Unequal Societies: Income Distribution and the Social Contract

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This paper develops a theory of inequality and the social contract aiming to explain how countries with similar economic and political "fundamentals" can sustain such different systems of social insurance, fiscal redistribution, and education finance as those of the United States and Western Europe. With imperfect credit and insurance markets some redistributive policies can improve ex ante welfare, and this implies that their political support tends to decrease with inequality. Conversely, with credit constraints, lower redistribution translates into more persistent inequality; hence the potential for multiple steady states, with mutually reinforcing high inequality and low redistribution, or vice versa. (*JEL* D31, E62, P16, O41, I22)

The social contract varies considerably across nations. Some have low tax rates, others a steeply progressive fiscal system. Many countries have made the financing of education and health insurance the responsibility of the state. Some, notably the United States, have left it in large part to families, local communities, and employers. The extent of implicit redistribution through labor-market policies or the mix of public goods also shows persistent differences. Can these societal choices be explained without appealing to exogenous differences in tastes, technologies, or political systems?

Adding to the puzzle is the fact that redistribution is often correlated with income inequality in just the opposite way than predicted by standard politico-economic theory: among industrial democracies the more unequal ones tend to redistribute less, not more. The archetypal case is that of the United States versus Western Europe, but the observation holds within the latter group as well; thus Scandinavian countries are both the most equal and the most redistributive. In the developing world a similar contrast is found in the incidence of public education and health services, which is much more egalitarian in East Asia than in Latin America (e.g., South Korea versus Brazil). Turning finally to time trends, it is rather striking that the welfare state is being cut back in most industrial democracies at the same time that an unprecedented rise in inequality is occurring.

The aim of this paper is to develop a joint theory of inequality and the social contract which can contribute to resolving some of these puzzles. In the process, it also seeks to reconcile certain empirical findings of the recent literature on political economy and growth. Several authors, such as Alberto F. Alesina and Dani Rodrik (1994) or Torsten Persson and Guido Tabellini (1994), have documented a negative relationship between initial disparities of income or wealth and subsequent aggregate growth. The proposed explanation is that greater inequality translates into a poorer median voter relative to the country's mean income, as in Alan H. Meltzer and Scott F. Richard (1981). This leads to increased pressure for redistributive policies, which in turn reduce incentives for the accumulation of physical and human capital. The cross-country data, however, do not seem very supportive of this explanation. Roberto Perotti (1994, 1996), and most

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of the other studies reviewed in Bénabou (1996c), find no relationship between inequality and the share of transfers or government expenditures in GDP. Among advanced countries the effect is actually negative, as suggested by the above examples (Francisco Rodriguez, 1998).[1] As to the effect of transfers on growth, most studies yield estimates which are in fact significantly positive.

The point of departure for this paper is a rather different view of both the role of the state and the workings of the political process. When capital and insurance markets are imperfect, a variety of policies which redistribute wealth from richer to poorer agents can have a *positive* net effect on aggregate output, growth, or more generally *ex ante* welfare. Examples considered here will include social insurance through progressive taxes and transfers, state funding of public education, and residential integration. Net efficiency gains lead to very different political economy consequences from those of standard models: popular support for such redistributive policies *decreases with inequality,* at least over some range. Intuitively, efficient redistributions meet with a wide consensus in a fairly homogeneous society but face strong opposition in an unequal one. Conversely, if agents engage in any type of investment, capital market imperfections imply that lower redistribution translates into more persistent inequality. The combination of these two mechanisms creates the potential for *multiple steady states:* mutually reinforcing high inequality and low redistribution, or low inequality and high redistribution. Temporary shocks to the distribution of income or the political system can then have permanent effects.

I formalize these ideas in a stochastic growth model with incomplete asset markets and heterogeneous agents who vote over redistributive policies, whether fiscal or educational. In the short run, redistribution is shown to be *U-shaped* with respect to inequality; in the long run they are *negatively* correlated across steady states. There are two important ingredients in the analysis. The first one is that redistribution enhance *ex ante* welfare, at least up to a point. I thus examine policies which reduce the variance and possibly increase the mean of family income, by providing insurance against idiosyncratic shocks and relaxing credit constraints. The second one is a simple extension of the standard voting model, reflecting the fact that some groups have more influence in the political process than others. I present extensive evidence that the propensities to vote, give political contributions, work on campaigns, and participate in most forms of political activity rise with income and education. In the model the pivotal agent is richer than the median, but need not be richer than the mean. It should be emphasized that I do not appeal to variations in political rights or participation to explain countries' different societal choices: this parameter is kept fixed across steady states. In the comparative statics analysis I vary the efficiency costs and benefits of redistribution on the one hand (via the elasticity of labor supply and the degree of risk aversion), the political system on the other, so as to identify their respective contributions to the results. In particular, I show that there exists a *critical level* for the gain in *ex ante* welfare from a redistributive policy, such that: (i) below this threshold, no allocation of political influence can sustain more than a single social contract; (ii) above this threshold, multiple steady states arise provided the political weight of the rich is neither too large nor too small.

When two "unequal societies" arise from common fundamentals, they cannot be Pareto ranked. As to macroeconomic performance, the trade-off between tax distortions and liquidity-constraint effects allows for two interesting scenarios. One, termed "growth-enhancing redistributions," is consistent with the positive coefficients of transfers in growth regressions, as well as the contributions of education and land policies in East Asian and Latin American countries to their respective developments (or lack thereof). The other, termed "eurosclerosis," explains how European voters can choose to sacrifice more employment and growth to social insurance than their American counterparts, even though both populations have the same basic preferences. Another prediction of the

[1] Using panel data for 20 OECD countries and controlling for national income, population, and the age distribution, Rodriguez finds that pretax inequality has a significantly negative effect on every major category of social transfers as a fraction of GDP, as well as on the capital tax rate.

model is that, depending on their source, exogenous shocks to income inequality will bring about sharply different evolutions of the social contract. Thus, an increased variability of sectoral shocks will lead to an expansion of the social safety net, while a surge in immigration may prompt large-scale cutbacks.

Some methodological features of the model may also be worth mentioning. The first is its analytical tractability. Individual transitions are linear, reflecting the absence of nonconvexities; yet there is multiplicity. The distribution of wealth remains lognormal, and closed-form solutions are obtained. The second is the progressivity of the redistributive schemes over which agents vote. The third is the intuitive formalization of political influence. These modelling devices could be useful in other settings.

The paper is related to three strands of literature. The first one emphasizes the political economy of redistribution (Giuseppe Bertola, 1993; Perotti, 1993; Gilles Saint-Paul and Thierry Verdier, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994, 1996). The second one is concerned with the financing and accumulation of human capital (Glenn C. Loury, 1981; Gerhard Glomm and B. Ravikumar, 1992; Oded Galor and Joseph Zeira, 1993; Bénabou, 1996b; Steven N. Durlauf, 1996a; Mark Gradstein and Moshe Justman, 1997; Raquel Fernández and Richard Rogerson, 1998). The third one stresses the wealth and incentive constraints which bear on entrepreneurial investment (Abhijit Banerjee and Andrew Newman, 1993; Philippe Aghion and Patrick Bolton, 1997; Thomas Piketty, 1997). Most directly related are the models in Bénabou (1996b, c), upon which I build, and the paper by Saint-Paul (1994), which identifies another mechanism through which capital market imperfections can lead to multiple politico-economic regimes.[2] A rather different explanation for international differences in redistribution is provided in Piketty (1995), where agents' imperfect learning of the social mobility process allows different views of the equity-efficiency trade-off to persist in the long run.[3]

The paper is organized as follows. Section I explains the main ideas using the simplest possible setup, which treats the aggregate impact of redistribution as exogenous. Section II presents the actual economic model, and Section III the political mechanism. Section IV focuses on an endowment economy to study whether social insurance will be more or less extensive in a more unequal country. Section V solves the full model with endogenous wealth dynamics. The range of economic and political "fundamentals" which allow multiple steady states is characterized, and the growth rates under alternative regimes are compared. Section VI recasts the model so as to explain differences in countries' systems of education finance. Section VII discusses other applications such as altruism, residential integration, and the mix of public goods. Section VIII concludes. All proofs are gathered in the Appendix.

I. A Simplified Presentation of the Main Ideas

As a prelude to the actual model, I present in this section a very stylized reduced form which provides a shortcut to the main intuitions and results.

A. *The Standard View*

Let there be a continuum of agents, $i \in [0, 1]$, with lognormally distributed endowments: $\ln y^i \sim \mathcal{N}(m, \Delta^2)$. The lognormal is a good approximation of empirical income distributions, leads to tractable results, and allows for an unambiguous definition of inequality, as increases in Δ^2 shift the Lorenz curve outward. This variance also measures the distance between median and per capita income: $m = \ln y - \Delta^2/2$, where $y \equiv E[y^i]$. Suppose now that

[2] Saint-Paul points out that increases in inequality whose adverse impact is concentrated in the lower tail of the income distribution may be accompanied by a rise in median income, relative to the mean, so that the politically decisive middle class will reduce its transfers to the poor. If exit from poverty requires some investment, credit constraints may then lead to multiple steady states: a large underclass which persists due to low redistribution, or one which is kept small by significant transfers.

[3] Piketty's mechanism is similar to a collective form of the bandit problem. Because individual experimentation is costly, agents never fully learn the extent to which income is affected by effort rather than predetermined by social origins. The citizens of otherwise identical countries may then end up with different distributions of beliefs over the disincentive effects of redistribution.

agents are faced with the choice between two stylized policies:

($\mathcal{P}$) *laissez-faire:* each consumes his own endowment, $c^i = y^i$, for all i.

($\hat{\mathcal{P}}$) *complete redistribution:* resources are pooled, and everyone consumes $c^i = y$.

In this benchmark case where redistribution has no aggregate impact, how many people are in favor of it? Clearly, all those with endowment below the mean, i.e., a proportion

$$(1) \qquad p = \Phi\left(\frac{\Delta^2/2}{\Delta}\right) = \Phi\left(\frac{\Delta}{2}\right),$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal. Because the income distribution is right-skewed, the median is below the mean, $p > 1/2$. A strict majority rule would thus predict that redistribution should always take place. In reality the poor vote with lower probability than the rich and, to some extent, money buys political influence. Therefore the relevant threshold for redistribution to occur may not be $p^* = 50$ percent $= \Phi(0)$, but $p^* = \Phi(\lambda)$, $\lambda > 0$. For instance if the π poorest agents never vote, $\Phi(\lambda) = (1 + \pi)/2$.[4]

What is important and robust in (1) is therefore not the level effect, $p > 1/2$, but the comparative statics, $\partial p/\partial \Delta > 0$: in a more unequal society there is greater political support for redistribution. For *any* degree of bias λ in the voting system, positive or negative, the likelihood that redistribution takes place increases with inequality—or more specifically, skewness.

This result is only reinforced under the standard assumption that redistribution entails some deadweight loss (endogenized later on), reducing available resources from y to ye^{-B}, $B > 0$. Given the choice between laissez-faire and sharing this reduced pie, the extent of political support for the *inefficient redistribution* is:

$$(2) \quad p = \Phi\left(\frac{-B + \Delta^2/2}{\Delta}\right) = \Phi\left(-\frac{B}{\Delta} + \frac{\Delta}{2}\right).$$

Note that now $p \gtrless 1/2$ but $\partial p/\partial \Delta$ is even more positive than before. As inequality increases, so does the likelihood that a policy which reduces aggregate income gets implemented. This is, in essence, the mechanism by which inequality reduces growth in models such as Alesina and Rodrik (1994), Persson and Tabellini (1994), or some cases of Bertola (1993) and Perotti (1993).[5] The idea that distributional conflict hampers economic performance appears to be reasonably well supported by the evidence: a number of studies have confirmed Persson and Tabellini's and Alesina and Rodrik's findings of a negative effect of inequality on growth.[6] This correlation, however, does not seem to arise through increased redistribution. Perotti (1994, 1996), Philip Keefer and Stephen F. Knack (1995), and Peter H. Lindert (1996) find no relationship between the income share of the middle class (which corresponds to the median voter) and any tax rate or share of government transfers in GDP. George R. G. Clarke (1992) finds no correlation between any measure of inequality and government consumption. As casual empiricism suggests, more unequal countries do not redistribute more. Among advanced nations, they typically redistribute *less* (Rodriguez, 1998). Furthermore, the coefficients on transfers in growth regressions are most often significantly positive: see, among others, Shantarayan Devarajan et al. (1993), Lindert (1996), Xavier Sala-i-Martin (1996), and especially Perotti (1994, 1996), who controls for the endogeneity of redistribution. While none of these findings should be viewed as definitive evidence, altogether they do suggest that something important may be missing from the traditional story.

B. *Efficiency Gains and Redistribution: The Static Case*

In a world of incomplete insurance and loan markets some policies with redistributive

[4] Section III will formalize in more detail—as well as present extensive evidence on—the influence of income and human wealth on most forms of political activity.

[5] Naturally, this simple reduced form fails to capture the richness of the original models.

[6] See Bénabou (1996c) for a survey. As with nearly all growth regressions this finding is robust to some changes in specification but not to others. This caveat should be kept in mind, but in any case such a correlation is not essential to my results, which primarily involve inequality and redistribution. Thus, Section V, subsection B, will identify parameter configurations such that inequality and growth are positively, or negatively, correlated across steady states.

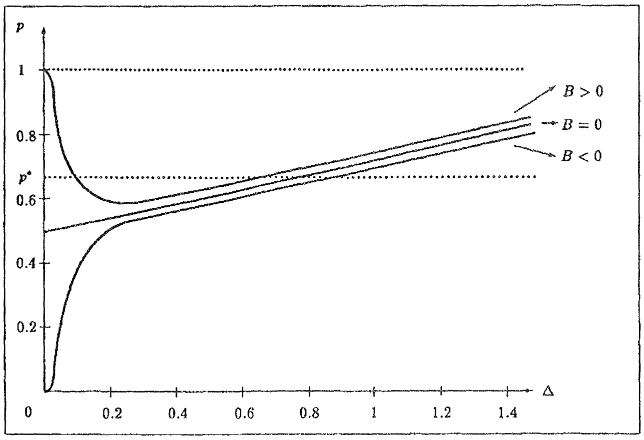

FIGURE 1. INEQUALITY AND POLITICAL SUPPORT FOR REDISTRIBUTION

Note: B = 5 percent, 0, −5 percent.

features can have a positive effect on total welfare, and even output (as the evidence on transfers and growth may suggest). *Ex ante* welfare gains, in turn, imply a political support that varies with inequality in a radically different way from the traditional one. Indeed, suppose that by redistributing resources agents achieve increased efficiency, so that each gets to consume ye^B. For now I continue to take $B > 0$ as exogenous, but later on I shall derive it from a variety of channels: insurance, altruism, or credit constraints on the accumulation of human and physical capital. The fraction of people who support an *efficient redistribution* is:

$$(3) \quad p = \Phi\left(\frac{B + \Delta^2/2}{\Delta}\right) = \Phi\left(\frac{B}{\Delta} + \frac{\Delta}{2}\right).$$

It is of course always higher than (1) and (2), but the important point is the one illustrated on Figure 1.

PROPOSITION 1: *When a redistributive policy generates gains in ex ante efficiency, political support for it initially declines with inequality. In the present framework, the relationship is U-shaped.*

The intuition is simple: when dispersion is relatively small compared to the average gain, there is near-unanimous support for the policy. As inequality rises, the proportion of those who stand to lose from the redistribution increases. At high enough levels of inequality, however, the standard skewness effect eventually dominates: there are so many poor that they impose redistribution no matter what its aggregate impact may be. There is thus *no monotonic relationship* between income inequality or the relative position of the median agent (both measured here by Δ^2) and the likelihood of redistribution.[7]

To relate the extent of popular support for a policy to actual outcomes one needs to specify the mechanism through which preferences are aggregated. I shall continue to assume that redistribution occurs if p exceeds a threshold $p^* \equiv \Phi(\lambda)$, where λ reflects the degree to which financial or human wealth contributes to political influence. More generally, the probability of implementation could be some increasing function of p. Figure 1 shows that *only if* redistribution's aggregate impact B is positive can the

[7] The fact that inequality affects political support for redistribution in opposite ways, depending on the sign of its aggregate impact, is essentially independent of distributional assumptions. For any symmetric distribution $F(x)$ with mean μ, a symmetric, mean-preserving spread leads to a decline in popular support $p = F(\mu + B)$ for all $B > 0$, and an increase for all $B < 0$. With lognormally distributed wealth, a rise in Δ combines this general effect of dispersion with a fall in $m = \mu - \Delta^2/2$ due to increased skewness; hence the U shape. These properties remain true when B is a function of Δ, as long as $\Delta B'(\Delta)/B(\Delta) < 1$. Such will be the case when B is endogenized later on.

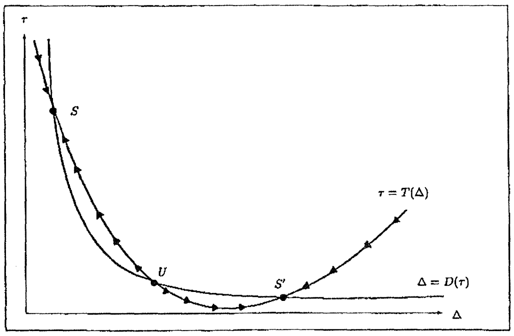

FIGURE 2. DYNAMICS AND STEADY STATES

policy be abandoned as the result of greater inequality—no matter how biased the political system might be. Conversely, a positive λ is needed for the political outcome to reflect the drop in popular support. The full model will confirm the joint importance of *efficiency gains* and *wealth bias* in shaping a declining relationship between inequality and redistribution. The arguments seen here for a zero-one policy choice will then apply to marginal changes in the tax rate, i.e., to every electoral contest between τ and $\tau + d\tau$.

Finally, consider the dynamic implications of Proposition 1. A society which starts with enough wealth disparity to find itself below p^* on the U-shaped curve of Figure 1 will not implement the redistributive policy; as a result, high inequality will persist into the next period or generation. Conversely, low inequality creates wide political support for efficient policies which prevent disparities from growing. The *dynamic feedback* operates whenever some form of investment is credit constrained, so that current resources affect future earnings. Thus the same type of market imperfections which can give rise to a (partly) decreasing relationship between inequality and transfers also provide the second ingredient required for *multiple steady states.* These dynamics are represented on Figure 2 (which will be formally derived later on), for a continuous rate of redistribution $\tau \in [0, 1]$. The U-shaped curve $\tau = T(\Delta)$ is similar to that of Figure 1, while the declining $\Delta = D(\tau)$ locus arises from the accumulation mechanism.

The main part of the paper, to which I now turn, will show that all the intuitions obtained in this section carry over to a fully specified intertemporal model of individual behavior and collective choice, where the welfare gains and losses from redistribution are endogenous.

II. Incomplete Asset Markets and Progressive Taxation

A. *Technology, Preferences, and Decisions*

The economy is populated by overlapping-generations families, $i \in I \equiv [0, 1]$. In generation t, adult i combines his (human or physical) capital endowment k_t^i with effort l_t^i to produce output, subject to an independently and identically distributed (i.i.d.) productivity shock z_t^i:

$$y_t^i = z_t^i (k_t^i)^{\gamma} (l_t^i)^{\delta}. \tag{4}$$

Taxes and transfers, specified below, transform this gross income y_t^i into a disposable income $\hat{y}_t^i$ which finances both the adult's consumption, c_t^i, and his investment or educational bequest, e_t^i:

$$\hat{y}_t^i = c_t^i + e_t^i \tag{5}$$

$$k_{t+1}^i = \kappa \xi_{t+1}^i (k_t^i)^{\alpha} (e_t^i)^{\beta}. \tag{6}$$

Capital depreciates geometrically at the rate $1 - \alpha \geq \beta\gamma$, and investment is subject to i.i.d. productivity shocks ξ_t^i. There is no loan market for financing individual investment or educational projects (e.g., children cannot be held responsible for the debts of their parents) and no insurance or securities market where the idiosyncratic risks z_t^i and ξ_{t+1}^i could be diversified away. These are extreme forms of market incompleteness, but all that really matters is that there be some imperfections.[8] Both shocks are assumed to be lognormal with mean one, and initial endowments lognormally distributed across families: thus $\ln z_t^i \sim \mathcal{N}(-v^2/2, v^2)$, $\ln \xi_t^i \sim \mathcal{N}(-w^2/2, w^2)$ and $\ln k_0^i \sim \mathcal{N}(m_0, \Delta_0^2)$.

[8] Perotti (1994) provides evidence that credit-market frictions reduce aggregate investment, especially where the income share of the bottom 40 percent is low. Additional evidence on asset-market incompleteness as a constraint on investment decisions in education and farming is discussed in Bénabou (1996c).

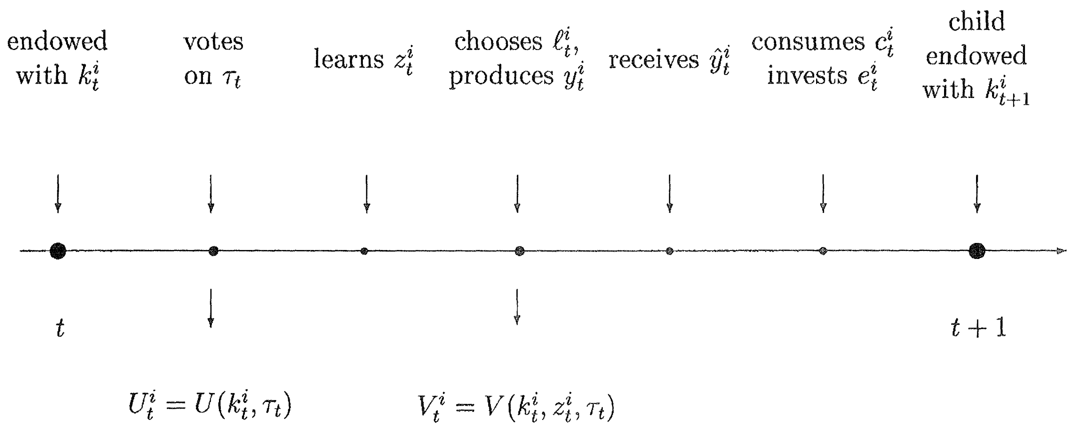

FIGURE 3. PREFERENCES AND THE TIMING OF DECISIONS

Adults have preferences defined over their own consumption and effort, as well as their child's endowment of capital. Following David M. Kreps and Evan L. Porteus (1979), Larry G. Epstein and Stanley E. Zin (1989), and Philippe Weil (1990), these preferences are defined recursively over an individual's lifetime; see Figure 3. Thus, upon discovering his productivity, z_t^i, agent i chooses effort, consumption, and savings so as to maximize:

$$\text{(7)}\quad \ln V_t^i \equiv \max_{l_t^i, c_t^i} \{(1-\rho)[\ln c_t^i - (l_t^i)^\eta] + \rho \ln ((E_t[(k_{t+1}^i)^{r'} | k_t^i, z_t^i])^{1/r'})\}.$$

The disutility of effort is measured by $\eta > 1$, which corresponds to an intertemporal elasticity of labor supply equal to $1/(\eta - 1)$. The discount factor ρ defines the relative weights of the adult's own felicity and of his bequest motive, while his (relative) risk aversion with respect to the child's endowment k_{t+1}^i is $1 - r'$. At the beginning of period t, however, when evaluating and voting over redistributive policies, the agent has not yet learned z_t^i. In other words, he knows his type (k_t^i, z_t^i) *imperfectly*.[9] The resulting uncertainty over his *ex post* utility level V_t^i is reflected in his *ex ante* preferences,

$$\text{(8)}\quad U_t^i \equiv \ln (E_t[(V_t^i)^r | k_t^i]^{1/r}),$$

with a risk-aversion coefficient of $1 - r$. When $r = 0$ preferences are time separable and $1/(1 - r)$ coincides with the intertemporal elasticity of substitution in consumption, which by (7) is fixed at one.[10] The more general specification allows $1 - r$ to parametrize the insurance value of redistributive policies, just as the labor-supply elasticity $1/(\eta - 1)$ parametrizes the effort distortions. Varying these parameters will make it possible to study how the set of politico-economic equilibria is shaped by the *costs and benefits of redistribution*—the central issue of this paper. Finally, it should be noted that while I have assumed overlapping genera-

[9] Therein lies the crucial difference between ξ_t^i and z_t^i, rather than in the fact that one affects human capital and the other production; the latter roles could be switched. Also, because the policies which I shall consider provide no insurance against ξ_{t+1}^i (only against z_t^i), the value of $1 - r'$ will turn out not to play an important role in the analysis. It could therefore be set (for instance) to zero, to one, or to $1 - r$, which is the more essential risk-aversion parameter defined in (8) below.

[10] This last assumption is made for analytical tractability, as in much of the literature on income distribution dynamics [e.g., Glomm and Ravikumar (1992), Banerjee and Newman (1993), Galor and Zeira (1993), Saint-Paul and Verdier (1993), Aghion and Bolton (1997), or Gradstein and Justman (1997)].

tions with "imperfect" altruism, many of paper's results can also be derived with infinitely lived agents.[11]

B. *Fiscal Policy*

Taxes and transfers map agent i's market income y_t^i into a disposable a income $\hat{y}_t^i$, according to the following scheme:

$$\hat{y}_t^i \equiv (y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}. \tag{9}$$

The break-even level $\tilde{y}_t$ is determined by the government's budget constraint: net transfers must sum to zero or, denoting per capita income by y_t:

$$\int_0^1 (y_t^i)^{1-\tau_t}(\tilde{y}_t)^{\tau_t}\, di = y_t. \tag{10}$$

The elasticity τ_t of posttax income measures the degree of *progressivity* (or regressivity) of fiscal policy.[12] An alternative interpretation is that of wage compression through labor-market institutions favorable to workers with relatively low skills. Confiscatory rates $\tau_t > 1$ must be excluded as not incentive compatible, but nothing in principle prevents a regressive tax, so I shall allow it. Restricting τ_t to be nonnegative would require dealing with corner solutions but would not change the nature of any result.

PROPOSITION 2: *Given a tax rate τ_t, agents in generation t choose a common labor supply and savings rate:*

$$l_t = \chi(1-\tau_t)^{1/\eta}$$

$$e_t^i = s\hat{y}_t^i$$

where $\chi^\eta \equiv (\delta/\eta)(1-\rho+\rho\beta)/(1-\rho)$ and $s \equiv \rho\beta/(1-\rho+\rho\beta)$.

Because taxes are progressive rather than merely proportional they affect effort, $l_t = l(\tau_t)$, in spite of the fact that utility is logarithmic in consumption.[13] I will refer from here on to $1/\eta$ as "the" elasticity of labor supply.[14]

C. *Redistribution and Accumulation*

Given Proposition 2, and substituting (9) into (6), capital accumulation simplifies to:

(11)

$$\ln k_{t+1}^i = \ln \xi_{t+1}^i + \beta(1-\tau_t)\ln z_t^i + \ln \kappa + \beta \ln s + (\alpha + \beta\gamma(1-\tau_t))\ln k_t^i + \beta\delta(1-\tau_t)\ln l_t + \beta\tau_t \ln \tilde{y}_t.$$

Due to the symmetry of agents' effort and savings decisions, wealth and income remain lognormally distributed over time. If $\ln k_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, the government's budget constraint (10) easily yields the break-even point of the redistributive scheme (see the Appendix):

$$\ln \tilde{y}_t = \gamma m_t + \delta \ln l_t + (2-\tau_t)\gamma^2\Delta_t^2/2 + (1-\tau_t)\vartheta^2/2. \tag{12}$$

[11] See Bénabou (1996a, 1999). The infinite-horizon version of the preferences used here is obtained by replacing k_{t+1}^i with V_{t+1}^i in (7), with $r' = r$ and (8) unchanged.

[12] When $\tau_t > 0$ the marginal rate rises with pretax income, and agents with average income are made better off: $\tilde{y}_t > y_t$. Measuring progressivity by the (local) elasticity of after-tax to pretax income was first proposed by Richard A. Musgrave and Tun Thin (1948). Ulf Jakobsson (1976) and Nanak C. Kakwani (1977) showed this to be the "right" measure of equalization: the posttax distribution induced by a fiscal scheme Lorenz-dominates the one induced by another (for *all* pretax distributions), if and only if the first scheme's elasticity is everywhere smaller. A "constant residual progression" scheme similar to (9) turns out to have been used in a couple of earlier but static models of insurance or risk taking (Martin S. Feldstein, 1969; S. M. Kanbur, 1979; Mats Persson, 1983).

[13] They would also affect savings, were it not for adults' simple bequest motive. Even with infinitely lived agents, however, the savings distortion can be fully offset by a balanced-budget combination of consumption taxes and investment subsidies; moreover, this can be shown to be Pareto optimal (Bénabou, 1996a, 1999). In any case, one distortion is enough to demonstrate how the trade-off between the costs and benefits of redistribution shapes the range of politico-economic equilibria.

[14] It is indeed the uncompensated elasticity to the net-of-tax rate $1-\tau_t$, and varies monotonically with the usual intertemporal elasticity of substitution for proportional variations in the real wage, $1/(\eta-1)$.

From (11), we then obtain two simple difference equations which govern the evolution of the economy:

$$m_{t+1} = (\alpha + \beta\gamma)m_t + \beta\delta \ln l_t + \beta\tau_t(2 - \tau_t)(\gamma^2\Delta_t^2 + v^2)/2 + \ln(\kappa s^\beta) - (w^2 + \beta v^2)/2 \tag{13}$$

$$\Delta_{t+1}^2 = (\alpha + \beta\gamma(1 - \tau_t))^2\Delta_t^2 + \beta^2(1 - \tau_t)^2 v^2 + w^2. \tag{14}$$

The effect of redistribution on the dynamics of inequality is clear: the progressivity rate τ_t determines the *persistence of family wealth,* $\alpha + \beta\gamma(1 - \tau_t)$. The impact on the dynamics of aggregate income is more complex, as it involves a trade-off between labor supply and credit-constraint effects:

PROPOSITION 3: *The distribution of pretax income at time t is* $\ln y_t^i \sim \mathcal{N}(\gamma m_t + \delta \ln l_t - v^2/2,\ \gamma^2\Delta_t^2 + v^2)$, *where* m_t *and* Δ_t^2 *evolve according to the linear difference equations (13)–(14) and* $l_t = \chi(1 - \tau_t)^{1/\eta}$. *The growth rate of per capita income is:*

$$\ln(y_{t+1}/y_t) = \ln\tilde{\kappa} - (1 - \alpha - \beta\gamma)\ln y_t + \delta(\ln l_{t+1} - \alpha \ln l_t) - \mathcal{L}_v(\tau_t)v^2/2 - \mathcal{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2, \tag{15}$$

where $\ln\tilde{\kappa} \equiv \gamma(\ln\kappa + \beta \ln s) - \gamma(1 - \gamma)w^2/2$ *is a constant and*

$$\mathcal{L}_v(\tau) \equiv \beta\gamma(1 - \beta\gamma)(1 - \tau)^2 > 0,$$

$$\mathcal{L}_\Delta(\tau) \equiv \alpha + \beta\gamma(1 - \tau)^2 - (\alpha + \beta\gamma(1 - \tau))^2 > 0.$$

The term in $-\ln y_t$ reflects the standard convergence effect; it disappears under constant aggregate returns, namely when $\alpha + \beta\gamma = 1$ or when κ is replaced by an appropriate spillover κ_t (see Section V, subsection B). The next term, capturing the effects of labor supply on accumulation, is also of a "representative agent" nature. The terms in $\mathcal{L}_v(\tau)$ and $\mathcal{L}_\Delta(\tau)$, on the other hand, represent growth losses specific to the heterogenous agent economy with imperfect credit markets. Suppose first that adults in generation t are *ex ante* identical ($\Delta_t^2 = 0$). Everyone then faces the same accumulation technology, with concavity (decreasing returns) $\beta\gamma(1 - \beta\gamma)$. The shocks z_t^i generate *ex post* income disparities (partially offset by redistribution), which credit constraints translate into inefficient variations in investment, reducing overall growth by $\mathcal{L}_v(\tau_t)v^2/2$. Consider now disparities in initial endowments, $\gamma^2\Delta_t^2$. When $\alpha = 0$ these have the same effect as income shocks: $\mathcal{L}_v \equiv \mathcal{L}_\Delta$. The marginal return to investment is higher for the poor than for the rich, because they are more severely liquidity constrained. When $\alpha > 0$, however, the preexisting capital stocks k_t^i represent *complementary inputs* which generate differential returns to investment, and thereby reduce the desirability of equalizing resources. Thus $\mathcal{L}_\Delta(\tau)$ is minimized for $\tau = (1 - \alpha - \beta\gamma)/(1 - \beta\gamma)$, which decreases with α.

D. *Individual Welfare*

I now turn from the evolution of the economy as a whole to individuals' evaluations of alternative policies. Given the optimal labor supply and savings responses to a tax rate τ_t, (7) gives agent i's *ex post* welfare V_t^i for any productivity realization z_t^i. Since he must vote before learning z_t^i, his preferences over τ_t are then defined by the *ex ante* utility U_t^i, according to (8).

PROPOSITION 4: *Given a rate of fiscal progressivity* τ_t, *agent i's intertemporal welfare is:*

$$U_t^i = \bar{u}_t + A(\tau_t)(\ln k_t^i - m_t) + C(\tau_t) - (1 - \rho + \rho\beta) \times (1 - \tau_t)^2(\gamma^2\Delta_t^2 + Bv^2)/2, \tag{16}$$

where $\bar{u}_t$ is independent of the policy τ_t and:

$$(17)\quad A(\tau) \equiv \rho\alpha + (1 - \rho + \rho\beta)\gamma(1 - \tau),$$

$$(18)\quad C(\tau) \equiv (1 - \rho)(\delta \ln l(\tau) - l(\tau)^{\eta}) + \rho\beta\delta \ln l(\tau),$$

$$(19)\quad B \equiv 1 - r + \rho r(1 - \beta) \geq 0.$$

The first term in U_t^i depends only on the state variables m_t and Δ_t^2 and on the (endogenous but constant) investment rate s. The second term, which disappears through aggregation, makes clear the *redistributive* effects of tax policy, including its impact on the persistence of social positions, $\alpha + \beta\gamma(1 - \tau)$. The last two terms represent the *aggregate welfare cost* and *aggregate welfare benefit* of a progressivity rate τ_t. Thus $C(\tau_t)$, which is maximized for $\tau_t = 0$, reflects the distortions in labor supply entailed by such a policy. Conversely, the term $-(1 - \tau_t)^2(\gamma^2\Delta_t^2 + Bv^2)$, which is maximized for $\tau_t = 1$, embodies the efficiency gains which arise from better insurance and the redistribution of resources from low to high marginal-product investments. Note that B is now *endogenous,* and monotonically related to *risk aversion,* $1 - r$. More generally, by varying $1 - r$, β, v^2, and $1/\eta$, the net *ex ante* efficiency gain, $(1 - \rho + \rho\beta)(1 - \tau)^2Bv^2/2 - C(\tau)$, can be made arbitrarily large or small relative to preexisting income inequality, $\gamma^2\Delta_t^2$.

To make the role of market incompleteness more explicit I consider again each source of heterogeneity in turn. By (16), idiosyncratic uncertainty lowers everyone's utility by $(1 - \rho + \rho\beta)B(1 - \tau)^2v^2/2$. When $\beta = 1$ this simplifies to $(1 - r)(1 - \tau)^2v^2/2$: with constant returns, the *ex ante* value of redistribution stems from the insurance it provides. In general, it also contributes to efficiency through the relaxation of credit constraints. This is best seen with risk-neutral parents who care only about their offspring: when $r = \rho = 1$ the utility loss is $\beta(1 - \beta)(1 - \tau)^2v^2/2$, which is the shortfall in expected (and aggregate) bequests resulting from idiosyncratic resource shocks and a concave investment technology. Turning now to preexisting inequality, the loss in aggregate welfare is $(1 - \rho + \rho\beta)(1 - \tau)^2\gamma^2\Delta_t^2/2$; it embodies two effects, only one of which is due to market incompleteness. First, reallocating investment resources towards the poor again increases the growth rate of total wealth, $\ln(k_{t+1}/k_t)$. Second, there is the standard effect of concave (logarithmic) utility functions, whereby average welfare increases when individual consumptions (of c_t^i and k_{t+1}^i) are distributed more equally.[15] Equivalently in this model, it captures the effect of *skewness,* as in Section I: the median agent, who is poorer than average, gains when consumption and bequests are redistributed progressively.

III. The Political System

I now turn to the determination of the equilibrium policy. Each generation chooses, before the individual productivity shocks z_t^i are realized, the rate of fiscal progressivity τ_t to which it will be subject. Agent i's ideal policy would thus be to maximize U_t^i.[16] These individual preferences are aggregated through a political process in which some groups have more influence than others.

[15] One can rewrite $(1 - \rho + \rho\beta)\gamma^2(1 - \tau)^2$ as $\rho[\beta\gamma^2(1 - \tau)^2 - (\alpha + \beta\gamma(1 - \tau))^2] + (1 - \rho)\gamma^2(1 - \tau)^2 + \rho(\alpha + \beta\gamma(1 - \tau))^2 = -\rho \ln(k_{t+1}/\ln k_t) + (1 - \rho)\mathrm{var}_{j\in I}[\ln c_t^j] + \rho\mathrm{var}_{j\in I}[\ln k_{t+1}^j] + \mu_t$, where $\mathrm{var}_{j\in I}[\,\cdot\,]$ denotes a cross-sectional variance, μ_t is independent of τ_t, and I have set $v = w = \delta = 0$ for notational simplicity.

[16] Due to the overlapping-generations structure, no intertemporal strategic considerations are involved. Infinite horizons, by contrast, would generate a dynamic game where voters try to influence future political outcomes τ_{t+k} by altering the evolution of the wealth distribution Δ_{t+k}^2 through their choice of τ_t [see (14)]. This problem is notoriously intractable, so the standard practice is to assume that voters are "myopic," either ignoring their influence on future outcomes or—as here—not caring about it due to a limited bequest motive. Notable exceptions are Saint-Paul (1994) and Gene M. Grossman and Elhanan Helpman (1996). Alternatively, Per Krusell et al. (1997) look for numerical solutions. In Bénabou (1996a) I considered a different form of political myopia and obtained results similar to those presented here. In short, agents with fully dynastic preferences choose *a constitution* (namely a constant sequence $\{\tau_{t+k} = \tau_t\}_{k=0}^{\infty}$), ignoring the fact that future generations may revise it. In any steady state, however, $[\Delta_t^2 = \Delta_{t+1}^2$ in (14)], every generation, given the same choice set as its predecessors, validates the existing social contract.

A. *Preferred Policies*

I first consider the simpler case where labor supply is inelastic, $1/\eta = 0$. The utility function U_t^i is then quadratic in τ_t, and maximized at:

$$(20)\quad \frac{1}{1-\tau_t^i} = \frac{\gamma^2\Delta_t^2 + Bv^2}{\gamma \max\{\ln k_t^i - m_t, 0\}}.$$

Voters below the median desire the maximum feasible tax rate, $\tau_t^i = 1$, which is also the *ex ante* efficient one in the absence of distortions. Voters above the median desire a tax rate $\tau_t^i < 1$ which decreases with their initial wealth and increases with the variance of productivity shocks, for both insurance and investment reasons (concavity of preferences and concavity of the technology).

With endogenous labor supply, complete redistribution is never chosen, as it would lead to zero effort and output. Agent i's desired policy is given by the first-order condition $\partial U_t^i/\partial\tau = 0$, or:

$$(21)\quad (1-\tau)(\gamma^2\Delta_t^2 + Bv^2) - \gamma(\ln k_t^i - m_t) - \frac{\delta}{\eta}\left(\frac{\tau}{1-\tau}\right) = 0.$$

This quadratic equation always has a unique solution less than 1, which will be denoted τ_t^i.

PROPOSITION 5: *Each agent's utility U_t^i is strictly concave in the policy τ_t. His preferred tax rate τ_t^i decreases with his wealth k_t^i and increases with Bv^2. Finally, $|\tau_t^i|$ decreases with the labor-supply elasticity $1/\eta$.*[17]

These results are intuitive. Lower personal wealth or greater *ex ante* efficiency benefits from redistribution increase an agent's demand for such policies. A more elastic labor supply increases the deadweight loss from taxes and transfers, whether progressive or regressive, which cause individuals to distort their labor supply away from the first-best level. Finally, as a prelude to the analysis of political equilibrium, note how (21) embodies the same intuitions as the stylized model of Section I. For any $\tau < 1$ the proportion of agents who would like further redistribution at the margin,

$$(22)\quad p(\tau, \Delta_t) \equiv \text{card}\left\{i \,\middle|\, \frac{\partial U_t^i}{\partial \tau} > 0\right\} = \Phi\left(\frac{(1-\tau)Bv^2 - \delta\tau/(\eta(1-\tau))}{\gamma\Delta_t} + (1-\tau)\gamma\Delta_t\right),$$

is U-shaped in Δ_t, provided the resulting gain in *ex ante* efficiency $(1-\tau)Bv^2$ dominates the distortion $\delta\tau/(\eta(1-\tau))$. Otherwise, $p(\tau, \Delta_t)$ is strictly increasing in Δ_t. Also as on Figure 1, variations in popular support for efficient increases in τ all take place above the 50-percent level, so they will influence policy outcomes only under some departure from the pure "one person, one vote" democratic ideal.

B. *Wealth and Political Influence*

It is well known that poor and less educated individuals have a relatively low propensity to register, turn out to vote, and give political contributions. These are among the facts documented in Tables 1 and 2, which present data from Steven J. Rosenstone and John M. Hansen (1993); see also Raymond E. Wolfinger and Rosenstone (1980), Thomas B. Edsall (1984), or Margaret M. Conway (1991). For each form of participation in electoral or governmental politics, the *representation ratio* of a given socioeconomic group is the ratio between its share of the population engaged in this activity and its share of the general population. Thus the poorest 16 percent account for only $0.76 \times 16 = 12.2$ percent of the votes and $0.25 \times 16 = 4.0$ percent of the number of campaign contributors, while the richest 5 percent account for $1.27 \times$

[17] More specifically, for $\ln k_t^i \le m_t + \gamma\Delta_t^2 + Bv^2/\gamma$ the ideal τ_t^i is positive and decreases towards zero as $1/\eta$ rises. For $\ln k_t^i > m_t + \gamma\Delta_t^2 + Bv^2/\gamma$, $\tau_t^i < 0$ and it increases towards zero as $1/\eta$ rises.

TABLE 1—POLITICAL PARTICIPATION BY INCOME

Political activity: *Electoral Politics, 1952–1988*	Total fraction taking part (in percent)	Representation ratios by percentile family income					Pivot p^* (in percent)
		0–16	17–33	34–67	68–95	96–100	
Vote	66.1	0.76	0.90	1.00	1.16	1.27	55.5
Try to influence others	26.7	0.63	0.79	0.98	1.25	1.54	60.6
Contribute money	8.9	0.25	0.51	0.98	1.54	3.25	73.6
Attend meetings	7.8	0.49	0.73	0.93	1.31	2.27	65.7
Work on campaign	4.6	0.48	0.74	0.85	1.37	2.42	67.6

Source: Rosenstone and Hansen (1993), Table 8-2, plus my computation of p^*.

TABLE 2—POLITICAL PARTICIPATION BY EDUCATION

Political activity:	Total fraction taking part (in percent)	Representation ratios by years of education (with corresponding percentage of population)					Pivot p^* (in percent)
		0–8 (20.1)	9–11 (16.3)	12 (32.8)	13–15 (16.8)	16+ (14)	
Electoral Politics, 1952–1988							
Vote	66.1	0.85	0.83	1.00	1.12	1.26	55.8
Try to influence others	26.7	0.61	0.75	0.94	1.33	1.61	63.5
Contribute money	8.9	0.33	0.51	0.87	1.37	2.41	73.7
Attend meetings	7.8	0.48	0.50	0.85	1.43	2.14	72.2
Work on campaign	4.6	0.48	0.50	0.87	1.33	2.25	72.0
Governmental Politics, 1976–1988							
Sign petition	34.8	0.34	[←	0.87	→]	1.44	69.5
Attend local meeting	18.0	0.31	[←	0.78	→]	1.46	73.3
Write Congress	14.6	0.38	[←	0.72	→]	1.56	75.9

Source: Rosenstone and Hansen (1993), Tables 8-1 and 8-2, plus my computation of p^*.

$5 = 6.4$ percent of the votes and $3.25 \times 5 = 16.3$ percent of contributors.[18]

The data in Tables 1 and 2 are striking in several respects. The propensity to participate in *every* reported form of political activity rises with income and education. For voting itself the tendency is relatively moderate, whereas for contributing to political campaigns it is drastic. In the latter case the actual bias is still understated since the data reflects only the number of contributions, and not their amounts. It is intuitive that the wealthy should be overrepresented in money-intensive channels of political influence: such lobbying is a form of collective investment where liquidity constraints are even more likely to bind than usual. One might have expected poorer, less skilled agents to have a countervailing advantage for attending meetings, working on campaigns, writing Congress, and other time-intensive activities for which they have a lower opportunity cost. But, remarkably, the pro-wealth (financial and human) bias is here again not only positive, but extremely strong.

I shall not seek to explain the source of these biases, only to model them in a plausible and convenient manner.[19] Let each agent's opinion be affected by a relative weight, or probability of voting, $\omega^i / \int_0^1 \omega^j \, dj$. If individual preferences are single-peaked and the preferred policy is monotonic in wealth, or more generally if pref-

[18] Put differently, the representation ratios are the slopes of the (piecewise linear) *Lorenz curve* which describes the concentration, by income or education, of a given form of political influence.

[19] John E. Roemer (1998) shows how the presence of a second dimension in the political game (morals, religion) can result in a similar kind of bias, by splitting the coalition which would naturally arise in favor of redistribution. It is worth emphasizing again that I shall not appeal to differences in the allocation of political power or influence to explain why redistribution varies across countries (although the model can readily incorporate this "easier" explanation).

erences satisfy a single-crossing condition (as in Joshua S. Gans and Michael Smart, 1996), a median-voter-type result applies, but where the median is computed on an appropriately renormalized population. With a lognormal distribution, the following schemes yield particularly simple results.

PROPOSITION 6: *Suppose that agents $i \in [0, 1]$ have preferences $U(k^i, \tau)$ over some policy variable $\tau \in \mathbb{R}$, such that: for all $k < k'$ and $\tau < \tau'$, if $U(k', \tau') > U(k', \tau)$ then $U(k, \tau') > U(k, \tau)$.*

(1) If an agent's political weight depends on his rank in the wealth distribution, $\omega^i = \omega(p^i)$, the pivotal voter is the one with rank $p^* = \Phi(\lambda)$ and (log) wealth $\ln k^* = m + \lambda\Delta$, where $\lambda \lesseqgtr 0$ is defined by $(\int_0^{\Phi(\lambda)} \omega(p)\, dp)/(\int_0^1 \omega(p)\, dp) = 1/2$.

(2) If an agent's political weight depends on the absolute level of his wealth, with $\omega^i = (k^i)^\lambda$ for some $\lambda \lesseqgtr 0$, the pivotal voter has rank $p^* = \Phi(\lambda\Delta)$ and (log) wealth $\ln k^* = m + \lambda\Delta^2$.

Ordinal schemes ensure that each person's weight and the identity of the pivotal voter remain invariant when the distribution of wealth shifts due to growth, or when it becomes more unequal.[20] Previous discussions have often assumed that political rights or influence depend on one's absolute level of wealth. This was motivated by historical examples such as voting franchises restricted to citizens owning enough property (Saint-Paul and Verdier, 1993; Persson and Tabellini, 1994) or costly membership in a ruling elite (Alberto Ades and Verdier, 1996). I find it more plausible that even such cutoffs should be relative ones, keeping up with aggregate growth and the competitive nature of bids for political influence. I shall therefore focus on ordinal schemes, but the second part of Proposition 6 shows that absolute income effects are just as easy to capture; the case $\lambda = 1$, for instance, corresponds to a "one dollar, one vote" rule. Moreover, this alternative formulation would only reinforce the paper's results, as it implies that the political system becomes more biased towards the wealthy as inequality rises.

In the last columns of Tables 1 and 2, I interpolated the empirical $\omega(p)$ function to compute the *position of the pivotal agent* p^* for each separate form of political participation, i.e., as if it were the only one that mattered.[21] No data exist which would allow me to weigh them by their relative importance in determining the political outcome. For voting, the wealth bias is moderate, with the pivot at the 56th percentile rather than the usually assumed median. For all other forms of influence it is much stronger, with the pivotal agent always above the 60th percentile, and quite often the 70th. From this evidence it seems safe to conclude that the decisive political group is located *above the median* in terms of income and human wealth. Depending on the relative efficacy of the different channels of political influence it may even be above the mean, which in the U.S. income distribution falls around the 63rd percentile.[22]

C. *Political Equilibrium, Inequality, and Redistribution*

We are now ready to establish the paper's first main result. By virtue of Propositions 5 and 6, the policy outcome is obtained by simply setting $\ln k_t^i - m_t = \lambda\Delta_t$ in the first-order condition $\partial U_t^i/\partial\tau = 0$, or equivalently $p(\tau, \Delta_t) = p^* = \Phi(\lambda)$ in (22). As before, I consider first the case where labor supply is inelastic ($1/\eta = 0$); for $\lambda > 0$, this yields:

[20] Also, for any ordinal scheme $\omega(\cdot)$ the associated λ is a sufficient statistic: it is as if the bottom $2\Phi(\lambda) - 1$ voters [or the top $1 - 2\Phi(\lambda)$ when $\lambda < 0$] systematically abstained. Note that even though $\lambda > 0$ is the more empirically relevant case, the model will be solved for all $\lambda \gtreqless 0$.

[21] The weights ω^i are obtained by rescaling each group's representation ratios by the population's average propensity to participate in the activity under consideration (given in column 1). Concerning the relationship between voters' income and their political preferences, Nolan M. McCarthy et al. (1997) provide substantial evidence that U.S. politics are highly—and increasingly—unidimensional, along the axis of rich/opposed to redistribution versus poor/favoring redistribution.

[22] We shall see below that the first condition, but not the second, is required for redistribution to decline with inequality in the model.

$$(23) \qquad \frac{1}{1-\tau_t} = \frac{\gamma^2 \Delta_t^2 + Bv^2}{\lambda \gamma \Delta_t}.$$

The equilibrium tax rate is clearly *U-shaped* in Δ_t, and minimized where $\gamma^2\Delta^2 = Bv^2$. Similarly, in the general case it is the unique solution $T(\Delta_t) < 1$ to the quadratic equation derived from (21):

$$(24) \qquad (1-\tau_t)\left(\frac{\gamma^2 \Delta_t^2 + Bv^2}{\gamma \Delta_t}\right) - \frac{\delta}{\eta \gamma \Delta_t}\left(\frac{\tau_t}{1-\tau_t}\right) = \lambda.$$

PROPOSITION 7: *The rate of fiscal progressivity $\tau_t = T(\Delta_t)$ chosen in generation t has the following features:*

(1) τ_t increases with the ex ante efficiency gain from redistribution (gross of distortions) Bv^2, and decreases with the political influence of wealth, λ.
(2) $|\tau_t|$ decreases with the elasticity of labor supply $1/\eta$.[23]
(3) For $\lambda > 0$, τ_t is U-shaped with respect to inequality Δ_t. It starts at the ex ante efficient rate $T(0) = 1 - 2(1 + \sqrt{1 + 4\eta Bv^2/\delta})^{-1}$, declines to a minimum at some $\underline{\Delta} > 0$, then rises back towards $T(\infty) = 1$. The larger Bv^2, the wider the range $[0, \underline{\Delta})$ where $\partial\tau_t/\partial\Delta_t < 0$. For $\lambda \leq 0$, τ_t is increasing in Δ_t.

The first two results show that the equilibrium tax rate depends on the costs and benefits of redistribution, as well as on the allocation of political influence, in a very intuitive manner. The third result provides an *endogenously derived* analogue to Figure 1, with the continuous policy τ now replacing the proportion of people supporting redistribution in a zero-one decision. It also confirms several claims made earlier about the result that *redistribution may decline with inequality.* First, it is not predicated on the pivotal agent being richer than the mean, or becoming richer relative to the mean. Second, it is more likely to occur the larger the *ex ante* welfare gain from redistribution, e.g., the larger Bv^2.[24] Third, some bias $\lambda > 0$ with respect to pure majority rule is needed as well, because the median agent always wants to push redistribution beyond its range of efficiency. As shown in (22), it is only within that range that political support for a tax increase, $p(\tau, \Delta_t)$, can decline with higher inequality.

It is worth noting that the distinctive nonmonotonic relationship predicted by the model has recently been tested by Paolo Figini (1999), who found in cross-country regressions a significant U-shaped effect of income inequality on the shares of tax revenues and government expenditures in GDP.

The above results apply equally in an endowment economy ($\beta = 0$) and in the presence of accumulation. In the first case the efficiency gains arise from insurance. In the second they also reflect the reallocation of resources to agents whose marginal product of investment is higher, due to tighter liquidity constraints. In studying which social contracts emerge in the long run, I shall consider each case in turn.

IV. Inequality and Social Insurance in an Endowment Economy

Should we expect a more generous welfare state in countries with greater disparities of income and wealth, as predicted by standard models, or a less generous one, as a comparison between Sweden and the United States would suggest? To study the political economy of pure social insurance, let us focus on an endowment economy ($\beta = 0$).[25] Each dynasty's endowment k_t^i then simply follows a geometric AR(1) pro-

[23] More specifically, if $Bv^2 > \lambda^2/4$ then $\tau_t > 0$ and $\partial\tau_t/\partial\eta > 0$. If $Bv^2 < \lambda^2/4$, then $\partial\tau_t/\partial\eta$ has the sign of τ_t.

[24] With respect to the first claim, note that $\ln k^* - \ln E[k] = \lambda\Delta - \Delta^2/2$ is increasing only on $[0, \lambda]$ and positive only on $[0, 2\lambda]$. Neither interval coincides with, nor contains, $[0, \underline{\Delta}]$. The second claim follows from Proposition 7, but even when $Bv^2 = 0$ one sees from (22) that for all $\Delta \in [0, \underline{\Delta}]$ a marginal rise in τ above $T(\Delta)$ increases *ex ante* efficiency. These gains now arise from lowering the effort distortions due to regressionary taxes, as $Bv^2 = 0$ implies $T(0) = 0$, hence $\tau_t < 0$ on $[0, \underline{\Delta}]$.

[25] The important assumption here is the absence of insurance markets. The incompleteness of the loan market is inessential, and indeed this section's results remain unchanged with $\rho = 0$. This one-shot model is close to that of

cess with serial correlation α [see (6)], and the equilibrium policy is $T(\Delta_t)$. In the long run inequality converges to $\Delta_\infty^2 \equiv w^2/(1-\alpha^2)$, and the tax rate therefore to $\tau_\infty \equiv T(\Delta_\infty)$. When $1/\eta = 0$, for instance,

$$(25)\quad \frac{1}{1-\tau_\infty} = \frac{w}{\lambda}\left[\frac{\gamma}{\sqrt{1-\alpha^2}} + (1-(1-\rho)r)\left(\frac{v}{w}\right)^2\left(\frac{\sqrt{1-\alpha^2}}{\gamma}\right)\right].$$

More generally, the following results are immediate (focussing on the empirically relevant case of $\lambda > 0$).

PROPOSITION 8: *The steady-state rate of fiscal progressivity τ_∞ increases with agents' degree of risk aversion $1 - r$ and with income uncertainty v^2, but is U-shaped with respect to the variability w^2 and the persistence α of the endowment process. It decreases with the political influence of wealth λ, and declines in absolute value with the labor-supply elasticity $1/\eta$.*

Recalling that steady-state income dispersion is $\gamma^2w^2/(1-\alpha^2) + v^2$, the above results indicate that the relationship between inequality and redistribution is not likely to be monotonic. What matters is not just the *amount* of income inequality, but also *its source.* To the extent that high income disparities in some countries reflect larger uninsurable shocks (or more imperfect insurance markets), higher taxes and transfers should be observed. But if greater inequality is due to more persistent wealth dynamics or to greater *ex ante* heterogeneity at the time of the policy decision—correlated for instance with ethnic or regional differences—the reverse correlation may be observed.[26] While greater persistence makes income more variable, it also increases the number of agents for whom the value of insurance is more than offset by their vested interest in the status quo.

V. History-Dependent Social Contracts

A. *Dynamics and Multiple Steady States*

I now turn to the paper's second main idea, sketched at the end of Section I: if more inequality leads to less redistribution, and if investment resources depend on past transfers, multiple steady states can arise. To demonstrate this point I solve the full model with capital accumulation subject to wealth constraints ($\beta > 0$). The critical difference with the endowment economy is that the wealth distribution is now endogenous, through the effect of fiscal policy on persistence, $\alpha + \beta(1-\tau_t)$. The joint evolution of inequality and policy is thus described by the recursive dynamical system:

$$(26)\quad \begin{cases} \tau_t = T(\Delta_t) \\ \Delta_{t+1} = \mathcal{D}(\Delta_t, \tau_t) \end{cases}$$

where $T(\Delta_t)$ is the unique solution less than one to (24), while $\mathcal{D}(\Delta_t, \tau_t)$ is given by (14). Under a time-invariant policy, in particular, long-run inequality decreases with redistribution:

$$(27)\quad \Delta_\infty^2 = \frac{w^2 + \beta^2(1-\tau)^2v^2}{1-(\alpha+\beta\gamma(1-\tau))^2} \equiv D^2(\tau).$$

A steady-state equilibrium is an intersection of this downward-sloping locus, $\Delta = D(\tau)$, with the U-shaped curve $\tau = T(\Delta)$ described in Proposition 7; see Figure 2. Substituting (27) into (24), this corresponds to a rate of tax progressivity solving the equation:

$$(28)\quad f(\tau) \equiv (1-\tau)\left(\frac{\gamma^2D(\tau)^2 + Bv^2}{\gamma D(\tau)}\right) - \frac{\delta}{\eta\gamma D(\tau)}\left(\frac{\tau}{1-\tau}\right) = \lambda.$$

Persson (1983), except that he assumes no initial heterogeneity ($\Delta_0 = 0$).

[26] A related result obtains in Persson and Tabellini (1996), where two regions bargain over the degree of risk sharing in the federal constitution. In both models the underlying assumption is the inability to make transfers contingent only on unpredictable innovations to individual or regional income, as distinguished from its permanent component. See also George Casamata et al. (1997) for a study of how *ex post* political equilibrium constrains the *ex ante* design of public health insurance systems.

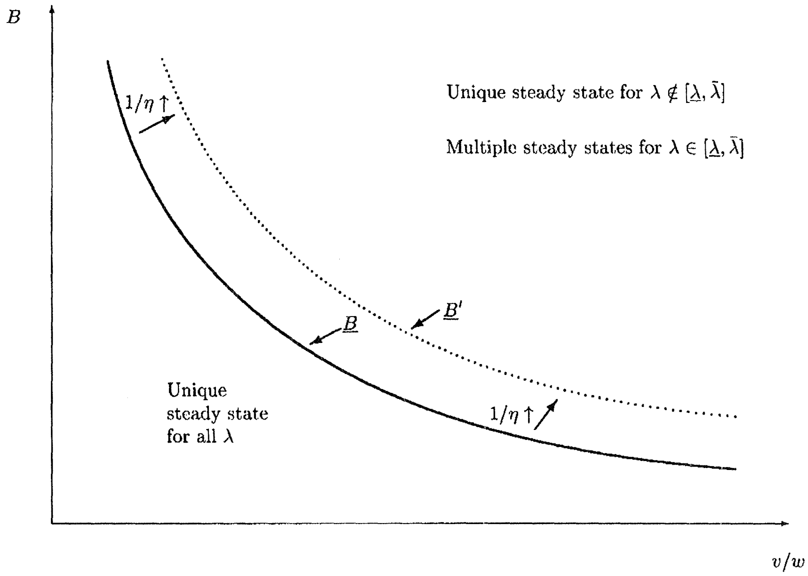

FIGURE 4. MULTIPLICITY OR UNIQUENESS OF STABLE STEADY STATES

Note: The dotted line corresponds to $1/\eta' > 1/\eta$.

This is a polynomial equation of degree eight, and therefore quite complex. Yet, by exploiting geometric intuitions on the shape of f, one can establish a series of results which formalize the paper's main ideas.[27] As illustrated on Figure 4, two countries with the same economic and political fundamentals can nonetheless evolve into *different societies,* provided:

(a) the *ex ante* welfare benefits of redistribution are high enough, relative to the costs;
(b) the political power of the wealthy lies in some intermediate range.

THEOREM 1: *Let* $1 - \alpha < 2\beta\gamma$. *When the normalized efficiency gain* $B \equiv 1 - r(1 - \rho + \rho\beta)$ *is below some critical value* $\underline{B}$ *(equivalently, when risk-aversion* $1 - r$ *is less than some* $1 - \underline{r}$*), there is a unique, stable, steady state. When* $B > \underline{B}$*, on the other hand, there exist* $\underline{\lambda}$ *and* $\bar{\lambda}$ *with* $0 < \underline{\lambda} < \bar{\lambda}$*, such that:*

(1) For each λ *in* $[\underline{\lambda}, \bar{\lambda}]$ *there are (at least) two stable steady states.*[28] *Inequality is lower, and social mobility higher, under a more redistributive social contract.*
(2) For $\lambda < \underline{\lambda}$ *or* $\lambda > \bar{\lambda}$ *the steady state is unique.*

Where multiple steady states occur, *history matters.* Temporary shocks to the distribution of wealth (immigration, educational discrimination, shifts in demand or technology) as well as to the political system (slavery, voting-rights restrictions) can permanently move society from one equilibrium to the other, or more generally have long-lasting effects on the economy and the social contract.

This history-dependence contrasts sharply with traditional politico-economic models,

[27] The intuition behind the proofs is the following. Consider $f(\tau)$ as a function of $1 - \tau \in (0, (1 - \alpha)/\beta\gamma)$. Since D is monotonic, the first fraction in (28) is U-shaped in $1 - \tau$. After multiplication by $1 - \tau$ the product typically becomes N-shaped, so that it will have three intersections with the horizontal λ, for some range of values of λ. The larger B, the more pronounced this N shape, which is also that of the vertical difference $T(\Delta) - D^{-1}(\Delta)$ on Figure 2. Conversely, the last term in (28), reflecting effort distortions, tends to make f strictly increasing in $1 - \tau$ (at least where $\tau > 0$), and therefore works towards uniqueness.

[28] Specifically, there are $2 \leq n \leq 4$ stable steady states. If $1/\eta$ is small enough then $n \leq 3$, and if α is also small enough then $n = 2$. See the theorem's proof in the Appendix.

where countries can deviate at most temporarily from a common steady-state level of inequality and redistribution (given stable "fundamentals"). In particular, fiscal policy operates there as a stabilizing force on the distribution of wealth: more inequality today means more redistribution, hence less inequality tomorrow.[29] Here, on the contrary, there emerges in the long run a *negative correlation between inequality and redistribution,* as indeed one observes between the United States and Europe, or among advanced countries in general (Rodriguez, 1998).[30]

Also of interest is the predicted negative correlation between inequality and *social mobility,* which is consistent with the results obtained by Robert Erikson and John H. Goldthorpe (1992) for a sample of 15 developed countries. But what of the conventional wisdom of the United States as an exceptionally mobile society? In fact, most econometric studies of intergenerational income mobility find either no significant difference, or even somewhat greater mobility in European "welfare states": see Anders Björklund and Markus Jäntti (1997a) for a survey.[31]

Indeed, the most extreme forms of social immobility at the lower end, such as urban ghettos or the persistence of welfare dependency across generations, do seem more exacerbated in American society. Things could well be different for the middle class, so a more satisfactory comparison across countries would take into account such nonlinearities in the mobility process (e.g., Suzanne J. Cooper et al., 1994). These remain beyond the scope of the present model and of most existing comparative studies.

Having demonstrated how the *sustainability of different societal choices* depends on the importance of risk aversion and credit constraints (both summarized in B), as well as on the allocation of power in the political system (λ), I now consider the role of income and endowment shocks (v^2 and w^2) and that of tax distortions ($1/\eta$). Due to the complexity of the problem, their effects on the multiplicity threshold $\underline{B}$ are studied under additional assumptions.

PROPOSITION 9: *Let* $1/\eta = 0$. *The efficiency threshold for multiplicity* $\underline{B}$ *is a decreasing function of* v/w, *with* $\lim_{v/w \to 0}(\underline{B}) = +\infty$ *and* $\lim_{v/w \to +\infty}(\underline{B}) = 0$.

This result appears on Figure 4. The intuition is that income uncertainty interacts with market incompleteness in generating efficiency gains from redistribution, as reflected by the term Bv^2 in (16). For a given B, multiplicity therefore occurs when v^2 is large enough compared to the variance w^2 of the shocks which agents learn prior to choosing policy. The concrete implications are important: depending on their source, changes in the economic environment which have similar short-run effects on income inequality can bring about radically *different evolutions of the social contract.* Thus, an increase in the variability of sectoral shocks (similar to v^2) may lead to an expansion of the welfare state and public education. Conversely, a surge in immigration which results in a greater heterogeneity of the population (similar to a rise in w^2) may lead to cutbacks or even a large-scale dismantling. In the first case the policy response will mitigate the shock's impact on

[29] In Persson and Tabellini (1994) income inequality, hence the equilibrium policy, depends only on the exogenous distribution of talent. In Bertola (1993) and Alesina and Rodrik (1994) the deterministic nature of the models allows any distribution of initial endowments to persist indefinitely. Incorporating idiosyncratic shocks would normally lead to a unique steady-state distribution. Uniqueness also obtains in Perotti (1993) and Saint-Paul and Verdier (1993). In Saint-Paul and Verdier (1992) greater inequality again results in higher taxes and spending on public education (which is problematic in view of the empirical evidence), but this stabilizing effect is more than offset by a divergence in the incentives of rich and poor to invest privately in additional human capital. Hence two possible steady states: high (low) inequality and public education expenditures, with private accumulation by the rich only (by both classes).

[30] Multiple steady states due to a negative impact of inequality on redistribution is the distinguishing feature which the present model shares with Saint-Paul (1994). Consistent with the general argument that this entails redistributions which increase the size of the pie, Saint-Paul's model has the property that transfers raise aggregate income in the long run.

[31] For instance, Kenneth A. Couch and Thomas A. Dunn (1997) find greater mobility—especially in terms of education—in Germany than in the United States, and Björklund and Jäntti (1997b) find similar results for Sweden. Aldo Rustichini et al. (1999), on the other hand, find lower mobility in Italy than in the United States.

long-run inequality, while in the second it will aggravate it.

Just as greater benefits of redistribution increase the likelihood of multiplicity, greater distortions reduce it. This is also illustrated on Figure 4, where the efficiency threshold $\underline{B}$ *shifts up* as $1/\eta$ rises. The formal proposition is somewhat more complicated, as it is only for positive values of τ that labor-supply distortions rise with $1/\eta$.[32] I shall not present it here due to space constraints, and because a somewhat similar result appears in Theorem 2 below. Basically, when regressive fiscal policy is ruled out, or more generally for economies that operate in a region where $\tau > 0$, distortions do rise monotonically with $1/\eta$, and the scope for multiple regimes correspondingly declines.

B. *Growth Implications of Different Social Contracts*

The steady states corresponding to two different social contracts are clearly not Pareto rankable. How do they compare in terms of aggregate performance? Recall from (15) that the effect of τ_t on short-run growth reflects the trade-off between tax distortions and credit-constraint effects; taking limits shows that this remains true for long-run income. Moreover, any comparison of long-run levels is easily transposed to *long-term growth rates,* through knowledge spillovers or public goods complementing private investment. For instance, let κ in (6) be replaced by $(\kappa_t)^{\phi}$, where the human or physical capital aggregate

$$\kappa_t \equiv \left(\int_0^1 (k_t^i)^{\gamma} di \right)^{1/\gamma} \tag{29}$$

captures external effects of the economic environment on accumulation, other than those of policy. As κ_t does not enter into the determination of the politico-economic equilibrium (Δ_t, τ_t), all previous results remain unchanged, with κ simply replaced by κ_t everywhere. The presence of the spillover only affects the growth rate along each equilibrium trajectory, transforming for instance finite steady states (when $0 \leq \phi < 1 - \alpha - \beta\gamma$) into endogenous growth paths (when $\phi = 1 - \alpha - \beta\gamma$).[33] The following results therefore apply equally to short- and to long-run economic growth.

PROPOSITION 10: *A more redistributive social contract*

(1) has higher income growth when $1/\eta = \alpha = 0$ and $\beta\gamma < 1$;
(2) has lower income growth when $1/\eta > 0$, $\alpha = 0$ and $\beta\gamma = 1$.

Since both conditions are compatible with Theorem 1's requirement that $1 - \alpha < 2\beta\gamma$, they allow the comparison of steady states corresponding to different, self-sustaining values of τ. Two interesting empirical scenarios can thus be accounted for by the model.

Case 1: Growth-enhancing redistributions.— The fact that all equilibria have (endogenously) the same savings rate makes clear that the faster growth under the more redistributive social contract arises from a more efficient allocation of investment expenditures.[34] Tax distortions, meanwhile, remain relatively small. This scenario is particularly relevant for human capital investment (which is considered in more detail in the next section) and public health expenditures, where the contrasted paths followed by East Asia and Latin America come to mind.

[32] Where effort is distorted by regressionary taxes and transfers, on the contrary, a rise in τ represents an efficiency gain which increases with $1/\eta$: recall that $C(\tau)$ is maximized for $\tau = 0$.

[33] Because it aggregates individual contributions with the same elasticity of substitution as total output, κ_t is *heterogeneity neutral,* in the sense that it does not introduce any additional effects of income distribution on growth. It just makes more permanent those due to imperfect credit markets, by reducing or even eliminating the "convergence" term $-(1 - \alpha - \beta\gamma) \ln y_t$ from (15). Alternative constant elasticity of substitution (CES) aggregates with elasticities other than $1/(1 - \gamma)$ could easily be dealt with, as in Bénabou (1996b).

[34] The equality of investment rates is true in the infinite-horizon version of the model as well. A higher τ implies a lower private savings rate, but this is exactly offset by a higher equilibrium rate of consumption taxation and investment subsidization.

More generally, it offers a potential explanation, in a context of endogenous policy choice, for the fact that regression estimates of the effects of social and educational transfers on growth are often significantly positive.

Case 2: Eurosclerosis and the welfare state.—In this converse case, the credit-constraint effect is weak compared to the tax distortions. European countries, it is often argued, have chosen a higher degree of social insurance and compression of inequalities than the United States, at the cost of higher unemployment and slower growth.[35] Whether this is viewed as enlightened policy or dismal "Eurosclerosis," it begs the question of why voters on both sides of the Atlantic would choose such different points on the equity-efficiency, or insurance-growth, trade-off. In my model, Europeans choose more redistribution than Americans not because they are intrinsically more risk averse, but because in more homogeneous societies there is less erosion of the consensus over social insurance mechanisms which, *ex ante,* would be valued enough to compensate for lesser growth prospects.

Consider finally average welfare, which here is also that of the median voter; see (16). Since multiplicity requires some political bias ($\underline{\lambda} > 0$) it is clear from (21) that, in each steady state, a *marginal* increase in redistribution would raise $\int_0^1 U_t^i\, di$. This corresponds to the requirement, in the stylized model of Section I, of an aggregate gain from the policy $\hat{\mathcal{P}}$ relative to $\mathcal{P}$. Comparing steady states, on the other hand, involves discrete variations in τ. When tax distortions are small enough ($1/\eta$ is low), a more redistributive steady state does have higher total welfare; but in general this need not be the case.

VI. Explaining International Differences in Education Finance

A. *Alternative Systems of School Funding*

Education finance provides perhaps the most compelling case of a redistributive policy with positive efficiency implications. Loan market imperfections are more likely to affect investment in human than in physical capital, which can serve as collateral. The same is true for decreasing returns. The financing of elementary and secondary education also constitutes a striking example of international differences in redistributive policy. Japan and most European countries have state-funded public schools, which essentially equalize expenditures across pupils. The United States, in contrast, relies in large part on local financing; because communities are heavily income segregated, expenditures reflect parental resources to a large extent, making education a quasi-private good. In Bénabou (1996b) I show how a move from local to state funding of schools can raise the economy's long-run income, or even its long-term growth rate. Calibrating a model with local funding to U.S. data, Fernández and Rogerson (1998) find that a move to state finance could raise steady-state GDP by about 3 percent. Whether or not one subscribes to this view, differences in national education systems which persist for over a century represent a puzzle.[36]

To examine this issue, let k_t^i now specifically represent human wealth. The term $(k_t^i)^\alpha$ in (6) captures the transmission of human capital or ability within the family, while the shocks ξ_{t+1}^i represent the unpredictable component of innate talent. Finally, instead of progressive taxes and transfers I now consider *progressive subsidies* to educational investment. Thus (9) is replaced by $\hat{y}_t^i = y_t^i$, while in (6) parental savings e_t^i are replaced by the net (after-tax) resources invested in their child's education, namely:

$$\hat{e}_t^i = e_t^i(\tilde{y}_t/y_t^i)^{\tau_t}, \tag{30}$$

with $\tilde{y}_t$ still determined by (10). Because agents

[35] It is only in recent years that European growth has fallen short of U.S. growth, but unemployment has been higher in Europe for nearly two decades.

[36] In Glomm and Ravikumar (1992) private finance of education leads to higher long-run growth than public funding, as it generates better incentives to accumulate human wealth. Bénabou (1996b) shows that allowing for idiosyncratic shocks (e.g., children's ability) tends to reverse this ranking, as do economy-wide spillovers. Gradstein and Justman (1997) study similar issues in a model with endogenous labor supply, then examine voters' choice among different funding regimes. They obtain a unique equilibrium.

will still choose a common savings rate, the government's budget constraint, which is now

$$\int_0^1 (\hat{e}_t^i - e_t^i)\, di = 0, \tag{31}$$

will again be satisfied. The progressivity rate τ_t is the *elasticity of the tax price* of education with respect to wealth. Given agents' savings behavior, $e_t^i = sy_t^i$, it also measures the extent to which education is *publicly and equally* provided: thus $\tau = 0$ corresponds to private finance, while $\tau = 1$ is equivalent to a European-style system where universal public education is funded by a proportional income tax.

PROPOSITION 11: *Given a rate of education finance progressivity τ_t, agents in generation t choose a common labor supply and savings rate: $e_t^i/\hat{y}_t^i = \rho\beta/(1 - \rho + \rho\beta) \equiv s$, as before, while:*

$$l_t = \left(\frac{\delta}{\eta}\right)^{1/\eta} \left(\frac{1 - \rho + \rho\beta(1 - \tau_t)}{1 - \rho}\right)^{1/\eta}.$$

The effort distortion is smaller than with fiscal redistribution, because the education-based policy leaves untouched the part of their income which adult agents consume. Up to that difference in $l_t = l(\tau_t)$, individual and aggregate wealth dynamics are exactly identical to (11)–(15), and the resulting *ex ante* utility function resembles closely the one which arose under fiscal redistribution.

PROPOSITION 12: *Given a rate of education finance progressivity τ_t, agent i's intertemporal welfare is:*

$$U_t^i = \bar{u}_t + A(\tau_t)(\ln k_t^i - m_t) + C(\tau_t) - \rho\beta(1 - \tau_t)^2\gamma^2\Delta_t^2 + B(\tau_t)v^2/2, \tag{32}$$

where $\bar{u}_t$ is independent of the policy τ_t and

$$A(\tau) \equiv \rho\alpha + (1 - \rho + \rho\beta(1 - \tau))\gamma \tag{33}$$

$$C(\tau) \equiv (1 - \rho)(\delta \ln l(\tau) - l(\tau)^\eta) + \rho\beta\delta \ln l(\tau) \tag{34}$$

$$B(\tau) \equiv -(1 - \rho + \rho\beta(1 - \tau)^2) + r(1 - \rho + \rho\beta(1 - \tau))^2. \tag{35}$$

Two differences with Proposition 4 are worth mentioning. On the cost side, $C(\tau)$ is the same function of effort $l(\tau)$ as before, but $l(\tau)$ is now different. In particular, distortions remain bounded, so U_t^i may be maximized at $\tau = 1$ for a poor enough agent. The other difference occurs in the benefits term. For $\rho = 1$, $B(\tau) = -\beta(1 - r\beta)(1 - \tau)^2$ as in (16), and with the same interpretation in terms of insurance and reallocation of liquidity-constrained investments. But in general $B(\tau)$ is no longer proportional to $-(1 - \tau)^2$, and when $r > 0$ it is not even monotonic in τ. While the education model is sufficiently close to the tax model to ensure that the political equilibrium remains qualitatively similar, the formal analysis is made more difficult by these differences. Moreover, deriving an exact analogue to Theorem 1 would be repetitious. I shall therefore focus instead on a simpler case, which yields new and more explicit comparative statics results.

B. *Sustainability of Centralized and Decentralized Education Systems*

From here on, policy is restricted to two options. Under laissez-faire or *decentralized funding*, $\tau = 0$, education expenditures are determined by family or community resources; the two are essentially equivalent when communities are stratified by socioeconomic status. *Public funding* of education corresponds to $\tau = 1$ or more generally to $\tau = \bar{\tau}$, where $0 < \bar{\tau} \le 1$. Given an initial distribution of human capital Δ_t, this system is adopted if $U_t^i(\bar{\tau}) > U_t^i(0)$ for at least a fraction $p^* \equiv$

$\Phi(\lambda)$ of the population. Setting $\ln k_t^i - m_t = \lambda\Delta_t$ in (32), this means:

$$(36)\quad \lambda < \left((B(\bar\tau) - B(0))\left(\frac{v^2}{2}\right) - (C(0) - C(\bar\tau))\right)\frac{1/\rho\beta\bar\tau}{\gamma\Delta_t} + \left(\frac{2-\bar\tau}{2}\right)\gamma\Delta_t.$$

Intuitively, the *political influence of wealth* must not be too large compared to the *aggregate welfare benefit* of redistributive education finance (relative to laissez-faire). Preexisting inequality *raises the hurdle* which public policy must overcome, as reflected by the term $1/\gamma\Delta_t$ multiplying the net welfare benefit. This effect tends to make adoption of state finance more difficult where it has not previously been in place (e.g., the United States), because of the greater human capital disparities which result over time from a decentralized system. Conversely, the term in $\gamma\Delta_t$ incorporates the combined effects of skewness and credit constraints, which both intensify the demand for redistribution. As a result of these offsetting forces the right-hand side of (36) has the usual U shape in Δ_t, and is in fact very similar to (3) in the stylized model of Section I. To focus now on the long run, let us replace Δ_t with $\Delta_\infty = D(\bar\tau)$, given by (27). Public funding of education (partial or complete) is thus a steady state when:

(37)

$$\lambda < \left((B(\bar\tau) - B(0))\left(\frac{v^2}{2}\right) - (C(0) - C(\bar\tau))\right) \times \left(\frac{1/\rho\beta\bar\tau}{\gamma D(\bar\tau)}\right) + \left(\frac{2-\bar\tau}{2}\right)\gamma D(\bar\tau) \equiv \bar\lambda.$$

Conversely, private or local financing is a steady state, with inequality $\Delta_\infty = D(0)$, when:

(38)

$$\lambda > \left((B(\bar\tau) - B(0))\left(\frac{v^2}{2}\right) - (C(0) - C(\bar\tau))\right) \times \left(\frac{1/\rho\beta\bar\tau}{\gamma D(0)}\right) + \left(\frac{2-\bar\tau}{2}\right)\gamma D(0) \equiv \underline{\lambda}.$$

The two regimes can coexist if and only if $\underline{\lambda} < \bar\lambda$, which occurs when the differential gain

$$(39)\quad B(\bar\tau) - B(0) = \rho\beta\bar\tau[2 - \bar\tau - r(2 - 2\rho + \rho\beta(2 - \bar\tau))]$$

exceeds the differential cost

$$(40)\quad C(0) - C(\bar\tau) = \frac{\delta}{\eta}\left[(1 - \rho + \rho\beta)\ln\left(\frac{1-\rho+\rho\beta}{1-\rho+\rho\beta(1-\bar\tau)}\right) - \rho\beta\bar\tau\right]$$

by a sufficient amount, specified below. It is easily seen that the distortion $C(0) - C(\bar\tau)$ is positive and increasing $1/\eta$. I will assume that $B(\bar\tau) > B(0)$, so that there is an actual gain to compare to this cost. Such is the case provided agents care enough about insurance $(1 - r \geq 1)$ or about investment in their children's education $(\rho(2 + \beta) \geq 1)$, or alternatively provided $\bar\tau$ is not too large.

THEOREM 2: *If the gain in ex ante welfare from progressive public financing of education, relative to private or decentralized financing, is large enough, namely*

$$(\bar\tau) - B(0) \geq (C(0) - C(\bar\tau))(2/v^2) + G(\bar\tau, v^2/w^2) \equiv \underline{B},$$

where $G(\bar\tau, v^2/w^2) > 0$ is given in the Appendix, then there exist $0 < \underline{\lambda} < \bar\lambda$ such that:

(1) For each λ in $[\underline{\lambda}, \bar\lambda]$ both public and decentralized school funding are stable steady states. The first regime has lower inequality and greater social mobility than second.
(2) For $\lambda < \underline{\lambda}$ public funding is the only steady state, while for $\lambda > \bar\lambda$ it is decentralized funding.

The efficiency threshold for multiplicity $\underline{B}$ decreases with income uncertainty v^2 and increases with endowment variability w^2, with

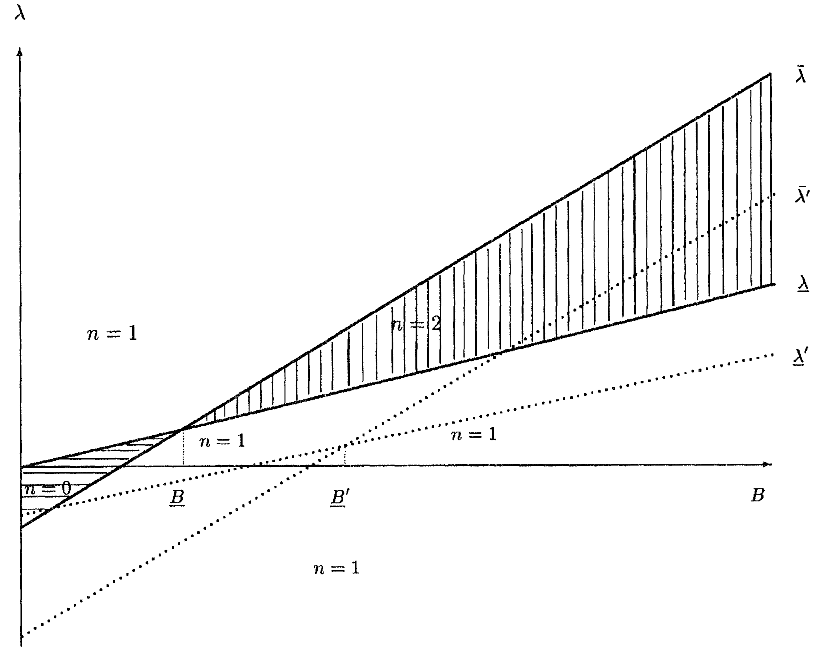

FIGURE 5. THE NUMBER OF STEADY STATES

Note: The dotted lines correspond to $1/\eta' > 1/\eta$.

$\lim_{v/w \to 0}(\underline{B}) = +\infty$. *It rises with the labor-supply elasticity* $1/\eta$.

These results demonstrate how such different systems of school finance as those of the United States and Western European countries can be self-perpetuating, once arisen from historical circumstances.[37] As to which one leads to faster growth, this depends once again on the trade-off between the positive impact of redistributive education finance on wealth constraints and its adverse effect on incentives: Proposition 10 applies unchanged. Because the distortions are less severe than with fiscal policy, however, the likelihood that state funding enhances growth is now greater.

Theorem 2 also makes clear show how the efficiency threshold for multiplicity $\underline{B}$ (or equivalently, the minimal risk aversion $1 - \underline{r}$) varies with the costs of redistributive school finance, as well as with different sources of income inequality. These results can again be represented on Figure 4, with $B \equiv B(\bar{\tau}) - B(0)$ now on the vertical axis, and the same interpretations as for Proposition 9. The only difference is that for $B < \underline{B}$ there might be no steady state, as $\bar{\lambda}$ is then less than $\underline{\lambda}$. The economy can instead be shown to cycle between the two regimes, as in Gradstein and Justman (1997).[38] Actual instances of such cycling are hard to come by, and indeed Theorem 1 indicates that nonexistence is largely an artifact of restricting the policy τ to discrete values.[39]

[37] Generalizing Theorem 2 to any policy pair $\{\underline{\tau}, \bar{\tau}\}$ with $0 \leq \underline{\tau} < \bar{\tau} \leq 1$ is straightforward.

[38] Equations (37)–(38) show that for $\lambda \notin [\bar{\lambda}, \underline{\lambda}]$ there is a unique (and intuitive) steady state, but for $\lambda \in [\bar{\lambda}, \underline{\lambda}]$ there is none. The model of Gradstein and Justman (1997) corresponds to the case where $1 - r = 1$ (time-separable, logarithmic utility), the disutility of effort $-l^{\eta}$ is replaced by $\ln(1 - l)$, $\tau \in \{0, 1\}$ (pure public or private system), $\lambda = 0$ (pure democracy), and $v^2 = 0$ (no uncertainty at the time of voting). From Theorem 2 we see that the restrictions $\lambda = 0$ and $v^2 = 0$ preclude multiple equilibria.

[39] With a continuous τ the analogue of equation (28) for education funding is easily shown to always have at least one stable steady state. Conversely, an analogue of Theorem 2 can be derived for *fiscal policy,* with the progressivity rate τ restricted to $\{0, \bar{\tau}\}$. This shows that the nonlinear redistributive schemes used in the paper, namely (9) and (30), are not driving the results: for $\tau = 0$ and $\tau = \bar{\tau} \equiv 1$ the

I now return to the central issue, namely the coexistence of multiple regimes. The *range of political systems* which allow this indeterminacy is illustrated on Figure 5.

PROPOSITION 13: *The scope for the political system to generate multiple equilibria increases with the efficiency benefits of redistribution and decreases with their efficiency costs: as $B \equiv B(\bar{\tau}) - B(0)$ increases, due to greater risk aversion or more decreasing returns, both $\underline{\lambda}$ and $\bar{\lambda}$ rise but the interval $[\underline{\lambda}, \bar{\lambda}]$ widens. A higher labor-supply elasticity $1/\eta$ has the opposite effects.*

VII. Other Applications

A. *Concern for Equity*

Apart from social insurance and capital market imperfections, one of the main reasons for income redistribution is simply that most people dislike living in a society which is too unequal. This may be due to pure altruism or to the fact that inequality generates social tensions, crime, and similar problems which have direct costs. To capture these ideas one can simply augment (7) as follows:

$$(7') \quad \ln \hat{V}_t^i = \ln V_t^i - (\mathcal{A}/2) \times [(1-\rho)\mathrm{var}_{j\in I}[\ln c_t^j - (l_t^j)^\eta] + \rho\ \mathrm{var}_{j\in I}[\ln k_{t+1}^j]].$$

The coefficient $\mathcal{A}$ represents everyone's aversion to disparities in felicity, measured by the cross-sectional variances of consumption (including leisure) and bequests. It is easily seen that the economy's laws of motion remain unchanged and the political equilibrium quite similar. In an endowment economy ($\beta = 0$), for instance, the only difference is that the ubiquitous $\gamma^2\Delta_t^2 + Bv^2$ is replaced by $(1+\mathcal{A})\gamma^2\Delta_t^2 + (B+\mathcal{A})v^2$: inequality aversion is equivalent to a simultaneous increase in risk aversion and in the concavity of aggregate welfare (previously logarithmic). Thus, with $1/\eta = 0$ the steady-state tax rate becomes

$$(25') \quad \frac{1}{1-\tau_\infty} = \frac{w}{\lambda}\left[(1+\mathcal{A})\left(\frac{\gamma}{\sqrt{1-\alpha^2}}\right) + (1+\mathcal{A}-(1-\rho)r)\left(\frac{v}{w}\right)^2\left(\frac{\sqrt{1-\alpha^2}}{\gamma}\right)\right].$$

If one observed two countries, the first with low pretax inequality yet extensive redistribution, the other with the reverse situation, one would indeed be tempted to conclude that the citizens of the first country were more altruistic, or their poor better organized politically. In fact it could be that preferences are identical and political institutions equivalent (same $\mathcal{A}$ and λ), but that the second country's more unequal distribution reflects a more persistent income process. This could be due to exogenous factors, as with α here, or be endogenous, as in the case of multiple steady states.

B. *The Mix of Public Goods*

Some public services such as the legal system, the protection of property, prisons, etc., benefit citizens largely in proportion to their levels of wealth or investment. Others have more uniformly or even regressively distributed benefits. Klaus Deininger and Lyn Squire (1995) find in cross-country regressions that public investment affects income growth equally for all quintiles, while public schooling benefits the bottom 40 percent most, the middle class to a lesser extent, and the rich not at all. Consider therefore a government choosing (at the margin) a single public good or service from a menu of options: if g_t is spent on a good with characteristics (κ, α, β, γ), the private sector faces the accumulation technology $k_{t+1}^i = \kappa\xi_{t+1}^i(k_t^i)^\alpha(e_t^i)^\beta(g_t)^\gamma$. A public good or institution with high $\alpha + \beta$ makes wealth more persistent, so relatively well-off agents may prefer it to an alternative which has higher overall productivity (larger κ or $\alpha + \beta + \gamma$). The problem is thus analogous to the earlier ones, implying that countries can sustain different choices without any underlying differences in tastes. In reality, many public goods are provided simultaneously and the debate is over the appropriate mix, but the same intuitions should remain applicable.

geometric specification and the standard linear one coincide exactly.

C. *The Socioeconomic Structure of Cities*

The presence in human capital accumulation of peer effects, role models, and other neighborhood interactions implies that residential stratification increases the *persistence* of income disparities across families (e.g., Bénabou, 1993; Durlauf, 1996a). Urban ghettos are but the most extreme example of this phenomenon, which is the subject of a large empirical literature.[40] Moreover, I show in Bénabou (1993, 1996b) that equilibrium segregation generally tends to be *inefficiently high.* The two conditions identified in this paper for multiple politico-economic regimes are thus again satisfied, leading to the following predictions: (a) more highly segregated cities are also those where socioeconomic disparities are greater; (b) in such cities public policy (on housing, schooling, transport, or infrastructure) will tend to accommodate and even facilitate segregation, while in better integrated (and more equal) cities more public support and resources will be mobilized to prevent further polarization.[41]

VIII. Conclusion

In this paper I have asked how countries with similar preferences and technologies, as well as equally democratic political systems, can nonetheless make very different choices with respect to fiscal progressivity, social insurance, and education finance. The proposed answer is a simple theory of inequality and the social contract, based on two mechanisms which arise naturally in the absence of complete insurance and credit markets. First, redistributions which would increase *ex ante* welfare command less political support in an unequal society than in a more homogeneous one. A lower rate of redistribution, in turn, increases inequality of future incomes due to wealth constraints on investment in human or physical capital. This leads to two stable steady states, the archetypes for which could be the United States and Western Europe: one with high inequality yet low redistribution, the other with the reverse configuration. These two societies are not Pareto rankable, and which one has faster income growth depends on the balance between tax distortions to effort and the greater productivity of investment resources (particularly in education) reallocated to more severely credit-constrained agents.

These ideas were formalized in a stochastic growth model with missing markets, progressive fiscal or education finance policy, and a more realistic political system than the standard median voter setup. The resulting distributional dynamics remain simple enough to allow a number of extensions. In Bénabou (1999) I develop and calibrate an infinite-horizon version of the incomplete markets model, then use it to quantify the effects of fiscal and educational redistribution on growth, risk, and welfare. Another interesting problem is to endogenize the kind of wealth-biased political mechanism used here, where those with more resources command more influence; François Bourguignon and Verdier (1997) and Rodriguez (1998) are recent examples of such models. Finally, the original question of why the social contract differs across countries, and whether these choices are sustainable in the long run, remains an important topic for further research.

[40] Recent references include Cooper et al. (1994), George J. Borjas (1995), and Giorgio Topa (1997). Christopher Jencks and Susan E. Mayer (1990) provide an extensive survey of earlier empirical studies, and Charles F. Manski (1993) a critical discussion of methodology. Indeed the identification of these social spillovers remains the subject of some controversy; see for instance William N. Evans et al. (1992).

[41] At the heart of this class of models (e.g., Bénabou, 1996b) lies a general law of motion for human capital of the form: $h^i_{t+1} = F(\xi^i_{t+1}, h^i_t, L^i_t, H_t)$, where L^i_t and H_t are human capital averages capturing respectively *local externalities* (e.g., peer effects) and *economy-wide* interactions (production complementarities, knowledge spillovers, etc.). Where families have sorted into socioeconomically homogeneous communities, $L^i_t = h^i_t$ for all i, so $h^i_{t+1} = F(\xi^i_{t+1}, h^i_t, h^i_t, H_t)$. Conversely, under perfect integration $L^i_t = L^j_t = L_t$, hence $h^i_{t+1} = F(\xi^i_{t+1}, h^i_t, L_t, H_t)$. The similarity with the cases $\tau = 0$ and $\tau = 1$ in the present model is readily apparent. An alternative source of multiplicity is explored in Durlauf (1996b), where it is only when socioeconomic disparities are not too large that rich families are willing to share with poorer ones the fixed costs of running a community and its schools.

APPENDIX

PROOF OF PROPOSITION 2:

Once agent i knows his productivity z_t^i, hence also his pre- and posttax incomes $y_t^i = z_t^i(k_t^i)^\gamma(l_t^i)^\delta$ and $\hat{y}_t^i = (y_t^i)^{1-\tau_t}(\tilde{y}_t)$, his decision problem takes the form:

$$\text{(A1)}\quad \ln V_t^i = \max_{l,\nu}\{(1-\rho)[\ln((1-\nu)\hat{y}_t^i) - l^\eta] + (\rho/r')\ln E_t[(k_{t+1}^i)^{r'}] \mid k_{t+1}^i = \kappa\xi_{t+1}^i(k_t^i)^\alpha(\nu\hat{y}_t^i)^\beta\}$$

$$= \max_{\nu}\{(1-\rho)\ln(1-\nu) + \rho\beta\ln\nu\} + \max_{l}\{-(1-\rho)l^\eta + (1-\rho+\rho\beta)(1-\tau_t)\delta\ln l\}$$

$$+ \rho(\ln\kappa - (1-r')w^2/2) + [\rho\alpha + (1-\rho+\rho\beta)\gamma(1-\tau_t)]\ln k_t^i$$

$$+ (1-\rho+\rho\beta)[(1-\tau_t)\ln z_t^i + \tau_t\ln\tilde{y}_t],$$

where $\nu_t^i \equiv e_t^i/\hat{y}_t^i$ is the savings rate. Strict concavity in ν and l is easily verified, and the first-order conditions directly yield the stated results.

PROOF OF PROPOSITION 3:

Let us start by computing the redistributive scheme's cutoff level $\tilde{y}_t$. If $\ln k_t^i \sim \mathcal{N}(m_t, \Delta_t^2)$, then (4) implies that aggregate income is:

$$\text{(A2)}\quad \ln y_t = \ln E[z_t^i] + \ln E[(k_t^i)^\gamma] + \delta\ln l_t = \gamma m_t + \delta\ln l_t + \gamma^2\Delta_t^2/2.$$

The level of transfers $\tilde{y}_t$ which satisfies the government budget constraint (10) is then given by:

$$\tau_t\ln\tilde{y}_t = \ln y_t - \delta(1-\tau_t)\ln l_t - \ln E[(z_t^i)^{1-\tau_t}] - \ln E[(k_t^i)^{\gamma(1-\tau_t)}]$$

$$= \ln y_t - \delta(1-\tau_t)\ln l_t + ((1-\tau_t) - (1-\tau_t)^2)v^2/2 - ((1-\tau_t)\gamma m_t + (1-\tau_t)^2\gamma^2\Delta_t^2/2),$$

since the z_t^i's and k_t^i's are independent. Thus:

$$\text{(A3)}\quad \tau_t\ln\tilde{y}_t = \gamma\tau_t m_t + \delta\tau_t\ln l_t + \tau_t(2-\tau_t)\gamma^2\Delta_t^2/2 + \tau_t(1-\tau_t)v^2/2,$$

as claimed in (12). Equation (14) follows from taking variances in (11), while (13) follows from taking averages with $\tau_t\ln\tilde{y}_t$ replaced by (A3). Finally, combining both laws of motions with (A2) yields:

$$\ln y_{t+1} = \delta\ln l_{t+1} + \gamma[(\alpha+\beta\gamma)m_t + \beta\delta\ln l_t + \beta\tau_t(2-\tau_t)(\gamma^2\Delta_t^2 + v^2)/2 + \ln(\kappa s^\beta) - (w^2+\beta v^2)/2]$$

$$+ \gamma^2[(\alpha+\beta\gamma(1-\tau_t))^2\Delta_t^2 + \beta^2(1-\tau_t)^2v^2 + w^2]/2$$

$$= \gamma(\ln\kappa + \beta\ln s - (1-\gamma)w^2/2) + \delta(\ln l_{t+1} - \alpha\ln l_t) + (\alpha+\beta\gamma)[\gamma m_t + \delta l_t + \gamma^2\Delta_t^2/2]$$

$$- \beta\gamma[1 - \tau_t(2-\tau_t) - \beta\gamma(1-\tau_t)^2]v^2/2$$

$$- [\alpha+\beta\gamma - (\alpha+\beta\gamma(1-\tau_t))^2 - \beta\gamma\tau_t(2-\tau_t)]\gamma^2\Delta_t^2/2$$

$$= \ln\tilde{\kappa} + \delta(\ln l_{t+1} - \alpha\ln l_t) + (\alpha+\beta\gamma)\ln y_t$$

$$- \beta\gamma(1-\beta\gamma)(1-\tau_t)^2v^2/2 - \mathcal{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2,$$

hence the result, given the definitions of $\ln\tilde{\kappa}$, $\mathcal{L}_v(\tau)$ and $\mathcal{L}_\Delta(\tau)$.

PROOF OF PROPOSITION 4:

Substituting the optimal $l_t^i = l_t$ and $\nu_t^i = s$ into (A1), and denoting $\ln\kappa' \equiv \ln\kappa - (1-r')w^2/2$, yields:

(A4) $$\ln V_t^i = \rho \ln \kappa' + (1-\rho)\ln(1-s) + \rho\beta \ln s - (1-\rho)l_t^\eta + (1-\rho+\rho\beta)(1-\tau_t)\delta \ln l_t$$
$$+ [\rho\alpha + (1-\rho+\rho\beta)\gamma(1-\tau_t)]\ln k_t^i + (1-\rho+\rho\beta)[(1-\tau_t)\ln z_t^i + \tau_t \ln \tilde{y}_t].$$

Thus *conditional* on k_t^i, $\ln V_t^i$ is normally distributed, with variance $(1-\rho+\rho\beta)^2(1-\tau_t)^2 v^2$. This implies:

(A5) $$U_t^i \equiv \frac{1}{r}\ln E[(V_t^i)^r | k_t^i] = E[\ln V_t^i | k_t^i] + r(1-\rho+\rho\beta)^2(1-\tau_t)^2 v^2/2.$$

Therefore, to obtain U_t^i one simply needs to replace in (A4) the term in $\ln z_t^i$ by $-(1-\rho+\rho\beta)(1-\tau_t)[1-r(1-\rho+\rho\beta)(1-\tau_t)]v^2/2$. Finally, substituting in the value of $\tau_t \ln \tilde{y}_t$ from (A3) yields the claimed result, with

(A6) $$\bar{u}_t \equiv \ln[(1-s)^{1-\rho}s^{\rho\beta}] + \rho(\ln \kappa - (1-r')w^2/2)$$
$$+ (\rho\alpha + (1-\rho+\rho\beta)\gamma)m_t + (1-\rho+\rho\beta)\gamma^2\Delta_t^2/2.$$

PROOF OF PROPOSITION 5:

We can rewrite (21) as a second-degree polynomial in $x \equiv 1-\tau$,

(A7) $$P(x) \equiv x^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma(\ln k_t^i - m_t) - \delta/\eta)x - \delta/\eta = 0,$$

which always has two real roots of opposite sign. Since $\tau_t^i \leq 1$ necessarily, the relevant root is $x_t^i = 1 - \tau_t^i > 0$, and at that point $P'(x_t^i) > 0$. It is easy to compute τ_t^i explicitly, but for comparative statics one can simply use the implicit function theorem. Since $\partial P/\partial(Bv^2) > 0 > \partial P/\partial \ln k_t^i$ and $P'(x_t^i) > 0$, the theorem implies that $\partial x_t^i/\partial \ln k_t^i > 0 > \partial x_t^i/\partial(Bv^2)$. Similarly, $\partial x_t^i/\partial \Delta_t < 0$. Finally, $-\partial P/\partial(1/\eta) = \delta(1-x)$, so $\partial x_t^i/\partial(1/\eta)$ has the sign of $1 - x_t^i = \tau_t^i$, or equivalently $\partial \tau_t^i/\partial(1/\eta)$ has the sign of $-\tau_t^i$. Finally, $\tau_t^i > 0$ if and only if $x_t^i < 1$, which means $P(1) > 0$, or $\gamma(\ln k_t^i - m_t) < \gamma^2\Delta_t^2 + Bv^2$.

PROOF OF PROPOSITION 6:

Let us index agents by their log-wealth, $\theta^i \equiv \ln k^i$, and denote its cumulative distribution function by $F(\theta)$. For any weighting scheme $\omega^i = g(\theta^i)$, the proportion of votes cast by agents with $\theta^i \leq \theta$ (more generally, their total political weight) is $G(\theta)/G(\infty)$, where $G(\theta) \equiv \int_{-\infty}^{\theta} g(z)\,dF(z)$. For an ordinal scheme, $g(z) = \omega(F(z))$, so $G(\theta) = \int_0^{F(\theta)} \omega(p)\,dp$. Given the single-crossing condition satisfied by preferences, the agent with log-wealth θ^* and rank $p^* = F(\theta^*)$ defined by $G(\theta^*)/G(\infty) = \frac{1}{2}$ is clearly pivotal (see Gans and Smart, 1996). In the lognormal case $F(\theta) = \Phi((\theta - m)/\Delta)$, so if we define $\lambda \equiv \Phi^{-1}(p^*)$ then $\theta^* = m + \lambda\Delta$. This wealth level is the same as if the whole distribution of $\theta = \ln k$ were shifted up by $\lambda\Delta$. Let us now turn to the cardinal scheme $g(\theta) = e^{\lambda\theta}$. Simple derivations show that

(A8) $$G(\theta) \equiv \int_{-\infty}^{\theta} e^{\lambda z}\,dF(z) = e^{\lambda(m+\lambda\Delta^2/2)} \cdot F(\theta - \lambda\Delta^2),$$

hence $G(\theta)/G(\infty) = F(\theta - \lambda\Delta^2)$. The whole distribution is thus shifted by $\lambda\Delta^2$, and so is the solution to $G(\theta^*)/G(\infty) = \frac{1}{2}$.

PROOF OF PROPOSITION 7:

The equilibrium tax rate is the one preferred by the agent with $\ln k_t^i - m_t = \lambda\Delta_t$, so claims (1) and (2) follow directly from the properties of τ_t^i established in Proposition 5. To establish the third claim let us rewrite that agent's first-order condition, $\partial U_t^i/\partial \tau = 0$, in terms of $x_t \equiv 1 - \tau_t$. By (A7), x_t is the unique positive root of the polynomial:

$$Q(x) \equiv x^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma\lambda\Delta_t - \delta/\eta)x - \delta/\eta = 0. \tag{A9}$$

Now, $Q'(x_t) > 0$ and $\partial Q/\partial\Delta_t = 2x^2\gamma^2\Delta_t - \gamma\lambda x$, so $\partial x_t/\partial\Delta_t$ has the sign of $\lambda - 2\gamma x_t\Delta_t$. Therefore $\partial\tau_t/\partial\Delta_t > 0$ if and only if $x_t > \lambda/2\gamma\Delta_t$. For $\lambda \leq 0$ this is always true, hence τ_t is strictly increasing in Δ_t. For $\lambda > 0$, on the other hand, the condition is equivalent to:

$$\Delta_t^2 Q(\lambda/2\gamma\Delta_t) = (\lambda/2\gamma)^2(\gamma^2\Delta_t^2 + Bv^2) - (\gamma\lambda\Delta_t - \delta/\eta)(\lambda\Delta_t/2\gamma) - (\delta/\eta)\Delta_t^2 < 0 \Leftrightarrow$$

$$R(\Delta_t) \equiv -(\lambda + 4\delta/\eta\lambda)\Delta_t^2 + 2(\delta/\eta\gamma)\Delta_t + \lambda Bv^2/\gamma^2 < 0.$$

This second-degree polynomial in Δ_t has two real roots of opposite sign. Denoting $\underline{\Delta}$ the positive one [with, clearly, $\partial\underline{\Delta}/\partial(Bv^2) > 0$], we conclude that $\partial\tau_t/\partial\Delta_t > 0$ if and only if $\Delta_t > \underline{\Delta}$. Thus τ_t is indeed U-shaped in Δ_t, and its limiting values at $\Delta_t = 0$ and as $\Delta_t \to \infty$ are readily obtained from (A9). Finally, recall that $\tau_t^i > 0$ if and only $\gamma(\ln k_t^i - m_t) < \gamma^2\Delta_t^2 + Bv^2$; therefore

$$\tau_t > 0 \Leftrightarrow \gamma^2\Delta_t^2 - \gamma\lambda\Delta_t + Bv^2 > 0. \tag{A10}$$

When $Bv^2 > \lambda^2/4$ the condition always holds, so $\tau_t > 0$. When $Bv^2 < \lambda^2/4$ there is a range $[\Delta', \Delta''] \subset (0, 2\lambda)$ such that $\tau_t < 0$ if and only if Δ_t is in that interval.

PROOF OF THEOREM 1:

Let us start with a lemma characterizing stable and unstable steady states.

LEMMA 1: *A stable steady state is a point τ^* where the function $f(\tau)$ cuts the horizontal λ from above, or equivalently a point (Δ^*, τ^*) where the curve $\Delta = D(\tau)$ cuts the curve $\tau = T(\Delta)$ from above. An unstable steady state corresponds in each case to an intersection from below.*

PROOF:

The dynamical system (26) reduces to a one-dimensional recursion: $\Delta_{t+1} = \mathcal{D}(\Delta_t, T(\Delta_t))$. A fixed-point $\Delta^* = \mathcal{D}(\Delta^*, T(\Delta^*))$ is stable if and only if $(d\mathcal{D}(\Delta_t, T(\Delta_t))/d\Delta_t)_{\Delta=\Delta^*} < 1$, or:

$$\mathcal{D}_1(\Delta^*, \tau^*) + T'(\Delta^*)\mathcal{D}_2(\Delta^*, \tau^*) < 1, \tag{A11}$$

where $\tau^* \equiv T(\Delta^*)$ and a j subscript denotes a jth partial derivative. Now, the function $\tau = T(\Delta)$ is implicitly given by the first-order condition (24), or:

$$\psi(\tau, \Delta) \equiv (1-\tau)\left(\frac{\gamma^2\Delta^2 + Bv^2}{\gamma}\right) - \frac{\delta}{\eta\gamma}\left(\frac{\tau_t}{1-\tau_t}\right) - \lambda\Delta = 0, \tag{A12}$$

therefore $T'(\Delta) = -(\psi_2/\psi_1)(T(\Delta), \Delta)$ and the stability condition becomes:

$$\mathcal{D}_1(\Delta^*, \tau^*) - \left(\frac{\psi_2(\tau^*, \Delta^*)}{\psi_1(\tau^*, \Delta^*)}\right)\mathcal{D}_2(\Delta^*, \tau^*) < 1. \tag{A13}$$

Next, recall from (28) that f is defined by: $f(\tau) - \lambda = \psi(\tau, D(\tau))/D(\tau)$, so that $f' < 0$ if and only if $\psi_1 + (\psi_2 - \psi/D)D' < 0$. Finally, $D(\tau)$ is defined by (27) as the (unique) fixed-point solution to $D(\tau) = \mathcal{D}(D(\tau), \tau)$; therefore: $D' = \mathcal{D}_2/(1 - \mathcal{D}_1)$. Substituting D' and using the fact that $\psi(\tau^*, \Delta^*) = 0$ at a steady state, this becomes:

$$f'(\tau^*) < 0 \Leftrightarrow \psi_1(\tau^*, \Delta^*)(1 - \mathcal{D}_1(\Delta^*, \tau^*)) + \psi_2(\tau^*, \Delta^*)\mathcal{D}_2(\Delta^*, \tau^*) < 0,$$

which is the same as (A13), hence the result in terms of the slope of f. Its translation into the condition that $T'(\Delta^*) > (D^{-1})'(\Delta^*)$ is immediate.

We now come to actually solving for steady states. It will be more convenient here to work with the variable $x \equiv \beta\gamma(1 - \tau) \in [0, \infty)$. Accordingly, let us define:

$$\text{(A14)} \qquad \Delta(x) \equiv \sqrt{\frac{w^2 + x^2 v^2/\gamma^2}{1 - (\alpha + x)^2}} = D(\tau),$$

and rewrite the equation $f(\tau) = \lambda$ as:

$$\text{(A15)} \qquad \varphi(x) \equiv x\left(\Delta(x) + \frac{Bv^2/\gamma^2}{\Delta(x)}\right) - \left(\frac{\beta\delta}{\eta\gamma\Delta(x)}\right)\left(\frac{\beta\gamma - x}{x}\right) = \lambda\beta.$$

A stable steady state is now an intersection of the function $\varphi(x)$ with the horizontal $\lambda\beta$, from below. Since $\varphi(0) \leq 0$ (it equals $-\infty$ for $1/\eta > 0$, or 0 for $1/\eta = 0$) while $\varphi(1 - \alpha) = +\infty$, for any $\lambda > 0$ there is always at least one stable equilibrium $x \in (0, 1 - \alpha)$, with $0 < \Delta(x) < +\infty$. Moreover, the total number of intersections must always be odd, with n intersections from below (stable equilibria), alternating with $n - 1$ intersections from above (unstable equilibria). Multiple intersections ($n > 0$) will actually occur, for some nonempty interval of values of λ, if and only if $\varphi(\cdot)$ is *nonmonotonic*. Indeed, since $\varphi'(0) > 0$ [this is easily verified from (A15)] and $\varphi(1 - \alpha) = +\infty$, nonmonotonicity is equivalent to the property of having at least one strict local maximum, followed by one strict local minimum, in $(0, 1 - \alpha)$. Given the boundary values of φ, multiple equilibria then occur *if and only if* λ belongs to the range $[\underline{\lambda}, \bar{\lambda}]$, where:

$$\text{(A16)} \qquad \begin{cases} \underline{\lambda} \equiv \beta^{-1} \min \{\varphi(x) | x \text{ is a strict local minimum of } \varphi(x)\} > 0 \\ \bar{\lambda} \equiv \beta^{-1} \max \{\varphi(x) | x \text{ is a strict local minimum of } \varphi(x)\} > 0. \end{cases}$$

That $\bar{\lambda}$ and $\underline{\lambda}$ are both always positive follows from the fact that for $1/\eta = 0$, $\varphi(x) > 0$ for all $x > 0$ [see (A15)], while for $1/\eta > 0$, if $\varphi(x) < 0$ then $\varphi'(x) > 0$ necessarily. This last property can be verified directly from (A15) and (A17) below, or more intuitively by observing that if it were not true, there would be a subinterval of values of x where $\varphi(x) < 0$ and φ is not monotonic. This, in turn, would imply that there exists values of $\lambda < 0$ for which (A15) has at least two solutions. But such solutions are also intersections of the curves $\Delta = D(\tau)$ and $\tau = T(\Delta)$; the former is always decreasing, and we saw earlier that, for all $\lambda < 0$, the latter is always increasing. Multiple intersections are therefore impossible.

Theorem 1 will now be proved by characterizing the set $\mathcal{B} \equiv \{B \geq 0 | \varphi \text{ is nonmonotonic on } (0, 1 - \alpha)\}$, then studying its variations with the parameters v, w, and η.

LEMMA 2: *Let $1 - \alpha < 2\beta\gamma$. The set $\mathcal{B} \equiv \{B \geq 0 | \exists\, x \in (0, 1 - \alpha), \varphi'(x) < 0\}$ is a nonempty interval of the form $\mathcal{B} = (\underline{B}, +\infty)$ with $\underline{B} > 0$, or $\mathcal{B} = [0, +\infty)$.*

PROOF:

Let us differentiate (A15):

$$\varphi'(x) \equiv \frac{\varphi(x)}{x} + x\Delta'(x)\left(1 - \frac{Bv^2/\gamma^2}{\Delta^2(x)}\right) - \frac{\beta\delta}{\eta\gamma}\left[\left(\frac{1}{x} - \frac{2\beta\gamma}{x^2}\right)\frac{1}{\Delta(x)} - \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta^2(x)}\right]$$

$$= \Delta(x) + \frac{Bv^2/\gamma^2}{\Delta(x)} - \left(\frac{\beta\delta}{\eta\gamma\Delta(x)}\right)\left(\frac{\beta\gamma - x}{x^2}\right) + x\Delta'(x)\left(1 - \frac{Bv^2/\gamma^2}{\Delta^2(x)}\right)$$

$$- \frac{\beta\delta}{\eta\gamma}\left[\left(\frac{1}{x} - \frac{2\beta\gamma}{x^2}\right)\frac{1}{\Delta(x)} - \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta^2(x)}\right].$$

Grouping terms, $\varphi'(x) < 0$ if and only if:

$$\text{(A17)} \quad \frac{Bv^2}{\gamma^2}\left(\frac{x\Delta'(x)}{\Delta(x)} - 1\right) > \left(\frac{x\Delta'(x)}{\Delta(x)} + 1\right)\Delta^2(x) + \left(\frac{\beta\delta}{\eta\gamma}\right)\left(\frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta(x)}\right).$$

We now establish the lemma through two intermediate claims.

Claim 1: If (A17) is satisfied for some $B \geq 0$ at some $x \in (0, 1 - \alpha)$, then $x\Delta'(x)/\Delta(x) > 1$. As a consequence, (A17) is satisfied at x for all $B' > B$.

PROOF:

If $\Delta'(x)/\Delta(x) - 1/x \leq 0$ the left-hand-side of (A17) is nonpositive, so on the right-hand side it must be that $\beta\gamma/x - 1 < 0$. But then:

$$\frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{\Delta'(x)}{\Delta(x)} > \frac{\beta\gamma}{x^2} + \left(\frac{\beta\gamma}{x} - 1\right)\frac{1}{x} = \frac{2\beta\gamma - x}{x^2}.$$

Since $x < 1 - \alpha < 2\beta\gamma$, this implies that the right-hand side of (A17) is positive, a contradiction.

Claim 2: There exists an $\hat{x} \in (0, 1 - \alpha)$ such that $x\Delta'(x)/\Delta(x) < 1$ on $(0, \hat{x})$ and $x\Delta'(x)/\Delta(x) > 1$ on $(\hat{x}, +\infty)$. As a consequence, for any $x > \hat{x}$, (A17) holds for B large enough.

PROOF:

Let us denote from here on $\omega \equiv v/\gamma w$. Since $\Delta^2(x) = w^2(1 + \omega^2x^2)/(1 - (\alpha + x)^2)$,

$$\text{(A18)} \quad \frac{\Delta'(x)}{\Delta(x)} = \frac{\omega^2 x}{1 + \omega^2 x^2} + \frac{\alpha + x}{1 - (\alpha + x)^2}.$$

Therefore $x\Delta'(x)/\Delta(x) > 1$ if and only if

$$\omega^2x^2(1 - (\alpha + x)^2) + x(\alpha + x)(1 + \omega^2x^2) > (1 + \omega^2x^2)(1 - (\alpha + x)^2) \Leftrightarrow$$

$$\omega^2x^3(\alpha + x) - (1 - (\alpha + x)^2) + x(\alpha + x) > 0 \Leftrightarrow$$

$$\omega^2x^3(\alpha + x) + (\alpha + x)(\alpha + 2x) - 1 > 0.$$

This last expression is clearly increasing in x on $(0, 1 - \alpha)$, from $\alpha^2 - 1 < 0$ at $x = 0$ to $\omega^2(1 - \alpha)^3 + 1 - \alpha > 0$ at $x = 1 - \alpha$. This proves Claim 2 which, together with Claim 1, establishes that the set $\mathcal{B}$ is a nonempty interval of the form $(\underline{B}, +\infty)$ or $[\underline{B}, +\infty)$. Moreover, note that its complement, $\mathbb{R}_+\backslash\mathcal{B} \equiv \{B \geq 0 | \forall\ x \in (0, 1 - \alpha),\ \varphi'(x) \geq 0\}$, is a closed set because φ' is continuous in B at every point. This implies that either $\mathcal{B} = (\underline{B}, +\infty)$ with $\underline{B} > 0$, or else $\mathcal{B} = [0, +\infty)$, and thereby finishes to establish Lemma 2. From (A17)–(A18), moreover, it is clear that $\underline{B}$ depends on v, w, and η only through v/w and ηv^2; let it therefore be denoted as $\underline{B}(v/w;\ \eta v^2)$.

To conclude the proof of Theorem 1 as well as the additional claims in footnote 28 concerning the exact number of steady states, we shall make use of a last lemma.

LEMMA 3: *For $B < \underline{B}(v/w;\ \eta v^2)$, there is a unique, stable, steady state. For $B > \underline{B}(v/w;\ \eta v^2)$ the same is true if $\lambda \notin [\underline{\lambda}, \bar{\lambda}]$, while for $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ there are $n \in \{2, 3, 4\}$ stable steady states. Moreover, if $1/\eta$ is small enough then $n \leq 3$, and if α is also small enough then $n = 2$.*

PROOF:

By definition, for $B < \underline{B}(v/w;\ \eta v^2)$ the function φ is strictly increasing everywhere, so the steady state is unique. The same is true for $B = \underline{B}(v/w;\ \eta v^2) > 0$, since then $\mathcal{B} = (\underline{B}, +\infty)$. The other measure-zero case, $B = \underline{B}(v/w;\ \eta v^2) = 0$, is too special to be of interest. Now, for $B > \underline{B}(v/w;\ \eta v^2)$, we saw earlier that there must be n stable equilibria and $n - 1$ unstable ones in the interval $(0, 1 - \alpha)$. To examine what values n can take, rewrite $\varphi(x) = \lambda\beta$ as:

$$S(x) \equiv \left[x^2\left(\Delta^2(x) + \frac{Bv^2}{\gamma^2}\right) - \left(\frac{\beta\delta}{\eta\gamma}\right)(\beta\gamma - x)\right]^2 - [\lambda\beta x\Delta(x)]^2 = 0. \tag{A19}$$

By (A14), $\Delta^2(x)$ is a polynomial fraction in x whose numerator and denominator are both of degree 2. Multiplying the whole equation (A19) by the squared denominator of $\Delta^2(x)$, we therefore obtain a polynomial $S^*(x)$ of degree $2 \times (2 + 2) = 8$, which can have at most 8 real roots. But we saw in the discussion following (A15) that $\varphi(x) = \lambda\beta$ must have an odd number of solutions on $(0, 1 - \alpha)$, with n intersections from above and $n - 1$ from below. The numbers of stable and unstable equilibria can therefore only be (1, 0), (2, 1), (3, 2), or (4, 3).

When $1/\eta = 0$ we can simplify (A19) by x^2, leaving for $S^*(x)$ only a polynomial of degree 6; this rules out $n = 4$. When, in addition, $\alpha = 0$, note from (A14) that $\Delta(x)$ depends on x only through x^2. As a consequence, the sixth-degree polynomial $S^*(x)$ is also a polynomial of degree 3 in x^2, so it has at most three real roots. This rules out $n = 3$. Finally, since the polynomial $S^*(x)$, like (A19), is continuous with respect to $1/\eta$ and α, so is (generically with respect to the other parameters) its number of real zeroes in the interval $(0, 1 - \alpha)$. The preceding results therefore also apply for $1/\eta$ and α small enough.

This concludes the proof of Theorem 1.

PROOF OF PROPOSITION 9:

When $1/\eta = 0$ the condition for $\varphi'(x) < 0$, namely (A17), becomes:

$$\frac{Bv^2}{\gamma^2} > \left(\frac{x\Delta'(x)/\Delta(x) + 1}{x\Delta'(x)/\Delta(x) - 1}\right)\Delta^2(x), \tag{A20}$$

with the requirement that the denominator must be positive. Using (A18), this can be rewritten as

$$B > \left(\frac{x^2 + \omega^{-2}}{x^3(\alpha + x) - \omega^{-2}(1 - (\alpha + x)(\alpha + 2x))}\right)$$
$$\times \left(\frac{x^2(2 - (\alpha + x)(\alpha + 2x)) + \omega^{-2}(1 - \alpha(\alpha + x))}{1 - (\alpha + x)^2}\right) \equiv \Gamma(x, \omega)$$

where $\omega \equiv v/\gamma w$. It is easily verified that the each of the bracketed functions is increasing in ω^{-2}, therefore:

$$\underline{B}(\omega) \equiv \inf\{\Gamma(x, \omega) | x \in (0, 1 - \alpha) \quad \text{and} \quad x^3(\alpha + x) > \omega^{-2}(1 - (\alpha + x)(\alpha + 2x))\} \tag{A21}$$

is strictly positive, and decreasing in ω. Observe next that, as ω tends to infinity, $\Gamma(x, \omega)$ approaches $\Gamma(x, +\infty) = x(2 - (\alpha + x)(\alpha + 2x))/(1 - (\alpha + x)^2)$, whose infimum value on $(0, 1 - \alpha)$ is zero; therefore, $\lim_{\omega \to +\infty} \underline{B}(\omega) = 0$. Finally, for any x with $x^3(\alpha + x) > \omega^{-2}(1 - (\alpha + x)(\alpha + 2x))$, note that $\Gamma(x, \omega) > \omega^{-2}/(1 - \alpha^2)$, which tends to infinity as ω tends to 0. Therefore $\lim_{\omega \to 0} \underline{B}(\omega) = +\infty$.

PROOF OF PROPOSITION 10:

Given lognormality, (29) becomes $\ln \kappa_t = m_t + \gamma\Delta_t^2/2 = (\ln y_t - \delta \ln l_t)/\gamma$, so substituting $\ln \tilde{\kappa}$ into the growth equation (15) yields:

$$\ln y_{t+1} - (\alpha + \beta\gamma + \phi)\ln y_t = \ln \tilde{\kappa} + \delta(\ln l_{t+1} - (\alpha + \phi)\ln l_t) - \mathcal{L}_v(\tau_t)v^2/2 - \mathcal{L}_\Delta(\tau_t)\gamma^2\Delta_t^2/2, \tag{A22}$$

with $\ln \bar{\kappa} \equiv \beta\gamma \ln s - \gamma(1-\gamma)w^2/2$. In a steady state, if $\alpha + \beta\gamma + \phi < 1$ the left-hand side equals $(1 - \alpha - \beta\gamma - \phi)$ times the output level $\ln y_\infty$; when $\alpha + \beta\gamma + \phi = 1$ it becomes equal to the asymptotic growth rate, $\lim_{t \to +\infty} \ln(y_{t+1}/y_t)$. As to the right-hand side, in a steady state with $\tau_t = \tau$ it becomes:

$$g_\infty(\tau) \equiv \ln \bar{\kappa} + \beta\gamma\delta \ln l(\tau) - \mathcal{L}_v(\tau)v^2/2 - \mathcal{L}_\Delta(\tau)\gamma^2 D(\tau)^2/2. \tag{A23}$$

For $\alpha = 0$ we saw earlier that $\mathcal{L}_\Delta(\tau) = \mathcal{L}_v(\tau) = \beta\gamma(1-\beta\gamma)(1-\tau)^2$; therefore $\mathcal{L}_v(\tau)v^2/2 + \mathcal{L}_\Delta(\tau)\gamma^2 D(\tau)^2/2$ is strictly decreasing in τ. Now, with $1/\eta = 0$ the labor-supply term is constant, therefore $g_\infty(\tau)$ is strictly increasing in τ; this proves the proposition's first claim. Conversely, when $\beta\gamma = 1$, then $\mathcal{L}_v(\tau) = \mathcal{L}_\Delta = 0$; with $1/\eta > 0$, $g_\infty(\tau)$, like $\ln l(\tau)$, is then decreasing in τ; hence the second claim.

PROOF OF PROPOSITION 11:

Once agent i knows his productivity z_t^i, hence also his income $y_t^i = z_t^i(k_t^i)^\gamma(l_t^i)^\delta$ and his investment subsidy rate $\hat{e}_t^i/e_t^i = (\tilde{y}_t/y_t^i)^{\tau_t}$, his decision problem takes the form:

$$\begin{aligned} \ln V_t^i &= \max_{l,\nu}\{(1-\rho)[\ln((1-\nu)y_t^i) - l^\eta] + (\rho/r')\ln[E_t(k_{t+1}^i)^{r'}] \mid k_{t+1}^i = \kappa\xi_{t+1}^i(k_t^i)^\alpha(\hat{e}_t^i)^\beta\} \\ &= \max_\nu\{(1-\rho)\ln(1-\nu) + \rho\beta\ln\nu\} + \max_l\{-(1-\rho)l^\eta + (1-\rho+\rho\beta(1-\tau_t))\delta\ln l\} \\ &\quad + \rho(\ln\kappa - (1-r')w^2/2) + [\rho\alpha + (1-\rho+\rho\beta(1-\tau_t))\gamma]\ln k_t^i \\ &\quad + (1-\rho+\rho\beta(1-\tau_t))\ln z_t^i + \rho\beta\tau_t \ln\tilde{y}_t, \end{aligned} \tag{A24}$$

where $\nu_t^i \equiv e_t^i/y_t^i$ is the savings rate. Strict concavity in ν and l is easily verified, and the first-order conditions directly yield the stated results.

PROOF OF PROPOSITION 12:

Substituting the optimal $l_t^i = l_t$ and $\nu_t^i = s$ into (A24) and denoting $\ln\kappa' \equiv \ln\kappa - (1-r')w^2/2$ yields:

$$\begin{aligned} \ln V_t^i &= \rho\ln\kappa' + (1-\rho)\ln(1-s) + \rho\beta\ln s \\ &\quad - (1-\rho)l_t^\eta + (1-\rho+\rho\beta(1-\tau_t))\delta\ln l_t \\ &\quad + [\rho\alpha + (1-\rho+\rho\beta(1-\tau_t))\gamma]\ln k_t^i \\ &\quad + (1-\rho+\rho\beta(1-\tau_t))\ln z_t^i + \rho\beta\tau_t\ln\tilde{y}_t. \end{aligned} \tag{A25}$$

Thus *conditional* on k_t^i, $\ln V_t^i$ is normally distributed, with variance $(1-\rho+\rho\beta(1-\tau_t))^2 v^2$. This implies:

$$U_t^i \equiv \frac{1}{r}\ln E[(V_t^i)^r \mid k_t^i] = E[\ln V_t^i \mid k_t^i] + r(1-\rho+\rho\beta(1-\tau_t))^2 v^2/2. \tag{26}$$

Therefore, to obtain U_t^i one simply needs to replace in (A25) the term in $\ln z_t^i$ by $-(1-\rho+\rho\beta(1-\tau_t))[1-r(1-\rho+\rho\beta(1-\tau_t))]v^2/2$. Finally, substituting in the value of $\tau_t \ln\tilde{y}_t$ from (A3) yields the claimed result, with:

$$\begin{aligned} \bar{u}_t &\equiv \ln[(1-s)^{1-\rho}s^{\rho\beta}] + \rho(\ln\kappa - (1-r')w^2/2) \\ &\quad + (\rho\alpha + (1-\rho+\rho\beta)\gamma)m_t + \rho\beta\gamma^2\Delta_t^2/2. \end{aligned} \tag{A27}$$

PROOFS OF THEOREM 2 AND PROPOSITION 13:

By (37) and (38), $\bar{\lambda} > \underline{\lambda}$ if and only if:

$$B(\bar{\tau}) - B(0) - (C(0) - C(\bar{\tau}))\left(\frac{2}{v^2}\right) > \rho\beta\bar{\tau}\left(\frac{2-\bar{\tau}}{v^2}\right)\gamma^2 D(0)D(\bar{\tau}) \tag{A28}$$

which yields Theorem 2, with:

$$G(\bar{\tau}, v^2/w^2) \equiv \rho\beta\bar{\tau}(2-\bar{\tau})\gamma^2\left(\sqrt{\frac{w^2/v^2+\beta^2}{1-(\alpha+\beta\gamma)^2}}\right)\left(\sqrt{\frac{w^2/v^2+\beta^2(1-\bar{\tau})^2}{1-(\alpha+\beta\gamma(1-\bar{\tau}))^2}}\right). \tag{A29}$$

Note that since $G(\bar{\tau}, v^2/w^2) > 0$, if $B(\bar{\tau}) - B(0) > \underline{B}$ then it must be that $\underline{\lambda} > 0$. Finally, both $\underline{\lambda}$ and $\bar{\lambda}$ are clearly increasing in $B \equiv B(\bar{\tau}) - B(0)$ and in $-(C(0) - C(\bar{\tau}))$ (hence in η). Since the common coefficient of these two terms is $1/(\bar{\tau}D(\bar{\tau}))$ in $\bar{\lambda}$ and $1/\bar{\tau}D(0)$ in $\underline{\lambda}$, Proposition 13 follows immediately.

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