Top incomes and the shape of the upper tail

Recent interest in top incomes has focused on the rise in top income shares, but it is also important to examine the distribution *within* the top income group. Has the changing composition of the top 1 per cent been accompanied by a less concentrated distribution? Or have those at the very top extended their lead? One answer to this question is provided by the Pareto coefficient, or, more intuitively, the inverse Pareto coefficient, Beta (β), which measures the relative advantage of those higher up the income scale. Where the upper tail is Pareto in form, the average income of those above income, y, is given by βy. From the WTID data, where β is calculated from the share of the top 0.1 per cent in the income of the top 1 per cent, it may be seen that in the US, β has risen from 2.42 in 1992 to 2.85 in 2007 and 2.93 in 2012. On this basis, not only has the share of the top 1 per cent risen, but it has also become more concentrated.

The validity of employing the inverse Pareto coefficient depends on the extent to which the upper tail is indeed well approximated by the Pareto distribution. This is a question that has been much-debated. Here I suggest one way of approaching an answer that has not, as far as I know, been used in the literature.

1. Investigating the Pareto distribution

The attractive feature of the Pareto distribution is that, wherever, one stands with income y, the average income of those above is equal to βy: i.e. if we denote the proportion of tax units with incomes of y or higher by 1-F and the total income received by these units, divided by the total population, by Ω(y), then Ω(y)/[1-F] = βy. In general, however, β is not constant but is a function of F, which we write as the function:

$$M\{F\} = \Omega(y\{F\})/(y\{F\}\ [1-F]) \qquad (1)$$

where y{F} inverts the cumulative distribution, and dy/dF = 1/f(y), where f(y) is the density of the distribution. The right hand side of (1) is the ratio of the "remaining" income to the amount that would remain if all those above y had exactly an income of y.

In what follows, a simple test of the Pareto assumption is conducted by calculating from the WTID the values of M at different percentiles: the top 1 per cent, top 0.5 per cent, top 0.1 per cent and top 0.01 per cent. From this, it is clear that there are certain countries, and certain periods, when the Pareto approximation seems quite acceptable. In France, shown in Figure 1, the average absolute difference between M calculated at the top 1 per cent and at the top 0.01 per cent over the period from 1946 to 1997 is 0.07. It exceeded 0.1 in only 10 of the 52 years. Clearly such a difference does not materially affect the comparison with the US (the Beta coefficient in France in 2007 was 1.82).

But there are also periods, and countries, where the Pareto assumption does not seem appropriate. In the case of France, we can see that, in recent years, the M values increase as one moves up the income scale. The average value for the years 2000 to 2007 were 1.78 for M evaluated at the top 1 per cent, 1.86 at the top 0.5 per cent, 1.90 at 0.1 per cent, and 1.97 at 0.01 per cent. In contrast, in the inter-war period, the curves were ranked in the reverse order, indicating that M decreased as one moved up the scale, from 2.48 at the top 1 per cent, averaging the years 1920 to 1939, to 2.35 at the top 0.5 per cent, 2.13 at 0.1 per cent, and 2.02 at 0.01 per cent. These figures demonstrate that the non-constancy of M may affect the conclusions drawn concerning change over time. Evaluated at the top 1 per cent, the years 2000 to 2007 have less concentration than the inter-war period (1.78 versus 2.48), but the figures at the top 0.01 per cent are close (1.97 and 2.02) and suggest no effective difference.

The variation in M may be illustrated graphically – see Figure 0. The horizontal axis measures y.[1-F]. It is assumed that this is strictly decreasing in y (and in F), and it approaches zero as we approach the top of the distribution. The vertical axis measures the amount of income received by those below y, expressed as a proportion of the total, which, if the mean is μ, is equal to (μ-Ω)/μ. As one moves to the left in Figure 0, one is effectively climbing the income "mountain". With a Pareto distribution, the climb is at a constant rate; the gradient does not change. But where M is decreasing, the slope becomes more gradual. The value of M is given by the slope of the chord joining the top of the mountain to the curve, and is decreasing as we move to the left in the case of the curve labelled "convex". This corresponds to the usage of geographers, who describe such slopes as convex, and such a mountain would be described as "dome-shaped". Where M is increasing, the curve is concave and the mountain would be described as "volcanic". In that sense, France has moved from having a dome-shaped upper tail in the inter-war period to having a volcanic shape in the 2000s.

If M is not constant, what does this imply about the functional form? If M were linear,

$$M = a + b(1-F)$$

then the distribution can be written as

$$y = A\,(1-F)^{-(a-1)/a}\,[a+b(1-F)]^{-(1+a)/a}$$

The first term in (1-F) is the standard Pareto distribution, but it is modified by the second term, which becomes less important as F approaches 1. I have not seen this functional form used before; indeed it is “dual” to the standard forms in that it treats y as a function of F rather than F as a function of y.

2. **Evidence from the WTID on 13 countries**

Figures 1 to 13 show the values of M calculated from the WTID at different percentiles: the top 1 per cent, top 0.5 per cent, top 0.1 per cent and top 0.01 per cent. The countries covered are those for which the income thresholds are included in the WTID. There are some obvious omissions, such as the UK and Norway, where the thresholds need to be added to the WTID.

The results are summarized in Table 1 in terms of three main periods: the inter-war period 1920 to 1939, the post-war years from 1950 to 1979; and the recent decades from 1980. In considering the fit of the Pareto distribution, I have distinguished between cases where the average absolute difference between M at the top 1 per cent and M at the top 0.01 per cent was less than 0.1, described as Pareto, and those where the average absolute difference was between 0.1 and 0.2, described as Approximately Pareto. As may be seen, there are three cases, all in the post-war period, when the Pareto fit satisfied the stricter condition. There are a further four cases, three in recent decades (Canada, US and Portugal), satisfying the weaker condition.

There are nine cases where the M curve slopes down, and eight cases where the M curve slopes up. What is striking is that all but one of the latter occurs in recent decades, whereas in all cases the inter-war figures show M sloping down. The shift is not universal, but there is a definite tendency for the distribution to change between the inter-war period and today. The distribution used to depart from the Pareto in one direction; it now departs in the opposite direction – just as we saw earlier to be the case in France. Whereas in the past the income mountain became less steep as one rose – a dome-shaped mountain – today the mountain is volcanic.

Conclusions

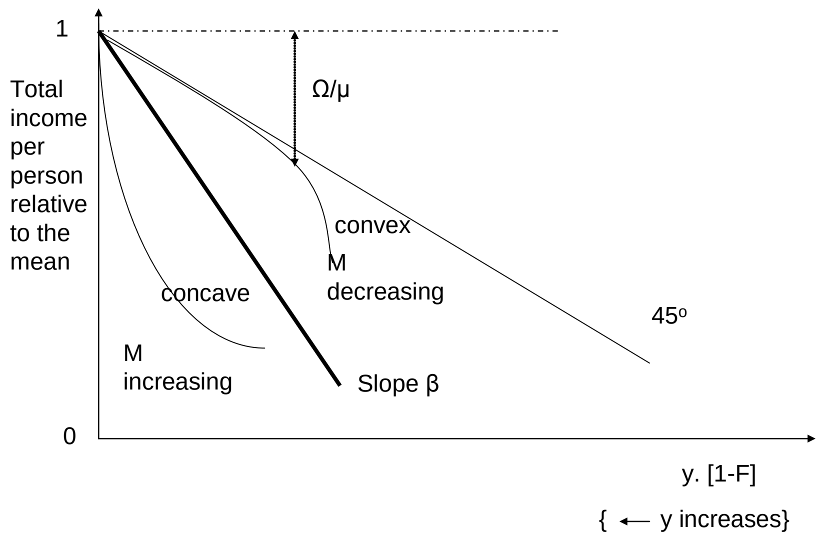
- The first conclusion is negative: we should be cautious about using the Beta (or Alpha) coefficients as summary statistics of concentration;
- The second conclusion is positive: investigation of the shape of the upper tail suggests that, in a number of countries, there is a distinct difference between the inter-war period and the recent decades. We have not simply returned to the earlier situation. The change in the shape of the distribution in turn raises questions for the explanation of the upper tail.

Table 1

	Inter-war period	Post-war period	Recent decades
France	M slopes down	Pareto (av difference 0.066)	M slopes up
Denmark	M slopes down	?	M slopes up
Canada	M slopes down	M slopes down	Approx Pareto (av difference 0.15)
Japan	n/a	Approx Pareto (av difference 0.145)	M slopes up
US	M slopes down (except 1929)	M slopes up	Approx Pareto (av difference 0.196)
Sweden	M slopes down	Pareto (av difference 0.055)	M slopes up
Australia	M slopes down	Pareto (av difference 0.083)	?
Italy	n/a	n/a	M slopes up, since 1988
Switzerland	n/a	n/a	M slopes up
Spain	n/a	n/a	M slopes up
Portugal	n/a	n/a	Approx Pareto (av difference 0.153)
India	M slopes down	M slopes down	?
Colombia	n/a	n/a	M slopes down to 2006, then close to Pareto

n/a denotes not available

Figure 0


1
Total
income
per
person
relative
to the
mean
Ω/μ
convex
M
decreasing
concave
M
increasing
Slope β
45º
0
y. [1-F]
{ ← y increases}

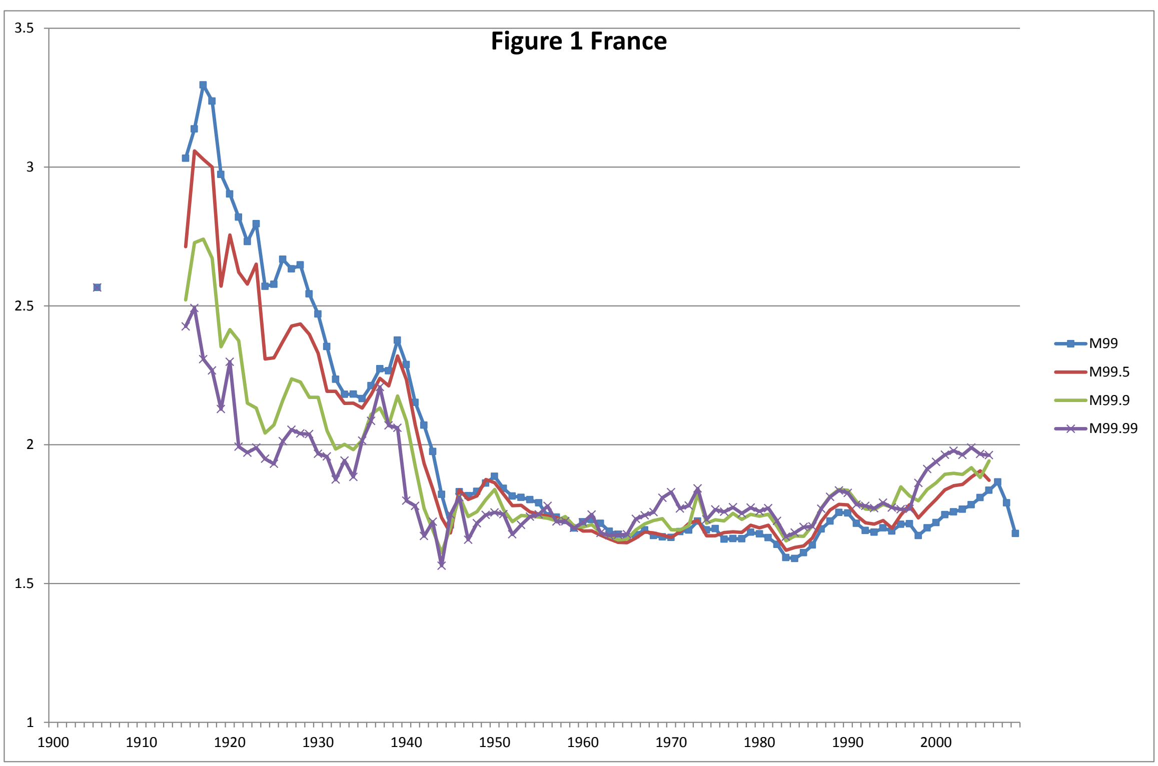


Figure 1 France
3.5
3
2.5
2
1.5
1
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
M99
M99.5
M99.9
M99.99

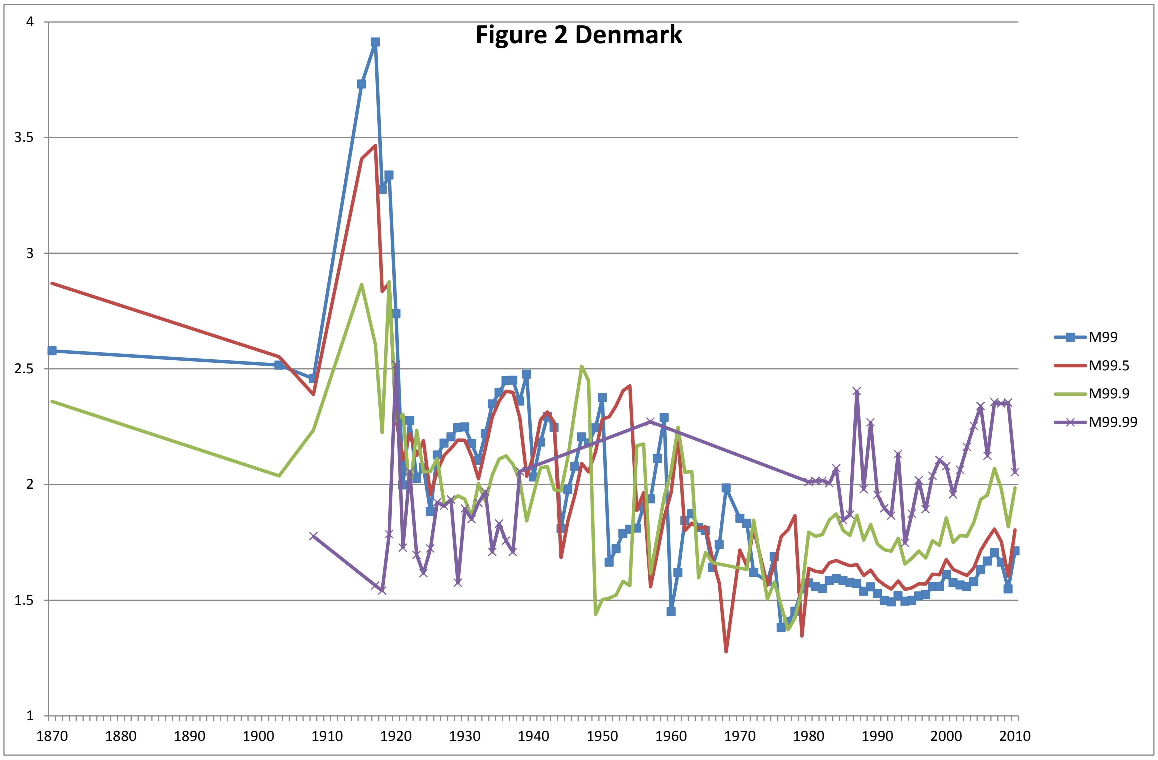


Figure 2 Denmark
4
3.5
3
2.5
2
1.5
1
1870
1880
1890
1900
1910
1920
1930
1940
1950
1960
1970
1980
1990
2000
2010
M99
M99.5
M99.9
M99.99

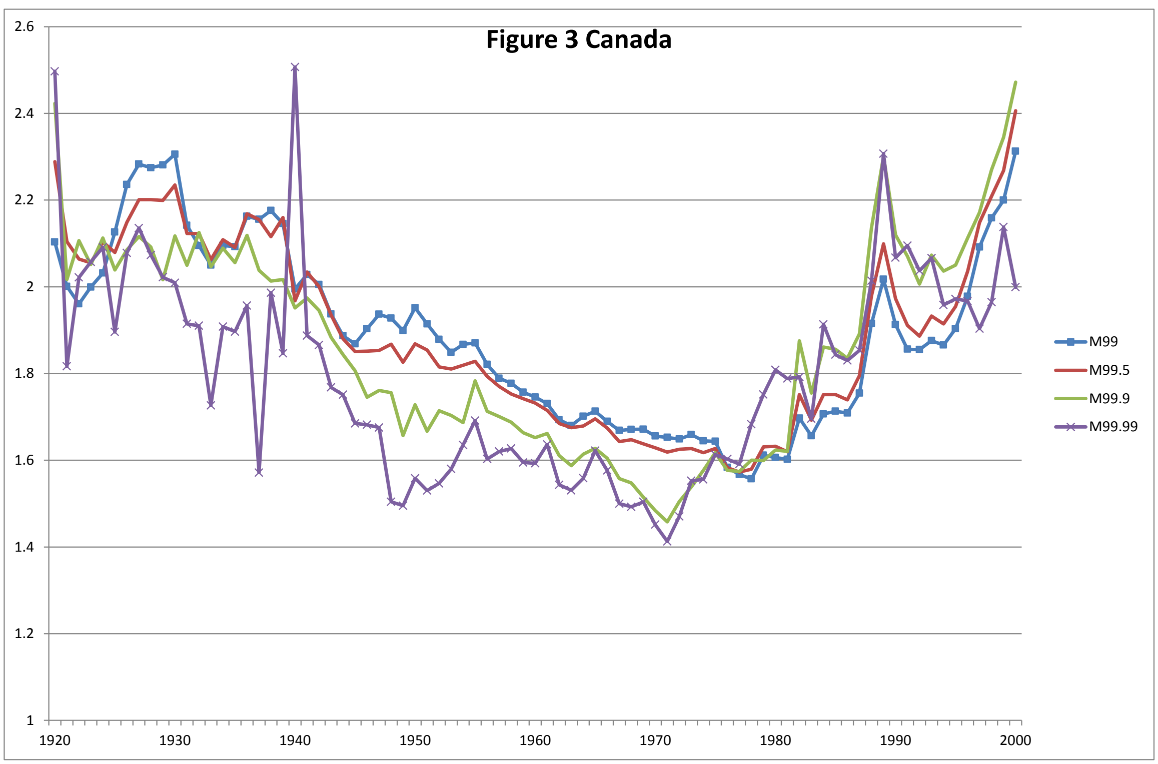


Figure 3 Canada
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1920
1930
1940
1950
1960
1970
1980
1990
2000
M99
M99.5
M99.9
M99.99

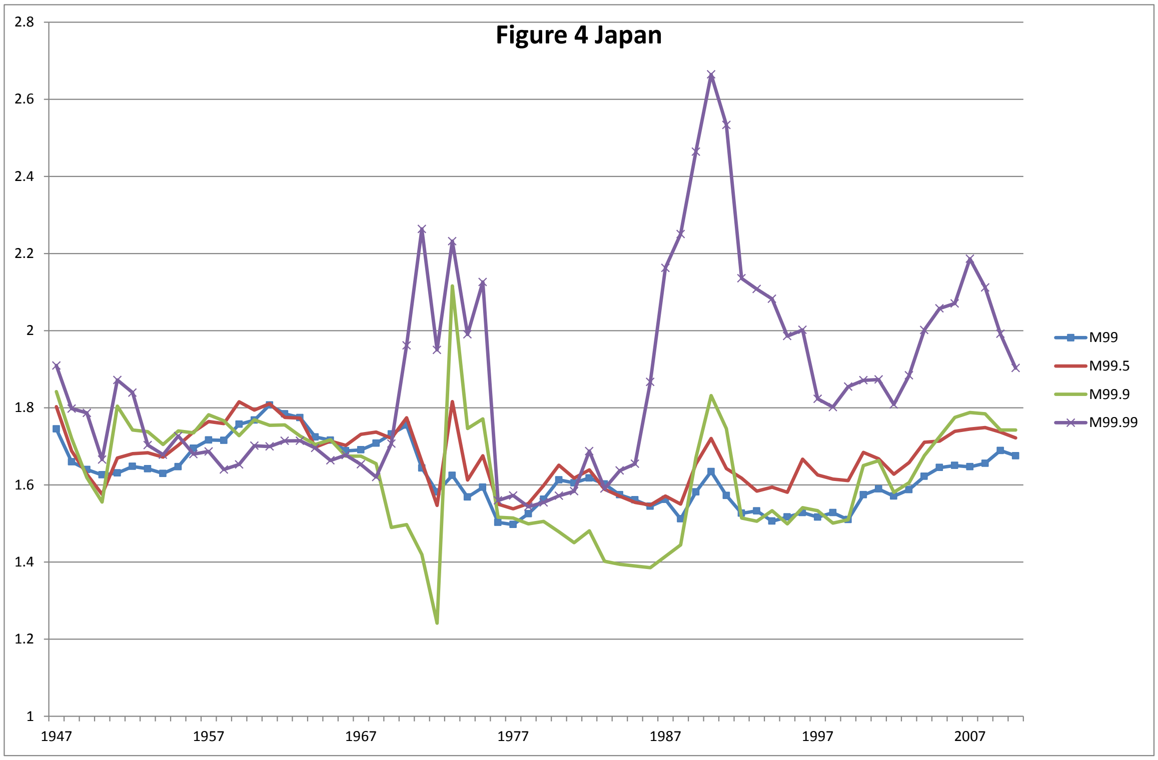
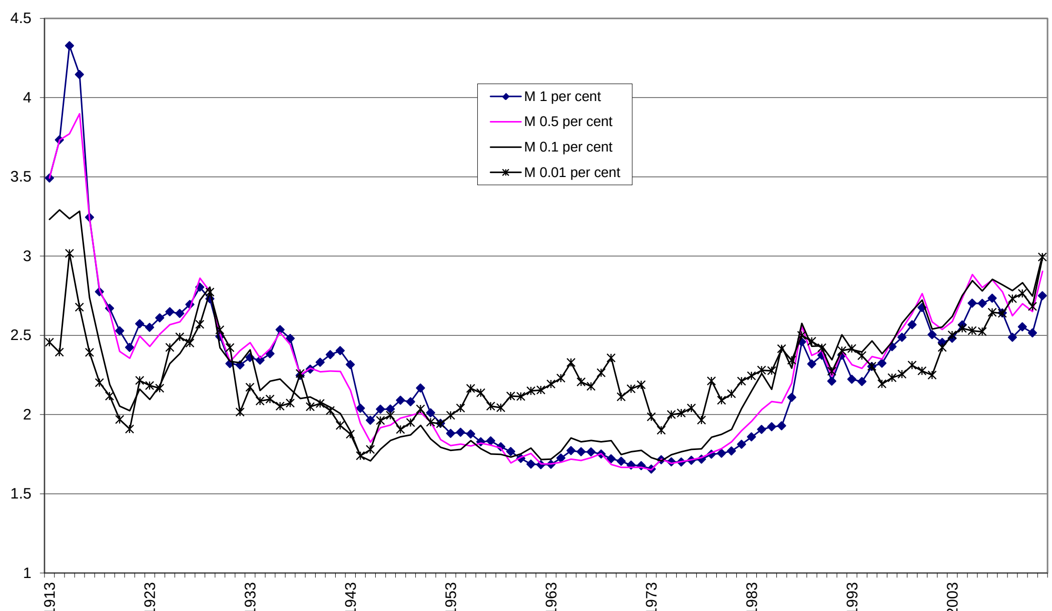


Figure 5 US Saez data



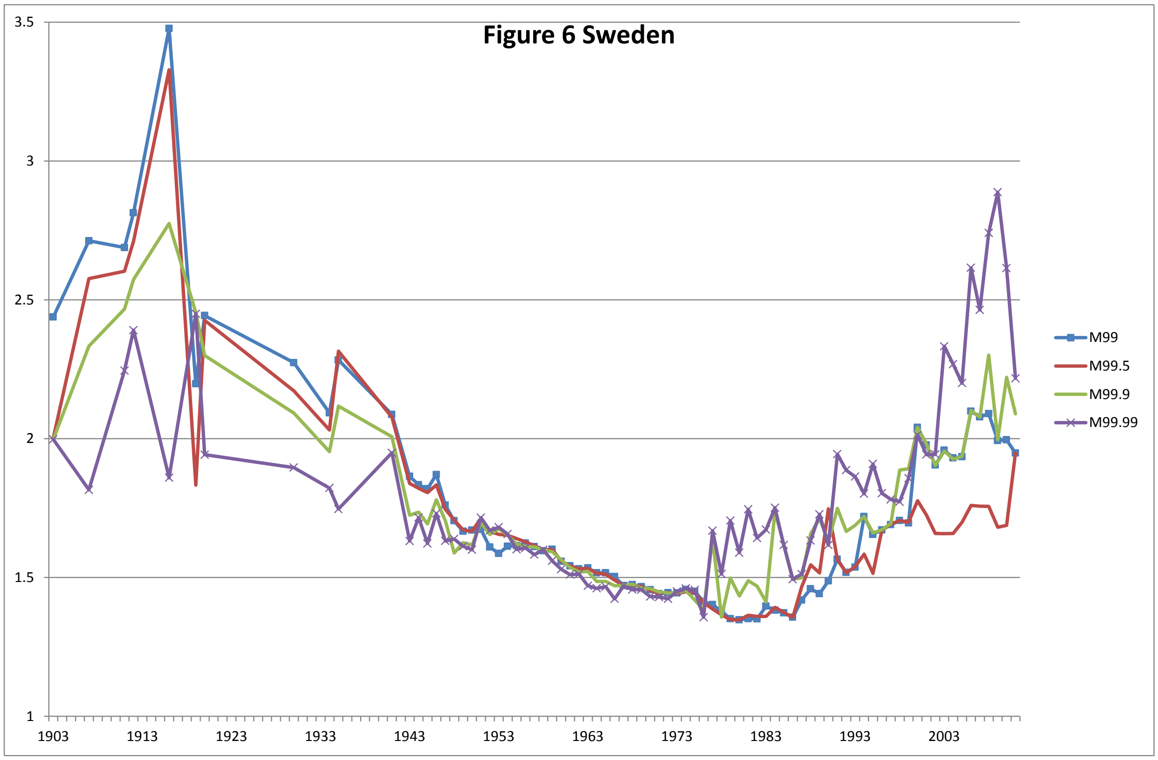


Figure 6 Sweden
3.5
3
2.5
2
1.5
1
1903
1913
1923
1933
1943
1953
1963
1973
1983
1993
2003
M99
M99.5
M99.9
M99.99

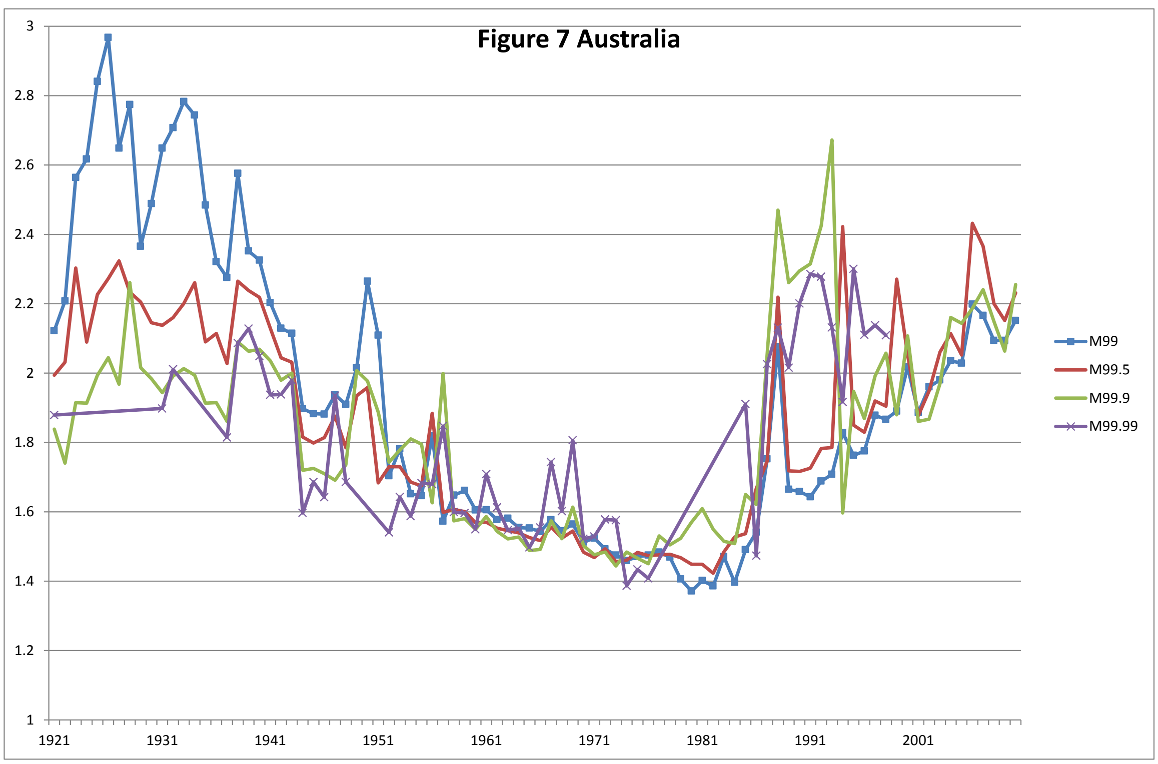


Figure 7 Australia
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1921
1931
1941
1951
1961
1971
1981
1991
2001
M99
M99.5
M99.9
M99.99

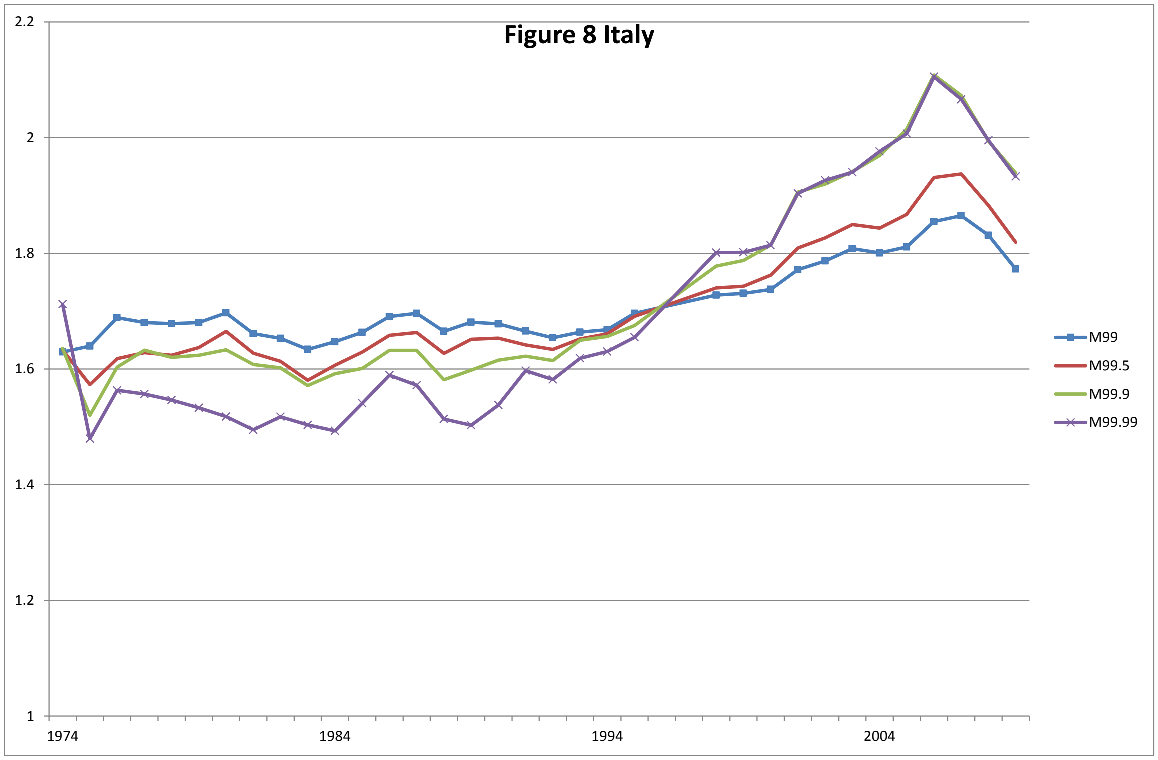


Figure 8 Italy
2.2
2
1.8
1.6
1.4
1.2
1
1974
1984
1994
2004
M99
M99.5
M99.9
M99.99

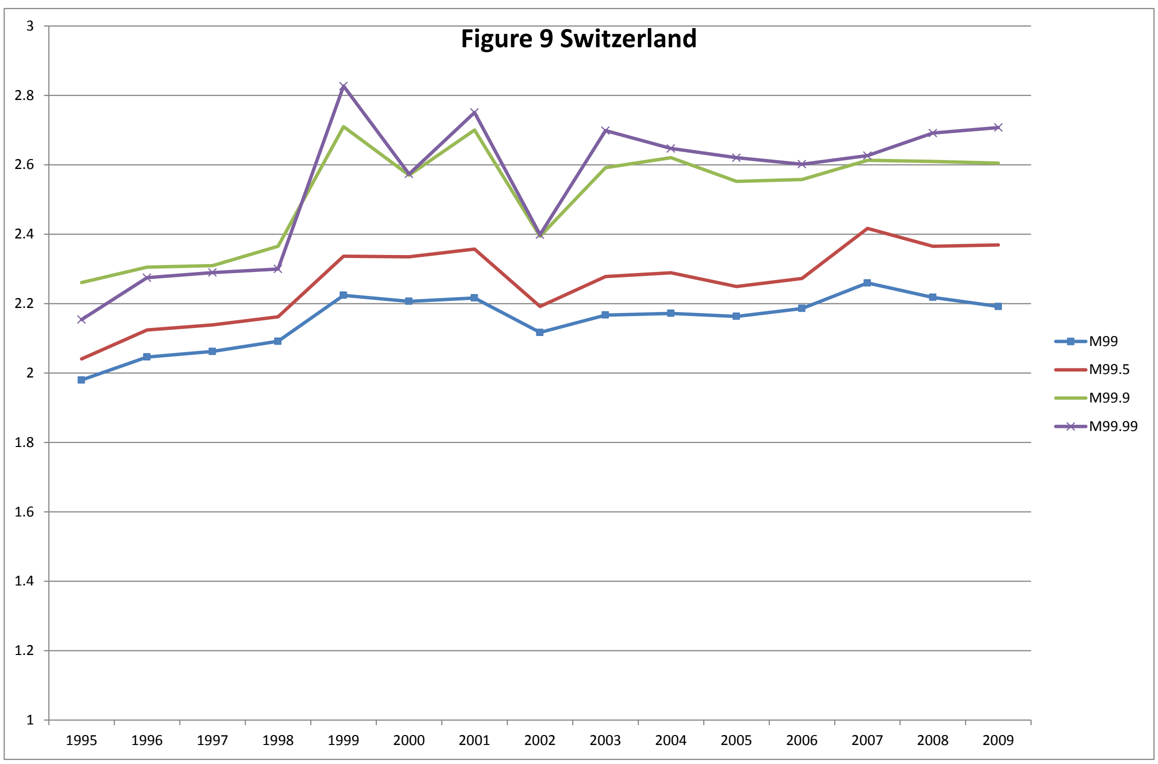


Figure 9 Switzerland
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
M99
M99.5
M99.9
M99.99

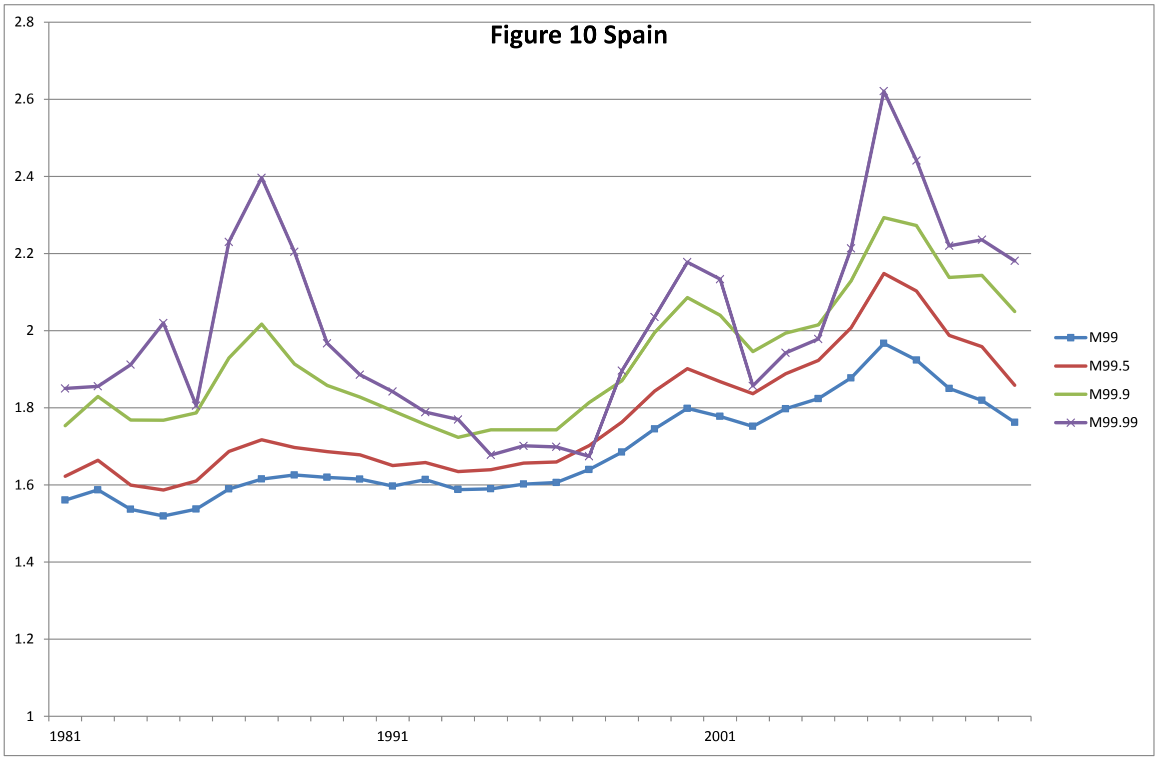


Figure 10 Spain
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1981
1991
2001
M99
M99.5
M99.9
M99.99

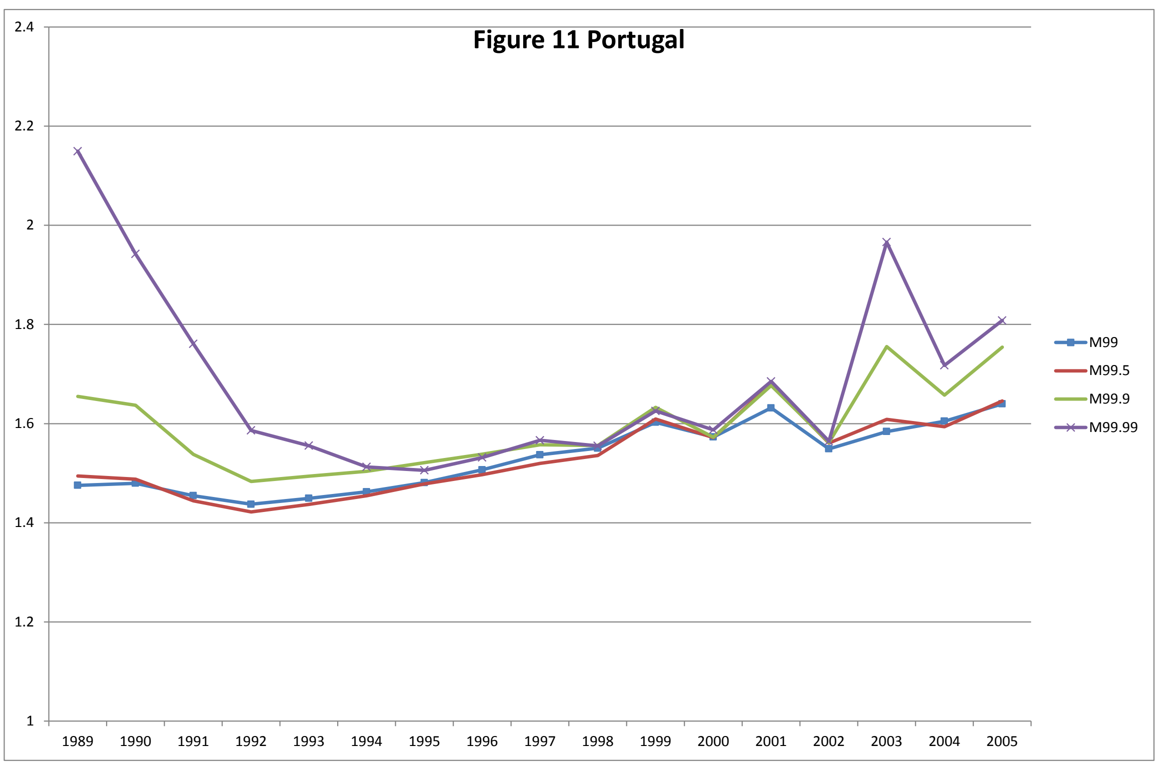


Figure 11 Portugal
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
M99
M99.5
M99.9
M99.99

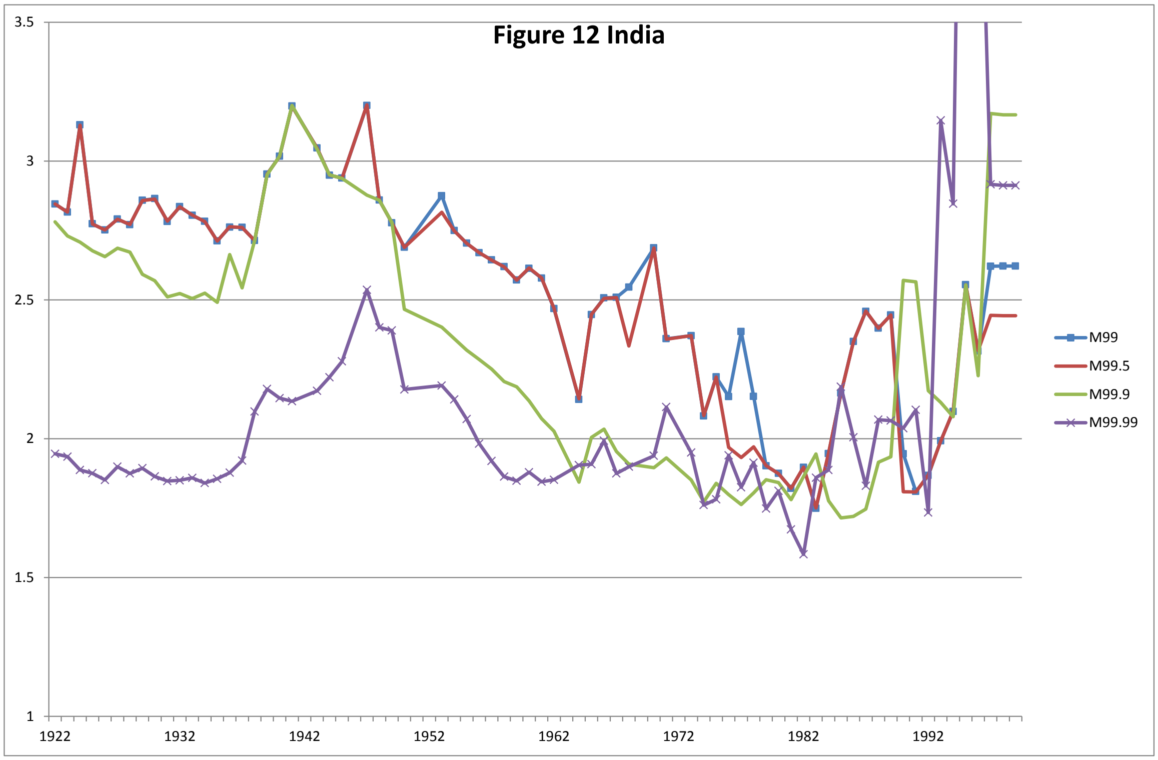


Figure 12 India
3.5
3
2.5
2
1.5
1
1922
1932
1942
1952
1962
1972
1982
1992
M99
M99.5
M99.9
M99.99

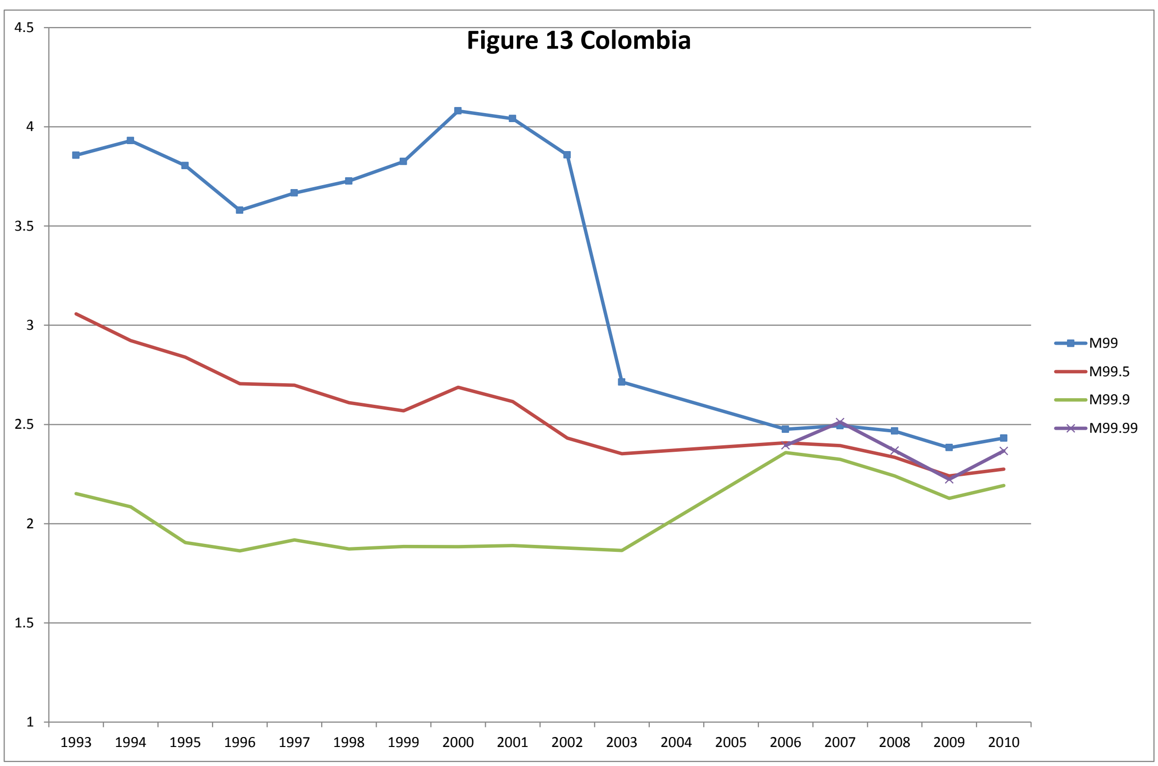


Figure 13 Colombia
4.5
4
3.5
3
2.5
2
1.5
1
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005
2006
2007
2008
2009
2010
M99
M99.5
M99.9
M99.99