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REDISTRIBUTION AND INSURANCE: MANDATORY ANNUITIZATION WITH MORTALITY HETEROGENEITY

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ABSTRACT

This paper examines the distributional implications of mandatory longevity insurance when there is mortality heterogeneity in the population. Previous research has demonstrated the significant financial redistribution that occurs under alternative annuity programs in the presence of differential mortality across groups. This paper embeds that analysis into a life cycle framework that allows for an examination of distributional effects on a utility-adjusted basis. It finds that the degree of redistribution that occurs from the introduction of a mandatory annuity program is substantially lower on a utility-adjusted basis than when evaluated on a purely financial basis. In a simple life-cycle model with no bequests, complete annuitization is welfare enhancing even for those individuals with much higher-than-average expected mortality rates, so long as administrative costs are sufficiently low. These findings have implications for policy toward annuitization, particularly as part of a reformed Social Security system.

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Most public pension systems combine elements of redistribution and insurance. For example, the U.S. Old Age Survivors Insurance program (OASI) uses a non-linear benefit formula that provides a higher replacement rate for lower income workers in an effort to make the system progressively redistributive. At the same time, OASI insures individuals against longevity risk through the provision of benefits in the form of life annuities.

For some types of risk, providing insurance and engaging in progressive redistribution are complementary activities. This is true, for example, with Disability Insurance. In the U.S., workers covered by the DI program are provided with insurance against income loss in the event of becoming disabled. Because individuals who are disabled have, by definition, diminished earnings capacity, this same program serves a progressively redistributive role. Even on an *ex ante* basis, if lower wage individuals have a higher probability of becoming disabled, then a disability insurance program would even redistribute from higher to lower income individuals in expectation.

For other types of risk, however, the provision of insurance can have regressive distributional effects. Longevity risk is one such case. In a life-cycle setting, individuals who do not know how long they will live are, in general, made better off by annuitizing their wealth. However, because high-income individuals have longer life expectancies, they will have a higher expected present value of annuity payments than will low income individuals, if everyone is required to annuitize at a uniform price as in most public pension plans.

There is a large literature focusing on measuring the insurance value of annuitization for representative life-cycle consumers (e.g., Mitchell, et al 1999, Brown 2001). These papers generally quantify the utility gains from access to actuarially fair annuity markets by finding how much incremental, non-annuitized wealth would be equivalent to providing access to an actuarially fair annuity market (sometimes called the "annuity equivalent wealth"). A standard result from this approach is that a 65-year old male with log utility, whose mortality expectations mirror that of the population average, would find annuities equivalent in utility terms to a 50%

increase in wealth. With few exceptions, however, these utility-based calculations have been conducted only for "average" consumers who have access to annuities that are actuarially fair, i.e., that are priced using the individual specific mortality rates. Little has been done to examine the utility implications of annuitizing in an environment of heterogeneous mortality.[1]

The contribution of this paper is to examine the distributional impact of alternative annuity designs in a framework that incorporates the utility value of the longevity insurance. In particular, it examines how the annuity equivalent wealth varies across socioeconomic groups when annuities are priced uniformly. Staying with the no bequest assumption, this approach provides answers to three types of questions. First, under what conditions are individuals, particularly those in high-mortality risk groups, made better off by annuitizing at a uniform price? Second, how much redistribution is there on a utility-adjusted basis? Third, how are the answers to the first two questions affected by alternative annuity designs? For example, would individuals with shorter life expectancies prefer constant real annuities or some other path of payments?

This approach yields several interesting findings. First, in the absence of administrative costs, uniform priced annuities can make all life-cycle consumers better off, even those with mortality rates that are substantially higher than those used to price the annuity. Second, the amount of redistribution that arises from mandatory annuitization is much smaller on a utility adjusted basis than on a financial basis. Third, even high mortality risk individuals generally prefer real annuities to nominal ones, despite the fact that nominal annuities "front-load" annuity payments and thus provide, in expectation, higher lifetime payments to these short-lived individuals.

[1] There is a related literature examining the distributional effects of the current U.S. Social Security system (e.g., Gustman & Steinmeier 2001, Liebman 2000, Coronado, Fullerton & Glass 2000), and a smaller but growing literature examining redistribution within an individual accounts system (e.g., Brown 2000, Feldstein & Liebman 2000). However, these analyses have focused on purely financial measures of redistribution, as opposed to the utility-based measure used in this paper.

These findings are relevant to the debate about how to reform the U.S. Social Security system. Mortality differentials can have a significant effect on the progressivity of mandatory annuitization schemes, including most public pension systems. Several recent papers have suggested that the progressivity of the OASI benefit formula, which provides a higher replacement rate for lower wage individuals, is at least partially offset by the fact that higher income individuals tend to live longer than lower income individuals (Gustman & Steinmeier 2001, Liebman 2000, Coronado, Fullerton, & Glass 2000, Cohen, Steuerle & Carasso 2001). While these studies differ in the degree of overall progressivity in the system, all of them find that mortality differentials contribute to a lessening of income-based redistribution.

Many commentators have expressed concern that supplementing or partially replacing the current Social Security system with a program of personal accounts would have regressive distributional implications. In particular, if there is no redistribution elsewhere in the program (such as in contribution rates or benefit offsets), then there will be no offset of any distributional effects that arise from mortality heterogeneity in the payout phase. For example, if all individuals were required to annuitize their retirement accounts at a uniform price upon reaching age 67, the expected present value of future annuity payments would be substantially smaller for individuals with higher mortality probabilities, even if the account balances were identical in size. This paper demonstrates that these redistributive effects are substantially mitigated when evaluated on a utility-adjusted basis, and that the gains from annuitization are significant even for groups with high mortality rates.

This paper proceeds as follows. Section 1 provides a review of the literature on why annuities are valuable to representative retirees. Section 2 presents evidence on the interaction between mortality and socioeconomic status using data from the National Longitudinal Mortality Study, and discusses the impact of this on financial measures of distribution. Section 3 uses a simplified, two-period model to provide intuition for how the utility value of an annuity is affected by differential mortality. Section 4 discusses the dynamic programming methodology for solving for annuity valuation in a multi-period problem with liquidity constraints. Section 5 reports dynamic programming simulation results of the annuity equivalent wealth for multi-period life cycle individuals with more realistic constraints on annuity payments. Section 6 concludes.

1. The Insurance Value of Annuitization

In a widely cited article, Yaari (1965) demonstrated that a risk averse, life-cycle consumer facing an uncertain date of death would find actuarially fair annuities of substantial value. In fact, under certain conditions, including the absence of bequests and the absence of other sources of uncertainty, life cycle consumers find it optimal to invest 100% of wealth into actuarial notes. More recent theoretical work indicates that annuities are often welfare enhancing in a broader set of cases than those allowed by Yaari, including in the presence of aggregate risk, adverse selection, and intertemporal non-additivity of the utility function (Davidoff et al 2001). Other extensions, such as allowing for precautionary savings and bequest motives, tend to reduce the value of annuitization.

Annuities derive their value from the elimination of longevity risk. In the absence of annuities, individuals facing an unknown date of death must allocate their wealth across an uncertain number of periods. Unless the individual lives to the maximum lifespan, following the optimal consumption path will result in the individual dying with positive financial wealth. Assuming the individual does not value bequests, the individual would have been better off, *ex post*, had she consumed more each period while alive. *Ex ante*, however, following a more aggressive consumption path would have exposed her to the risk of having very low consumption levels in the event that she lived longer than expected. This problem arises in the absence of annuities because the individual is unable to allocate wealth in a state contingent manner. Instead, she must, for any given future period, set aside an equal amount of wealth for the state in which she is alive, and thus values consumption, and the state in which she is dead and does not value consumption.

Annuities partially complete the market by allowing an individual to make future resources survival-state contingent. In particular, annuities allow the individual to increase the income available in future periods conditional on being alive, in return for accepting zero resources in the event that she dies. If an individual has no bequest motive and therefore cares only about future states in which she is alive, this enables her to consume more each period while alive and completely eliminate the risk of living "too long" with resources insufficient to support desired consumption levels.

Previous work indicates that for a 65-year old man with average U.S. population mortality and log utility, gaining access to an actuarially fair real annuity market is equivalent to a 50% increase in wealth (Brown, Mitchell & Poterba, 2001). However, all of these studies have assumed that individuals have access to annuity markets that are actuarially fair, i.e., that the annuity is priced according to each individual's own mortality rates. In most realistic policy settings, such as public or private pension systems, individuals with heterogeneous mortality are pooled into a common annuity market. As such, very few individuals have access to annuities that are priced in a manner that is actuarially fair at the individual level, even if the system is actuarially fair on average. As such, the utility gains from annuitization in such a setting will vary across individuals.

2. Mortality Heterogeneity and Annuity Prices

There is substantial heterogeneity in expected lifetimes in the U.S. population. In addition to differences by age and gender, it has been substantially documented that mortality rates are correlated with race (Preston et al 1996, Sorlie et al 1992), ethnicity (Sorlie et al 1993), income (Deaton & Paxson 2001), wealth (Attanasio & Hoynes 2000), marital status (Brown & Poterba 2001), and educational attainment (Kitawaga & Hauser 1972, Deaton & Paxson 2001, Lantz et al 1998). In general, these correlations work in the direction that individuals of higher socioeconomic status live longer than those in lower socioeconomic groups. For example, whites live longer than blacks, higher income and higher wealth individuals live longer than individuals with less wealth, married people live longer than singles, and more highly educated individuals live longer than less educated individuals. There is also controversial evidence suggesting that Hispanics live longer than whites in the U.S., though this appears to be more true for foreign born than U.S. born Hispanics, and may be due to data contamination.

To evaluate the effect of mortality differentials on annuity valuation, it is necessary to construct a set of mortality tables that are differentiated based on demographic characteristics. This paper will use mortality estimates that are differentiated by age, gender, educational attainment, race and ethnicity. Age and gender are obvious characteristics to condition on, given the near universal pattern of adult mortality rates rising with age and the fact that females have lower mortality rates than males.

The level of lifetime financial resources available to an individual is clearly of policy interest in evaluating the distributional implications of annuity policy. Unfortunately, solid measures lifetime resources are not always available to researchers. One widely used measure is the current income of an individual or family. Current income, however, is a poor measure of lifetime resources. The most important criticism of this measure is the problem of simultaneous causation between income and health. Low-income individuals are more likely to suffer from health problems and thus experience higher mortality rates. But it is also true that individuals in poor health may be unable to earn a high income, in which case the causality is reversed.

Another frequently used measure is wealth. Attanasio & Hoynes (2000), Menchik (1993) and Palmer (1989) all provide compelling evidence that wealth and mortality are inversely correlated. The use of wealth partially addresses the simultaneity problem that arises when using current income. However, as noted by Attanasio & Hoynes, it cannot be considered exogenous either because wealth accumulation behavior of individuals with different life expectancies is likely different.

A third measure, and the one used in this study, is educational attainment. A significant negative correlation between education and mortality has been well documented (Deaton & Paxson 2001, Lantz et al 1998, Kitawaga & Hauser 1973). Education is a reasonable proxy for lifetime resources because more highly educated individuals have, on average, higher incomes. In addition, education is a pre-determined variable for most retired individuals. These benefits, combined with the fact that in the data used here the income data is of poor quality and the wealth data is not available, is the primary motivation for using educational status as a measure of economic status.

The primary motivation for examining results by race and ethnicity is that these measures are directly relevant to the politics of the Social Security debate in the U.S. For example, the Interim Report of the President's Commission to Strengthen Social Security specifically highlighted the impact of Social Security on African-Americans and Hispanics. While no racial or ethnic group is monolithic, there is tremendous political interest in how racial and ethnic sub-groups will fare under various proposals, as this could, in part, determine the viability of any reform effort. Thus, the ability to examine the effect of annuity policy on racial and ethnic groups is relevant to public policy. In addition, race and ethnicity serve as additional, albeit imperfect, proxies for economic status when combined with other measures such as educational attainment. However, it is important to remember that the differences in mortality rates across racial and ethnic groups presented in this paper are not necessarily caused by racial and ethnic differences. Rather, these differences likely reflect a combination of factors, including unobserved differences in economic status that are not conditioned out by the educational measures. However, to the extent that one is interested in how racial and ethnic groups fare, on average, under various reform options, the measures presented below are quite useful.

The group specific mortality differentials are estimated using data from the National Longitudinal Mortality Study (NLMS). The NLMS is a survey of individuals who were originally included in the Current Population Survey and/or Census in the late 1970s and early

1980s. Throughout the 1980s, death certificate information from the National Death Index was merged back into the survey data, allowing researchers to compare the death rates of individuals on the basis of demographic characteristics at the time of the interview.

Age specific mortality rates are constructed from the NLMS for black, white and Hispanics males and females, a total of six groups. The white and black groups are then further differentiated based on education, less than high school, high school plus up to three years of college, and college graduates. Due to small sample sizes, it is not possible to differentiate Hispanics along educational lines.

Several steps are required to turn these NLMS estimates into cohort mortality tables for specific groups. First, the NLMS sample is split into groups based on the gender, race, ethnic, and educational categories. For each group g, the age-specific, non-parametric (np) mortality rate, $q^{np}_{x,g}$, is calculated as the fraction of those individuals age x who die before attaining age x+1. This procedure provides a simple, non-parametric estimate of the age specific mortality rate for individuals with the characteristics of group g.

In order to correct for non-monotonicity that occasionally arises due to small cell sizes in some populations, the non-parametric estimates, $q^{np}_{x,g}$, are treated as the independent variable in a non-linear least squares regression on age x. The non-linear regression is used to estimate three parameters of a Gompertz/Makeham survival function, as explained in Jordan (1991). The Gompertz/Makeham formula used is:

$$l_x = ks^x g^{c^x} \qquad (1)$$

$$\text{where } k = \frac{l_0}{g} \text{ and } q_x = \frac{l_{x+1} - l_x}{l_x}$$

x is age, and g, c, and s are the parameters to be estimated. Note that if l_0 is set equal to one, then l_x is simply the cumulative survival probability to age x. Using the NLLS estimates of g, c, and s, one then has a "Makeham formula" that gives mortality q_x as a function of x. Let us denote these fitted values of mortality for group g at age x as $q^{fit}_{x,g}$. An important feature of this approach is that fitted mortality rates are a monotonically increasing function of age x. It also allows one to create out-of-sample estimates of mortality. Therefore, while only data from age 25 to 84 is used to fit the curve, the formula can provide estimates of mortality for ages outside of this range. This model has been found to describe human mortality patterns quite accurately up to the age of 96 (Riggs and Millecchia 1992). It has, however, had difficulty accounting for mortality patterns among the very oldest individuals (Witten 1988, Riggs & Millecchia 1992). Fortunately, the results in this paper are very insensitive to the mortality rates at these extreme old ages because the contribution of consumption to lifetime utility at these ages is being heavily discounted both by interest rates and mortality rates.

Once these predicted mortality rates are in hand, the next step is to convert them into cohort life tables for each group. This requires two related assumptions. The first is that the ratios of a group's age-specific mortality to that of the population as a whole ($q_{x,g}/q_x$) in the NLMS sample is an accurate portrayal of these ratios in the full population in 1980. The second assumption is that these ratios are constant over time. By invoking these two assumptions, it is possible to then construct a group specific cohort life tables for any year.

Specifically, let $q^{fit}_{x,g}$ be the fitted value of the mortality rate for an individual age x belonging to group g, and let q^{fit}_{x} be the mortality rate for an individual age x for the population as a whole, both from the fitted NLMS data. Let q^{SSA}_{x} be the age-specific mortality rate from the 1978 birth cohort table from the Social Security Administration, which represents individuals turning age 22 in the year 2000 (the group of study in this paper). Then the cohort, group specific mortality rates are constructed as follows:

$$q_{x,g}^{SSA} = q_x^{SSA} \frac{q_{x,g}^{fit}}{q_x^{fit}} \tag{2}$$

The one exception to this methodology is that in the case of college educated black males and females, the mortality ratio between college and high school is assumed to be the same for blacks as for whites. This ratio is applied to the fitted q's for blacks with a high school education in order to construct the estimate for a college educated black. This was done because the sample sizes at many ages were too small for college-educated blacks to reliably construct an independent estimate.

Table 1 reports how the life expectancy of a 22-year-old in the year 2000 varies by the gender, race, ethnicity, and education as calculated using the above methods. The average 22 year old male can expect to live to age 77.4, while the average 22 year old woman can expect to live to age 83.4. However, these estimates vary widely by race. White, black, and Hispanic 22-year old males have life expectancies of 78.3, 71.8 and 77.7 years respectively, while white, black and Hispanic females have life expectancies of 84.0, 80.0, and 85.2 years respectively. Life expectancy at age 22 also varies substantially by education level. White men with less than a high school education have a life expectancy at age 22 of 75.3 years, a full 5.2 years less than that of a white male with a college degree. Low educated black males have by far the lowest age 22 life expectancy of any group examined, at 68.1 years. The highest life expectancy is college educated white women, who can expect to live to age 85.1.

Table 1 also reports the life expectancy as of age 67 for this same cohort. As can be seen, there is still a substantial range in the estimates, although the differential in years is not as large as at age 22. This is because much of the life expectancy difference that arises for 22 year olds is due to higher mortality probabilities in the pre-retirement period. Conditional on reaching age 67, these differences are diminished. The numbers suggest that a 67-year-old white can expect to live approximately 16 months longer than a 67-year-old black. When further differentiating by educational attainment, the difference is naturally larger, with a 3.4-year difference between college educated white men and less than high school educated black men.

The general racial and ethnic patterns in this data set are consistent with other sources of mortality patterns. For example, the U.S. Census Bureau reports life expectancies at birth that are higher for Hispanics than for non-Hispanic whites, which in turn are higher than those for blacks.

It should be noted, however, that there is controversy about the nature of the mortality differences for Hispanics. The limited research available suggests that U.S. Hispanics have lower mortality rates than non-Hispanic whites, despite a greater proportion living in poverty, lacking health insurance, and having more limited access to health care (Sorlie et al 1993). Hispanics tend to have lower rates of heart disease, cancer and pulmonary disease, although these differences do not seem to be explained by the major known risk factors for these diseases. There are reasons to be cautious in interpreting the Hispanic results. First, if sampling techniques tend to under-sample less healthy Hispanics (e.g. migrant workers), this would bias mortality rates down. In addition, studies like the National Longitudinal Mortality Study obtain death statistics by linking to the National Death Index. This means that deaths outside of the U.S. are not recorded, and thus some individuals' deaths will be missed. This is particularly likely to occur among immigrants to this country, some of whom may return to their country of origin at the end of their lives. A final reason that the Hispanic results should be interpreted cautiously is that there is very substantial heterogeneity in this population. Of particular importance is the fact that foreign-born persons tend to have lower mortality risk than native-born persons (Sorlie et al 1993). This "health migrant effect" would also help explain lower mortality rates among Hispanics.

One can use these mortality estimates to construct a "money's worth" of an annuity that is priced based on the average mortality in the population. A money's worth measure is simply the expected present value of annuity payments per dollar spent to purchase the annuity, and has been used in many past studies of annuity prices (Friedman & Warshawsky 1988 and 1990, Warshawsky 1988, Mitchell et al 1999). Table 2 reports the money's worth ratio for the cohort entering the workforce in the year 2000. This purely financial measure indicates that the money's worth of an inflation indexed life annuity for a 67 year old black male with less than a high school education would be only 0.800, while a white woman with a college education would have a money's worth of 1.106. Viewed solely from this financial perspective, mandating annuitization at a uniform price is tantamount to a system of taxes and transfers that takes resources from poorly educated black men and gives it to highly educated white woman. This is due to the fact that an annuity, by design, serves to transfer resources from shorter-lived to longer-lived individuals, combined with the fact that there is heterogeneous mortality in the population.

Brown (2000) explores the money's worth of a richer set of annuity options, and finds that the dispersion in money's worth across groups can be substantially reduced by considering annuities that "front-load" annuity payments or offer bequest options. As reported in Table 2, an annuity that declines in real value by 3% per year increases the money's worth for low educated black men to 0.83. Even more striking, offering an annuity with a 20-year period certain guarantee[2] increases the money's worth to black men with less than a high school education to 0.955, as indicated in column 3.

The money's worth, however, is purely a financial measure, and as such it ignores the insurance value that individuals derive from the elimination of longevity risk. To assess the welfare effect of differential mortality, it is necessary to embed the heterogeneous mortality into a utility-based model.

3. Annuity Valuation with Heterogeneous Populations

Previous studies have used an "annuity equivalent wealth" measure to quantify the gains from actuarially fair annuitization. Intuitively, the annuity equivalent wealth is a dollar measure of how much value an individual places on access to an annuity market. It is closely related to the measure of "equivalent variation" used in standard welfare analysis. It proceeds by asking the question "what increment to and individual's wealth would make that person as well off as if she had access to an actuarially fair annuity market?" For a given lifetime utility function, this is calculated first by finding the utility level associated with full annuitization of the individual's wealth. One can then calculate how much additional wealth would be required to attain this same level of utility in the absence of annuitization. This annuity equivalent wealth measure uses the full annuitization utility level as its baseline. Alternatively, one could use the non-annuitized level of utility as a baseline, and then calculate how much wealth one could take away if access to annuities were provided and still leave the person at the same utility level. This latter approach, sometimes called "wealth equivalence" (Mitchell et al 1999) is similar to the compensating variation measure in welfare analysis. Both approaches lead to quite similar conclusions about the welfare gains of annuitization. The annuity equivalent wealth measure has been used more often in previous studies due to a slight computational advantage, and is used here to make results more comparable to these other studies.[3]

To understand this approach analytically, it is useful to examine a much-simplified problem. Consider a two-period model for a single consumer, with additively separable log utility of consumption, and the interest rate and time preference rate both equal to zero. Let P be the probability that the individual will survive to period 2, and let ϕ be the period 1 price of consumption in period 2. Then the consumer's problem is:

$$\underset{\{C_1, C_2\}}{Max} \;\; \ln C_1 + P \ln C_2 \tag{3}$$

subject to:

$$C_1 + \phi C_2 = W \tag{4}$$

Taking first order conditions, we find that

$$C_2 = \frac{P}{\phi} C_1 \tag{5}$$

Note that if no annuities are available, $\phi=1$, and the optimal consumption path is declining proportionally with the probability of survival. If annuities are actuarially fair, then $\phi=P$, and the

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[2] A life annuity with a 20-year period certain guarantee means that the annuity will make a minimum of 20 years worth of payments to either the insured or the named beneficiary. If the annuitant is still alive at this point, payments continue for life, otherwise, the payments end.

[3] Readers interested in a more detailed discussion of these differences will find it in Brown, Mitchell, Poterba and Warshawsky (2001).

individual wishes to perfectly smooth consumption over the life cycle. If annuities are available, but are more expensive than actuarially fair, then 1>ϕ>P, and consumption will decline at an intermediate rate.

Solving (2) and (3) and plugging into (1), we find that the indirect utility function V(P,ϕ,W) is:

$$V(P,\phi,W) = (1+P)\ln\left(\frac{W}{1+P}\right) + P\ln\left(\frac{P}{\phi}\right) \tag{6}$$

Denote the Annuity Equivalent Wealth as α, which is implicitly defined as:

$$V(P,1,\alpha W) = V(P,\phi,W) \tag{7}$$

The left hand side of equation 7 is the utility level achieved when the individual does not have access to annuities, so that the price of second period consumption is equal to one, but has additional wealth. The right hand side of equation 7 is the utility level achieved when an individual has access to an annuity with a price of ϕ. The Annuity Equivalent Wealth, α, is a measure of the additional wealth that must be given to the individual in the absence of annuities to be as well off as if the individual could annuitize at a price of ϕ.

Solving equation 7, we find that $\alpha = \phi^{-\left(\frac{P}{1-P}\right)}$. Naturally, when no annuities are available, ϕ=1, and therefore α=1. When the annuity is actuarially fair, ϕ=P, and $\alpha = P^{-\left(\frac{P}{1-P}\right)}$. For example, if ϕ=P=.5, then α=1.26, indicating that an individual would be indifferent between \$1.26 of non-annuitized wealth, and \$1.00 of annuitized wealth. Therefore, access to actuarially fair annuity markets can be said to be worth a 26% increase in wealth.

This highly simplified framework allows one to immediately see several stylized results.[4]

[4] Readers interested in a more general and more rigorous theoretical treatment of annuities will find it in Davidoff, et al, 2002.

Result 1: $\alpha>0$ for all $0<P<1$ and $0<\phi<1$. So long as there are no additional administrative costs of annuitization, all consumers are made better off by annuitization. Importantly, even if individuals with very short life expectancies are required to annuitize in a market where pricing is based on high survival probabilities (i.e., low P and high ϕ), the annuity equivalent wealth exceeds 1.0. Thus, the oft-used assumption that annuities be priced actuarially fair is overly restrictive. The intuition for this is straightforward – with no loading costs, the availability of annuities that are not actuarially fair for an individual still have the effect of reducing the price of future consumption for that individual, and thus making the consumer better off.

Result 2: For fixed P ($0<P<1$), $\frac{\partial \alpha}{\partial \phi}<0$. Any individual with an uncertain lifespan values annuities more highly when they are priced using lower survival probabilities. This is quite intuitive, since it simply states that all individuals are better off when the price of future consumption falls.

Result 3: For fixed ϕ, $\frac{\partial \alpha}{\partial P}>0$. For a fixed price, increasing an individual's survival probability makes the annuity more valuable because they are more likely to survive to consume the annuity.

Result 4: When $P=\phi$, $\frac{\partial \alpha}{\partial P}>0$ up to some $\overline{P}$, and then $\frac{\partial \alpha}{\partial P}<0$. When annuities are actuarially fair for each individual, increasing P from 0 to 1 has a positive and then negative effect on annuity valuation. This is because a change in P now has two effects. First, individuals with longer life expectancies (higher P) are more likely to survive to the second period and thus consume the annuity, which makes annuities more valuable. Second, high P individuals must pay more for second period consumption, which makes annuitization less valuable. These effects work in opposite directions, and thus a plot of α against P is hump shaped. In this simple example given here, the value of annuitization peaks at P=.278, a relatively low survival probability. If we were to compare a cross-section of individuals, all of whom had a P>.278, we would find that individuals with shorter life expectancies value individually priced annuities more highly than those with longer life expectancies! This is in contrast to the intuition of result 2, which is the standard intuition about annuity valuation. The reason for this result is that individuals with lower survival rates are rewarded with a lower price of consumption for the second period.

In this two-period problem, P is a sufficient statistic for both the life expectancy (e.g., the mean) and the degree of longevity risk (e.g., higher moments). When one moves to a multi-period problem, life expectancy is no longer a sufficient statistic for how much mortality risk one faces, and therefore, for how much one values an annuity. Life expectancy is an average, and the simple economics of risk suggests that the degree of uncertainty around the mean also matters. In fact, it is possible for a person with a longer life expectancy to value an identical annuity less than someone with a shorter life expectancy. For a trivial (and very hypothetical) example, consider a 65 year-old man who knows he will live exactly 20 more years and die on his 85th birthday. This person has no risk to insure against, and the annuity is worth no more to him than the simple discounted value of the 20 years of payments. If a second man has an identical 20-year life expectancy, but substantial risk around this mean, he will value the annuity more highly. As such, there exists some $\varepsilon > 0$ such that we can reduce this second person's life expectancy to $20-\varepsilon$, and still have a higher annuity equivalent wealth than the person who will live to 20 for sure, due to the uncertainty around this mean. As such, it is not accurate to claim that an individual with a longer life expectancy will always value annuities more highly. A multi-period model will be discussed in the next section.

So far, we have been assuming that there are no mark-ups of price over marginal cost. If there are mark-ups, such as in the form of administrative costs, the budget constraint in (4) can be rewritten as:

$$C_1 + \theta \cdot \phi \cdot C_2 = W \tag{8}$$

where $\theta>1$ for a positive mark-up. This leads us to another straightforward result:

Result 5: If $(\theta \cdot \phi)>1$, then $\alpha<1$. If administrative costs are high enough to completely offset the price reduction that arises from the mortality rates used to price the annuity, then this has the effect of making second period consumption more expensive, and annuities become less valuable.

This two-period model is useful insofar as it builds some simple intuition for how the value of annuitization is related to the price of annuities, survival probabilities and administrative costs. These results generally apply to multi-period models in which the individual has the ability to choose survival contingent consumption in each period separately, i.e., Arrow-Debreu markets are complete. For example, if an individual finds that his own survival probabilities in some periods are higher than those used in the pricing of annuities, he would choose to consume more in those states. In most "real world" annuity markets, however, the structure of annuity payments typically constrains one's ability to do this. For example, in the U.S. Social Security system, individuals are forced to annuitize in a constant real annuity, and are constrained against borrowing from future annuity payments. They must therefore purchase units of consumption across periods in fixed proportions, and this means they are unable to precisely match the annuity to their preferred consumption profile. When these constraints bind, this will have the effect of reducing annuity value. Importantly, these constraints will have differential effects on groups with different mortality expectations. Therefore, the differential utility impact of multi-period annuity contracts needs to be examined in a multi-period setting to determine if the basic results of the two-period model go through.

4. Multi-Period Annuity Valuation

Calculating the Annuity Equivalent Wealth (α) for more realistic, multi-period settings with constraints on the annuity payments can in some cases be solved in closed form. Generally, however, the presence of liquidity constraints imposed by the annuity structure makes closed form solutions difficult to obtain. In such cases, one way to solve for the α is to use dynamic programming techniques.

To generalize the problem, let $U(C_t)$ represent the one-period utility function defined over real consumption, ρ the utility discount rate, and T the maximum possible life-span of an individual. Then the consumer's problem, assuming additive separability over time, is:

$$Max_{\{C_t\}} \left(\sum_{t=1}^{T-age+1} \frac{P_t \cdot U(C_t)}{(1+\rho)^t} \right\} \tag{9}$$

where P_t is the probability of surviving to period t, subject to the following constraints:

$$\begin{aligned} &(i)\ W_0\ \ given \\ &(ii)\ W_t \geq\ 0,\ \forall t \\ &(iii)\ W_{t+1}\ = (W_t - C_t + A_t)(1+r) \end{aligned} \tag{10}$$

In these constraints, W_t is non-annuitized wealth in period t, C_t is consumption, and A_t is the annuity payment that can be purchased when annuity markets are available. Assume that the individual, prior to any annuitization, has financial wealth W^*. Then for the case in which no annuities are available, $W_0 = W^*$, and $A_t=0$, $\forall$t. In the case in which the individual fully annuitizes all financial assets, then $W_0 = 0$, and A_t is determined by the pricing in the annuity market. For the special case in which the annuity is actuarially fair for the individual, A_t is determined by the equation:

$$W^* = \sum_{t=1}^{T-age+1} \frac{A_t \cdot P_t}{(1+r)^t (1+\pi)^t} \tag{11}$$

In equation (8), the real interest rate is represented by r, and the inflation rate by π. Note that this formula determines the nominal value of a fixed nominal annuity. The real value of this annuity declines by the factor $1/(1+\pi)$ each period. By setting $\pi=0$, equation (3) can be used to determine the starting value of a real annuity as well. In the simulations that follow, it will be assumed that $r=\rho=.03$. By replacing the individual's P_j with the average P_j for the annuitizing population, we can construct the annuity payments available in a uniform price system. It is also straightforward to incorporate administrative loading costs into the calculation by multiplying the right-hand side of equation 11 by one minus the load factor.

In order to use dynamic programming techniques to solve for the optimal consumption path, it is useful to introduce a value function $V_t(W_t)$, which is defined as:

$$V_t\left(W_t\right) = Max_{\{C_t\}}\left\{\sum_{t=1}^{T-age+1} \frac{P_t U\left(C_t\right)}{\left(1+\rho\right)^t}\right\} \tag{12}$$

subject to the constraints in equation (10).

The value function at time t is the present discounted value of expected utility evaluated along the optimal path. This value function satisfies the following recursive Bellman equation:

$$Max_{\{C_t\}} V_t\left(W_t\right) = Max_{\{C_t\}} U\left(C_t\right) + \frac{\left(1-q_{t+1}\right)}{\left(1+\rho\right)} V_{t+1}\left(W_{t+1}\right) \tag{13}$$

where q_{t+1} is the one period mortality probability, i.e., the probability of dying in period t+1 conditional on surviving through period t. The relationship between q and P is:

$$P_t = \prod_{j=1}^{t}\left(1-q_j\right) \tag{14}$$

The Bellman equation reduces the full maximization problem to a series of 2-period problems that can be solved numerically by solving back from the final period. This maximization is subject to the constraints in equation (10). I use standard methods of discretizing the wealth space to closely approximate the solution.

To calculate α, the Annuity Equivalent Wealth, one must first find the maximum utility V^* for the case in which the individual has the ability to fully annuitize W^*. Because this individual fully annuitizes, he starts off with zero non-annuitized wealth, $W_0 = 0$. One then solves for the case in which annuities are not available. That is, A_t is constrained to be zero for all t. It is then possible to solve for the amount of additional wealth, ΔW, which must be given to the individual in the absence of annuities such that the utility without annuities is equal to V^*.

That is, ΔW is defined such that:

$$V(W^{*}+\Delta W \mid A_t = 0, \forall t) = V^{*} \qquad (15)$$

Annuity equivalent wealth is then defined as:

$$\alpha = \frac{W^{*} + \Delta W}{W^{*}} \qquad (16)$$

5. Results

The Annuity Equivalent Wealth is calculated for individuals retiring at age 67, which is the Normal Retirement Age that the existing U.S. OASI system is transitioning towards. The cohort chosen for this study is that which enters the workforce at age 22 in the year 2000.[5] Results are quite similar for other cohorts. Within this cohort, we consider the mortality differentials across the gender, racial, ethnic and education groups described in section 2. While it is true that individuals in these demographic groups may enter retirement with substantially different levels of wealth, the CRRA utility function used in the simulations is invariant to the scale of wealth and therefore the annuity equivalent wealth measure, which is stated as a percentage of initial wealth, is unaffected by the differences in wealth levels across groups.

The value of annuitization is, however, related to the degree of risk aversion. In particular, more risk averse individuals will value annuities more highly than less risk averse individuals. While there is some evidence that risk aversion differs across segments of the population (Eisenhower & Halek *forthcoming*, Barsky, et al 1997), it is difficult to pin down these differences in a precise manner. Therefore, annuity equivalent wealth values are reported for all demographic groups for CRRA coefficients one through five. A risk aversion of one corresponds to log utility, a value that is often found to be the average risk aversion in many studies of consumption (Laibson, Repetto, & Tobacman 1998). Higher levels of risk aversion have been

[5] This is a cohort that has been used in several other studies of Social Security reform (Feldstein & Ranguelova 2000, Brown 2000).

found in other studies, particularly those examining the equity premium puzzle, and thus annuity equivalent wealth results are reported for higher levels as well.

Table 3 reports the annuity equivalent wealth for the case of a constant real annuity that is uniformly priced for all individuals. There are several aspects of these numbers that are worth noting. First, as has been found in previous studies focusing on representative individuals, the utility gains from annuitization are quite high. Focusing on average men, the annuity equivalent wealth ranges from 1.35 at log utility to 1.546 for a risk aversion coefficient of 5. Second, even poorly educated black men, those with the worst mortality prospects of all the groups represented, have an annuity equivalent wealth of 1.296 when evaluated using log utility. Thus, even though the money's worth calculation indicates that poorly educated black men receive negative transfers on the order of –20% from being required to annuitize at a uniform price, the utility gains are still substantial. Third, as expected, annuity valuation is rising with risk aversion for all individuals.

Fourth, there is a surprising lack of significant dispersion in the annuity equivalent wealth figures across demographic groups. The largest effects are between men and women. With log utility, the difference between the utility gain to average women and that to average men is 11.5% of wealth. This should be contrasted with the 15.6% difference when evaluated on purely a financial basis in Table 2. This difference shrinks to only 4.2% of wealth at risk aversion of 5. Within genders, there is very little difference. For example, the difference in annuity equivalent wealth of college educated white men and less than high school educated black men is only 6.5% of wealth when risk aversion is 1, and only 1.2% of wealth when risk aversion is 5. Again, this stands in stark contrast to the results when reported on purely a financial basis, where the difference in money's worth between these two groups was 16.7% of wealth.

These results may seem somewhat surprising given that the financial transfers are so large. It is important to realize, however, that much of the utility value of annuitization comes from the fact that it eliminates the risk of running resources down to a very low level in the event that one lives longer than expected. Even high mortality risk groups have a non-zero probability of living to advanced ages, and the utility gains from avoiding states of low consumption are quite large. In the absence of annuitization, the optimal consumption path requires a high-mortality risk individual to set aside money for the low probability event that he will lived to be age 100. In most cases, this money will be "wasted," since the individual is likely to die much earlier and, in this model, does not value bequests.

As an interesting comparison, annuity equivalent wealth results are next computed for the case in which annuities are priced for each demographic group on an actuarially fair basis. In other words, the annuity is "risk-class" priced, so groups with lower mortality rates receive lower annuity payments. From a financial perspective, the money's worth for every group is equal to one. Table 4 reports the difference in annuity payouts that arise under this pricing assumption. Note that the monthly payment ranges from a low of $553.08 for college educated white women, to a high of $776.92 for black men with less than a high school education.

Table 5 indicates that high mortality risk individuals value actuarially fair annuities far more highly than low mortality risk individuals. For example, with a risk aversion coefficient of one, a black male with less than a high school education has an Annuity Equivalent Wealth of 1.632, meaning that gaining access to actuarially fair annuity markets is equivalent to a 63.2% increase in non-annuitized wealth. This represents a doubling of the 32.1% increase in wealth for a college educated white female, despite the fact that the payments are only 40% higher. Once again, at higher levels of risk aversion, the annuity equivalent wealth rises for all demographic groups. At a risk aversion coefficient of 5, for example, the annuity equivalent wealth ranges from a low of 1.435 to a high of 1.929.

Results thus far suggest several interesting conclusions. First, even in an environment in which annuities are uniformly priced, if administrative costs are zero, all consumers are made better off by availability of the annuity. Second, while the degree of redistribution appears large when measured on a financial basis, the degree of redistribution when measured on a utility-adjusted basis is substantially smaller. Third, if annuities are not uniformly priced, but rather are priced based on the mortality experience of each risk class, the high mortality risk groups benefit the most from annuitization.

High mortality risk groups experience a low money's worth due to the fact that they are less likely to be alive in future periods to consume the annuity payments. As such, as was shown in table 2, they are better off from a financial perspective if the annuity is declining in real terms. This is because a declining real annuity front-loads payments into early periods, when the individual is more likely to be alive. As was also demonstrated in section 2, if an individual's mortality rate is higher than that used in the pricing of annuities, he will prefer a consumption path that with a downward tilt. Fixing annuities in nominal terms and letting inflation erode its real value over time is an example of a product that would provide such a downward slope. Table 6 reports annuity equivalent wealth results for the case of an annuity that declines at a real rate of 3% per annum. This rate is roughly consistent with the average historical rate of inflation in the U.S. Comparing the results from table 6 with those of table 3 (constant real annuities), it is clear that most individuals are made worse off by having the annuity decline in real terms. In fact, for risk aversion of 2 or greater, every group is better off with constant real annuities. Only in the case of log utility is any group made better off by declining annuities, and one might expect, these are high mortality risk groups. Specifically, whites with less than a high school education, and black men of all education levels, are the only groups to do better under a declining real annuity with log utility. While the dispersion in annuity equivalent wealth does decrease, thus decreasing the amount of redistribution, it does so mainly by depressing the value of annuities for most groups, rather than raising it for many. It should also be noted that these are results for a declining real annuity. A true nominal annuity that is subject to inflation risk would lower annuity values for everyone. Thus, from a utility standpoint, it seems that front-loading payments through imperfect inflation indexing is not a satisfactory way to handle distributional concerns.

Another option for lessening the degree of financial redistribution is the period certain guarantee. For example, an annuity contract that is for "life plus 20 years certain" pays off the longer of 20 years or the insured's life. Period certain guarantees, usually 10 or 20 years, are commonly attached to life annuity products. In the context of the present utility-based valuation model, in which the insured individual is assumed not to value bequests, a period certain product would not be optimally chosen. Nonetheless, given their popularity, I report results for a 20 year period certain product in table 7. As one would expect, the overall valuation of the annuity is lower for all households for the simple reason that a life plus 20-year period certain product has lower payouts than a straight life annuity. If one does not value a bequest motive, then the use of a period certain payout operates like a load factor,[6] reducing the payouts with no corresponding utility benefit. As such, the annuity is equivalent to only an increase in non-annuitized wealth of between 8 and 18 percent, depending on the degree of risk aversion. Consistent with earlier estimates, however, the degree of dispersion across groups is remarkably small.

All of the above results assume that annuities do not have any additional costs, i.e., that they are actuarially fair for the average annuitant. However, it is unlikely that annuities can be provided with no administrative costs. For example, private annuity markets in the U.S. are estimated to have administrative costs of approximately 8% (Mitchell, et al 1999). Table 8 shows annuity equivalent wealth results for the case of a uniform price, constant real annuity with 8% administrative costs. Not surprisingly, all the annuity valuations fell relative to table 3 by approximately 8%, and the basic finding that there is limited redistribution on a utility-adjusted basis still holds.

7. Conclusions

Annuities provide valuable longevity insurance to individual with uncertain lifetimes. However, mandating that all individuals annuitize at a uniform price also has distributional implications. When measured on a financial basis, these transfers can be quite large and often away from economically disadvantaged groups and towards groups that are better off financially.

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[6] Under the assumption of a 3 percent real interest rate and a 3 percent inflation rate, and using the unisex mortality tables, the switch from a straight life annuity to a life + 20 year certain annuity reduces the annual

This paper indicates, however, that the insurance value of annuitization is sufficiently large that, relative to a world with no annuities, all groups can be made better off through a mandatory annuitization system, so long as administrative costs are kept low. In particular, even groups with mortality rates far higher than those used to price the annuities are made better off than in the absence of annuities. Furthermore, there appears to be far less redistribution when evaluated on a utility adjusted basis.

These results are based on a counterfactual world in which no annuities are available. This is not as extreme a counterfactual as it may at first appear, given that outside of Social Security and some defined benefit plans, annuity markets in the U.S. are quite thin (Brown et al 2001). It is important to note, however, that the distributional consequences of mandating annuitization in an individual accounts system, for example, will differ depending on the counterfactual. If, for example, individuals are already fully annuitized in a system that prices annuities uniformly, then moving to a system of individual accounts that does the same thing will not have any new distributional effects arising from annuitization (though differences may still arise from other features of the system). If individuals are already annuitized in a system that prices annuities separately for each demographic group, then the move to a uniform priced annuity system would clearly represent a shift in resources away from high risk individuals to low risk individuals. The story becomes even more complex if, in the counterfactual world, annuities are available on a voluntary basis only, and at high cost (a fair representation of the individual annuity market in the U.S.) Mandating annuitization can raise the average payout rate by forcing individuals into the market, improving the welfare of those who had been annuitizing previously. It would also improve the welfare of those that would have liked to annuitize but did not due to the cost structure.

It should also be noted that a full social welfare comparison of alternative annuity systems would require the specification of an explicit social welfare function. Recent work by

payout by approximately 18 percent.

Sheshinski (1999) demonstrates conditions under which uniform annuity pricing can in fact be social welfare maximizing, and conditions under which it is not.

This paper has focused exclusively on longevity insurance. One potentially fruitful area for future research would be an investigation of the utility value of other insurance aspects of public pension systems, such as disability, survivor and dependent benefits. To the extent that these programs have substantial insurance value, previous studies of the distributional effects of Social Security that have ignored this value may not tell the complete story about the distributional effects of social insurance programs.

Future research could also extend the annuity valuation framework to include bequest motives. While the economics profession is far from a consensus about the importance of bequests or how to model them, additional work would be useful for understanding the value of annuities with bequest options.

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TABLE 1
Conditional Life Expectancy by Gender, Race, Hispanic Status, and Education

	Conditional Life Expectancy at age 22		Conditional Life Expectancy at age 67	
	Men	Women	Men	Women
All	77.4	83.4	83.5	87.2
All Whites	78.3	84.0	83.6	87.4
All Blacks	71.8	80.0	82.3	86.1
All Hispanics	77.7	85.2	84.8	88.3
Whites: College +	80.5	85.1	84.4	87.8
Whites: HS +	77.8	83.9	83.4	87.3
Whites: < HS	75.3	82.1	82.3	86.5
Blacks: College +	75.7	81.9	83.4	86.8
Blacks: HS +	71.6	80.0	82.2	86.1
Blacks: < HS	68.1	77.5	81.0	85.1

Notes: "Conditional Life Expectancy" is used to describe the age to which an individual can expect to live, conditional on attaining age 22 or 67.

Source: Author's calculations as described in text.

TABLE 2
Money's Worth of Annuities

	Real Annuity r = .03	Nominal Annuity r = π = .03	Real Annuity with 20-year Period Certain
MEN			
All	0.920	0.938	0.972
All Whites	0.927	0.944	0.973
All Blacks	0.862	0.886	0.964
All Hispanics	0.988	0.998	0.980
Whites: College +	0.967	0.980	0.978
Whites: HS +	0.916	0.934	0.973
Whites: < HS	0.865	0.889	0.964
Blacks: College +	0.916	0.935	0.970
Blacks: HS +	0.857	0.881	0.964
Blacks: < HS	0.800	0.830	0.955
WOMEN			
All	1.076	1.059	1.026
All Whites	1.084	1.067	1.027
All Blacks	1.022	1.011	1.018
All Hispanics	1.123	1.097	1.042
Whites: College +	1.106	1.086	1.030
Whites: HS +	1.080	1.063	1.027
Whites: < HS	1.044	1.031	1.022
Blacks: College +	1.055	1.041	1.023
Blacks: HS +	1.022	1.011	1.018
Blacks: < HS	0.976	0.970	1.011

Source: Author's calculations as described in text

TABLE 3
Annuity Equivalent Wealth Under Uniform Pricing

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.350	1.449	1.497	1.527	1.546
All Whites	1.352	1.450	1.498	1.528	1.546
All Blacks	1.328	1.437	1.488	1.522	1.542
All Hispanics	1.362	1.449	1.495	1.523	1.543
Whites: College +	1.361	1.452	1.498	1.527	1.546
Whites: HS +	1.351	1.451	1.499	1.529	1.548
Whites: < HS	1.325	1.434	1.486	1.520	1.540
Blacks: College +	1.343	1.443	1.492	1.523	1.542
Blacks: HS +	1.328	1.437	1.488	1.523	1.543
Blacks: < HS	1.296	1.415	1.472	1.511	1.534
WOMEN					
All	1.465	1.531	1.560	1.577	1.588
All Whites	1.465	1.531	1.560	1.577	1.588
All Blacks	1.459	1.529	1.560	1.577	1.588
All Hispanics	1.487	1.545	1.570	1.585	1.597
Whites: College +	1.466	1.530	1.559	1.576	1.588
Whites: HS +	1.465	1.531	1.561	1.577	1.588
Whites: < HS	1.463	1.531	1.562	1.578	1.589
Blacks: College +	1.462	1.530	1.560	1.577	1.588
Blacks: HS +	1.459	1.529	1.561	1.577	1.588
Blacks: < HS	1.453	1.526	1.560	1.577	1.587

Source: Author's calculations as described in text

BC1.g

TABLE 4
Monthly Income from $100,000 Policy if Priced Based on Group Specific Mortality

	Monthly Income	
	Men	Women
All	$675.36	$577.36
All Whites	670.42	572.90
All Blacks	720.83	608.15
All Hispanics	629.12	553.08
Whites: College +	642.73	561.83
Whites: HS +	678.25	575.13
Whites: < HS	718.40	595.19
Blacks: College +	678.22	589.01
Blacks: HS +	725.13	608.01
Blacks: < HS	776.92	636.84

Source: Author's calculations as described in text

TABLE 5
Annuity Equivalent Wealth with Actuarially Fair Risk Class Pricing

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.471	1.578	1.633	1.665	1.688
All Whites	1.462	1.568	1.622	1.653	1.675
All Blacks	1.548	1.675	1.737	1.774	1.799
All Hispanics	1.381	1.469	1.515	1.544	1.563
Whites: College +	1.409	1.504	1.553	1.582	1.601
Whites: HS +	1.479	1.587	1.643	1.674	1.697
Whites: < HS	1.539	1.666	1.728	1.766	1.791
Blacks: College +	1.470	1.578	1.635	1.668	1.691
Blacks: HS +	1.557	1.686	1.748	1.786	1.810
Blacks: < HS	1.632	1.783	1.859	1.900	1.929
WOMEN					
All	1.359	1.421	1.499	1.465	1.476
All Whites	1.349	1.410	1.437	1.454	1.464
All Blacks	1.427	1.496	1.527	1.543	1.553
All Hispanics	1.318	1.372	1.396	1.410	1.419
Whites: College +	1.321	1.380	1.408	1.424	1.435
Whites: HS +	1.354	1.416	1.443	1.460	1.470
Whites: < HS	1.399	1.466	1.495	1.512	1.521
Blacks: College +	1.384	1.449	1.478	1.495	1.505
Blacks: HS +	1.426	1.496	1.526	1.543	1.553
Blacks: < HS	1.489	1.565	1.599	1.615	1.629

Source: Author's calculations as described in text

BC2.g

TABLE 6
Annuity Equivalent Wealth Under Uniform Pricing
Nominal (Declining Real) Annuity

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.350	1.419	1.441	1.446	1.447
All Whites	1.351	1.419	1.440	1.446	1.447
All Blacks	1.339	1.419	1.441	1.447	1.446
All Hispanics	1.353	1.411	1.434	1.444	1.449
Whites: College +	1.354	1.416	1.438	1.445	1.449
Whites: HS +	1.351	1.421	1.442	1.447	1.447
Whites: < HS	1.338	1.418	1.441	1.447	1.446
Blacks: College +	1.346	1.416	1.439	1.446	1.447
Blacks: HS +	1.340	1.420	1.442	1.447	1.446
Blacks: < HS	1.321	1.414	1.439	1.449	1.445
WOMEN					
All	1.408	1.441	1.447	1.448	1.450
All Whites	1.408	1.441	1.447	1.448	1.450
All Blacks	1.409	1.444	1.477	1.448	1.451
All Hispanics	1.417	1.443	1.448	1.448	1.450
Whites: College +	1.406	1.439	1.477	1.449	1.449
Whites: HS +	1.408	1.441	1.447	1.448	1.450
Whites: < HS	1.410	1.443	1.447	1.448	1.451
Blacks: College +	1.408	1.442	1.447	1.448	1.451
Blacks: HS +	1.409	1.444	1.447	1.448	1.451
Blacks: < HS	1.409	1.445	1.446	1.448	1.451

Source: Author's calculations as described in text

BC3.g

TABLE 7
Annuity Equivalent Wealth Under Uniform Pricing
Life Annuity with 20-Year Period Certain Guarantee

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.105	1.159	1.175	1.181	1.179
All Whites	1.106	1.159	1.175	1.180	1.179
All Blacks	1.096	1.160	1.175	1.182	1.180
All Hispanics	1.105	1.152	1.170	1.177	1.178
Whites: College +	1.107	1.156	1.173	1.179	1.178
Whites: HS +	1.106	1.161	1.176	1.181	1.179
Whites: < HS	1.094	1.159	1.174	1.182	1.180
Blacks: College +	1.101	1.157	1.173	1.181	1.179
Blacks: HS +	1.096	1.161	1.175	1.182	1.181
Blacks: < HS	1.082	1.155	1.174	1.183	1.183
WOMEN					
All	1.150	1.176	1.179	1.179	1.181
All Whites	1.150	1.175	1.179	1.179	1.181
All Blacks	1.151	1.177	1.180	1.179	1.180
All Hispanics	1.157	1.177	1.179	1.179	1.181
Whites: College +	1.149	1.174	1.179	1.179	1.180
Whites: HS +	1.151	1.176	1.179	1.179	1.181
Whites: < HS	1.151	1.177	1.180	1.179	1.181
Blacks: College +	1.150	1.1776	1.180	1.179	1.181
Blacks: HS +	1.151	1.177	1.181	1.179	1.180
Blacks: < HS	1.150	1.178	1.182	1.180	1.180

Source: Author's calculations as described in text

BC8.g

TABLE 8
Annuity Equivalent Wealth Under Uniform Pricing with 8% Load Factor

	CRRA=1	CRRA=2	CRRA=3	CRRA=4	CRRA=5
MEN					
All	1.243	1.332	1.380	1.407	1.426
All Whites	1.245	1.333	1.381	1.408	1.427
All Blacks	1.223	1.320	1.373	1.402	1.421
All Hispanics	1.254	1.333	1.377	1.405	1.424
Whites: College +	1.252	1.335	1.381	1.408	1.428
Whites: HS +	1.244	1.334	1.382	1.409	1.428
Whites: < HS	1.221	1.318	1.371	1.400	1.419
Blacks: College +	1.237	1.326	1.375	1.403	1.423
Blacks: HS +	1.223	1.321	1.374	1.403	1.421
Blacks: < HS	1.194	1.303	1.359	1.392	1.411
WOMEN					
All	1.349	1.410	1.438	1.455	1.465
All Whites	1.349	1.410	1.438	1.455	1.465
All Blacks	1.343	1.410	1.438	1.455	1.466
All Hispanics	1.368	1.423	1.448	1.462	1.470
Whites: College +	1.349	1.409	1.437	1.453	1.464
Whites: HS +	1.349	1.411	1.438	1.455	1.466
Whites: < HS	1.347	1.412	1.439	1.456	1.467
Blacks: College +	1.346	1.410	1.438	1.455	1.466
Blacks: HS +	1.343	1.410	1.438	1.455	1.466
Blacks: < HS	1.337	1.408	1.437	1.455	1.465

Source: Author's calculations as described in text

BC4.g