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The current account and precautionary savings for exporters of exhaustible resources ☆

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ABSTRACT

Exporters of exhaustible resources have historically exhibited higher income volatility than other economies, suggesting a heightened role for precautionary savings. This paper uses a parameterized small open-economy model to quantify the role of precautionary savings for exporters of exhaustible resources, when the only source of uncertainty is the price of the exhaustible resource. The parameterized model fares moderately well at capturing current account balances in both cross-section and time-series data. The results show that the precautionary motive can generate sizable external sector savings, the more so the greater the weight of exhaustible resource revenues in future income.

1. Introduction

Exporters of exhaustible resources have historically exhibited considerably higher income volatility than other economies, suggesting a heightened role for the precautionary motive and related accumulation of buffer stock savings. Indeed, these exporters have accumulated sizable external savings over the last decade, contributing significantly towards global current account imbalances. How much of these savings can be attributed to the precautionary motive?

This paper documents the relevant empirical regularities and constructs a model that allows for estimation of the size of precautionary savings for exporters of exhaustible resources. To gauge the link between income volatility and savings, we use a representative agent small open economy model modified to account for an exhaustible resource sector with the price of the exhaustible resource as the only source of uncertainty. To focus on the external savings problem, the model abstracts from domestic investment and resource extraction decisions as well as from explicit insurance against shocks to export prices. We argue that these considerations are not of primary importance, when estimating precautionary savings for exporters of exhaustible resources. The representative agent then solves a self-insurance problem, whereby accumulation of foreign assets diversifies income away from the volatile exhaustible resources. Precautionary savings are represented by the change in (external) savings due to this self-insurance motive. The model is parameterized to replicate the relevant country-specific characteristics of thirteen oil and gas exporting economies whose exhaustible resource sectors account for a significant share of economic activity.

The framework is shown to yield quantitative predictions about a range of phenomena.

The model fares moderately well at capturing historical external savings behavior in both time-series and cross-section data. Annual changes in current account balances in the model and data exhibit a positive correlation with a mean of 0.70. During the post-2000 price boom, the model generates 2/3 of the accumulated current account surpluses in data. In a cross-section, the correlation between changes in net foreign assets, as a share of GDP, in the model and data is 0.73.

The modeling framework allows for a decomposition of external savings into an intertemporal consumption smoothing component, which is present even in the absence of uncertainty, and a precautionary savings component. We find that the consumption smoothing motive is the main determinant of the size of the current account. At the same time, the precautionary motive can generate sizable additional external savings and improves the model's fit with data. These findings are robust to the choice of parameter values and

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modeling assumptions. In terms of quantitative results, precautionary savings in 2007 add up to 1% of the sample countries' GDP or 36 billion dollars. Results for other years are similar in magnitude. The importance of the precautionary motive varies considerably across countries and is driven by the weight of exhaustible resource revenues in expected discounted future income.

In addition to providing estimates for the size of precautionary savings, the paper contributes to the literature by developing a small open economy model with *aggregate* uncertainty, where the net foreign asset position in the stationary equilibrium *is not* restricted to some exogenous value. As has been repeatedly noted (e.g., Schmitt-Grohé and Uribe, 2003; Ghironi, 2007), the representative agent small open economy model with aggregate uncertainty exhibits unit root dynamics in net foreign assets and therefore does not harbor a well-defined stationary equilibrium. The model of this paper avoids this problem by assuming that the only source of aggregate uncertainty is the price of the exhaustible resource. Once the resource is exhausted, the model becomes deterministic. As a result, the value of net foreign assets in the stationary equilibrium is determined endogenously and depends on the model's initial conditions as well as the history of subsequent shocks.

This paper deviates from the extensive literature on precautionary savings (e.g., Caballero, 1990; Carroll, 2001 and references therein) along several dimensions. First, it deals with aggregate, as opposed to idiosyncratic, shocks. Second, the focus is on open economies, with savings taking place through the external sector. Third, while the previous literature has concentrated exclusively on advanced economies, the sample in this paper includes both developed and developing countries.

Two previous studies are closely related to our work. Ghosh and Ostry (1997) find that aggregate income uncertainty increases current account balances in a group of advanced economies. Fogli and Perri (2006) show that differences in precautionary savings, induced by lower income volatility in the US relative to the rest of the world, can explain a significant share of the US current account deficits since early 1980s. Both studies find that the link between aggregate income volatility and external savings is economically significant. For the more volatile and open exporters of exhaustible resources, this channel is likely to have added relevance.

Another strand of related literature deals with the intergenerational equity in exhaustible resource countries. In essence, consumption smoothing considerations in the deterministic version of our modeling framework are used to determine the intertemporal allocation of exhaustible resource income.[1] Such a model can generate the large current account surpluses observed in exhaustible resource countries. Our paper differs from this literature, since we explicitly model the effects of uncertainty on the external sector dynamics. In the presence of a precautionary motive, the deterministic model overestimates consumption and therefore underestimates the size of savings and the current account balance in the short run, more so the greater the uncertainty attached to the exhaustible resource wealth and the larger the weight of exhaustible resources in economic activity.

The structure of the rest of the paper is as follows. The next section summarizes the empirical regularities that motivate our study. Section 3 presents the model and its optimal solution. Section 4 discusses the parameterization procedure and examines the fit between the model outcomes and historical data. Section 5 decomposes external savings in the model into two components — consumption smoothing and precautionary savings. Section 6 performs extensive sensitivity analysis of the model results. Finally, Section 7 concludes.

2. Income volatility and channels of diversification

This section documents the high income volatility in exhaustible resource countries and the extent to which high income volatility is transmitted into volatility of consumption. We also examine the role of various channels in reducing the volatility of consumption. Throughout the paper the definition of exhaustible resources is restricted to oil and gas, for which there are country-level estimates available on historical resource extraction and the size of the remaining stock of resources.

The volatility of income in exhaustible resource countries has been 2–3 times higher than in other economies (see panel a in Fig. 1). This finding remains valid in sample sub-periods (see panel a in Fig. 2). Although income volatility has decreased in recent decades, exhaustible resource countries continue to stand out. As of 2007, the 10-year rolling income volatility in exhaustible resource countries was more than twice the volatility in other economies and substantially larger than in major non-fuel commodity exporting countries.

The finding of high income volatility in exhaustible resource countries is consistent with other results in the literature. Easterly et al. (1993) and Broda (2004) among others report a significant correlation between output volatility and terms of trade volatility, while Baxter and Kouparitsas (2006) find that terms of trade volatility is the highest in fuel-exporting countries. Heightened income volatility in exhaustible resource countries stems from a combination of more volatile exhaustible resource prices, even when compared to other commodities, and a larger share of the volatile oil and gas component in total income. It should also be noted that exhaustible resource prices exhibit considerably larger variation than extraction quantities. At an annual frequency, prices over the most recent oil price cycle, i.e., 1980–2007, were 2–3 time more volatile than country-level extraction quantities.[2]

In contrast to income volatility, differences in the volatility of consumption for exhaustible resource countries are less pronounced. Panels a–b in Fig. 1 show that over the last half-century, the difference in the volatility of income of exporters of exhaustible resources and other countries has been larger than the difference in the volatility of consumption. Moreover, the difference in the volatility of consumption has altogether disappeared from 1990s onwards (see panel b in Fig. 2), as the volatility of consumption in exhaustible resource countries decreased to levels comparable to the other economies. Panels c in Figs. 1 and 2 present further quantitative evidence for the gap between income and consumption volatilities in exhaustible resource countries. This group of countries has exhibited historical ratios of consumption volatility to income volatility in the range of 0.5–0.7. In other economies the same ratios have been close to 1. Finally, panel d in Fig. 1 reports the correlation between income and consumption. Again, exhaustible resource countries exhibit lower income–consumption correlations than other economies. These empirical results indicate that countries with exhaustible resources have been able to smooth consumption in the face of large income shocks.

Next we investigate the channels through which volatility of consumption is mitigated. In theory, a country could insure against exhaustible resource income volatility by locking in future sale prices or by altogether selling the remaining stock of the resource. But usage of such tools remains very limited. Contrary to the prescriptions of economic theory, domestic ownership of exhaustible resources in major oil producing countries has been on the rise in recent decades and markets for hedging resource price fluctuations remain very small. As of April 2009, the total open interest in exchanges and over-the-counter markets for hedging resource price fluctuations was estimated at only 4 weeks of world oil consumption and 0.18% of proven oil reserves (see Borensztein et al., 2009). Daniel (2001)

[1] This modeling framework has been advocated in Davis et al. (2003) and applied by numerous studies (see, e.g., de Carvalho Filho, 2007 for an application to Trinidad and Tobago, Takizawa, 2005 for Kuwait, Bailen and Kramarenko, 2004 for Iran and Thomas et al., 2008 for a cross-country study).

[2] If one excluded the two Gulf Wars, prices are 4 times more volatile than extraction quantities (British Petroleum, 2008).

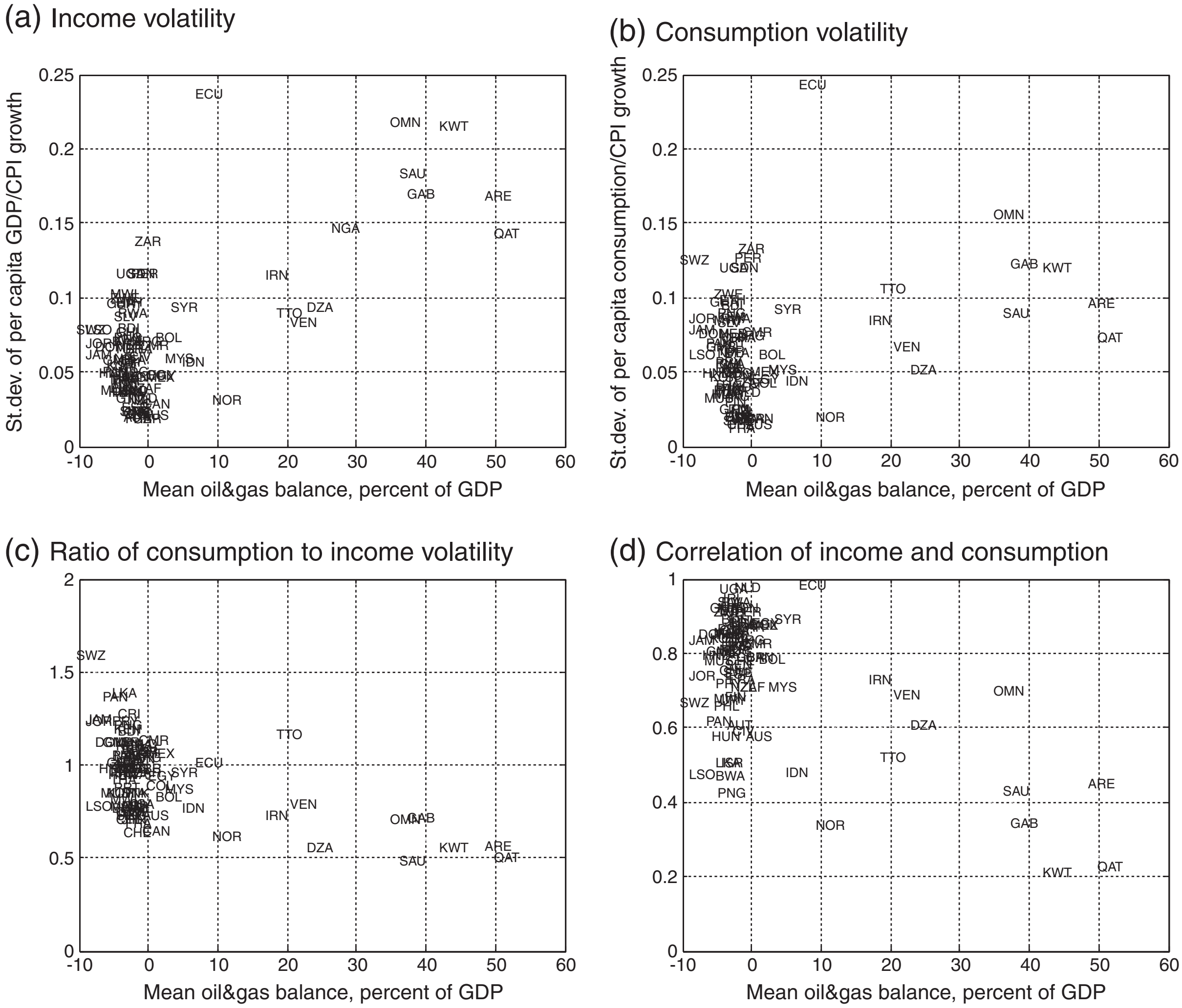

Fig. 1. Historical income and consumption volatility in exhaustible resource countries, 1964–2008. Sources: World Bank, 2008 World Development Indicators. Notes: Sample restricted to countries with at least 25 annual observations and a population of at least 1 million, which limits the sample to 90 countries.

argues that political constraints have held back governments from using explicit insurance.

Another approach would be to vary resource extraction quantities in a manner that offsets the effect of price changes on exhaustible resource revenues or reduces the dependence on such resources. An extensive literature has studied the supply-side of exhaustible resources under different market structures, but has not reached any consensus.[3] For the purpose of this paper, we note that price variation accounts for 85–90% of the 1-year changes in exhaustible resource revenues.[4] Longer horizons exhibit higher cross-country variation for contributions of prices and quantities, but average contributions for the sample of oil countries remain unchanged — price changes account for 80–95% of the variation in revenues for horizons of up to 10 years.

To further assess the relationship between the exhaustible resource price and extraction quantities, we consider the response of extraction quantities to the oil price boom of 1999–2008, which was not expected prior to 1999. Energy Information Administration (1998) forecasted that the real price of oil in 2005 would remain broadly unchanged at 24 USD/barrel (in 2005 prices), while the actual price in 2005 was 56 USD/barrel (in 2005 prices). Despite the surge in the price, Energy Information Administration (1998) projections for total extraction of oil in the sample countries were only 1% below the observed extraction quantities for 2005. Over that 7-year horizon, oil production exhibited very limited response to the changes in the price. A similar comparison of forecasts and realizations for the period from 1998 to 2008 would suggest an even smaller price elasticity of supply.

A possible explanation for the lack of response in extraction quantities – consistent with the model of this paper – is that changes in extraction capacity are very costly and their implementation requires large time lags. This property of resource extraction, when combined with the observation that historically exhaustible resource price follows a persistent trendless process, implies that extraction quantities cannot diversify exhaustible resource risk either at short or long horizons. Such a diversification strategy is prohibitively expensive in the short run, while the nature of the resource price limits its benefits in the long run.

An alternative to explicit insurance and adjustments in extraction quantities is income diversification, with an aim to counter income fluctuations over the oil price cycle and reduce exposure to the volatile exhaustible resource revenues over longer horizons. One potential channel for such diversification is external savings, whereby conversion of exhaustible resource revenues into foreign assets smoothes

[3] See, e.g., Griffin (1985) and Ramcharran (2002). A survey of the literature on the structure of the oil market by Cremer and Salehi-Isfahani (1991) concludes that "…the level of disagreement is unsettling" (p. 93).

[4] Computed for country i and year t as $\ln\left(\frac{price^i_{t+1}}{price^i_t}\right) \Big/ \ln\left(\frac{revenues^i_{t+1}}{revenues^i_t}\right)$.

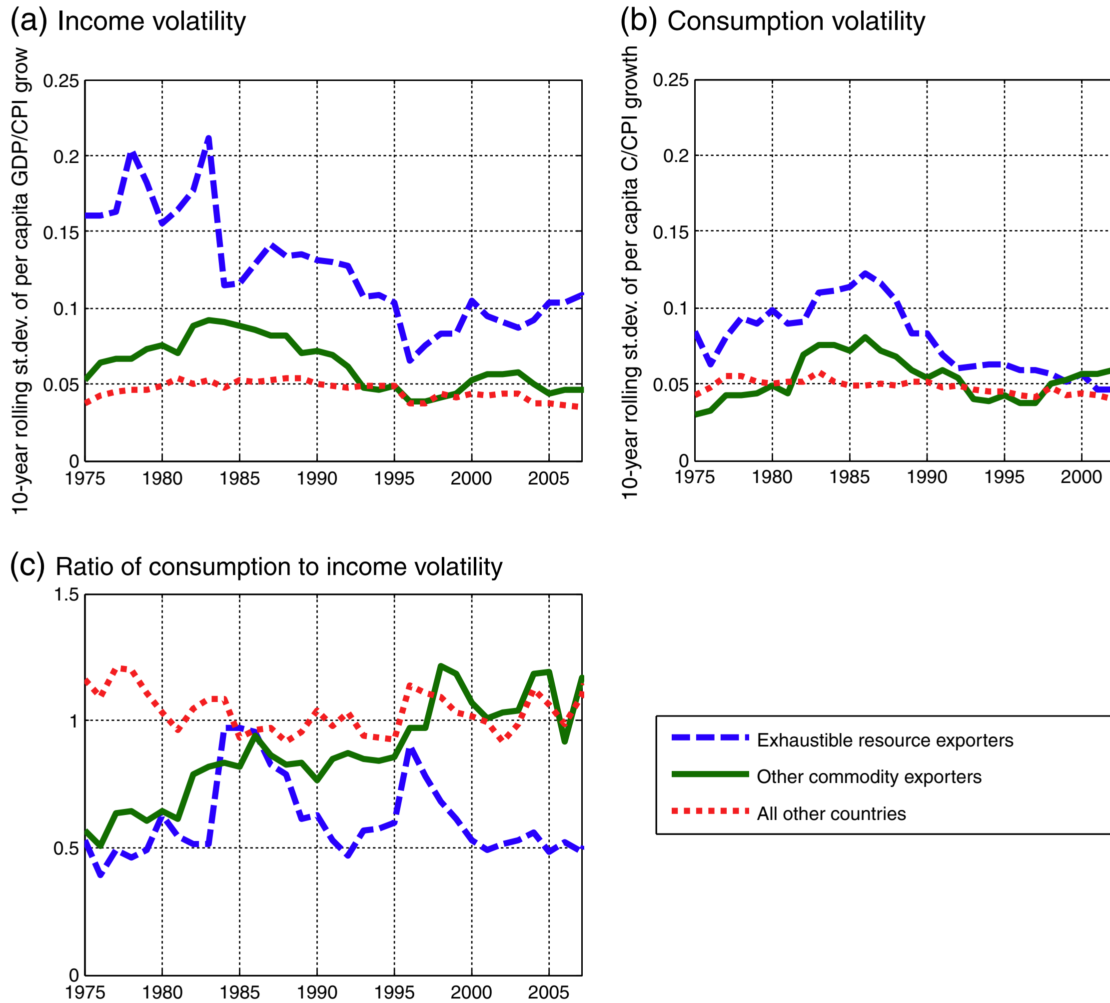

Fig. 2. Output volatility by decade. Sources: World Bank, 2008 World Development Indicators. Notes: 'Exhaustible resource exporters' are defined as economies, where median trade surplus for oil and gas during the period 1964–2007 was at least 10% of the GDP. This procedure identifies 11 exhaustible resource exporters — Algeria, Gabon, Iran, Kuwait, Nigeria, Oman, Qatar, Saudi Arabia, United Arab Emirates, Venezuela and Trinidad and Tobago. Yemen and Norway are excluded from the group, since in both economies sizable oil and gas exports started only during the 1990s. Due to the lack of data prior to 1990, CIS countries (e.g., Russia, Kazakhstan, and Azerbaijan) are also excluded from the sample. 'Other commodity exporters' include Iceland, New Zealand, Chile, Costa Rica, Ecuador, Honduras, Malaysia, Ghana, and Côte d'Ivoire.

consumption and gradually increases the share of income from foreign sources. The presence of this diversification channel would be consistent with exhaustible resource countries on average running current account surpluses, with larger surpluses when exhaustible resource prices are high.

In the absence of reliable data on the evolution of net foreign asset positions, we examine this channel using current account balances, depicted in Fig. 3. Exhaustible resource countries have been running sizable current account surpluses in periods with high exhaustible resource prices and less-than-offsetting deficits when prices are low. In particular, an average current account deficit of 1.2% of GDP during the 90s was followed by an average surplus of 13.7% of GDP in 2000–2007. Foreign asset accumulation during the price boom of 1974–1981 was similar in magnitude, with an average surplus of 15.4% of GDP.

Another potential diversification channel operates through the 'conventional' domestic economic activity that is not directly related to the extraction of exhaustible resources. Expansion of conventional economic activity lowers the share of volatile exhaustible resources in total income and may provide additional diversification, depending on the correlation of conventional income with the price of exhaustible resources.

As shown in Fig. 4, with the exception of Venezuela and Libya, exhaustible resource countries have experienced significant growth in conventional economic activity over the most recent oil price peak-to-peak cycle, i.e., 1980–2007. Panel b in Fig. 4 reports a decomposition of the growth rate into contributions of labor input and labor productivity. All exhaustible resource countries in our sample have experienced a sizable increase in labor force, which accounts for the bulk of observed output growth. At the same time, there is no systematic positive contribution from labor productivity to output growth for this group of countries. The latter result is in line with the findings of the 'resource curse' literature (see e.g. Sachs and Warner, 2001), recaptured in Fig. 5. Over the most recent oil price cycle, per capita income in exhaustible resource countries on average grew at a slower pace than in other economies. Furthermore, for the group of countries heavily dependent on exhaustible resources, the average growth in per capita income was close to zero. We also find that annual growth rates in the conventional sector exhibit a small positive correlation with growth rates in the exhaustible resource sector (see panel c in Fig. 4). The average correlation in the sample of twelve exhaustible resource countries is 0.19. Thus, the conventional sector does not contribute to the diversification of income over the oil–price cycle.

Overall, empirical evidence indicates that income diversification in exhaustible resource countries takes place through two main channels. First, exhaustible resource countries accumulate foreign assets. By doing so they smooth consumption relative to income and reduce the share of volatile exhaustible resources in aggregate income. Second, domestic 'conventional' economic activity in countries with exhaustible resources

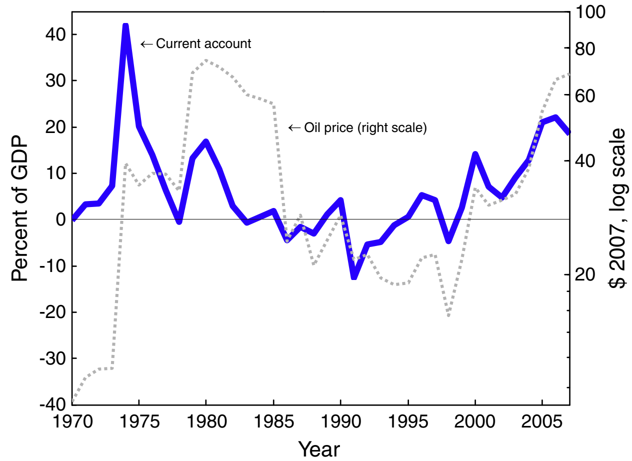

Fig. 3. The current account balance and the oil price. Sources: International Monetary Fund (2009) World Economic Outlook and British Petroleum (2008). Notes: 'Current account' defined as the sum of CA balances in exhaustible resource countries, normalized by the sample countries' GDP, all in current prices. For the definition of exhaustible resource countries see notes in Fig. 2.

is increasing in tandem with the expanding labor force, reducing the share of volatile exhaustible resources in aggregate income. In our study of precautionary savings, we include both of these channels.

To gauge the importance of precautionary savings, while controlling for the other important drivers of savings behavior – the desire to smooth consumption of the temporary exhaustible income over time regardless of uncertainty – we need a model. The construction and application of such a model is the central task of the rest of the paper.

3. Modeling framework

This section presents a model that captures the savings decision facing producers of exhaustible resources with volatile prices.

3.1. Small open endowment economy

Consider a small open economy with two sectors: one is conventional and the other is an exhaustible resource extracting sector. The aggregate production technology for the conventional sector is:

$$Y_t = AL_t, \tag{1}$$

where Y_t is output, A is a measure of productivity and L_t is labor input, which grows at a constant rate:

$$L_{t+1} = (1+n)L_t, \tag{2}$$

and is the only source of growth in the conventional sector. In the baseline model the conventional sector facilitates income diversification only through the exogenous growth in the labor force. The model abstracts from domestic investment/savings decision and the only endogenous channel of diversification is the external sector.[5] The effect of time-varying productivity is examined in a subsequent section on sensitivity analysis.

In the exhaustible resource sector, the extraction technology requires no factor inputs and output follows an exogenously specified sequence:

$$Z_t \begin{cases} \geq 0 & \text{for } t \leq T \\ = 0 & \text{for } t > T \end{cases}, \tag{3}$$

where Z_t is the quantity extracted. Future extraction quantities, $\{Z_t\}_{t=0}^{T} \geq 0$, are known at time t and such resources are exhausted in period T. The model does not address the question of optimal extraction quantities and instead takes the available projections for future extraction as given. The focus is on the uncertainty surrounding the prices of the exhaustible resource instead of its extraction quantities, which historically have exhibited a considerably smaller variation.

The aggregate per-period resource constraint of the economy is:

$$C_t + B_{t+1} = (1+r)B_t + p_t Z_t + Y_t. \tag{4}$$

In Eq. (4) C_t represents aggregate consumption. The difference between aggregate production and absorption is the trade balance, $B_{t+1} - (1+r)B_t$, where B_t stands for the stock of net foreign assets as of the end of period $t-1$ and r is the risk-free rate of return. We assume, in line with the empirical evidence, that the economy cannot lock in future prices of the exhaustible resource in insurance markets and the ownership of the stock of the exhaustible resource is non-transferable. Hence, in each period the extracted amount, $p_t Z_t$, is the only source of exhaustible resource related revenue.

The relative price of the exhaustible resource – which is determined outside this economy – follows a multiplicative error process:

$$p_{t+1} = ((1-\rho)\overline{p} + \rho p_t)\varepsilon_{t+1}, \tag{5}$$

where p_0 is given, $\overline{p}$ is the unconditional mean of p_t, ε_t is log normally distributed with mean 1 and variance σ_ε^2 and $\rho<1$.[6] In line with empirical findings (e.g., Krautkraemer, 1998; Lin and Wagner, 2007, and references therein), the relative price of the exhaustible resource in the model is assumed to be trendless in the long run.

The economy is inhabited by a representative household with the following CRRA preferences:

$$\sum_{t=0}^{+\infty} \beta^t L_t U(C_t/L_t), \tag{6}$$

where

$$U(C_t/L_t) = \begin{cases} \dfrac{(C_t/L_t)^{1-\sigma}}{1-\sigma} & \text{for } 0<\sigma<1 \text{ or } \sigma>1 \\ \log(C_t/L_t) & \text{for } \sigma=1 \end{cases}. \tag{7}$$

In Eq. (6), β is a subjective discount factor, σ determines the degree of risk aversion and intertemporal elasticity of substitution and L_t is the number of members in the representative household.

3.2. Optimal solution

To solve the model, it is instructive to recast all quantities in terms of ratios to the non-exhaustible resource sector output, where variables expressed in terms of effective units of labor are denoted

[5] In a more general model with endogenous capital, investment would provide an alternative diversification channel only when the domestic return on capital exceeds the return on foreign assets. This could be the case if, for example, the economy is credit constrained or experiences a positive productivity shock. Exhaustible resource revenues can then be used to exploit the temporarily higher domestic returns. However, on the balanced growth path, the return on domestic capital has to equal the world interest rate, limiting the long-run impact of this diversification channel.

[6] This process is borrowed from Borensztein et al. (2009). We do not use an AR(1) in logs, because it complicates the interpretation of precautionary savings (due to $E(e^{p_t}) > e^{E(p_t)}$). AR(1) in levels is not used to avoid negative prices.

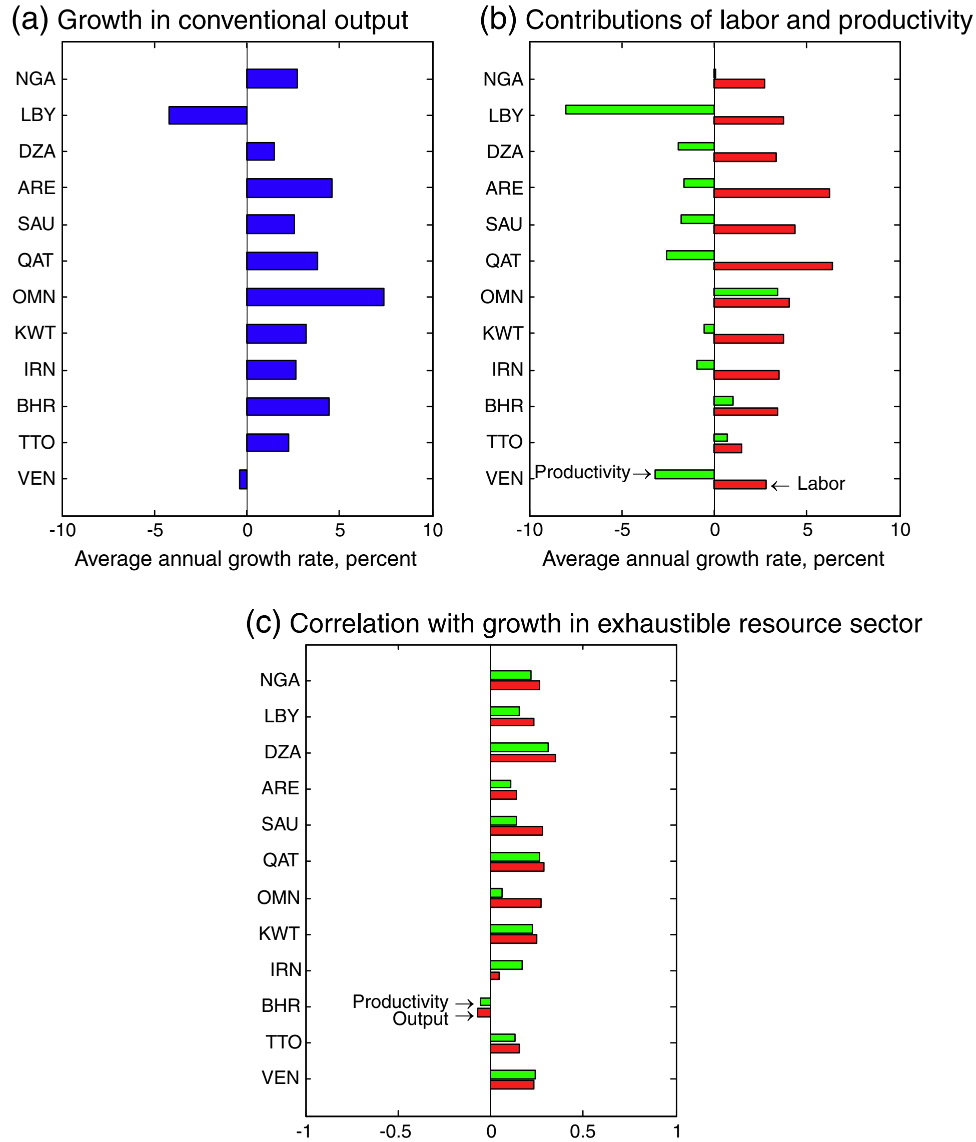

Fig. 4. Growth in conventional output, contributing factors and correlation with exhaustible resources (1980–2007). Sources: World Bank, 2008 World Development Indicators, International Monetary Fund (2009) World Economic Outlook and United Nations (2008). Notes: Due to lack of sectoral data on economic activity 'Conventional output' defined as GDP less exports of oil and gas. Growth in labor is defined as growth in population of age 15–64 from UN data.

by lowercase letters, e.g. $y_t = \frac{Y_t}{AL_t}$. Then the maximization problem can be formulated as:

$$\max_{\{c_t, b_{t+1}\}} E_0 \sum_{t=0}^{+\infty} \tilde{\beta}^t U(c_t), \tag{8}$$

subject to:

$$\begin{aligned} c_t + (1+n)b_{t+1} &= (1+r)b_t + p_t z_t + y_t, \\ p_{t+1} &= \left((1-\rho)\,\overline{p} + \rho p_t\right)\varepsilon_{t+1}, \\ \lim_{S\to+\infty} \left(\frac{1+n}{1+r}\right)^S b_S &= 0, \end{aligned} \tag{9}$$

where $\tilde{\beta} \equiv \beta(1+n)$ and b_0, p_0 are given.[7]

It is further assumed that $r = 1/\beta - 1$, which puts the model solution on a balanced growth path after exhaustible resources run out. The balanced growth path is characterized by a common constant growth rate, n, for consumption, output and the net foreign asset position, which allows output shares, c_t/y_t and b_t/y_t, to remain constant over time.

3.2.1. Deterministic case

In the deterministic case, the expression for optimal consumption is:

$$c = y + (r-n)\,b_0 + \frac{r-n}{1+r} \sum_{t=0}^{T} \left[\frac{1+n}{1+r}\right]^t p_t z_t. \tag{10}$$

Per-period consumption equals the sum of conventional output, the annuity value from the initial net foreign assets and the annuity value from the discounted steam of future exhaustible resource revenues. For a given income stream, the consumption–output ratio is therefore set at a constant optimal level and the external balance is used to smooth out any variation in income over time. In particular, the representative household accumulates wealth in periods with exhaustible resource revenues, and consumes interest income from the accumulated assets

[7] The problem in Eqs. (8)–(9) has a solution if the growth rate of the non-exhaustible resource sector is less than the real interest rate: n<r.

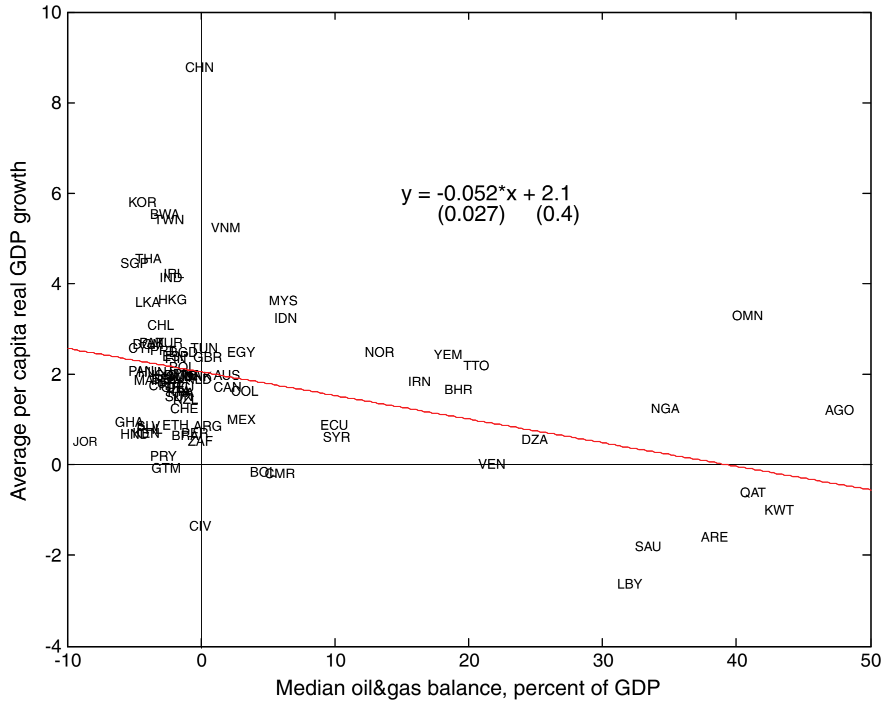

Fig. 5. Per capita real GDP growth rates, 1980–2007. Sources: World Bank, 2008 World Development Indicators and International Monetary Fund (2009) World Economic Outlook.

once exhaustible resource revenues run out. The ratio of foreign assets-to-output increases during the period of asset build-up and stays constant thereafter. To achieve this with a growing labor force, the economy must run current account surpluses on its balanced growth path. Since constant per capita consumption is achieved, the curvature in the utility function does not play any role in this deterministic case.

3.2.2. Stochastic case

In addition to considerations covered by the deterministic case, the risk-averse representative household now faces uncertainty about the future price of the exhaustible resource and wants to insure against variation in future income. In the absence of explicit insurance, the household solves a simple self-insurance problem. Holdings of net foreign assets are used to lower the exposure to the uncertain income from exhaustible resources.

In the model, *precautionary savings* are defined as the change in net foreign assets due to this self-insurance motive — and for any given period they can be calculated as the difference in net foreign asset positions between the deterministic and stochastic versions of the model.[8]

The exhaustible nature of the resource plays a crucial role in the model's solution. It makes the model deterministic from period T onwards and ensures a finite optimal value for expected consumption at the infinite horizon. To accommodate the precautionary savings motive, the optimal consumption in the stochastic model is upward tilting (until resources are exhausted and uncertainty is resolved) and accompanied by gradual accumulation of a buffer stock of net foreign assets. In initial periods, consumption is lower than in the deterministic case, but as savings accumulate and the interest income from foreign assets grows, it eventually exceeds the consumption path from the deterministic case. The stochastic version of the model does not have a closed form solution, but it can be solved numerically. Details of the solution method are presented in Appendix A.

[8] This definition of precautionary savings differs from the one used by studies of precautionary savings in models with idiosyncratic shocks, as in e.g. Carroll (2001), where precautionary savings are usually defined as change in the distribution of savings in a stationary steady state.

4. Parameterization and fit with historical data

This section applies the model to selected exporters of exhaustible resources in a historical context. The gist of the exercise is to specify the required model inputs and then solve for the optimal model responses in terms of consumption and external savings to a sequence of actual oil price realizations (i.e., shocks). The results of this section focus on the fit between country-specific model outcomes and historical data.

4.1. Model parameterization

The model is applied to thirteen exhaustible resource economies — Algeria, Iran, Kazakhstan, Kuwait, Libya, Nigeria, Norway, Oman, Qatar, Russia, Saudi Arabia, United Arab Emirates and Venezuela. Model economies differ in terms of their labor growth rate, stock of initial net foreign assets, share of exhaustible resource revenues in total output and extraction path for exhaustible resources. All other inputs of the exercise are identical across model economies.

To parameterize the model, one period is taken as one year in the data. For all sample economies the subjective discount rate is assumed to take a value of $\beta = 1/(1.04)$ and the curvature in CRRA utility is set to $\sigma = 2$. Both values are in the range of what is considered standard in the literature.

Parameters of the price process in Eq. (5) are based on a maximum likelihood estimation of the process using annual oil price data for 1970–2007, depicted in Fig. 3.[9] The relevant parameter values are $\rho = 0.89$ [S.E. $= 0.12$] and $\sigma_\varepsilon^2 = 0.092$ [S.E. $= 0.016$]. The estimated unconditional mean price, expressed in 2007 US dollars, is $p = 39.1$. All countries in the sample face the same price process.

The remaining model inputs are country-specific and are summarized in Tables 1 and 2. Oil and gas reserves are obtained from United States Geological Survey (2000), which estimates the two stocks as of end-of-1995 for each sample country (see column 5 in Table 1). The stock is defined as the sum of “known oil/gas”, “undiscovered oil/gas”

[9] For details on the estimation procedure see Borensztein et al. (2009).

Table 1
Country-specific model parameters and initial values.

	Labor force growth rate (%)		End-of-1995 NFA position (% of GDP)	Oil and gas revenues (% of GDP in 2007)	End-of-1995 oil and gas reserves (bn brls oil equivalent)	Projected exhaustion year for resources
	1996–2006	post-2006				
Algeria	4.2	0.7	−71	44	98	2077
Iran	3.4	0.3	−9	27	432	2171
Kazakhstan	0.8	0.2	−16	24	91	2105
Kuwait	6.0	1.0	181	54	113	2129
Libya	2.5	0.9	50	59	71	2117
Nigeria	2.8	2.0	−105	32	128	2108
Norway	1.4	0.3	4	24	115	2091
Oman	4.6	1.3	−6	45	29	2057
Qatar	6.5	0.7	985	56	148	2148
Russia	0.5	−0.9	−3	19	1012	2163
Saudi Arabia	0.6	1.4	40	53	707	2221
United Arab Emirates	8.1	1.2	277	41	152	2095
Venezuela	3.5	0.9	−1	27	149	2159

Sources: International Monetary Fund (2009) World Economic Outlook, British Petroleum (2009), United States Geological Survey (2000), Energy Information Administration (2008), United Nations (2008), Lane and Milesi-Ferretti (2007).
Note: Parameters and initial values not reported in the table are identical for all sample countries.

and “reserve growth”, less “cumulative oil/gas” extracted as of end-of-1995. Consistent with the dating of the reserve estimates, we set 1996 as the first year in the parameterized model.[10] Paths for oil and gas extraction quantities for 1996–2008 are obtained from British Petroleum (2009). Projections for the subsequent 2010–2030 period are taken from Energy Information Administration (1998). To project beyond 2030, we assume that resource extraction proceeds at a constant level until reserves are exhausted. The calculation is done separately for oil and gas and resulting country-specific extraction paths are summarized in Table 2. Annual labor force growth rate, n, is obtained from United Nations (2008) data and projections for working age population. For 1996–2006 average growth rate from historical series is used, while from 2007 onwards n is set equal to each country's average projected growth rate over 2010–2050. The initial net foreign asset stock, b_{1996}, is calculated as the end-of-1995 share of net foreign assets over the total output and obtained from the External Wealth of Nations data set (Lane and Milesi-Ferretti, 2007). Finally, the size of the exhaustible resource sector, $p_t z_t$, in the model is normalized to match the trade balance for oil and gas, as a share of GDP, which is our proxy for the size of the exhaustible resource sector in data. This normalization is done for the base year, which we set to 2007. For all other years the size of the exhaustible resource sector in the model is then determined by the evolution of L_t and $p_t z_t$.[11]

Country-specific model inputs exhibit considerable variation across the 13 sample countries. For example, with an exhaustible resource output share of 0.19, Russia in 2007 is the least dependent on exhaustible resource revenues. At the other end of the spectrum, Libya's exhaustible resource output share exceeds that of the conventional sector. In terms of the exhaustible resource lifespan, Oman is assessed to be the first to run out of exhaustible resources, by 2057. On the other end of the spectrum, in Russia, Iran, Saudi Arabia and Venezuela, extraction is projected to continue beyond 2150. Similarly, there is significant variation in the extraction time profiles in Table 2. By 2030 extraction quantities in Norway are projected to decrease by 20% relative the peak in 2007, while in Kazakhstan extraction quantities are projected to more than double over the same period.

4.2. Model results

To obtain time-series of optimal model outcomes, the infinite horizon problem is solved from the perspective of each year from 1996 till 2008. For each optimization, the initial price of exhaustible resources, p_0, is taken from the annual data used in the estimation of the price process. Historical price realizations can then be interpreted as positive or negative shocks to the exhaustible resource price, where shock is defined as the difference in the *path* of the conditional mean price between the current and previous year's price realizations. Optimizations from the perspective of adjacent years are connected though the choice of net foreign assets. In particular, the optimal choice of b_{t+1} from the perspective of year t is taken as the initial

Table 2
Summary of historical and projected extraction path for liquids (oil and gas, billion barrels oil equivalent per year).

	1996	2000	2005	2007	2010	2015	2020	2025	2030	2050
Algeria	0.9	1.1	1.3	1.3	1.5	1.7	1.9	2.0	2.2	0.8
Iran, I.R. of	1.6	1.8	2.2	2.3	2.4	2.5	2.7	2.9	3.2	3.2
Kazakhstan	0.2	0.3	0.6	0.7	0.9	1.2	1.3	1.5	1.7	1.7
Kuwait	0.8	0.9	1.0	1.0	1.0	1.2	1.2	1.3	1.3	1.3
Libya	0.6	0.6	0.7	0.8	0.9	0.8	0.8	0.8	0.8	0.8
Nigeria	0.8	0.9	1.1	1.1	1.2	1.4	1.5	1.7	1.8	1.8
Norway	1.4	1.5	1.6	1.6	1.5	1.4	1.3	1.3	1.2	1.2
Oman	0.4	0.4	0.4	0.4	0.4	0.5	0.6	0.6	0.7	0.3
Qatar	0.3	0.4	0.7	0.8	1.0	1.3	1.6	1.8	2.0	0.8
Russia	5.7	5.7	7.1	7.3	7.5	7.9	8.2	8.4	8.8	8.9
Saudi Arabia	3.7	3.8	4.5	4.4	4.5	5.1	5.5	5.8	6.1	6.1
United Arab Emirates	1.1	1.2	1.3	1.4	1.4	1.5	1.6	1.7	1.7	1.7
Venezuela, Rep. Bol.	1.3	1.4	1.2	0.9	0.8	0.9	1.0	1.2	1.2	1.2

Source: Data for 1996, 1997,…, 2007 refers to the actual extraction as reported in British Petroleum (2009). Data for 2010, 2015,…, 2030 are projections from Energy Information Administration (2008). Quantities for years between 2007 and 2030 were extrapolated using a linear trend. After 2030 production quantities kept constant until reserves, as reported in column 5 of Table 1, are exhausted.

[10] The exercise is not extended to earlier years because of lack of consistent reserve estimates. For the majority of sample countries data on the stock of reserves prior to 1996 show significant reserve growth. To allow for reserve growth in our model, one would need to introduce shocks to the stock of reserves, which is beyond the scope of the current paper. Results of an extended exercise for Norway, covering 1974–2007, are reported in Bems and de Carvalho Filho (2009).

[11] The model parameterization in this paper contains several differences from an earlier working paper (Bems and de Carvalho Filho, 2009). First, the assumed curvature in the utility function is lowered from $\sigma=6$ to $\sigma=2$ to reflect the standard choice in the RBC literature. Second, the amount of remaining reserves is based on estimates from the United States Geological Survey (2000) rather than British Petroleum (2008). We judge the former source's definition of reserves to be more appropriate for the purpose of this paper. Third, the price process for exhaustible resources is changed from AR(1) to a multiplicative error process. Fourth, the base year is moved from 2006 to 2007 and the sample is extended to include Oman and Russia. These changes did not alter the main findings of the paper.

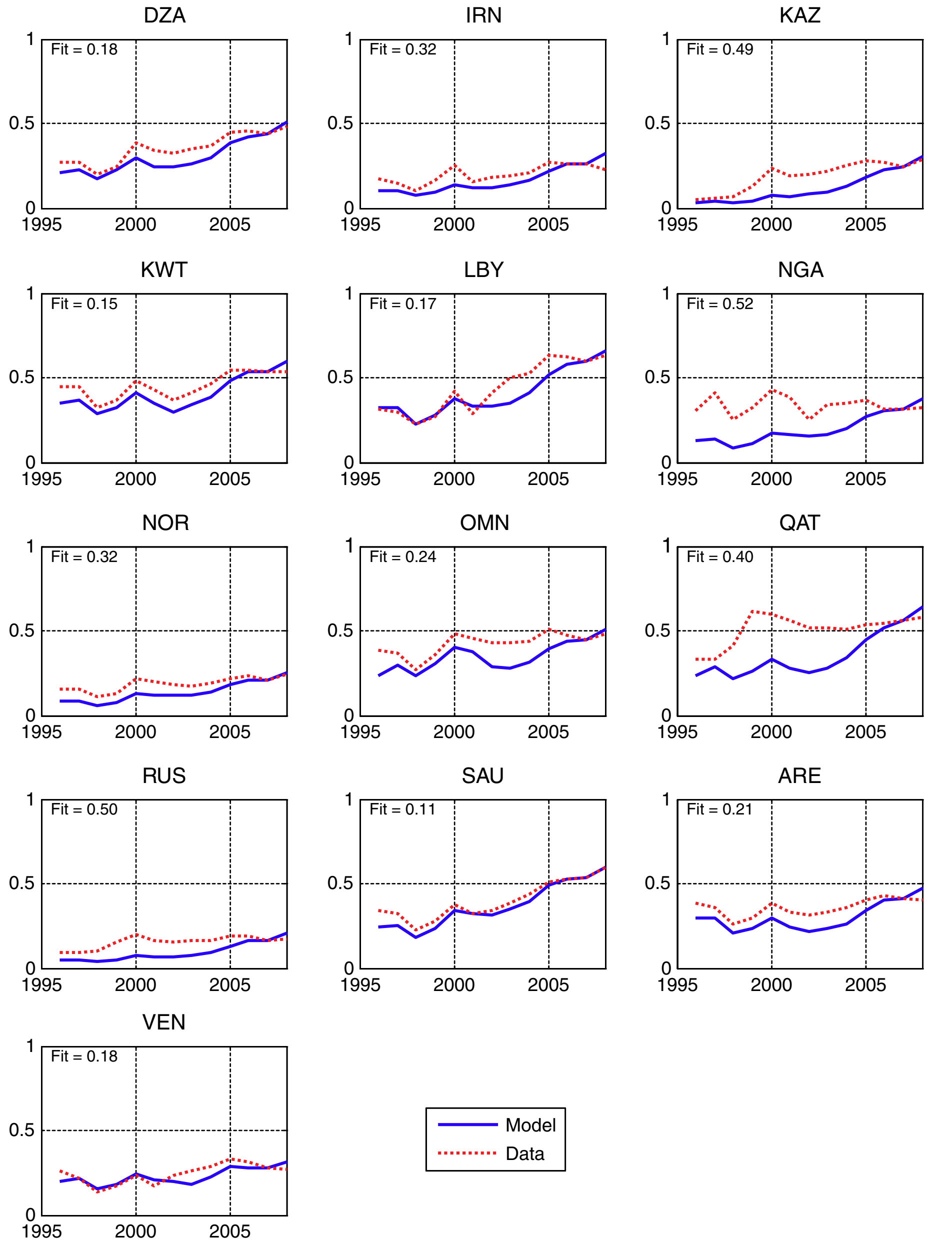

Fig. 6. Exhaustible resource share in economic activity in the model and data. Notes: "Fit" for each country calculated as root mean square error normalized by the average size of the exhaustible resource sector.

value, b_0, for the subsequent year's problem. Notice that the setup of the exercise implies that – consistent with the model of Section 3 – in the absence of price shocks the model is deterministic and needs to be solved only once.

It is instructive to first compare the share of exhaustible resources in total output in the model and data, reported in Fig. 6. In the data this share is approximated by net exports of oil and gas, as a share of GDP. By construction, the two shares coincide in 2007. However, for other years they can deviate to the extent that changes in the price of the exhaustible resource, extraction quantities and labor input in the model do not capture all the variation in economic activity in data. Results show that for majority of the sample countries the model can capture the trend in data. This finding supports our earlier empirical claim that changes in the labor force and oil and gas revenues account for the bulk of growth in countries with exhaustible resources.

For several countries the fit with data is more problematic. This is the case for Russia and to a lesser extent Norway, which have experienced significant growth in labor productivity, not captured by the model. These two economies are also the least dependent on exhaustible resources, further limiting the applicability of the model. Other major discrepancies between model outcomes and data in Fig. 6, i.e., Qatar, Nigeria and Kazakhstan, are driven by differences in exhaustible resource revenues as measured by the oil price and extraction quantities in the model and net exports of oil and gas in aggregate data. Model results for these countries should be interpreted with caution.

Per capita growth rates for output and consumption in the model are summarized in Fig. 7. Per capita output growth in the conventional sector is by assumption zero, therefore any variation in aggregate output is driven entirely by changes in per capita revenues in the exhaustible resource sector. As expected, aggregate volatility increases with the share of exhaustible resources in economic activity.

The optimal consumption choice smoothes the deterministic change in per capita extraction quantities, so that the remaining variation in

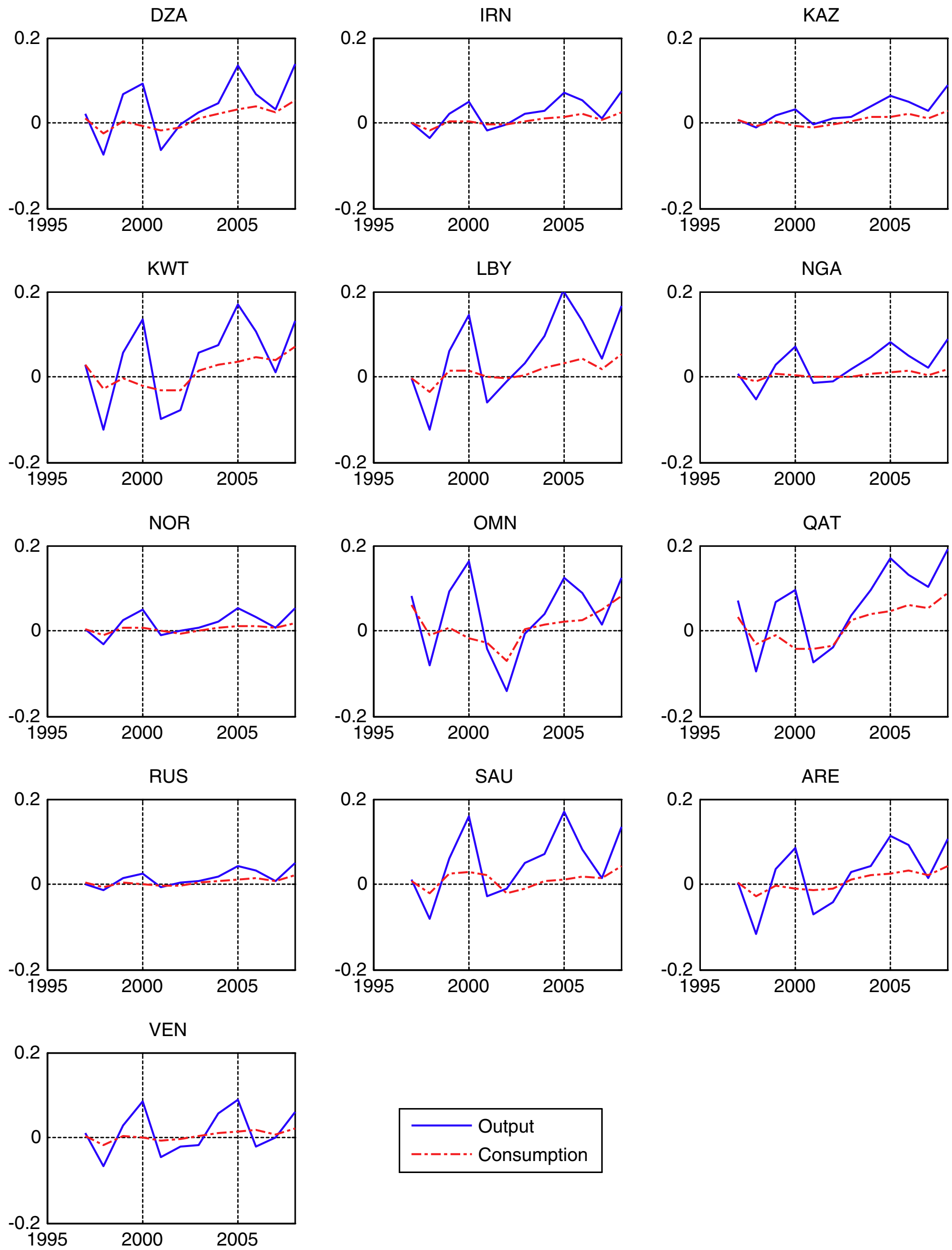

Fig. 7. Per capita growth rates for output and consumption in the model.

consumption is due to the effect of resource price shocks on the discounted future income. With the assumed price process for exhaustible resources, such variation is a fraction of the current period variation in the price of the exhaustible resource. Allowing for uncertainty in the resource price adds precautionary savings considerations to the deterministic model, which lowers the level of per capita consumption, but does not have a quantitatively significant effect on consumption growth rates. Cross-country differences in consumption growth rates are driven by several factors, including the share of the exhaustible resource sector in GDP and the time-profile of exhaustible resource extraction quantities. As a result, countries with equal current share of exhaustible resources in GDP can exhibit different consumption growth rates in response to the same price shock. For example, given a price shock and an equal current share of exhaustible resources in GDP, consumption will respond more in a country that expects to exhaust the resource sooner because of a larger effect of the price shock on discounted future wealth.

Table 3 compares standard deviations for output and consumption growth rates as well as the correlation between the two in the model and data. The variation in the model for both output and consumption is a fraction of the variation in data. This is to be expected, given that shocks in the model are restricted to exhaustible resource prices. We find that the model explains more of the consumption volatility for countries in which exhaustible resources dominate economic activity. There is also more consumption smoothing taking place in the model than in the data. Consumption variation in the model is in the range of 20–50% of the variation in output, with a median of 35%. In the data, the median value for the same ratio is 54%. Correlations between output and consumption in data are in line with our earlier findings (see Fig. 1), even though the length of time-series in Table 3 is considerably shorter. Comparable correlations in the model are uniformly higher, ranging from 0.69 to 0.97. However, as results in Fig. 7 indicate, high correlation in the model does not preclude consumption smoothing.

Table 3
Comparison of per capita output and consumption volatility and correlation in the model and data (1997–2008).

	Data			Model		
	std(Y)	std(C)	corr(Y,C)	std(Y)	std(C)	corr(Y,C)
Algeria	0.08	0.02	−0.03	0.07	0.02	0.79
Iran, I.R. of	0.07	0.03	0.42	0.03	0.01	0.89
Kazakhstan	0.07	0.09	0.59	0.03	0.01	0.85
Kuwait	0.13	0.05	0.25	0.10	0.03	0.68
Libya	0.16	–	–	0.10	0.02	0.90
Nigeria	0.13	–	–	0.04	0.01	0.88
Norway	0.05	0.01	−0.06	0.03	0.01	0.86
Oman	0.10	0.07	0.18	0.09	0.04	0.62
Qatar	0.12	0.10	0.16	0.09	0.05	0.80
Russia	0.09	0.09	0.77	0.02	0.01	0.88
Saudi Arabia	0.09	0.05	0.75	0.08	0.02	0.64
United Arab Emirates	0.10	0.04	0.63	0.07	0.02	0.80
Venezuela, Rep. Bol.	0.12	0.09	0.82	0.05	0.01	0.61
Sample mean	0.10	0.06	0.41	0.06	0.02	0.78
Sample median	0.10	0.05	0.42	0.07	0.02	0.80

Source: World Bank, 2008 World Development Indicators.

Next, we turn to the model results for the external sector. In the absence of exhaustible resources, external balances are trivially determined by the initial net foreign asset position. Countries with positive (negative) net foreign assets would run persistent current account surpluses (deficits) that ensure a constant NFA-to-output ratio.

Exhaustible resources affect external savings in the model through several channels. First, the extraction path for exhaustible resources plays a crucial and complex role, as it can increase or decrease external savings in a given period. For example, *ceteris paribus*, a longer lifespan of exhaustible resources decreases the size of current external savings, while a higher dependence on such resource increases external savings. Furthermore, a sufficiently back-loaded extraction path can generate negative initial savings. This is the case for Kazakhstan in our model parameterization. Similarly, the exhaustible resource price induces additional external savings when it is above its estimated mean and reduces savings when it is below the mean. The mechanism through which the path of prices affects external savings is the same as the mechanism for extraction profiles, because the two jointly determine the path for exhaustible resource revenues. Thus, to make an analogy with the back-loaded extraction path, if the initial price was sufficiently low relative to the mean, the model economy would initially run current account deficits. Finally, in the stochastic version of the model uncertainty about future exhaustible resource prices and accompanying precautionary motive lower current period consumption and increase external savings. This factor increases current account balances in the model for as long as there is uncertainty attached to a fraction of income.

How do external savings in the model economies compare to external savings behavior in the data? We find that the model fits the data moderately well, although there are large variations across sample countries. Changes in current account balances exhibit a positive correlation in the 0.33–0.94 range, with a mean of 0.70. The coefficients are higher for countries for which the model can capture a larger share of exhaustible resources in GDP. Correspondingly, Russia, Qatar and Kazakhstan exhibit the lowest “fit” in Fig. 6 and the lowest correlation coefficients in Fig. 8. In level terms, the model fits data better in the post-2002 period, during which it generates 2/3 of accumulated current account surpluses in data. In contrast, the pre-2002 period is characterized by an aggregated current account deficit in the model, while there is a surplus in the data. Excluding Russia, Qatar and Kazakhstan from the sample significantly improves the model's fit with the data both before and after 2002.

To further examine the fit, panel a in Fig. 9 compares changes in net foreign asset positions, as a share of GDP, in the model and data over the sample period. This variable summarizes the model's fit in terms of both accumulated external savings and output changes.[12] The correlation between the model and the data is 0.73 and for a majority of countries the changes in the data exceed the changes in the model. Panels b and c show the results separately for the pre and post-2002 periods. In line with results from Fig. 8, the fit, as measured by correlation and mean square error, is considerably better for the post-2002 period.

Overall, we find that the model replicates the relevant historical data moderately well. For a majority of the model economies, the share of exhaustible resources in economic activity captures the trend in data and changes in external balances correlate closely with the data. In a cross section, changes in net foreign assets also exhibit a strong positive correlation with the data over the sample period.

5. Role of consumption smoothing and precautionary savings

This section decomposes external savings in the model into two underlying motives — consumption smoothing and precautionary savings. For the purpose of tractability, we start by examining the results from the perspective of the base year — 2007. Focus on a single year simplifies the discussion of the determinants of savings, since we avoid dealing with multiple price shocks. Subsequently, we present decomposition results for the whole historical episode of 1996–2008 and contrast them to the results for 2007.

External savings are decomposed into consumption smoothing and precautionary savings components by separately solving the model with and without uncertainty. The model without uncertainty is the deterministic case, where all savings represent the consumption smoothing motive. When solving this deterministic model from the perspective of each year $t=\{1996,\ldots,2008\}$, the sequence of prices fed into the model is the conditional mean of the price process, given the observable initial price p_t. We use conditional rather than unconditional mean price because the goal of the exercise is to obtain a meaningful decomposition of savings into precautionary and consumption smoothing components. In this case, the initial observable price should be taken into account by the deterministic model, so that the difference in model-derived savings between the parameterizations with unconditional and conditional mean price sequences represents consumption smoothing of observed (and expected) price fluctuations. Precautionary savings are then defined as the difference in external savings between the stochastic and deterministic solutions and are limited to the model response to the stochastic nature of the price process.

The decomposition of model derived optimal 2007 current accounts for sample countries is summarized in Table 4. The first column reports the current account balance for year 2007.[13] The ‘consumption smoothing’ component of the current account – presented in the second column – is the optimal current account from the deterministic model, while the ‘precautionary savings’ component – presented in the third column – is calculated as the difference between optimal external savings in stochastic and deterministic versions of the model.

As already discussed in the previous section, the large variation in the size of the consumption smoothing component across countries is explained – in addition to dependence on exhaustible resources and their lifespan – by the variation in the current and future exhaustible resource weights in output. The focus on a single year allows us to

[12] Its dynamics are also affected by the initial net foreign asset position, reported in Table 1. In particular, large positive initial NFA-to-output ratios for Qatar and United Arab Emirates, combined with growth in output, induce downward trends in NFA-to-output that are only partly off-set by current account surpluses.

[13] The exercise is identical to the one of the previous section, except for the initial NFA, for which we use end-of-2005 figures from the data. This deviation has a negligible effect on the derived current accounts.

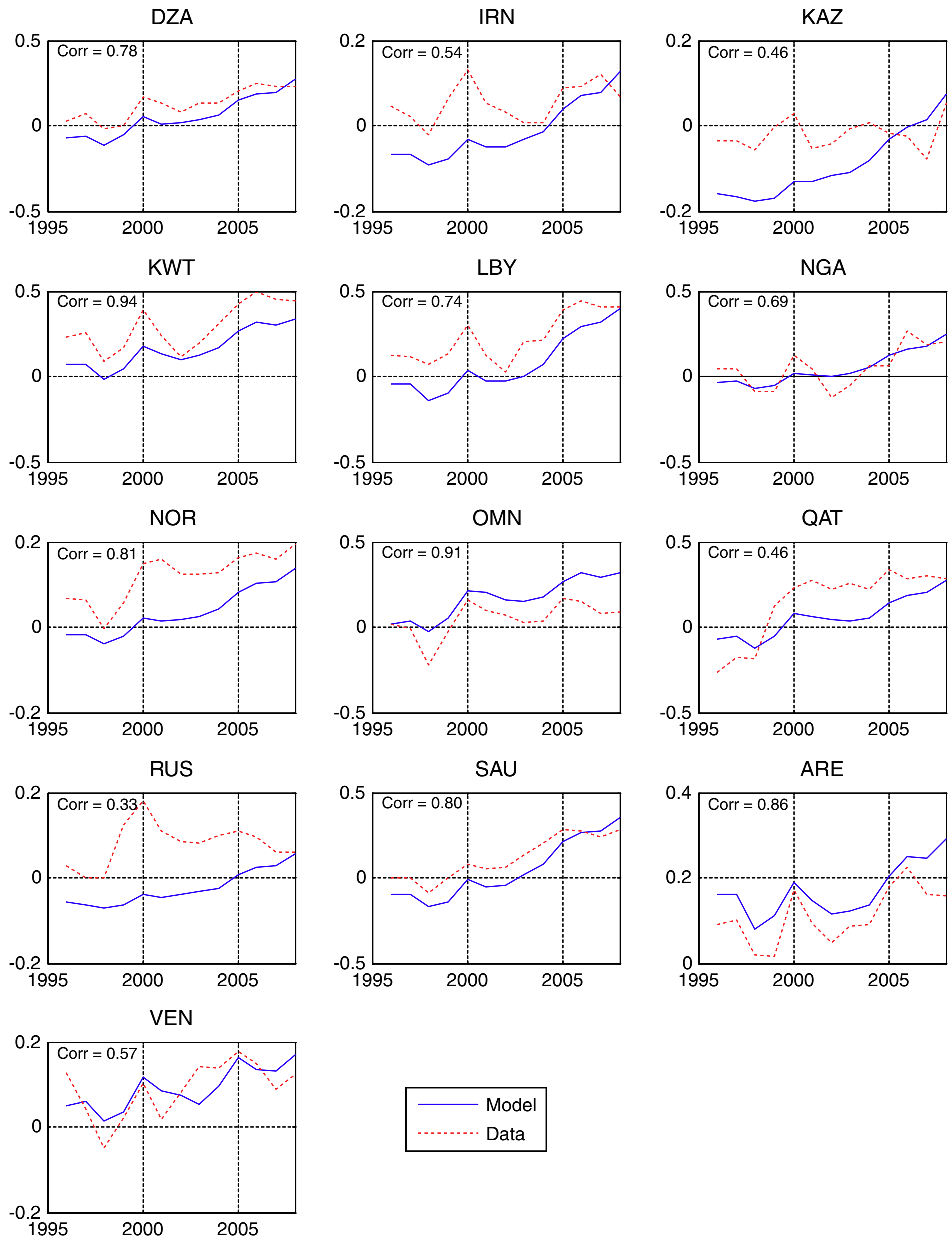

Fig. 8. Current account time series in the model and data, share of GDP. Notes: "Corr" refers to correlation, based on changes in the time-series.

examine these factors in more detail, with Fig. 10 depicting the path of the relevant weights for exhaustible resources. Compare, for example, the case of Norway and Kazakhstan. In 2007 the two economies exhibit similar dependence on exhaustible resources as well as expected lifespan of resources. Also, both countries face the same sequence of exhaustible resource prices. Nevertheless, the model prescribes a broadly balanced external account for Kazakhstan, while Norway is prescribed a surplus of 11.6% of GDP (see column 2 in Table 4). The difference is explained by the divergence in the future dependence on exhaustible resources. Results in Table 4 and Fig. 10 show that assumed trends in extraction profiles are a major determinant of the 'consumption smoothing' component of the current account.

Precautionary savings in the model cannot be derived analytically, but are largely also driven by the current and future exhaustible resource shares in output, as depicted in Fig. 10. In this case, both larger output shares and longer lifespan of exhaustible resources increase the share of a household's total wealth that is exposed to the uncertain exhaustible resource income. Consequently, precautionary savings increase in both factors — the output share and the lifespan of the exhaustible resources. Applying these criteria to Table 4 explains why among the thirteen sample countries, precautionary savings as a share of output are the largest in Qatar and the smallest in Norway and Nigeria.

Quantitatively, external savings in 2007 are dominated by the consumption smoothing motive. Precautionary savings add between 0.3 and 3.9% of GDP to the optimal current account balances and, when aggregated over the thirteen sample countries, add up to 1% of the GDP or 36 billion dollars. This is a fraction of the surpluses induced by the consumption smoothing motive, which amounts to 11% of the GDP. Nevertheless, allowing for precautionary savings improves the model's fit with data. Fig. 11 depicts the cross-section comparison for both the deterministic and stochastic versions of the model. The latter model generates higher correlation coefficient with data and also a smaller mean square error. Given the relatively small size of precautionary savings, for both statistics the improvement is marginal.

How does the size of precautionary savings in 2007 compare to other years? The comparison can be made with each of the years

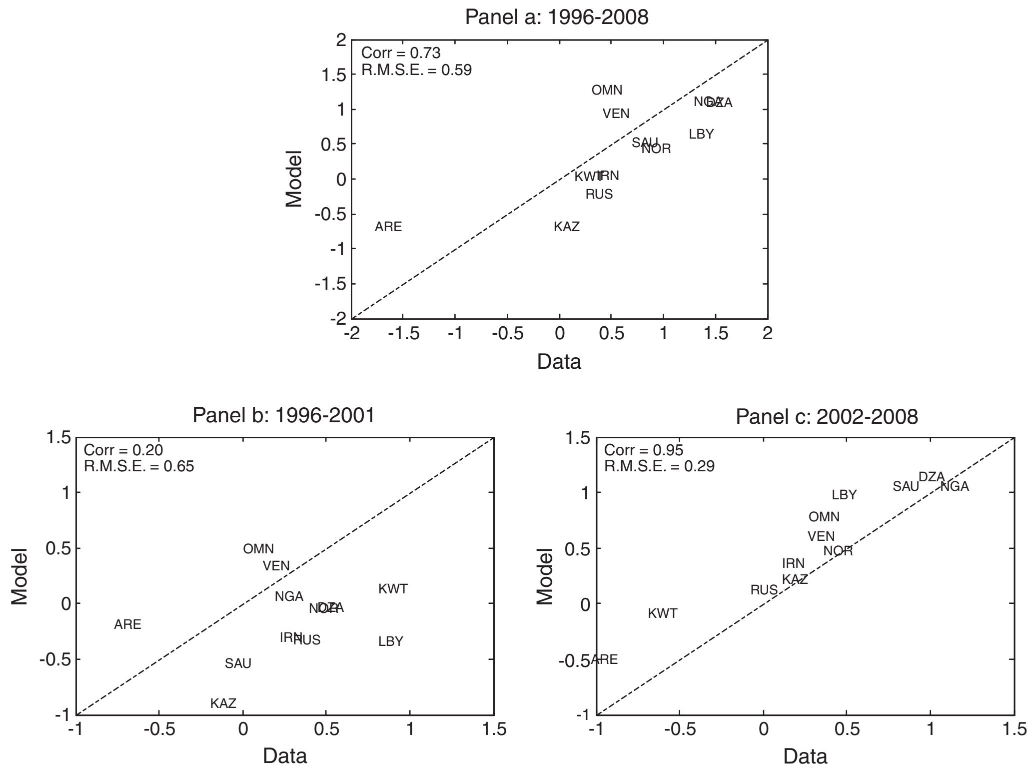

Fig. 9. Changes in net foreign asset positions in the model and data, as share of GDP. Notes: "R.M.S.E." calculated as root mean square error between model and data. "Corr" refers to correlation between model and data. Qatar is excluded from the figure, because it is an outlier. Its coordinates in panel a are (−8.1; −7.3), in panel b (−5.0; −2.8) and in panel c (−3.0; −4.5).

covered in the previous section. Similar to decomposition for 2007, the consumption smoothing component is obtained by sequentially solving the deterministic model for each year covered in Section 4, while the precautionary savings component is the difference between current accounts in the deterministic and stochastic models.

Results in Fig. 12 show that the size of precautionary savings, relative to GDP, is close to constant over time and directly comparable to the estimate for 2007. Each country's exposure to the volatile income from exhaustible resources changes only incrementally from year to year, resulting in similar amounts of precautionary savings. Over a longer horizon, as income is diversified away from the exhaustible resources and such resources draw closer to exhaustion, precautionary savings converge to zero. Notice that the relative importance of the consumption smoothing and precautionary savings motives varies considerably over the sample years. This is because precautionary savings by definition are positive, while the consumption smoothing component of external savings is decumulated during the period of low exhaustible resource prices. Hence, results for 2007 – a year with an above average resource price – underestimate the long-term role of the precautionary motive in accumulated external savings. We find that over 1996–2008 the contribution of the precautionary motive to the total external savings increases to 35%, from 9% in 2007.

Table 4
Optimal 2007 current accounts in the baseline model.

	CA in 2007, % of GDP	Of which: Consumption smoothing	Precautionary savings
	(1)	(2)	(3)
Algeria	20.4	18.6	1.9
Iran, I.R. of	7.9	7.0	0.9
Kazakhstan	1.7	0.2	1.5
Kuwait	30.3	28.4	1.9
Libya	33.2	31.1	2.1
Nigeria	19.0	18.6	0.4
Norway	10.7	10.4	0.3
Oman	28.5	26.6	1.9
Qatar	20.1	16.2	3.9
Russia	2.2	1.6	0.6
Saudi Arabia	28.6	26.8	1.8
United Arab Emirates	23.8	22.7	1.0
Venezuela, Rep. Bol.	13.1	12.6	0.6
Sample mean	18.4	17.0	1.4
Sample median	20.1	18.6	1.5

Overall, results from this section offer several key findings. First, the main driver of external sector balances for exhaustible resource countries is the consumption smoothing motive. Second, despite its lesser role, optimal outcomes from the parameterized model prescribe economically significant precautionary savings for exporters of exhaustible resources, amounting to around 1% of the GDP per year. Furthermore, adding precautionary savings motive to the deterministic model increases its fit with data. Finally, we show that the size of precautionary savings is close to constant over time, while its relative contribution to external savings over the resource price cycle is significantly underestimated in years with high resource price, such as 2007.

6. Sensitivity analysis

The modeling framework of this paper is a relatively simple one. The price of this simplicity comes in the form of several exogenous sequences (i.e., labor force and exhaustible resource quantities) and an exogenous stochastic process for the price of exhaustible resources, all of which can significantly affect model results. Another important parameter with scarce empirical motivation is the curvature in the CRRA utility, which governs the size of precautionary savings. In view of such concerns, our approach in parameterization has been to lean on the available data as much as possible.

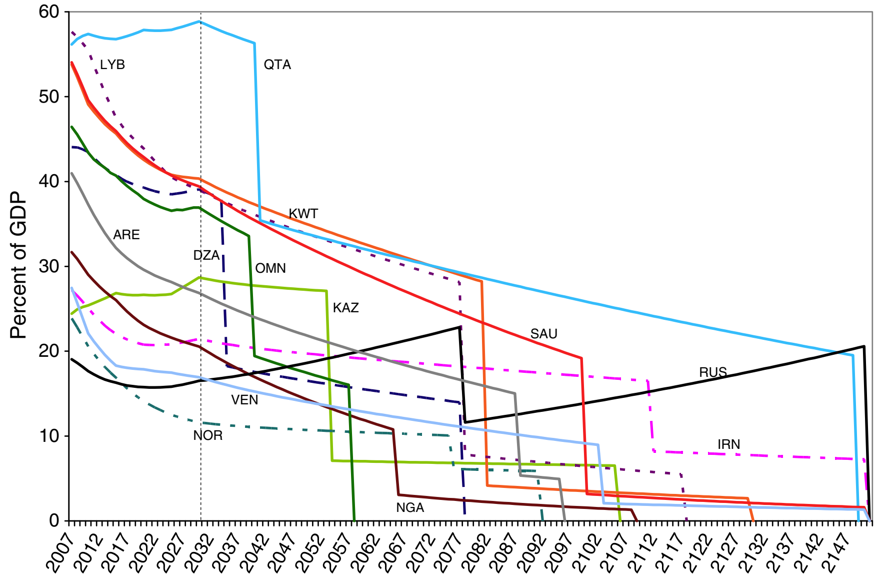

Fig. 10. 'Output at risk': expected share of exhaustible resource revenues in GDP.

This section, in turn, presents extensive sensitivity analysis of the model results. In order to interpret model responses to various deviations in inputs, the examination is focused on the base year (i.e., 2007). The examination covers four areas: (i) consumer preference parameters, (ii) the source and the size of growth in the non-exhaustible resource sector, (iii) exhaustible resource extraction quantities and (iv) parameters of the exhaustible resource price process. The results of sensitivity tests are summarized in Table 5. The first column in this table numbers the sensitivity test. The second column describes the type of deviation from the baseline that a particular sensitivity test considers. For example, row 2 looks at the case when risk-aversion parameter is higher than in the baseline while all other parameters and initial values are kept unchanged. The remaining columns report the size of consumption smoothing and precautionary savings components of the current account for each sensitivity test. Results are compared with the baseline case, reported in row 1. To conserve space, for each test only the mean and median values for sample countries are reported.[14]

6.1. Preference parameters

The size of precautionary savings depends crucially on the value of the coefficient of relative risk aversion in the utility function, σ. To see this, note that if $\sigma=0$, utility is linear and the optimal model solution exhibits no precautionary savings. In order to assess the sensitivity of baseline results to this curvature parameter, the model is solved for cases of $\sigma=4$ and $\sigma=1$. As expected, changes in precautionary savings are substantial. The lower curvature parameter decreases the size of the mean and median precautionary savings for sample countries from 1.4 and 1.5% of the GDP to 1.0 and 1.1% of the GDP respectively. The higher curvature parameter increases the same statistics to 2.8 and 2.8% of the GDP respectively (see rows 1, 2 and 3 in Table 5). Since curvature of the utility function does not affect the optimal solution in the deterministic case, the consumption smoothing component of the current account is unaltered and any change in the total 2007 current account is entirely due to changes in precautionary savings.

Next, consider the effect of a change in the subjective discount factor, β, reported in rows 4 and 5 of Table 5. Heavier discounting lowers the net present value of exhaustible resource wealth, but at the same time increases the risk-free interest rate that puts the economy on a stable growth path. The net effect for model economies is an increase in the annuity value of exhaustible resource wealth, which increases consumption and lowers savings, as captured by the lower consumption smoothing component of the current account (see row 5 in Table 5). Higher levels of consumption and lower savings imply that a larger share of total income is derived from the uncertain exhaustible resources (instead of the risk free foreign asset). In response to this increased uncertainty about future income, the optimal level of precautionary savings rises.

Within the range of considered values of the discount factor, sensitivity tests show relatively minor changes in the consumption smoothing and precautionary savings components of the current account. Furthermore, since the effect on the two current account components comes with the opposite sign, the overall impact on the current account balance is muted.

6.2. Growth in conventional output

In the model of Section 3, labor is the only source of growth in the conventional sector. The effect of changes in the growth rate of the labor force on the two current account components is similar to that of the subjective discount factor (see rows 6–7 in Table 5). In the deterministic model, lower labor force growth rate increases the per worker return on exhaustible resource wealth, thus raising the level of consumption and lowering the consumption smoothing component of the current account. At the same time, it increases the share of future income from the uncertain exhaustible resources, which leads to larger precautionary savings. The opposite forces are at work when the labor force growth rate is reduced.

How sensitive are the model's results to the assumption of zero productivity growth in the non-exhaustible resource sector? During the period 1970–2006, the average growth rate for real output per worker in Norway was close to 2%, suggesting that this assumption is restrictive. To address such concerns, row 8 in Table 5 presents results for the case when time-varying productivity is added to the modeling framework of Section 3. This is done by substituting A with A_t in Eq. (1) and assuming that $A_{t+1}=(1+g)A_t$, where $g=0.02$ is the productivity growth rate. With this specification, the interest rate on the stable growth path needs to satisfy $R=(1+g)^{\sigma}/\beta$, which for baseline parameter values implies an 8% risk free annual return. Although productivity growth affects external savings though several channels, quantitatively, for this specification, the higher output

[14] More detailed tables with country-by-country results for each sensitivity test are available from the authors upon request.

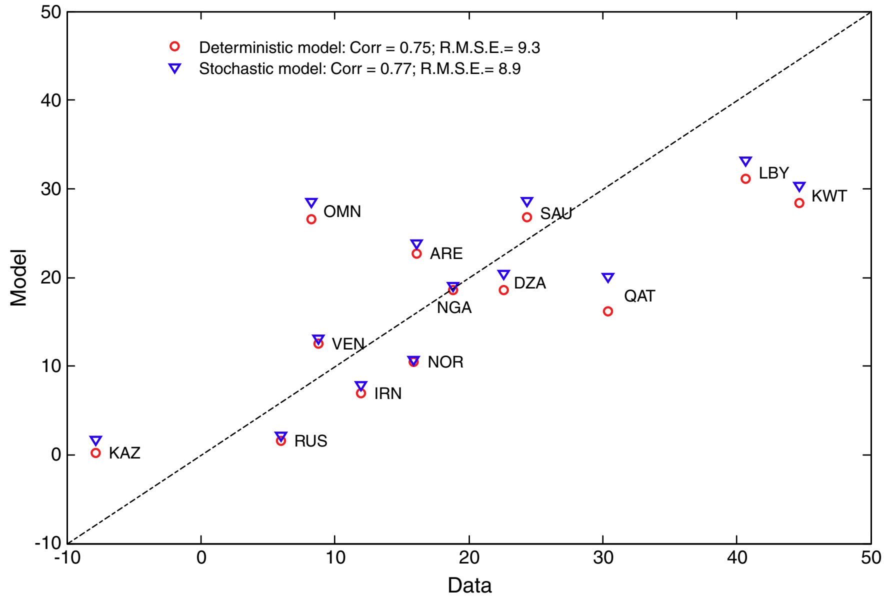

Fig. 11. Actual and model-based 2007 current account balances, in percent of GDP. Notes: "R.M.S.E." calculated as root mean square error between model and data. "Corr" refers to correlation between model and data.

growth rate on the stable growth path induces additional savings so as to sustain a constant consumption–output ratio. As a result, the consumption smoothing component of the current account increases. In contrast, the precautionary savings component decreases because of the reduced weight for exhaustible resources in future income.

Since it is hard to empirically justify an 8% risk-free interest rate, an alternative specification is also considered. Assume that instead of the utility specified in Eq. (6), the household maximizes consumption per effective unit of labor, $U(C_t/Y_t)$. Under this specification, the interest rate on the stable growth path satisfies $R = (1+g)/\beta$, which amounts to 6%. Results for this sensitivity test are presented in row 9. In this case, the lower interest rate further increases the amount of additional savings that are required to attain a constant consumption–output ratio on the balanced growth path. As a result, we see a further increase in the 2007 consumption smoothing component and a further decrease in the 2007 precautionary savings component of the current account (see row 9 in Table 5).

Both 'productivity growth' scenarios show that, although levels of precautionary savings can be significantly altered, the main finding from the baseline model – economically non-negligible levels of precautionary savings – survives introduction of productivity growth into the model. Furthermore, Norway has outperformed the rest of the sample in terms of labor productivity growth. For the sample as a whole, in line with the findings of the literature on the 'resource curse' (see e.g. Sachs and Warner, 2001 and Fig. 5), the average labor productivity growth rate over the most recent oil price cycle is close to zero and, thus, in line with the baseline parameterization.

6.3. Path and lifespan of exhaustible resource extraction

The exhaustible resource extraction path in the model is obtained by combining estimates from the 2000 US Geological Survey for oil and gas reserves and EIA's projected future extraction quantities until 2030. These underlying estimates and projections are subjected to significant uncertainty, primarily with regards to reservoir characteristics, future extraction technologies and costs, but also related to limitations in the coverage of available exhaustible resource reserve estimates.

The sensitivity analysis with respect to exhaustible resource quantities covers two scenarios. In the first one, the size of the exhaustible resource reserves are changed by varying the weight of exhaustible resource income in total output in all periods, keeping the lifespan of resources fixed. In the second scenario, the size of the exhaustible resource reserves is changed by varying the lifespan of the reserves, while keeping the exhaustible resource share in output constant. In both cases we consider a 10% increase/decrease in total reserves of exhaustible resources.

Results for the two scenarios are reported in rows 10–13 of Table 5. As already conveyed in the previous section, an increase in per-period quantities of exhaustible resources should increase precautionary savings as well as the consumption smoothing component of the current account. In contrast, an increase in the lifespan of exhaustible resources should increase precautionary savings, but decrease the consumption smoothing component. Overall, a 10% change in total exhaustible resource reserves leads to relatively minor deviations from the baseline results. Notice that a change in reserves has a smaller effect when introduced by varying the lifespan of exhaustible resource, because intertemporal discounting substantially lowers the net present value of additional revenues to be extracted from period T onwards.

6.4. Process for exhaustible resource prices

The parameterized process for the exhaustible resource price is another crucial input in the baseline model. Taking as given the assumed price process, rows 14–17 in Table 5 report sensitivity results with respect to two key inputs — the persistence of the process and its expected mean value.

A higher expected mean value of the price process decreases the 2007 consumption smoothing component of the current account, but increases precautionary savings. The consumption smoothing component decreases because higher expected mean price implicitly lowers the size of the initial positive 2007 price shock, or could even reverse its sign. With a smaller positive shock it is optimal to consume more of the current period's exhaustible resource income. In contrast, the precautionary savings component increases, as a higher expected mean price raises the weight of exhaustible resources in total output and thus increases uncertainty about the future wealth.

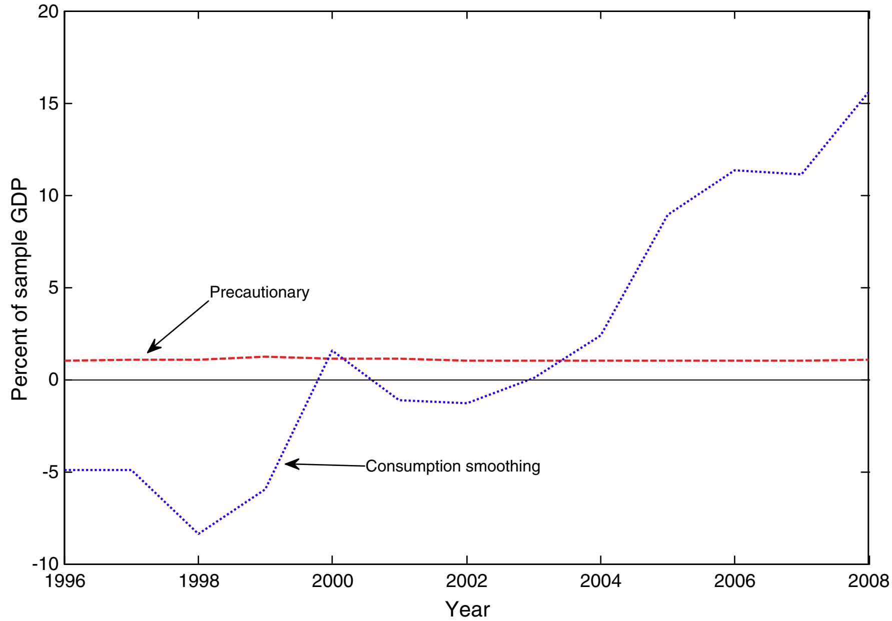

Fig. 12. Decomposition of model's external savings into precautionary and consumption smoothing motives (aggregated sample of 13 countries).

This result offers an interesting insight into the optimal behavior for exhaustible resource exporters. The recent increase in the price of exhaustible resources has motivated exporting countries (correctly or incorrectly) to increase the expected long-run price of the resource, which in turn justifies higher current consumption levels. However, it should be borne in mind that the higher consumption levels need to be adjusted downwards to account for the implicitly assumed increase in the long run weight of the more volatile exhaustible resource revenues in total income, brought about by the assumed increase in the long run price of the resource.

A more persistent price process makes the reversion to the mean following the initial positive price shock more gradual. As a result, the 2007 consumption smoothing component of the current account decreases and precautionary savings increase. The quantitative effects from altering the persistence of the price process can be substantial. For example, an increase in the persistence parameter from $\rho = 0.89$ to $\rho = 0.94$ triples the mean and median sizes of precautionary savings in the sample, while a reduction to $\rho = 0.84$ cuts precautionary savings by half.

We also examine if the model results under various sensitivity tests preserve the correlation with data observed in Fig. 11. In this regard two results are noteworthy. First, for all sixteen sensitivity tests the correlation between the current account in the model and data remains above 0.67 and in the case of the added productivity growth (rows 8 and 9), the correlation increases to 0.82. Second, in all sixteen cases the stochastic model exhibits higher correlation and lower mean square error between current accounts in the model and data than the deterministic model. This result reiterates our earlier finding that the precautionary motive is a significant determinant of current account balances.

Table 5
Summary of sensitivity analysis for current account components.

Row number	Deviation from baseline	Consumption smoothing, % of GDP		Precautionary savings, % of GDP	
		Mean	Median	Mean	Median
		(1)	(2)	(3)	(4)
1	Baseline	17.0	18.6	1.4	1.5
2	$\sigma' = \sigma + 4$	17.0	18.6	2.8	2.8
3	$\sigma' = \sigma - 1$	17.0	18.6	1.0	1.1
4	$\beta' = \beta + 0.005$	18.9	21.1	1.2	1.2
5	$\beta' = \beta - 0.005$	15.6	16.8	1.5	1.7
6	$n_i' = n_i + 0.005\,\forall i$	21.2	23.0	1.0	1.1
7	$n_i' = n_i - 0.005\,\forall i$	12.9	14.5	1.8	1.9
8	$A_{t+1}/A_t = 0.02\,\forall t$ (Case 1)	20.3	20.2	1.2	1.3
9	$A_{t+1}/A_t = 0.02\,\forall t$ (Case 2)	25.1	26.1	0.8	1.0
10	$z_t' = 1.1 z_t\,\forall t$	18.7	20.5	1.6	1.7
11	$z_t' = 0.9 z_t\,\forall t$	15.5	17.0	1.2	1.3
12	$T_i' = 1.1 T_i\,\forall i$	16.7	17.8	1.4	1.5
13	$T_i' = 0.9 T_i\,\forall i$	17.5	19.9	1.3	1.4
14	$\bar{p}' = \bar{p} + 10$	12.7	14.0	1.8	1.9
15	$\bar{p}' = \bar{p} - 10$	20.7	22.2	1.0	1.1
16	$\rho' = \rho + 0.05$	11.3	12.0	4.1	4.2
17	$\rho' = \rho - 0.05$	19.2	21.3	0.7	0.8

Note: In the column reporting the type of deviation from baseline 'primed' parameters represent the deviation and subscript i indexes sample countries. For definitions of Case 1 and Case 2 in rows 8–9 see Section 6.2.

7. Conclusions

This paper introduces the precautionary savings motive into the framework of a deterministic small open economy model to study the optimal consumption–savings choices in economies with exhaustible resources. The model fares moderately well at capturing savings/current account behavior in exhaustible resource countries in both cross-section and time-series data. According to the model, the sizable contribution of the exhaustible resource exporters to global imbalances over the last decade is justified on the grounds of the exhaustible nature of the resource, historically high resource prices and uncertainty about future prices.[15]

The model results show that, while the desire to smooth consumption is the main determinant of external sector dynamics, allowing for uncertainty in the price of exhaustible resources can generate sizable

[15] This benign view of current account surpluses in oil exporting countries is also espoused by Blanchard and Milesi-Ferretti (2009).

external savings, more so for countries where extraction of exhaustible resources dominates economic activity. When aggregated over all sample countries, precautionary savings in 2007 add up to 36 billion dollars, which amounts to 1% of the sample's GDP. The size of precautionary savings for other years is similar in magnitude. We also find that the precautionary motive improves the model's fit with data. Extensive sensitivity analysis shows that the main finding from the model – economically non-negligible levels of precautionary savings – is not driven by the parameterization of the baseline exercise.

There are several important issues that are left for future research. First, the 'small open economy' assumption is employed throughout the paper. When taken separately, each of the exhaustible resource economies is likely too small to affect outcomes in the rest of the world. However, when taken as a group, it is possible that the savings behavior of exhaustible resource countries affects the world interest rate.

Second, we have abstracted from the investment requirements for the extraction of exhaustible resources. For economies in the process of expanding their exhaustible resource output, e.g., Kazakhstan, this assumption might be problematic, since expansion of the extraction capacity requires large upfront investments. In such instances, current account deficits might be driven not only by the consumption smoothing and precautionary savings motives, but also by the required investment in the exhaustible resource sector.

Finally, several aspects of the paper's exercise suggest that the estimated precautionary savings represent a lower bound. There might be more uncertainty about the price of the exhaustible resource at distant horizons than the assumption of stationarity implies. There is also considerable uncertainty about the remaining stock of exhaustible resources, which we do not model. As rows 14–17 in Table 5 show, added uncertainty can significantly increase the size of precautionary savings.

Appendix A. Solution Method (case of $n=0$)

The problem we want to solve is:

$$\max_{\{C_t\}_{t=0}^{\infty}} E_0\left[\sum_{t=0}^{+\infty}\beta^t U(C_t)\right]$$
$$s.t.\quad B_{t+1} = B_t(1+r) + Y + p_t Z_t - C_t,$$

where notation is the same as in Section 3. B_0 is given and p_t is a random sequence. This problem does not have a steady-state solution, so one needs a shortcut to solve it. The shortcut we use is to assume that there is no randomness beyond the exhaustible resource depletion date T. Then the problem can be solved recursively. In the first period after depletion, $T+1$, the consumption–savings decision going forward is:

$$V(B_{T+1}) = \max_{\{C_t\}_{T+1}^{+\infty}} E_{T+1}\left[\sum_{t=T+1}^{+\infty}\beta^{t-T-1}U(C_t)\right]$$
$$s.t.\quad B_{t+1} = B_t(1+r) + Y - C_t.$$

With CRRA preferences this problem has a closed-form solution:

$$V(B_{T+1}) = \frac{(Y + rB_{T+1})^{1-\sigma}}{(1-\beta)(1-\sigma)},$$

where for each possible value on a grid for B_{T+1} (it can be a large positive or negative number), we can calculate the value of the program going forward. The problem can then be solved recursively back to the initial period. For T, the last period with positive output in the exhaustible resource sector, the problem to be solved numerically over a grid on B_T, p_T with a finite number of nodes is:

$$V_T(B_T, p_T) = \max_{C_T}\ \left[U(C_T) + \beta V_{T+1}(B_{T+1})\right]$$
$$s.t.\quad B_{T+1} = B_T(1+r) + Y + p_T Z_T - C_T,$$

For all periods up to T-1, the decision problem incorporates the uncertainty over prices going forward:

$$V_t(B_t, p_t) = \max_{C_T}\left[U(C_t) + \beta E_t V_{t+1}(B_{t+1}, p_{t+1})\right]$$
$$s.t.\quad B_{t+1} = B_t(1+r) + Y + p_t Z_t - C_t.$$

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