s}$ over all the exogenous variables (g, z, z^*, h, h^*) among individuals characterized by (i, s).

We have

$$
\begin{aligned}
m_3^{i+1,s}&\big(G' \times Z' \times (Z^*)' \times H' \times (H^*)'\big) \\
&= \int_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}} \Bigg[I_{\{g=\text{married}\}} Q(Z',z) Q\big((Z^*)',z^*\big) I_{\{\tilde{H}(h^*,e(i,s^*,g,z^*))\in(H^*)'\}} \\
&\qquad + I_{\{g=\text{single}\}} Q(Z',z) \tilde{\delta}^{i+1,s^*}\big((Z^*)' \times (H^*)'\big) \\
&\qquad \times \frac{\Pi_{i,s}(G'|g)}{1+\chi} I_{\{\tilde{H}(h,e(i,s,g,z))\in H'\}} \Bigg] \, dm_3^{i,s}.
\end{aligned}
$$

Note that if an individual of age i and sex s is currently single, then the distribution of a prospective partner's z and h in the subsequent period is given by $\tilde{\delta}^{i+1,s^*}$. On the other hand, if the individual was previously married, then the distribution of the partner's z and h depends on the current values of these variables. Again, we normalize so that

$$\sum_{i,s} m_3^{i,s}(\mathcal{G} \times \mathcal{Z} \times \mathcal{Z} \times \mathcal{H} \times \mathcal{H}) = 1.$$

A consistency requirement is that

$$
\begin{aligned}
m_3^{i,\text{male}}&\big(\{\text{married}\} \times Z \times Z^* \times H \times H^*\big) \\
&= m_3^{i,\text{female}}\big(\{\text{married}\} \times Z^* \times Z \times H^* \times H\big)
\end{aligned}
$$

for all $Z, Z^* \subset \mathcal{Z}$ and all Borel sets $H, H^* \subset \mathcal{H}$.

2.6. State Vector

An individual is characterized by eight characteristics, namely, (i) age, (ii) sex, (iii) marital status, (iv) asset holdings, (v) own stochastic productivity component, (vi) own pension claims, (vii) partner's stochastic productivity component, and (viii) partner's pension claims. We write $x = (i, s, g, a, z, h, z^*, h^*)$. Of course, the final two components only matter for a married individual and may be defined arbitrarily for single individuals.

2.7. Markets

The market structure, designed to be as simple as possible for our purposes, is as follows. For single households, there is an annuities market where claims to next-period consumption are traded at prices $q_{i,s,g}$ which reflect the agent's survival probability. For married households, we assume that at most one member of a married couple can die in any given year and that there is no market for life insurance. This means that married households trade only one-period risk-free bonds.

The expected return on an annuity r is given exogenously by a world capital market. All other financial markets are closed. There is a domestic spot market for labor and a world spot market for capital. There are no borrowing constraints, except that an individual of age 99, who (together with any possible partner) dies with probability 1, is not allowed to borrow. This implies that there exists a lower bound for assets, $\underline{a}(h, h^*)$, such that if, at age 99, $a \leq \underline{a}(h, h^*)$, then consumption is 0 or negative and hence marginal utility is infinite. This rules out Ponzi schemes.

2.8. A Single Individual's Decision Problem

Let $v(x)$ denote the maximized expected utility of an individual characterized by x. In this case, $x = (i, s, \text{single}, a, z, h, z^*, h^*)$, where z^* and h^* are irrelevant since there is no partner. Note that the distinction among being currently unmarried, widowed, or divorced is irrelevant from the point of view of decision making. We have

$$v(x) = \max_{c \geq 0} \left\{ u\left(\frac{c}{\eta(x)} \right) + \beta E\big[v(x')|x\big] \right\}$$

subject to

$$c + q_{i,\,s,\,\text{single}} d = (1 - \tau^n) e(i, s, \text{single}, z) + a \quad \text{if } i < 65$$

or

$$c + q_{i,\,s,\,\text{single}} d = p(\text{single}, h, \tau^n) + a \quad \text{if } i \geq 65,$$

where the price of a claim to next period's consumption good q is given by

$$q_{i,\,s,\,\text{single}} = (1 + \tau^k) \frac{\gamma_{i,\,s,\,\text{single}}}{1 + r}.$$

The evolution of a and h is described by

$$a' = \begin{cases} d & \text{if } g' = \text{single}, \\ d + d^* & \text{if } g' = \text{married}, \end{cases}$$

$$h' = \widetilde{H}\big(h, e(i, s, \text{single}, z)\big).$$

Meanwhile, the probability distributions over g', z', and $((z^*)', d^*, (h^*)')$ are mutually independent, so we will describe these probability distributions separately. The required joint probabilities are then found by just multiplying the marginal probabilities.

The survival probabilities as well as the distribution of g' conditional on g are described in Section 2.1. The distribution of z' conditional on z is described in Section 2.3. The distribution of $((z^*)', d^*, (h^*)')$ conditional on x is not exogenously specified, but is a part of the equilibrium. We impose exogenously, however, that the conditional distribution of $((z^*)', d^*, (h^*)')$ depends only on age and sex.

2.9. A Married Household's Decision Problem

In our economy, husbands and wives share assets and income and jointly enjoy the consumption of the household. By assumption, then, there is no conflict of interest with respect to the distribution of current consumption across household members. However, the partners will typically not agree on the savings choice. This is because of the possibility of separation and death. For example, women have lower pension entitlements and expect to live longer than men, so they would want to save more. As in Cubbedu and Ríos-Rull (1997), we assume that the savings choice maximizes a weighted sum of the two partners' expected utility. We also assume that if a married individual dies, assets are retained by the surviving spouse. In case of separation, assets (or debts) are split 50–50.

Denote a married man's state by $x = (i, \text{male}, \text{married}, a, z, h, z^*, h^*)$. Then his wife's state is $x^* = (i, \text{female}, \text{married}, a, z^*, h^*, z, h)$. The savings choice solves the following maximization problem, where κ is the weight assigned to men,

$$\max_{c\geq 0}\left\{u\left(\frac{c}{\eta(x)}\right)+\beta E\big[\kappa v(x')+(1-\kappa)v\big((x')^*\big)|x\big]\right\}$$

subject to

$$\begin{aligned}c+q_{i,\cdot,\text{married}}\quad d = {} & (1-\tau^n)\big[e(i,\text{male},\text{married},z)\\ & + e(i,\text{female},\text{married},z^*)\big]+a\end{aligned}$$

if $i < 65$ or

$$c+q_{i,\cdot,\text{married}}\quad d = p(\text{married},h,\tau^n)+p(\text{married},h^*,\tau^n)+a$$

if $i \geq 65$, where

$$q_{i,\cdot,\text{married}} = \frac{1+\tau^k}{1+r},$$

where d is household savings and

- if the marriage survives,

$$a' = (a^*)' = d;$$

- if there is a divorce,

$$a' = (a^*)' = \frac{d}{2};$$

- if the husband dies,

$$(a^*)' = d;$$

- if the wife dies,

$$a' = d;$$

and where

$$h' = \widetilde{H}\big(h, e\ (i, \text{male}, \text{married}, z)\big),$$
$$(h^*)' = \widetilde{H}\big(h^*, e\ (i, \text{female}, \text{married}, z^*)\big).$$

Note that it is not possible to separate and remarry within a single year. The probability distributions over g', z', $(z^*)'$ are mutually independent and are described above. The required joint probabilities are then found by just multiplying the marginal probabilities.

2.10. Equilibrium

A steady-state equilibrium is a decision rule $\tilde{d}$ for savings and a set of stationary measures $m^{i,s} : i \in \mathcal{I}$, $s \in \mathcal{S}$, such that the following conditions hold.

1. The savings function $\tilde{d}(x)$ solves the decision problem.
2. The measures $m^{i,s}$ are consistent with the exogenous dynamics and the decision rule, i.e.,

$$\begin{aligned} & m^{i+1,s}\big(G' \times Z' \times (Z^*)' \times H' \times (H^*)' \times A'\big) \\ &\quad = \sum_{g' \in G'} \bigg\{ \int_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}\times\mathcal{A}} \frac{\Pi_{i,s}(G'|g)}{1+\chi} I_{\{\widetilde{H}(h,e(i,s,g,z))\in H'\}} Q(Z', z) \\ &\qquad \times \Big[I_{\{g=\text{married}\}} Q\big((Z^*)', z^*\big) I_{\{\widetilde{H}(h^*,e(i,s^*,g,z^*))\in (H^*)'\}} \\ &\qquad\quad \times I_{\{(\tilde{d}(x)(1-0.5*I_{\{g'=\text{divorced}\}}))\in A'\}} + I_{\{g=\text{single}\}} \int_{\mathcal{Z}\times\mathcal{H}\times\mathcal{A}} I_{\{(z^*)'\in (Z^*)'\}} \\ &\qquad\quad \times I_{\{(h^*)'\in (H^*)'\}} I_{\{(\tilde{d}(x)+I_{\{g'=\text{married}\}} d^*)\in A'\}}\, d\varphi^{i,s} \Big]\, dm^{i,s} \bigg\} \end{aligned}$$

where the probability measure $\varphi^{i,s}$, which describes the distribution of characteristics among new spouses, is consistent with the decision rule $\tilde{d}$ and the distribution $\tilde{\delta}^{i,s^*}$ of $((h^*)', (z^*)')$ among newlyweds of age i and sex s^*.

3. The government budget balances

$$\begin{aligned} & C_g + \sum_{i,s} \int_{\mathcal{G}\times\mathcal{Z}\times\mathcal{H}} I_{\{i>64\}}\, p(g, h, \tau^n)\, dm_2^{i,s} \\ &\quad = \tau^n N + \sum_{i,s} \int_{\mathcal{G}\times\mathcal{Z}\times\mathcal{Z}\times\mathcal{H}\times\mathcal{H}\times\mathcal{A}} \big[I_{\{s=\text{male}\}} + I_{\{s=\text{female}\}\cap\{g=\text{single}\}} \big] \tau^k \\ &\qquad \times \frac{\gamma_{i,s,g}}{1+r}\, a\, dm^{i,s}, \end{aligned} \tag{4}$$

where C_g is government purchases.

What we have defined above is the distribution across individuals. To derive the distribution across households, we just exclude the married men (or women, but not both).

3. CALIBRATION

3.1. Marriage, Survival, and Household Preferences

Half of the individuals enter the economy as men and half as women. A certain fraction enter as married. In each period, an individual can transit from being single to married, from being married to divorced or widowed, and from being alive to dead. The fraction of 20-year-olds who are married as well as the transition probabilities are set according to numbers reported by Statistics Sweden (see SCB (1993a)). Marriage and separation probabilities depend on age and sex. The probability of dying depends on age, sex, and marital status. To make sure that these probabilities are consistent with one another, we do the following. The transition probabilities pertaining to men as well as the survival probability for single women are all taken from the data. The remaining transition probabilities are calculated as follows, using the notation of Section 2.1.[10]

$$\begin{aligned}
\pi_{i,\,\mathrm{female}}(\text{widowed}|\text{married}) &= \pi_{i,\,\mathrm{male}}(\text{dead}|\text{married}),\\
\pi_{i,\,\mathrm{female}}(\text{dead}|\text{married}) &= \pi_{i,\,\mathrm{male}}(\text{widowed}|\text{married}),\\
\pi_{i,\,\mathrm{female}}(\text{divorced}|\text{married}) &= \pi_{i,\,\mathrm{male}}(\text{divorced}|\text{married}),\\
\pi_{i,\,\mathrm{female}}(\text{married}|\text{single}) &= \pi_{i,\,\mathrm{male}}(\text{married}|\text{single})\frac{\mu_{i,\,\mathrm{male,single}}}{\mu_{i,\mathrm{female,single}}}.
\end{aligned}$$

To keep the number of possible ages finite, we set the probability of dying at age 99 to 1. Using data from 1912 to 1992, we find that the rate of population growth χ is approximately 0.005.

Recall that the period utility function takes the form $U(c/\eta)$. We calibrate the value of η as the number of consumer equivalents in the household.[11] For example, a single individual with no children is counted as 1.15, and a couple without children is counted as 1.9. A child adds between 0.45 and 0.75 consumer equivalents, depending on its age. Each household has a number of consumer equivalents that is given by a deterministic function of the household's age, marital status, and, if applicable,

[10] Given that $\mu_{20,s,g}$ are taken from the data, we can derive all the other $\mu_{i,s,g}$ etc. recursively using Eq. (1).

[11] More precisely, we use the notion of “normer för baskonsumtion” as it is defined by Socialstyrelsen.

TABLE I
Average Consumption Equivalents by Age, Sex, and Marital Status

	Single		
Age	Male	Female	Married
20–24	1.15	1.21	2.13
25–29	1.16	1.38	2.50
30–34	1.19	1.65	2.92
35–39	1.22	1.79	3.15
40–44	1.25	1.65	2.99
45–49	1.22	1.38	2.51
50–54	1.19	1.28	2.13
55–59	1.17	1.16	1.98
60–64	1.15	1.16	1.92
65+	1.15	1.15	1.90

sex. These numbers are averages across households taken from the Household Income Survey (HINK; see Domeij and Klein (1998)). For details, see Table I.

The coefficient of relative risk aversion, σ, is set to 1.5. The subjective discount factor β is set so as to match the ratio of average earnings among 20- to 64-year-olds to average wealth. This number is 1.71 in the data.[12] This means that the model with pensions exhibits the same amount of total wealth as the one without pensions, but that the two models have different subjective discount factors; see Table II for the numbers.

For simplicity, we set $\kappa = 1/2$, reflecting an even balance of power within households. We also conduct some sensitivity analysis with respect to this parameter.

3.2. Interest Rate

The return on assets is given exogenously by the world market and is calibrated at 3% on an annual basis. For sensitivity analysis, see Section 4.5.2.

3.3. Earnings Process

Each individual under the age of 65 supplies one unit of labor inelastically. Labor earnings are given by an exogenous idiosyncratic stochastic process. In the data, we interpret earnings to mean income from work plus transfer payments except the pensions of those of age 65 and over. The

[12]This may seem to be a rather low number. In a closed-economy representative agent growth model with a labor share of 0.75 and a capital/output ratio of 2.5, the ratio of wealth to earnings is 3.33. The reason our number is as low as 1.71 is that a fairly large fraction of the Swedish capital stock is owned by foundations, foreign residents, and the super-rich.

TABLE II
Patience Rate, Labor Tax Rate, and Foreign Debt/GDP Ratio

	Models with	
	Pensions	No pensions
β	0.980	0.937
τ^n	0.509	0.430
$\frac{\bar{a}}{\bar{e}(i<65)}$	1.71	1.71

earnings process is modeled as follows. The natural log of earnings at time t of individual j, e_t^j, consists of two components. The first, α_t^j, depends deterministically on age, sex, and marital status. The second, z_t^j, follows an AR(1) process (which, for computational purposes, we approximate by a nine-state Markov process). Following Flodén and Lindé (2001), we estimate the following equations using the generalized method of moments

$$e_t^{j,\,\mathrm{obs}} = \alpha_t^j + z_t^j + \xi_t^j \tag{5}$$

and

$$z_t^j = \rho z_{t-1}^j + \varepsilon_t^j, \tag{6}$$

where $e_t^{j,\,\mathrm{obs}}$ is observed log earnings, ξ_t^j is a white-noise measurement error, ε_t^j is a white-noise process, and α_t^j are the fitted values from regressing $e_t^{j,\,\mathrm{obs}}$ on age, age squared, and dummies for sex, marital status, and an interaction term that captures the fact that unmarried women earn much more than married women. We write

$$\begin{aligned}\alpha_t^j = {} & \beta_0 + \beta_1(\mathrm{AGE}_t^j - 20) + \beta_2(\mathrm{AGE}_t^j - 20)^2 + \beta_3 I_{\{\mathrm{SEX=female}\}} \\ & + \beta_4 I_{\{\mathrm{MARITAL\ STATUS=married}\}} \\ & + \beta_5 I_{\{\mathrm{SEX=female\ and\ MARITAL\ STATUS=married}\}},\end{aligned}$$

where β_0 is adjusted so that the sample average of $\exp(\alpha_t^j)$ is 1. The data are taken from HINK ($t = 1988, 1989, 1992$). Our estimate of ρ is 0.928, and our estimate of σ_ε^2 is 0.0498. When approximated by a nine-state Markov process and average earnings are normalized to 1, the process for z is characterized by

$$z \in \{-1.583, -1.232, -0.881, -0.529, -0.178, 0.173, 0.524, 0.876, 1.227\}$$

and a probability transition matrix with second greatest eigenvalue equal to 0.9249 (see Table III) and whose stationary probabilities are given by

$$(0.018, 0.055, 0.120, 0.193, 0.227, 0.193, 0.120, 0.055, 0.018).$$

This approximation is constructed so that the mean of $\exp(z_t)$ is equal to 1.

TABLE III
Probability Transition Matrix for the Productivity Shock z

0.644	0.336	0.019	0.000	0.000	0.000	0.000	0.000	0.000
0.114	0.573	0.298	0.014	0.000	0.000	0.000	0.000	0.000
0.003	0.136	0.590	0.261	0.011	0.000	0.000	0.000	0.000
0.000	0.004	0.163	0.600	0.226	0.008	0.000	0.000	0.000
0.000	0.000	0.006	0.193	0.603	0.193	0.006	0.000	0.000
0.000	0.000	0.000	0.008	0.226	0.600	0.163	0.004	0.000
0.000	0.000	0.000	0.000	0.011	0.261	0.590	0.136	0.003
0.000	0.000	0.000	0.000	0.000	0.014	0.298	0.573	0.114
0.000	0.000	0.000	0.000	0.000	0.000	0.019	0.336	0.643

This means that the earnings function is defined via $e(i, s, g, z) = \exp(\alpha(i, s, g) + z)$ where

$$\alpha(i, s, g) = -0.252 + 0.028(i - 20) - 0.0007(i - 20)^2 + 0.0056 I_{\{s=\text{female}\}} + 0.265 I_{\{g=\text{married}\}} - 0.536 I_{\{s=\text{female and } g=\text{married}\}}.$$

3.4. The Public Pension Scheme

The model's pension scheme, which is a somewhat stylized version of the Swedish one, has three components: one earnings-based benefit, one common benefit, and a housing subsidy.[13] Henceforth, all numbers are multiples of pre-tax earnings and transfers per adult below the age of 65 (in the data, this number is SEK 171000; in the model, it is normalized to 1).

Each individual has a pension claim based on lifetime earnings in the following way. Each year, 18.5% of earnings up to a cutoff level is added to an account. The cutoff level is 1.42. After age 65, the account pays interest where the rate of return is stipulated by law to be 1.6%. Retirees then receive an amount which is such as to exhaust the account exactly if the remaining lifetime is equal to its expected value at retirement. At age 65, the average expected remaining lifetime is 17 years.

On top of that, each individual receives a common benefit of 0.38 if married and 0.42 if single. However, this common benefit is reduced one for one by the earnings-based benefit when the earnings-based benefit is in the interval [0, 0.22] if married and [0, 0.25] if single. When the earnings-based

[13]There was a reform of the Swedish pension system in 1998. Its main features were (i) pension benefits were made contingent on lifetime earnings rather than just the 15 years with the highest earnings and (ii) benefits were made contingent on the rate of growth of the economy. Keeping track of the best 15 years is computationally very costly because the state space becomes enormous. We therefore assume, in our models, that pension benefits are based on lifetime earnings as in the new system. We leave for future research the issue of whether the reform is important for the issues that we study; there is no particularly strong reason to think that it would be.

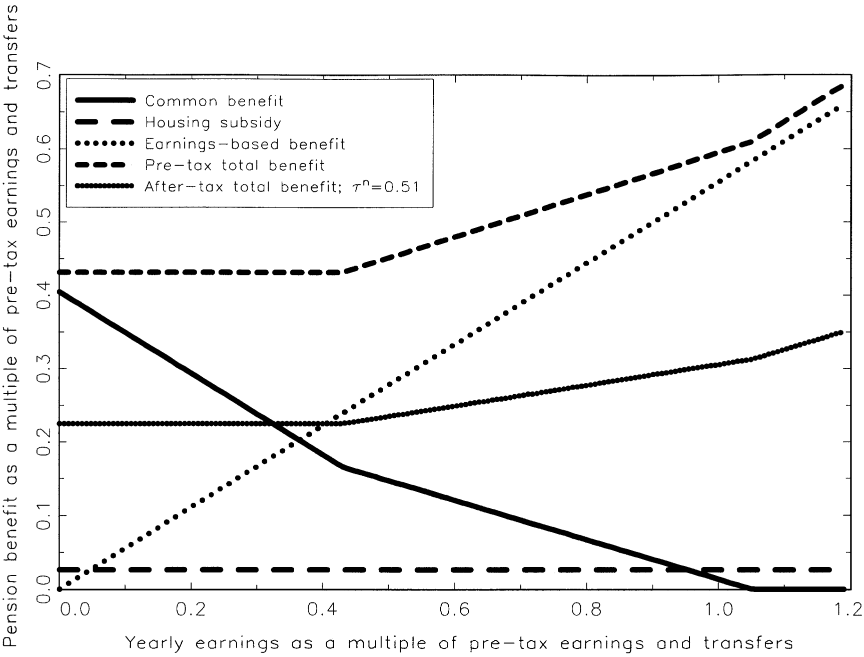

FIG. 1. Public pension scheme for a single individual.

benefit is in the interval [0.22, 0.55], and [0.25, 0.60] for singles, then the common benefit is reduced by 48% of the difference between the earnings-based benefit and 0.22 (0.25) for married (single) households.

The common benefit and the earnings-based benefit are considered as labor income and are taxed at rate τ^n. Retired households also receive a housing subsidy of 0.03 which is not taxed. This somewhat intricate system is illustrated in Fig. 1. It is clear from the figure that the pension system is very redistributive indeed. The mapping from past earnings to after-tax pension benefits is nearly flat at a rather high level. Our sources for these numbers are Swedish Law (Lag 1998:674 om inkomstgrundad ålderspension) and SCB (1993b).

3.5. Government Purchases and Taxation

Government purchases are set so as to match the 1960–1996 average ratio of general government purchases to earnings, which is 35%. The annuity tax τ^k is set to 0.01 which corresponds to a capital income tax of 30%, which is the rate legally payable on personal income from capital and capital gains in Sweden. The tax τ^n on labor income and pension benefits is set so as to balance the budget. See Table II for details on the numbers.[14]

4. RESULTS

4.1. Main Results

In what follows, we will be talking about two distinct model economies. The baseline economy is the one described in Section 2. The second one has no public pensions.

Our main results are the following. In the first place, we are able to replicate the degree of disposable income inequality with a reasonable degree of accuracy. In the baseline economy, the Gini coefficient for disposable income is 0.34 (it is 0.31 in the data). Second, the pension system goes a very long way in accounting for wealth inequality. In the baseline economy with pensions, it is 0.66 (it is 0.79 in the data). Meanwhile, in the model without pensions, the model Gini coefficient for disposable income is 0.45 and for net wealth it is 0.61. The important thing here is that the model with pensions does much better at replicating the *gap* between the Gini coefficient for disposable income and the Gini coefficient for net wealth.

In addition to this, we are able to replicate fairly well the fraction of households with zero or negative net wealth. The baseline economy features 22% of households with non-positive net wealth (this number is 24% in the data).

These results are in line with the fact noted by Quadrini and Ríos-Rull (1997) that OLG models where agents receive income throughout their lives generate a lot of wealth inequality by predicting a large fraction of households with very low wealth. With U.S. data, this is a problem, because wealth inequality there is, to a large extent, driven by the very rich. But with Swedish data, a large fraction of households with very low wealth is precisely what we observe. Note that, in the economy without pensions, households have no income apart from capital income after the age of 64. This explains why so few households have zero or negative wealth in this economy.

[14] Of course, the Swedish tax structure is slightly progressive. We have experimented with various modifications of the tax system, for example, to include a progressive tax on labor income and pension benefits. It turns out that our main results do not hinge substantially on the exact specification of the tax system.

TABLE IV
Distribution of Disposable Income

	Models with		
	Pensions	No pensions	Data
Gini coefficient	0.34	0.45	0.31
Decile shares			
1st	0.03	0.01	0.05
2nd	0.04	0.01	0.06
3rd	0.05	0.03	0.06
4th	0.07	0.05	0.07
5th	0.08	0.08	0.08
6th	0.09	0.10	0.10
7th	0.11	0.12	0.11
8th	0.13	0.15	0.13
9th	0.17	0.19	0.15
10th	0.24	0.28	0.20
Percentile share			
100th	0.04	0.04	0.04

4.2. Detailed Properties of the Income and Wealth Distributions

Looking at Table IV, we see two things. In the first place, the baseline economy captures the salient properties of the income distribution. In particular, it very nearly matches the Gini coefficient and comes very close to matching all the decile shares as well as the top percentile share. Second, the model without pensions does worse. In particular, it predicts more income inequality than there is in the data. The reason for this, of course, is that the pension system redistributes income.

A nice feature of the model with pensions is that it captures the double-peakedness of the disposable income distribution; see Fig. 2. This is, of course, intimately related to the phenomenon of marriage. The two peaks represent the modes of the distributions among singles and married couples, respectively.

Capturing the features of the disposable income distribution is not quite as trivial a task as one might think. In the first place, the demographic structure of our economy is based on the entire Swedish population, while that of our database for income and wealth is based on a sample of about 15,000 randomly selected households. Moreover, taxes, capital income, and pension benefits are endogenously determined components of disposable income. This may account for the fact that the model Gini coefficient for disposable income is slightly higher than the corresponding number in the data.

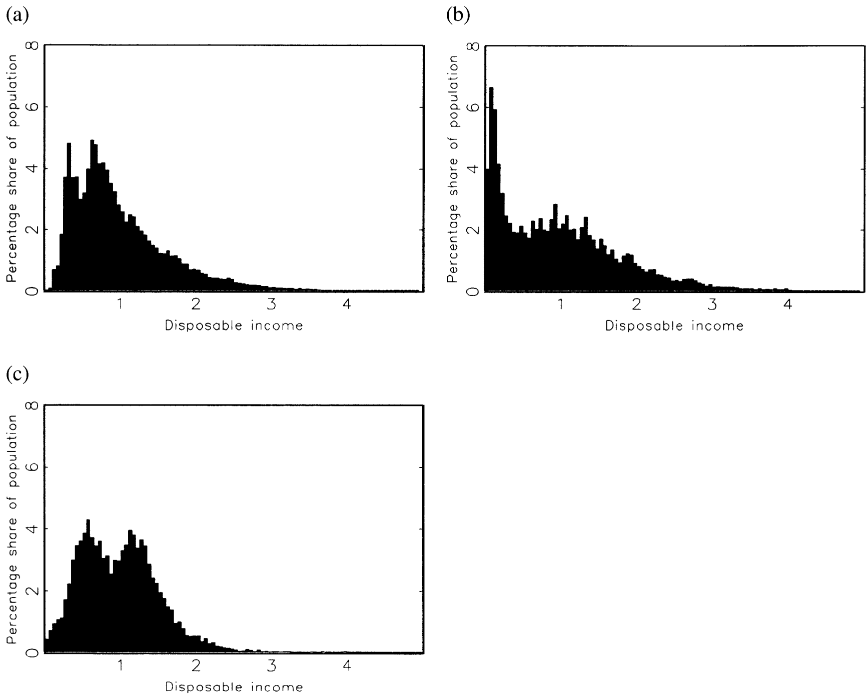

FIG. 2. Disposable income distribution. (a) Disposable income distribution, pensions. (b) Disposable income distribution, no pensions. (c) Disposable income distribution Swedish data.

Looking at Table V, we see that the results for net wealth are qualitatively similar to those pertaining to disposable income. In the first place, the baseline economy captures the salient properties of the wealth distribution. Again, the model without pensions does somewhat worse. In particular, it predicts a lower Gini coefficient and a lower share for the top decile. Notice also that the baseline economy comes very close to capturing the fraction of households that have zero or negative net wealth, whereas the model without pensions is further from the facts in this respect.

However, as Fig. 3 shows, we are not able to capture the detailed features of the lower tail of the wealth distribution. Although the model with pensions exhibits a similar fraction of households with zero or negative net wealth, the models tend to exaggerate the bunching around 0. With respect to the upper tail, the baseline model comes close to capturing the share of the top decile and even top percentile. However, it should be noted that our data exhibit no oversampling of the extremely rich. Thus the share of the top percentile is understated. (On this point, see also footnote 12.)

TABLE V
Distribution of Net Wealth

	Models with		
	Pensions	No pensions	Data
Gini coefficient	0.66	0.61	0.79
Decile shares			
1st	−0.01	−0.01	−0.06
2nd	−0.00	−0.00	−0.01
3rd	0.00	0.01	0.00
4th	0.02	0.02	0.01
5th	0.04	0.04	0.03
6th	0.07	0.07	0.06
7th	0.10	0.11	0.10
8th	0.15	0.16	0.15
9th	0.22	0.22	0.22
10th	0.41	0.38	0.50
Percentile share			
100th	0.07	0.06	0.13
Share($a \leq 0$)	0.22	0.18	0.24
$\rho(d, a)$	0.66	0.32	0.31

Note: Share($a \leq 0$) is the share of households with nonpositive net wealth. $\rho(d, a)$ is the correlation between disposable income and net wealth.

The model with pensions tends to overpredict the correlation between income and wealth (see Table V). This is a phenomenon common to models of this sort with exogenous labor supply; see Domeij and Heathcote (2001). If labor supply is endogenous, the predictions are typically closer to the facts in this respect; see Flodén and Lindé (2001).

4.3. Income and Wealth by Age and Marital Status

Figure 4 displays the average disposable income of households by age (in the data, the age is that of the head of household). The baseline economy is able to capture the inverted U-shape of the age profile of income. The model without pensions also exhibits an inverted U-shape for working-age households, but has higher disposable income for working-age households and features a much larger drop in income at retirement than we observe in the data.

There are two main reasons for these discrepancies. First, the model without pensions features counterfactually low taxes; this accounts for a large fraction of the difference in disposable income. Second, the precipitous fall in income at age 65 in the model without pensions occurs because we assume that retirees have zero earnings and receive no pension benefits.

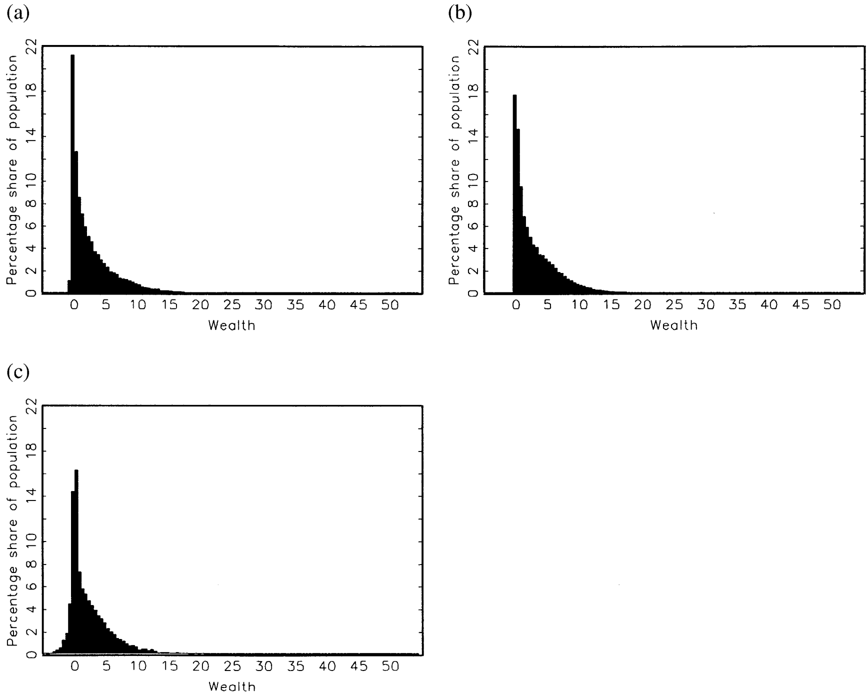

FIG. 3. Wealth distribution. (a) Wealth distribution, pensions. (b) Wealth distribution, no pensions. (c) Wealth distribution, Swedish data.

Tables VI and VII show how disposable income varies with marital status as well as age and how these aspects interact. We find that the baseline model comes very close to replicating the relative disposable incomes of single men, single women, and married couples, respectively.

Figure 5 displays age–wealth profiles for our two model economies and the data. The benchmark model (with pensions) matches the age–wealth profile fairly well. However, predicted asset holdings peak earlier than in the data.

In Tables VIII and IX, we see that the baseline model is able to replicate two important facts: single men have higher wealth than single women, and married couples have more than twice as much wealth as single-person households.

4.4. Within-Group Inequality

In Tables X and XI, we display the Gini coefficients for disposable income by age, sex, and marital status. For those below the age of 65, the models predict no strong relationship between age and inequality, and this

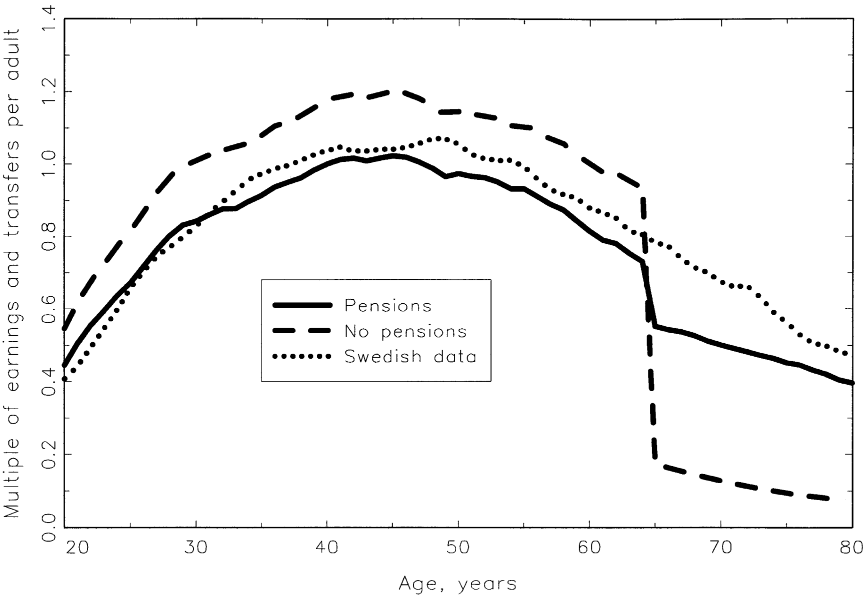

FIG. 4. Average disposable income by age.

is in line with the data. Meanwhile, in the marital status dimension, we are able to capture the fact that inequality is highest among single men and lowest among married couples.

For those above 65, the model with pensions underpredicts the degree of income inequality. One source of this discrepancy is that, in the data,

TABLE VI
Disposable Income by Age, Sex, and Marital Status—Swedish Data, 1992

Age	Single men	Single women	Married	All
20–24	0.43	0.41	0.94	0.55
25–29	0.56	0.56	1.14	0.86
30–34	0.58	0.65	1.28	1.05
35–39	0.65	0.80	1.34	1.16
40–44	0.68	0.70	1.40	1.21
45–49	0.70	0.69	1.39	1.23
50–54	0.65	0.61	1.36	1.19
55–59	0.72	0.62	1.24	1.09
60–64	0.55	0.55	1.12	0.97
65+	0.47	0.43	0.92	0.76
All	0.56	0.54	1.24	1.00

Note: Numbers represent mean within group as fraction of population mean.

TABLE VII
Disposable Income by Age, Sex, and Marital Status—Baseline Model with Pensions

Age	Single men	Single women	Married	All
20–24	0.50	0.53	1.12	0.72
25–29	0.57	0.61	1.28	1.00
30–34	0.64	0.68	1.38	1.16
35–39	0.68	0.73	1.46	1.27
40–44	0.79	0.77	1.53	1.35
45–49	0.77	0.74	1.51	1.34
50–54	0.69	0.71	1.43	1.28
55–59	0.68	0.66	1.31	1.19
60–64	0.61	0.59	1.15	1.03
65+	0.44	0.38	0.75	0.59
All	0.58	0.53	1.24	1.00

Note: Numbers represent mean within group as fraction of population mean.

some households with heads over 65 have members who are still working. If we remove these households, the Gini coefficient for the over-65s shrinks to 0.25. Nevertheless, it seems clear that our models exaggerate the redistributive effects of public pensions somewhat.

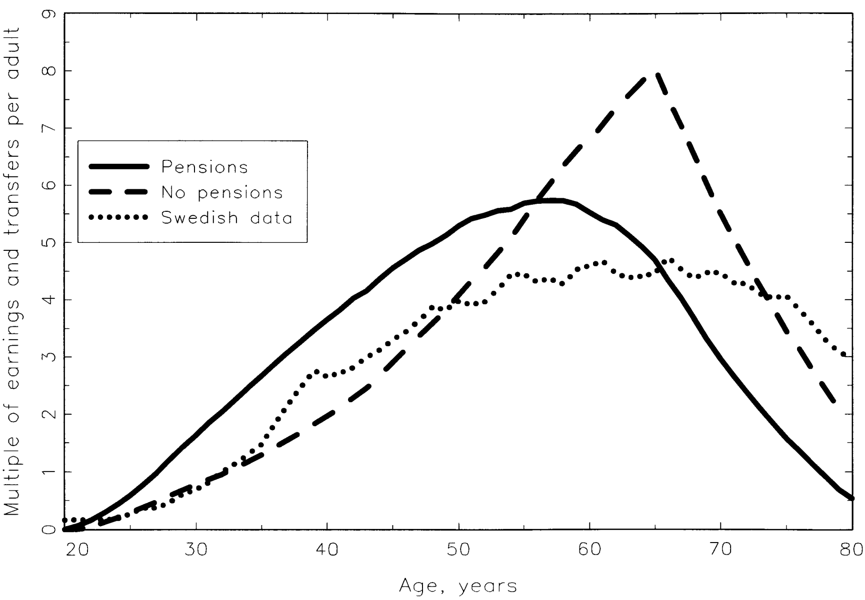

FIG. 5. Average net wealth by age.

TABLE VIII
Net Wealth by Age, Sex, and Marital Status—Swedish Data, 1992

Age	Single men	Single women	Married	All
20–24	0.07	0.02	0.04	0.05
25–29	0.17	0.11	0.15	0.15
30–34	0.24	0.07	0.44	0.35
35–39	0.44	1.05	0.86	0.81
40–44	0.71	0.34	1.14	0.98
45–49	0.67	0.46	1.51	1.29
50–54	0.75	0.58	1.71	1.48
55–59	0.91	0.68	1.81	1.55
60–64	0.75	0.72	1.91	1.61
65+	1.07	0.60	1.85	1.47
All	0.47	0.41	1.30	1.00

Note: Numbers represent mean within group as fraction of population mean.

Concerning the Gini coefficients for net wealth by age, sex, and marital status, we can see in Tables XII and XIII, that the data exhibit a decline in the degree of wealth inequality in age. The baseline economy replicates this decline among working-age households, although the level of inequality is lower than in the data. However, for the over-65s, the baseline economy predicts an increase in wealth inequality, which is out of line with the data.

In the marital status dimension, the baseline model captures the fact that wealth inequality is higher among single-person households than among married couples.

TABLE IX
Net Wealth by Age, Sex, and Marital Status—Baseline Model with Pensions

Age	Single men	Single women	Married	All
20–24	0.02	0.01	0.14	0.06
25–29	0.14	0.15	0.49	0.35
30–34	0.33	0.32	0.90	0.73
35–39	0.46	0.48	1.30	1.09
40–44	0.71	0.65	1.63	1.41
45–49	0.86	0.76	1.97	1.70
50–54	0.93	0.85	2.18	1.91
55–59	0.95	0.84	2.25	1.98
60–64	0.86	0.82	2.07	1.80
65+	0.25	0.20	0.95	0.63
All	0.35	0.31	1.36	1.00

Note: Numbers represent mean within group as fraction of population mean.

TABLE X
Disposable Income, Gini Coefficients by Age, Sex, and Marital Status—Swedish Data, 1992

Age	Single men	Single women	Married	All
20–24	0.23	0.19	0.15	0.30
25–29	0.21	0.17	0.14	0.26
30–34	0.20	0.21	0.16	0.25
35–39	0.28	0.27	0.18	0.26
40–44	0.27	0.21	0.22	0.28
45–49	0.29	0.22	0.21	0.26
50–54	0.37	0.22	0.22	0.29
55–59	0.39	0.18	0.22	0.28
60–64	0.38	0.23	0.23	0.29
65+	0.23	0.19	0.23	0.29
All	0.30	0.27	0.22	0.31

As Tables XIV and XV show, in the data it is mainly the young (under 35) that have zero or negative net wealth. The model with pensions predicts this, too. However, the model has two counterfactual predictions. First, the middle-aged are more indebted in the data than in the model. Second, the model with pensions predicts a somewhat higher fraction of indebted households among those over 65 than we observe.[15]

Moreover, consider the working-age households (those with heads under 65). In the data, the Gini coefficient for net wealth among working-age households is 0.83, and the baseline model predicts 0.63. Meanwhile, the fraction of households with zero or negative net wealth in this group is 0.27, and the baseline model predicts 0.18. In summary, it may be said that we underpredict inequality and indebtedness among working-age households and overpredict it for the retirees.

4.5. Sensitivity Analysis

4.5.1. *The Weights of the Partners in the Utility Function*

Because women live longer and face more downside income risk than men, they have a larger incentive to save. Therefore the savings decision will depend on the relative utility weight assigned to each partner (see Section 2.9). To see how big this effect is, we re-solve the model with a larger relative weight on wives; in particular, we set $\kappa = 0.25$. As can be seen in Table XVI, the effects are not quantitatively significant in any dimension.

[15]The low net assets of the old is best understood by keeping in mind that we assume that there is no altruistic bequest motive. See de Nardi (1999).

TABLE XI
Disposable Income, Gini Coefficients by Age, Sex, and Marital Status—Baseline Model with Pensions

Age	Single men	Single women	Married	All
20–24	0.34	0.33	0.25	0.37
25–29	0.34	0.35	0.25	0.34
30–34	0.35	0.33	0.26	0.32
35–39	0.34	0.32	0.26	0.31
40–44	0.35	0.32	0.26	0.30
45–49	0.36	0.33	0.26	0.30
50–54	0.33	0.32	0.26	0.30
55–59	0.30	0.32	0.25	0.29
60–64	0.31	0.30	0.26	0.29
65+	0.15	0.14	0.11	0.21
All	0.33	0.31	0.27	0.34

4.5.2. *The Rate of Return*

Here we consider a higher rate of return and set $r = 0.05$. This does not affect the results much, as can be seen in Table XVI. However, the wealth Gini and the fraction of households with non-positive net wealth both increase somewhat.

4.5.3. *Lump-Sum Pension Benefits*

In this section, we consider a pension scheme where pension benefits do not depend on past earnings. The lump-sum benefit is calibrated so that total government spending on pensions is the same as in the benchmark model. This implies giving every retiree a lump-sum benefit of 0.27. The

TABLE XII
Net Wealth, Gini Coefficients by Age, Sex, and Marital Status—Swedish Data, 1992

Age	Single men	Single women	Married	All
20–24	1.37	4.15	4.99	2.56
25–29	1.72	2.03	2.74	2.31
30–34	1.64	3.46	1.25	1.40
35–39	1.15	1.09	0.87	0.95
40–44	0.97	1.10	0.76	0.82
45–49	0.90	1.21	0.64	0.70
50–54	0.88	0.79	0.57	0.63
55–59	0.72	0.85	0.53	0.60
60–64	0.71	0.59	0.50	0.55
65+	0.59	0.66	0.50	0.56
All	1.00	1.02	0.69	0.79

TABLE XIII
Net Wealth, Gini Coefficients by Age, Sex, and Marital Status—Baseline Model with Pensions

Age	Single men	Single women	Married	All
20–24	2.95	2.11	0.88	1.30
25–29	1.23	1.13	0.66	0.79
30–34	0.79	0.76	0.56	0.62
35–39	0.71	0.62	0.51	0.56
40–44	0.62	0.58	0.448	0.52
45–49	0.56	0.55	0.45	0.49
50–54	0.56	0.55	0.42	0.46
55–59	0.53	0.50	0.41	0.45
60–64	0.48	0.49	042	0.46
65+	0.83	0.97	0.60	0.72
All	0.85	0.85	0.56	0.65

effects of this modification are remarkably small, indicating that Swedish public pensions are, for our purposes, very well approximated by a system with lump-sum benefits. For details, see Table XVI.

5. CONCLUSION

Using a calibrated overlapping generations model with a realistic demographical structure and idiosyncratic earnings risk, we found strong support

TABLE XIV
Fraction of Households with Non-positive Net Wealth by Age, Sex, and Marital Status—Swedish Data, 1992

Age	Single men	Single women	Married	All
20–24	0.40	0.41	0.52	0.43
25–29	0.52	0.54	0.51	0.52
30–34	0.53	0.56	0.36	0.42
35–39	0.47	0.44	0.25	0.31
40–44	0.40	0.34	0.20	0.24
45–49	0.33	0.37	0.15	0.19
50–54	0.27	0.25	0.11	0.15
55–59	0.23	0.23	0.09	0.12
60–64	0.23	0.14	0.07	0.10
65+	0.12	0.09	0.04	0.06
All	0.37	0.31	0.19	0.24

Note: Numbers represent the sum of net wealth within the group divided by the sum of disposable income within the group.

TABLE XV
Fraction of Households with Non-positive Net Wealth by Age, Sex, and Marital Status—Baseline Model with Pensions

Age	Single men	Single women	Married	All
20–24	0.69	0.66	0.38	0.57
25–29	0.44	0.43	0.21	0.30
30–34	0.31	0.30	0.13	0.18
35–39	0.26	0.20	0.08	0.12
40–44	0.22	0.17	0.04	0.08
45–49	0.15	0.15	0.03	0.06
50–54	0.13	0.15	0.02	0.04
55–59	0.12	0.11	0.02	0.04
60–64	0.11	0.13	0.02	0.04
65+	0.41	0.53	0.16	0.31
All	0.40	0.42	0.11	0.22

Note: Numbers represent the sum of net wealth within the group divided by the sum of disposable income within the group.

for the view that the pension system goes a very long way in accounting for the difference between income and wealth with respect to inequality. It is particularly encouraging that the model seems to generate wealth inequality via the right channel: in both the model and the data, wealth inequality is

TABLE XVI
Sensitivity Analysis, the Distribution of Net Wealth

	Models with pensions				
	Baseline	Low κ	High r	Lump sum	Data
Gini coefficient	0.66	0.66	0.66	0.65	0.79
Deciles					
1st	−0.01	−0.01	−0.01	−0.01	−0.06
2nd	−0.00	−0.00	−0.01	−0.00	−0.01
3rd	0.00	0.00	0.00	−0.00	0.00
4th	0.02	0.02	0.01	0.02	0.01
5th	0.04	0.04	0.04	0.04	0.03
6th	0.07	0.06	0.06	0.06	0.06
7th	0.10	0.10	0.10	0.10	0.10
8th	0.15	0.15	0.15	0.15	0.15
9th	0.22	0.22	0.23	0.23	0.22
10th	0.41	0.42	0.42	0.42	0.50
Percentile share					
100th	0.07	0.07	0.07	0.07	0.13
Share($a \leq 0$)	0.22	0.20	0.23	0.20	0.24

Note: Share($a \leq 0$) is the share of households with non-positive net wealth. Low κ refers to the case where κ, the relative weight on husbands, is equal to 0.25. High r refers to the case where the world interest rate is set to 0.05. Lump sum refers to the case where pension benefits are lump sum.

mainly driven by the large fraction of households with zero or negative wealth.

The feature of the public pension system that is important for these results is the great extent to which it is redistributive. In fact, the results are almost unchanged when we assume that pension benefits are lump sum (and not based on past earnings).

However, it is worth stressing that even though the pension system is a key factor behind Swedish wealth inequality, it is not the only factor. Therefore, it is not an embarrassing failure for our model that it does not deliver *all* the wealth inequality that we observe in the data. Rather, it leaves an interesting open question for further research.

APPENDIX A

Data

Our main data source is the 1992 version of Hushållens inkomster (HINK), published by Statistics Sweden. It features 12,484 households. A household is either a single person or a co-habiting couple, possibly with children. Only children 17 years old or younger count as such; 18-year-olds living with their parents are considered separate households.

The data on wealth are based on data collected by the tax authorities, but for the most part values have been adjusted to reflect market prices. The exception is condominiums, where no attempt to adjust for market prices has been made.

In a recent report (SCB, 2000) on the distribution of wealth from Statistics Sweden, data on car ownership have been added and condominiums are valued at estimated market prices. This turns out to have an insignificant effect on wealth; the fraction of households with zero or negative net wealth remains very high at 30%.

1992 was an unusual year in that the Swedish currency collapsed and the economy went into a severe recession. Nevertheless, it was not an exceptional year from the point of view of wealth inequality. The HINK has been published since 1978, and there have been only small changes in the degree of wealth inequality since then. From 1982 to 1992, the Gini coefficient for wealth ranged from 0.74 (1982) to 0.81 (1991).

Klevmarken and Bager-Sjögren (1998) are critical of the HINK, claiming that the interview-based HUS database is superior. However, we take the view that the shortcomings of HUS are more serious than those of HINK. In particular, the sample size is just 1150 households, individuals above the age of 75 are omitted, and a large number of records are incomplete (59% of the sample). See also Davies and Shorrocks (2000).

APPENDIX B

Computation of the Equilibrium

We fix grids for marital status, age, assets, own productivity, partner's productivity, own pension claims, and partner's pension claims. We use piecewise multilinear interpolation to evaluate the decision rule at points not on the grid.[16]

The algorithm is initialized by guessing the labor tax rate τ^n, the patience rate β, and the beliefs about the distribution of prospective partners' assets, productivity, and pension claims $\varphi^{i,s}$. In the latter case, we simplify matters by imposing that beliefs over a prospective partner's assets and pension claims are a deterministic function of sex and age and that a prospective partner's productivity is distributed according to the stationary distribution θ.

The decision rules are then solved for and the guesses are updated by simulating the economy. In our simulated economies, 1000 individuals of each sex enter at age 20. This implies that, at any point in time, the economy is inhabited by approximately 100,000 individuals.

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[16]The grids for marital status, assets, age, own productivity, partner's productivity, own pension claims, and partner's pension claims consist of 3, 18, 21, 9, 9, 3, and 3 points, respectively. This means that, for the baseline economy, each iteration requires us to solve 826,686 first-order conditions.

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