Rare Disasters, Asset Markets, and Macroeconomics

Assess implications of neoclassical growth model for real rates of return. In steady state (i.e. long run), real rates of return on assets (claims to capital and internal loans) exceed growth rates of real GDP and consumption. Does this prediction hold up for long-run averages of real rates of return and growth rates?

11 OECD countries have long-term data on real stock returns, along with real returns on short-term bills (usually Treasury Bills) and long-term bonds (usually 10-year government bonds). In NGM, comparison with growth rate of levels matters. Look at levels and per capita here. Note: real bill or bond returns not risk-free. Risk-free rates would be lower.

Growth Rates and Rates of Return for OECD Countries, 1870-2006 (or shorter samples)

	Growth rates				Real rates of return		
Country	**Δc/c**	**Δy/y**	**ΔC/C**	**ΔY/Y**	**stocks**	**bills**	**bonds**
Australia	0.015	0.016	0.032	0.033	0.103	0.013	0.035
Canada	0.019	0.021	0.036	0.038	0.074	0.013	0.038
Denmark	0.016	0.019	0.024	0.027	0.071	0.032	0.039
France	0.016	0.019	0.020	0.023	0.060	-0.008	0.007
Germany	0.019	0.021	0.025	0.027	0.076	-0.015	-0.001*
Italy	0.017	0.021	0.023	0.027	0.053	0.005	0.017
Japan	0.025	0.028	0.035	0.038	0.093	0.004	0.031
Norway	0.019	0.023	0.027	0.031	0.072	0.021	0.028
Sweden	0.021	0.023	0.027	0.029	0.092	0.025	0.032*
U.K.	0.015	0.016	0.019	0.020	0.064	0.018	0.028
U.S.	0.019	0.022	0.033	0.036	0.083	0.020	0.027
Means	**0.018**	**0.021**	**0.027**	**0.030**	**0.076**	**0.012**	**0.026**

Average real rate of return on stocks—7.6% per year—applies to levered equity (equity and debt finance). Let rates of return be r^u on unlevered equity, r^λ on levered equity, r^b on bonds. If λ is debt-equity ratio (around 0.5 in U.S.), Modigliani-Miller theorem (total value of firm independent of equity-debt composition) implies

$$r^\lambda = (1+\lambda)\cdot r^u - \lambda\cdot r^b$$

$$r^u = [1/(1+\lambda)]\cdot(r^\lambda + \lambda r^b)$$

$$\text{equity premium} = r^u - r^b = [1/(1+\lambda)]\cdot(r^\lambda - r^b)$$

$r^\lambda = .076$, $\lambda = 0.5$, $r^b = .010$ implies $r^u = .054$.

- Therefore, data imply unlevered equity premium around 0.044. Risk-free rate (short term) likely less than 0.010.

- Other conclusions from long-term data: real stock returns exceed real growth rates, real bill returns fall short of real growth rates, real bond returns similar to real growth rates. Pretty sure that, on average in the long run, risk-free real rates less than real growth rates. Pattern conflicts with neoclassical growth model?

- Modify neoclassical growth model to include stochastic shocks to assess predictions for different rates of return. To get insights from simple closed-form results, dispense with diminishing productivity of capital and assume stochastic GDP shocks all permanent to levels.

- Two models work to get into ballpark for explaining equity premium. First is Lucas (1978) fruit-tree model with stochastic productivity and rare disasters, as in Rietz (1988) and Barro (2006). Fruit from tree corresponds to GDP and consumption.

Second is AK, one-sector production model with stochastic depreciation (disasters). Model has endogenous saving/investment but not varying stock prices. (Price of K pegged at one. Need adjustment costs for K or varying degrees of monopoly power to change this.) Work through Lucas-tree model here. (Can readily add variable labor supply.) AK model in problem set.

Let Y_t be real GDP. No investment or government; closed economy. Consumption, C_t, equals Y_t. Evolution of Y_t:

(1) $$\log(Y_{t+1}) = \log(Y_t) + g + u_{t+1} + v_{t+1}$$

g: exogenous, deterministic part of growth rate,
u_{t+1}: normal with s.d. σ (economic fluctuations),
v_{t+1}: rare disasters (Rietz 1988, Barro 2006):

probability 1-p, $v_{t+1} = 0$,
probability p, $v_{t+1} = \log(1-b)$, $0<b<1$.

- p (per year) small, but b (fraction of output lost in disaster) large. Treat p as constant, although time evolution of p important for some analysis. Treat b as having fixed frequency distribution of sizes. No bonanzas here. Finite duration for disasters allowed in extension.

- i.i.d. assumptions for u_{t+1} and v_{t+1}. Shocks have permanent effects on levels, not fluctuation around deterministic trend. Extension allows for recoveries from disasters.

Disaster probability and sizes gauged in *QJE* 2006 paper (restricting $b \geq 0.15$) using Maddison long-term GDP data for 35 countries during 20th century. Disasters gauged by cumulative (peak-to-trough) declines in GDP of 15% or more. Ursua and I (2008) analyzed with long-term real consumer expenditure, C, and revised & extended GDP data back to 1870 (allowing for $b \geq 0.10$).

- With expanded data (and $b \geq 0.10$) found 95 C crises for 24 countries and 152 GDP crises for 36 countries over periods as long as 135 years. Got p around 3.5% per year—about 4 events per country back to 1870.

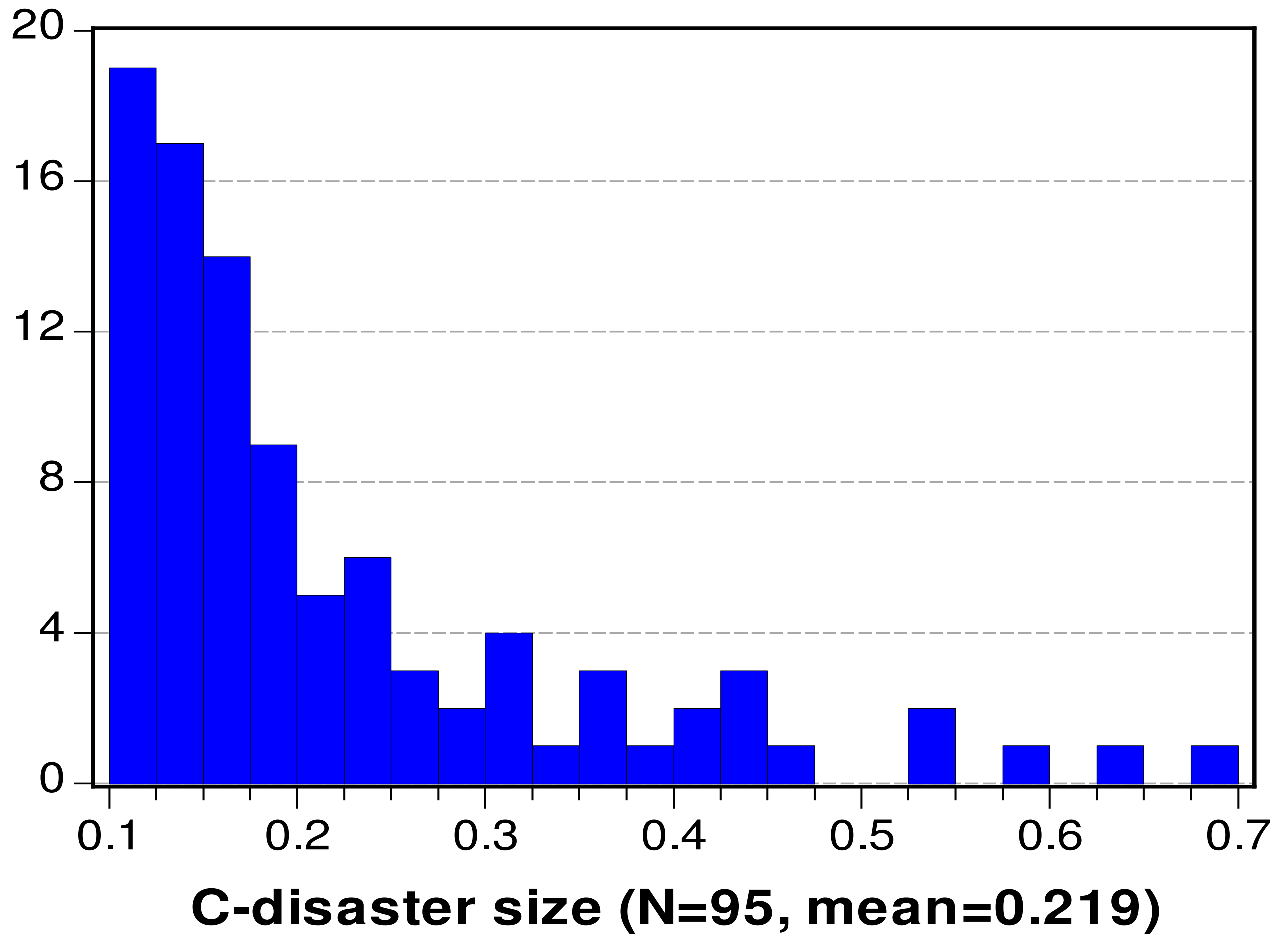
- Given unusual nature of disasters, to use history to gauge probability and size distribution, cannot rely on single country, such as U.S., even if we assume economic structure fixed. Long time series for broad international sample has enough disaster realizations to allow reasonably accurate inferences about disaster probabilities and sizes. (No longer a "peso problem.") Underlying the calculations is the assumption that probability distributions are reasonably similar across countries and over time.

Main Economic Crises of 20th Century (before 2008)

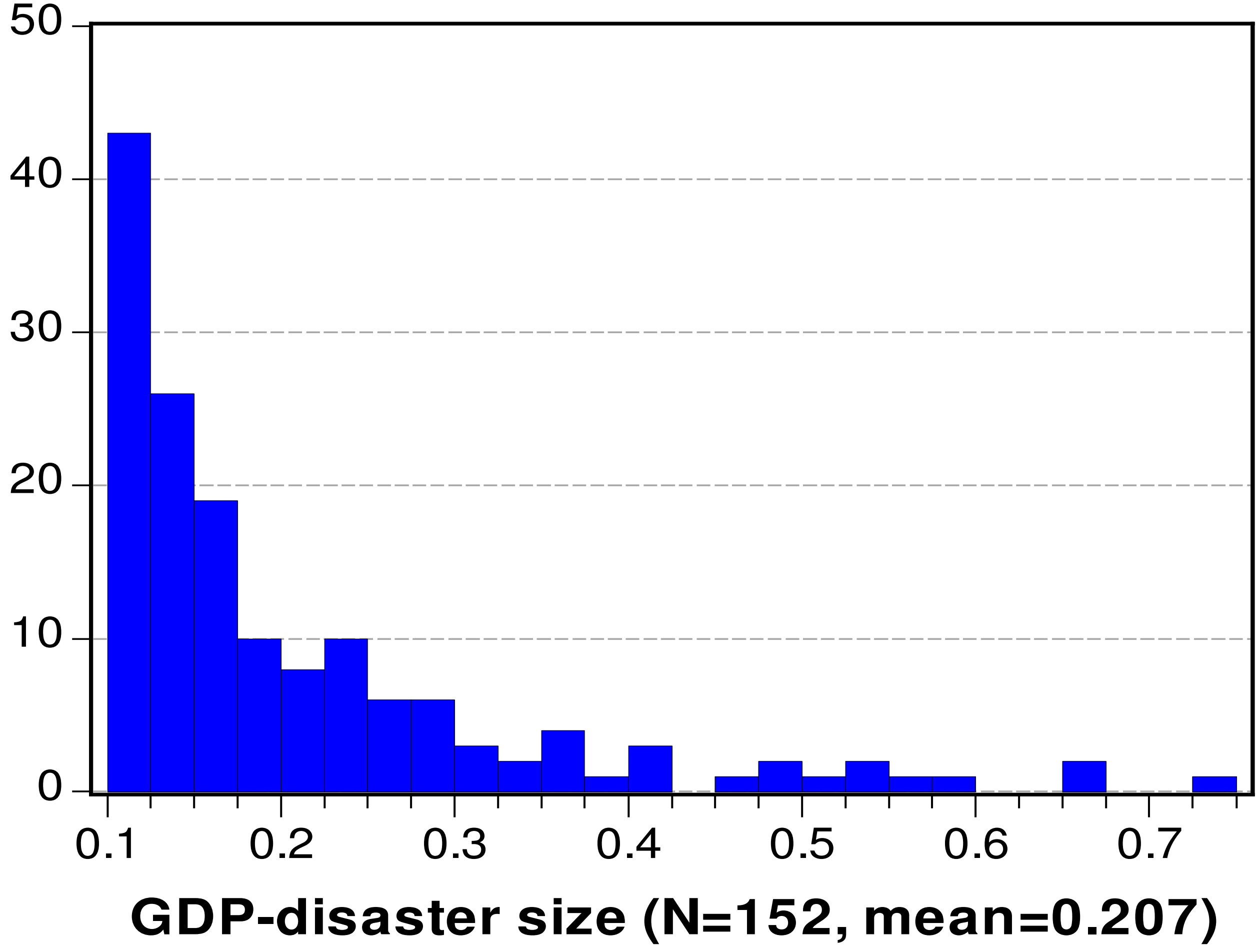
For real per capita consumer expenditure:

- WWII: 23 cases, average 34%
- WWI: 20 cases, average 24%
- Great Depression: 18 cases, average 21%
- 1920s (influenza): 11 cases, average 18%
- Post-WWII: 38 cases, average 18% (only 9 in tranquil OECD)
- Pre-1914: 21 cases, average 16%

Distribution of C Disasters



Distribution of GDP Disasters



In Eq. (1), expected growth rate of Y and C is $g^*=E_t[(Y_{t+1}/Y_t)-1]$. As period length approaches 0:

$$(2) \qquad g^* = g + (1/2)\cdot\sigma^2 - p\cdot Eb$$

$(1/2)\cdot\sigma^2$ is quantitatively trivial, using typical annual σ of 0.02-0.03. $p\cdot Eb$ matters more, with p around 0.035 and Eb around 0.22. Given g=0.025, get g^*=0.018.

Want to price asset claims. Start with power utility:

(3) $$U_t = E_t \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} \cdot [(C_{t+i})^{1-\gamma} - 1]/(1-\gamma)$$

As is well known, power utility implies that $\gamma>0$ represents coefficient of relative risk aversion (CRRA) and reciprocal of intertemporal elasticity of substitution (IES). Restriction generates counter-factual predictions about asset prices, as argued by Bansal and Yaron (2004). Soon generalize to preference formulation—Epstein and Zin (1989) and Weil (1990)—that de-links CRRA from IES.

Eq.(3) leads, using perturbation approach, to usual first-order condition for asset pricing:

(4) $$C_t^{-\gamma} = (\frac{1}{1+\rho}) \cdot E_t(R_t \cdot C_{t+1}^{-\gamma})$$

where R_t is gross return on any asset from t to t+1.

A key variable is market value, V, of tree that initially produces one unit of fruit. Determine V by summing prices for each "dividend," using FOCs for C over time:

$$(5) \qquad 1/V = \rho + (\gamma-1)g^* - (1/2)\gamma(\gamma-1)\sigma^2 - p\cdot[E(1-b)^{1-\gamma} - 1 - (\gamma-1)Eb]$$

V is P-D ratio for unlevered equity claim on tree. Right side of (5) is difference between expected rate of return on unlevered equity,

$$(6) \qquad r^e = \rho + \gamma g^* - (1/2)\gamma(\gamma\text{-}1)\sigma^2 \\ \qquad - p\cdot[E(1\text{-}b)^{1-\gamma} - 1 - (\gamma\text{-}1)Eb]$$

and expected growth rate, g*, from (2).

Transversality condition, which guarantees that market value of tree is positive and finite, is that right side of (5) be positive—$r^e > g^*$.

Key term is $E(1-b)^{1-\gamma}$: crisis expectation of product of relative marginal utility, $(1-b)^{-\gamma}$, and gross return on unlevered equity, $R=1-b$.

Risk-free rate, r^f, is:

$$(7) \qquad r^f = \rho + \gamma g^* - (1/2)\gamma(\gamma+1)\sigma^2 - p\cdot[E(1-b)^{-\gamma} - 1 - \gamma Eb]$$

which depends on $E(1-b)^{-\gamma}$. In deterministic neoclassical growth model, $\sigma = p = 0$, and $r^e = r^f = \rho + \gamma g$.

More uncertainty (higher σ, p, or b) lowers r^f in (7). (Demand for risk-free claims rises.) Two offsetting effects on r^e. Substitute away from risky claims but raise demand for assets overall (precautionary saving). Net effect is r^e down if $\gamma>1$ (more on this later). In any event, more uncertainty raises equity premium, given by

$$(8) \qquad r^e - r^f = \gamma\sigma^2 + pE\{b\cdot[(1-b)^{-\gamma}-1]\}$$

First term, $\gamma\sigma^2$, in (8) negligible and corresponds to Mehra and Prescott (1985).

Second term proportional to p. Disaster size, b, enters as expectation of product of b and proportionate excess of marginal utility in disaster, $[(1-b)^{-\gamma} - 1]$. Term large with historical distribution of b. Need γ around 3-4 to get unlevered equity premium of 0.05.

Problem: if $\gamma>1$, (5) implies that V rises with one-time increase in uncertainty (σ, p, b) and falls with one-time rise in g^* (Bansal-Yaron, 2004). These counter-intuitive results can be eliminated with Epstein-Zin-Weil (EZW) preferences.

Using minor modification of Weil (1990) formulation, extended utility formula is

(9)

$$U_t = \frac{\left\{ C_t^{1-\theta} + (\frac{1}{1+\rho}) \cdot [(1-\gamma) \cdot E_t U_{t+1}]^{(1-\theta)/(1-\gamma)} \right\}^{(1-\gamma)/(1-\theta)}}{(1-\gamma)}$$

γ still coefficient of relative risk aversion; θ=1/IES, not constrained to equal γ.

EZW preferences do not generally allow for simple, closed-form formulas for pricing assets. However, when underlying shocks are i.i.d., as already assumed, analysis simplifies dramatically.

Key property of solution under i.i.d. shocks is that attained utility, U_t, ends up as simple function of consumption, C_t:

$$(10) \qquad U_t = \Phi C_t^{1-\gamma}$$

where constant Φ depends on parameters of model.

Using (10), get F.O.C.'s for C from standard perturbation arguments. Result looks familiar:

$$C_t^{-\gamma} = (\frac{1}{1+\rho *}) \cdot E_t (R_t \cdot C_{t+1}^{-\gamma}) \qquad (11)$$

Important result: with i.i.d. shocks, conditions for asset pricing under EZW preferences look similar to those with power utility.

Two points: exponent, γ, in (11) is CRRA, not θ=1/IES. ρ*, effective rate of time preference, ≠ ρ unless γ=θ. Formula for ρ*:

(12) ρ* = ρ -

$$(\gamma-\theta)\cdot\left\{g^{*}-(1/2)\cdot\gamma\sigma^{2}-(\frac{p}{\gamma-1})\cdot[E(1-b)^{1-\gamma}-1-(\gamma-1)\cdot Eb]\right\}$$

ρ^* in (12) depends not only on preference parameters—ρ, γ, and ϑ—but also parameters for expected growth and uncertainty—g^*, σ, p, and b distribution.

Previous asset-pricing formulas remain valid if ρ^* replaces ρ.

For P-D ratio, V, from (5):

(13) $$1/V = \rho + (\theta-1)\cdot g^* - (1/2)\cdot\gamma\cdot(\theta-1)\cdot\sigma^2$$

$$-\,p\cdot\left(\frac{\theta-1}{\gamma-1}\right)\cdot[E(1-b)^{1-\gamma}-1-(\gamma-1)\cdot Eb]$$

$\theta<1$ (IES>1) gives all the “right” signs in (13). Once-and-for-all increase in uncertainty parameter (higher σ or p or a shift of b-distribution toward higher values) reduces stock prices (as seems plausible) if and only if $\vartheta<1$, so that IES>1. Also, V rises if g^* rises.

Formula for equity return, from (6), is now

(14) $r^e = \rho + \theta g^* - (1/2)\gamma(\theta-1)\sigma^2$

$$- p \cdot (\frac{\theta-1}{\gamma-1}) \cdot [E(1-b)^{1-\gamma} - 1 - (\gamma-1) \cdot Eb]$$

Corresponds to 1/V in (13).

Formula for equity premium same as before:

$$(8) \qquad r^e - r^f = \gamma\sigma^2 + pE\{b\cdot[(1-b)^{-\gamma}-1]\}$$

In calibration, set ρ to get right level of rates, $r^f=0.010$. Requires $\rho^*=0.029$, $\rho=0.045$. Key is whether equity premium in (8) is correct, around 0.05. Use disaster experience for p and distribution of b—gives $p=0.036$ ($b\geq 0.10$), $Eb=0.22$, $E(1-b)^{-\gamma}=3.9$. Equity premium (unlevered) around 0.05 if $\gamma=3.5$ (Barro & Ursua, 2008, Tables 10-11).

Can match observed volatility of V_t if p_t moves around in nearly random-walk-like manner, as in Gabaix (2008). Alternatively, g* may move around (Bansal & Yaron, 2004).

Given GDP process in (1), data on rates of return, such as r^e and r^f, and price-dividend ratio, V, pin down γ and effective rate of time preference, ρ^*. Since ρ^* depends on combination of ρ and ϑ (in [12]), data would not allow separate identification of ρ and ϑ

Parameters ρ and ϑ separately identified from other information; for example, how V responds to one-time changes in uncertainty parameters—σ, p, and distribution of b—or expected growth rate, g^*, in (13). Alternatively, in model with endogenous saving (e.g. AK model) identification follows from how saving ratio reacts to changes in σ, p, and distribution of b.

Bottom Line on NGM and Long-Run Rates of Return

- Expanded NGM is okay in according with long-run properties of rates of return and growth rates with addition of stochastic shocks to GDP/consumption if:

- Uncertainty/risk aversion enough to accord with observed equity premium (e.g. with rare disasters calibrated to disaster data).

- In this case, expected rate of return on (unlevered) equity claim exceeds expected growth rates of GDP and C (levels and per capita).

- Risk-free rate is below the expected growth rates—and this is okay.