Capital Accumulation, Technological Change, and the Distribution of Income during the British Industrial Revolution

by

Robert C. Allen

Nuffield College
New Road
Oxford OX1 1NF

Department of Economics
Oxford University
Manor Road
Oxford OX1 3UQ

email: bob.allen@nuffield.oxford.ac.uk

2005

Abstract

The paper reviews the macroeconomic data describing the British economy during the industrial revolution and shows that they contain a story of dramatically increasing inequality between 1800 and 1840: GDP per worker rose 37%, real wages stagnated, and the profit rate doubled. They share of profits in national income expanded at the expense of labour and land. A "Cambridge-Cambridge" model of economic growth and income distribution is developed to explain these trends. An aggregate production function explains the distribution of income (as in Cambridge, MA), while a savings function in which savings depended on property income (as in Cambridge, England) governs accumulation. Simulations with the model show that technical progress was the prime mover behind the industrial revolution. Capital accumulation was a necessary complement. The surge in inequality was intrinsic to the growth process: Technical change increased the demand for capital and raised the profit rate and capital's share. The rise in profits, in turn, sustained the industrial revolution by financing the necessary capital accumulation.

key words: British industrial revolution, kuznets curve, inequality, savings, investment

JEL classification numbers: D63, N13, O41, O47, O52

An early version of this paper was presented to the TARGET economic history conference at St Antony's College, Oxford in October, 2004, and I thank those present for their feedback. Peter Temin commented incisively on several drafts of the paper, for which I am very grateful. I thank Paul David for suggesting that I make savings a function of property income and call the theory a 'Cambridge-Cambridge' model. I am also grateful to Tony Atkinson and Andrew Glyn for their encouragement and suggestions. I would also like to thank Victoria Annable, Stan Engerman, Marcel Fafchamps, Tim Guinnane, Knick Harley, David Hendry, Brett House, Jane Humphreys, Ian Keay, Peter Lindert, Jim Malcolmson, Natalia Mora-Sitja, Tommy Murphy, Patrick O'Brien, Fraser Thompson, David Vines, and Gaston Yalonetzky for helpful discussions and comments on earlier drafts. This research has been funded by the Canadian SSHRCC Team for Advanced Research on Globalization, Education, and Technology and by the US NSF Global Prices and Incomes History Group. I am grateful for that support.

> "Since the Reform Act of 1832 the most important social issue in England has been the condition of the working classes, who form the vast majority of the English people...What is to become of these propertyless millions who own nothing and consume today what they earned yesterday?...The English middle classes prefer to ignore the distress of the workers and this is particularly true of the industrialists, who grow rich on the misery of the mass of wage earners."
>
> –Frederick Engels, The Condition of the Working Class in England in 1844, pp. 25-6.

Our knowledge of the macroeconomics of the industrial revolution is fuller than it was fifty years ago thanks to the work of Deane and Cole (1969), Wrigley and Schofield (1981), McCloskey (1981), Crafts (1976, 1983, 1985), Harley (1982, 1993), and Crafts and Harley (1992), and, most recently, Antràs and Voth (2003). We now have well researched estimates of the growth of real output, the three main inputs (land, labour, and capital), and overall productivity. More recent research by Feinstein (1998), Allen (1992), Turner, Beckett, and Afton (1997), and Clark (2002) has filled in our knowledge of input prices--real wages and land rents. Despite these advances, there is scope for progress in two areas. One is measurement and consists in drawing out the implications of the macroeconomic data for the history of inequality. The second is explanation. What was the interplay between growth and inequality during the British industrial revolution?

This paper continues the tradition of aggregate, macro-economic analysis. Some historians (e.g. Berg and Hudson 1992) have argued that the aggregate approach is a blind alley, and even some of its architects have moved away from it: Crafts and Harley (2004), for instance, have developed a model of the economy distinguishing agriculture and industry. This approach was initiated by Williamson (1985) and is an objective of other researchers. An open economy framework has also been explored in these models and others (e.g. O'Rourke and Williamson 2005). While disaggregation is essential to study the interactions between sectors, key questions can be answered without that detail. It is the contention of this paper that there is still much life left in the aggregate approach–if it is pushed a bit further.

The present essay aims to extend the macro-economic literature in two ways. The first is to chart the history of inequality by relating the evolution of factor prices and quantities to the growth in real output. Whether inequality rose or fell has, of course, been fiercely debated. What has not yet been recognized, however, is that the macro-economic data now at hand contain a story of dramatically rising inequality in the first four decades of the nineteenth century. The tip-off is the disjunction between Feinstein's real wage series, which shows negligible growth in this period, and the Craft-Harley GDP data which show a rise of 37% per head. That increase in output accrued to someone as income–and it wasn't workers. This finding would not have surprised Engels.

The second aim is to provide an integrated account of how technical change, capital accumulation, and inequality were interconnected. The existing literature has two features I want to amend. The first feature is treating growth and distribution as separate issues. I will argue that they were fundamentally intertwined because investment depended on inequality and inequality depended on the balance between technical progress and aggregate savings. The second feature is analysing technical progress and capital accumulation in a growth accounting framework that assigns separate, additive, and independent values to their 'contributions to growth.' In this framework, we can imagine changing one contribution without changing the others. Economic historians have been schizophrenic about this methodology. On the one hand, they usually regard its assumptions as odd (how could a new technique be adopted without erecting the equipment that embodied it?); on the other hand, no account of an industrial revolution is now complete without a table summarizing the growth rates of GDP, the aggregate inputs, and residual productivity. I aim to go beyond the artificialities of growth accounting by developing a model that identifies the effects of productivity and savings behaviour while recognising the complementarity between technical change and capital accumulation.[1]

The macro-economic record

First, I will summarize what is known about the evolution of the macro economy, and then draw out its implications for the history of inequality.

Most research has aimed to measure GDP and the aggregate inputs. A major finding is that the rate of economic growth was slow but still significant. Indeed, each revision of the indices of industrial output or GDP from Hoffmann (1955) to Deane and Cole (1969) to Harley (1982) to Crafts (1985) to Crafts and Harley (1992) has seen a reduction in the measured rate of economic growth. Likewise, real wage growth has decelerated from Lindert and Williamson (1983) to Crafts (1985) to Feinstein (1998). The latest estimates of GDP growth, on a per head basis, are shown in Table 1. Between 1761 and 1860, output per worker in Great Britain rose by 0.6% per year. Growth was slower before 1801 and accelerated thereafter. Even the fastest growth achieved (1.12% per year) was very slow by the standards of recent growth miracles where rates of 8% or 10% per year have been achieved. Nevertheless, between 1760 and 1860, per capita output increased by 82%. This was an important advance in the history of the world.

The record of growth in the British industrial revolution, thus, poses two questions. How do we explain the growth that occurred? Why wasn't growth faster? The first has received the most systematic attention, although debates about the second will be considered later.

Explanations of growth are based on theories. The growth theories popular in the early 1950s attributed economic expansion to a rise in the investment rate:

> The central problem in the theory of economic development is to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more. This is the central problem because the central fact of economic development is rapid capital accumulation. (Lewis 1954.)

Testing this theory required Feinstein's (1978, p. 91) estimates of investment and Craft's (1985, p. 73) estimates of GDP. Dividing one by the other showed that gross investment rose from 6% of GDP in 1760 to 12% in 1840. Although the pace of the increase was modest, the change in the British investment rate during the industrial revolution was consistent with the

---

[1] Crafts (2004) has explored the complementarity of investment and productivity growth in an expanded growth accounting framekwork.

Lewis model.

The analysis of growth was transformed with the publication of Solow's (1957) justification of growth account and his application of the methodology to twentieth century America, for which he claimed that technical progress, rather than capital accumulation, was the main cause of growth. To see whether industrializing Britain was the same, measures of the capital stock, labour force, and land input were needed as well as a series of real GDP.

Table 1 shows Solow's growth accounting model applied to Britain. In this approach, some of the rise in output per capita is attributed to the increase in the capital-labour ratio and some to the growth (in this case decline) in the ratio of land to labour. The second factor was negative and the first slight. The growth in total factor productivity more than equalled the growth in per capita output in 1800-30 and 84% of its growth in 1830-60. The overall conclusion is that the growth in income per head was almost entirely the result of technological progress–just like twentieth century America. Further research indicates that productivity growth in the famous, 'revolutionized' industries and in agriculture was enough to account for all of the productivity growth at the aggregate level (Crafts 1985a, Harley 1993). Productivity growth was negligible in other sectors of the economy.[2]

Technological progress has eclipsed capital accumulation as a source of growth. Indeed, capital exists in a kind of limbo without an important role to play in the industrial revolution. I will argue that this is an artificiality of residual productivity calculations and not a fundamental feature of the industrial revolution.

The macro-economic record of inequality

While there is considerable consensus about the evolution of aggregate inputs and outputs, confusion reigns in so far as income distribution is concerned. Much research has been guided by Kuznets' (1955) conjecture that inequality rises during early industrialization and declines as the economy matures (although it has risen again in the last few decades). Did Britain exhibit a Kuznets curve with growing inequality at the beginning of the industrial revolution and falling inequality later? If so, why?

The first question can only be answered through careful measurement. The most direct approach would compare inequality indices like Gini coefficients at different points in the industrialization process. Lindert and Williamson (1983b) and Williamson (1985) tried this, but the comparability of the data has been questioned (Feinstein 1988a). A less direct approach is necessary, and that is to study the prices of land, labour, and capital and the shares of national income accruing to these factors of production. Ownership of land and capital were concentrated in industrializing Britain, so a rise in property income signals an increase in inequality. This focus is also appropriate if one approaches inequality from the perspective of Victorian debates, which emphasized distribution between social classes defined by ownership of factors of production: In the Ricardian analysis of the corn laws, for instance, the gains from growth accrued to landlords, while in the work of Marx and Engels the free enterprise system directed income from workers to capitalists.

We have a much clearer idea of what happened to the distribution of income after 1860 than before. Feinstein's (1972) construction of the national income accounts of the UK from 1856 onwards shows that inequality declined in the long term. Labour's share (broadly defined) of the national income increased from about 55% in the late nineteenth century to

---

[2] This view has been disputed by Berg and Hudson (1992) and Temin (1997).

73% in the 1970s. At the same time, the rate of profit fell, and the average real wage increased (Matthew, Feinstein, and Odling-Smee 1982, p. 164, 187). Most of these changes occurred as discontinuous jumps after each world war. Lindert (2000) and Atkinson (2005) corroborate this decline while adding important nuances and also highlighting the increase in inequality since 1980. In terms of factor shares and profit rates, the period 1856-1913 was a plateau of constant–and elevated–inequality. The only moderating factor was the rise in real wages that occurred in this period.

The question is: what happened before 1856? Did inequality increase or was it always at the high late nineteenth century level? Williamson argued for a Kuznets curve in the wage distribution (as the earnings of skilled workers rose relative to those of the unskilled in the first half of the nineteenth century and declined thereafter) and in the income distribution as a whole. However, these hypotheses have been disputed by Jackson (1987) and Feinstein (1988a). The latter believed that "the best conclusion one can draw from the very imperfect evidence is that the nineteenth century exhibited no marked fluctuations in inequality" (Feinstein 1988a, p. 728).

This judgement is too pessimistic. Certainly, the data are not robust and more work in reconstructing national income and its components is warranted for 1760-1860. Nevertheless, latent in the information at hand, is a story of rapidly increasing inequality during the industrial revolution. The situation was more complex than the Kuznets curve allows. Several indicators point to falling inequality in the late eighteenth century. This moderating trend was followed by a sharp rise in inequality between 1800 and 1840. This rise was the ascending part of the Kuznets curve that finally descended after the world wars in the twentieth century. Feinstein's work on the real wage, the capital stock, and GDP are important components of this reinterpretation.

The main indicators of inequality trends are factor prices and factor shares. These are graphed in Figures 1-2 and 4-7 along with simulated values. All values are real returns and real shares measured in the prices of the 1850s. I consider them in the order in which they were constructed.

The average real wage, as calculated by Feinstein (1998, pp. 652-3), shows a significant upward trend from 1770 to 1800, then a plateau until about 1840, when the index resumes its ascent. The eighteenth century rise contributed to reduced inequality, while the early Victorian plateau contributed to rising inequality.

The real rent of land rose slowly over the century from 1760 to 1860 (Clark 2002, p. 303). *Pace* Ricardo, it does not play a major role in the surges of inequality.

By multiplying the real wage by the occupied population and the real rent by the cultivated land, one obtains the wage bill and total rent. Dividing these by real GDP gives the shares of labour and land. Subtracting these from one gives capital's share. Capital income ("profits") in this context includes the return to residential land, mines, entrepreneurship, and the premium of middle class salaries over the wages of manual workers[3] as well as profits or interest narrowly defined.

The shares are graphed in Figure 1. The share of rent in national income declined gradually over the century. The shares of wages and profits exhibited conflicting trends. Labour's share rose slightly between 1770 and 1800 and then declined significantly until 1840 when the situation stabilized. Capital's share moved inversely, falling in the 1780s and 1790s and then surging upward between 1800 and 1840. Capitalists gained at the expense of both landlords and labourers. The former advance may not have increased inequality, but the latter certainly did.

---

[3] As explained in the Appendix, Feinstein's real wage index is normalized to equal Deane and Cole's (1969, pp. 148-53) estimate of average earnings in 1851. Their earnings estimate is based on the wages of manual workers and so excludes the higher salaries of the middle class.

Finally, one can calculate the gross profit rate from equation 6 by multiply capital's share by real GDP and then dividing the product by Feinstein's estimate of the real capital stock (Figure 2). Also shown are analogous profit rates computed from Deane and Cole (1969, pp. 166-7) for 1801 onwards.[4] The gross profit rate was low and flat in the eighteenth century and jumped upwards between 1800 and 1840. Interest rates do not show the same increase, but they were so heavily regulated as to be unreliable indicators of the demand for capital. Temin and Voth (2005) found that Hoare's bank rationed credit instead of raising interest rates. Figure 2 is a more reliable indicator of the return on capital than interest rate series.

The case for rising inequality between 1800 and 1840 is, thus, based on three pieces of macro evidence: The stagnation of the real wage, the rise in the gross profit rate, and the shift of income from labour to capital. In addition, Lindert and Williamson's (1983b) reworking of the social tables of the period point to the same conclusion, although Lindert (2000) has equivocated on the matter. While more research on the measurement of national income might overturn these findings, they are implicit in the macro economic data as they stand today.

A Model of Growth and Income Distribution

While we have a clearer understanding of the trends in the British economy between 1760 and 1860, there are still important questions about the economic processes that governed its evolution. How were technical progress and capital accumulation interconnected? What determined the rate of investment? Why did inequality increase after 1800? Was the rise in inequality an incidental feature of the period or a fundamental aspect of the growth process? To answer these questions, we need a model of the economy. The model proposed here is of the simplest sort. Only one good (GDP) is produced, and it is either consumed or invested. Agriculture and manufacturing are not distinguished. As a result, issues like the inter-sectoral terms of trade are not modelled. Important social processes like urbanization lurk in the aggregates, however, as the population grows and capital is accumulated without much increase in cultivated land.

The model is a 'Cambridge-Cambridge' model.[5] Growth and income distribution are governed by a neoclassical production function (as in Cambridge, Massachusetts), while savings depends on the distribution of income (as in Cambridge, England). I begin with the three equations that comprise the heart of the Solow (1956) growth model:

---

[4] They estimated nominal property income, mainly from tax sources. Subtracting nominal agricultural rent gives an estimate of nominal profits at ten year intervals. Dividing each year's profits by the value of the capital stock series (recalculated in the prices of that year) gives the historical profit rates shown in Figure 2.

[5] The term is Paul David's. David (1978) calls the model a 'Cantabridgian Synthesis." Samuelson and Modigliani (1966) analysed the model theoretically. They referred to "A

$$Y = f(AL, K, T) \tag{1}$$

$$K_t = K_{t-1} + I_t - \delta K_{t-1} \tag{2}$$

$$I = sY \tag{3}$$

The first is a neoclassical production function in which GDP (Y) depends on the aggregate workforce (L), capital stock (K), and land area (T). The latter is not normally included in a Solow model but is added here due to its importance in the British economy during the Industrial Revolution. A is an index of labour augmenting technical change. Technical change of this sort is necessary for a continuous rise in per capita income and the real wage.

The second equation defines the evolution of the capital stock. The stock in one year equals the stock in the previous year plus gross investment (I) and minus depreciation (at the rate δ) of the previous year's capital stock.

The third equation is the savings or investment function according to which investment is a constant fraction (s) of national income. Equation (3) is the very simple Keynesian specification that Solow used. In some simulations, I will use it to set the economy-wide savings rate. However, equation (3) is not descriptive of industrializing Britain where all saving was done by landlords and capitalists. This idea is incorporated into the model with a savings function along the lines of Kalecki (1942) and Kaldor (1956):

$$I = (s_K \phi_K + s_T \phi_T) Y \tag{4}$$

In this specification, capitalists and landowners do all the savings since s_K is the propensity to saving out of profits and ϕ_K is the share of profits in national income. Likewise, s_T is the propensity to saving out of rents and ϕ_T is the share of rents. The economy-wide savings rate $s = (s_K \phi_K + s_T \phi_T)$ depends on the distribution of income. With equation 4, accumulation and income distribution are interdependent and cannot be analysed separately. In other words, one cannot first ask why income grew and then ask how the benefits of growth were distributed. Each process influenced the other.

Usually, a growth model also includes an equation specifying the growth in the work force or population (assumed to be proportional) at some exogenous rate. Since the model is being applied here to past events, the work force is simply taken to be its historical time series. There was some variation in the fraction of the population that was employed. I will ignore that, however, in this paper and use the terms output per worker and per capita income interchangeably.

Three more equations model the distribution of income explicitly. The derivatives of equation (1) with respect to L, K, and T are the marginal products of labour, capital, and land,

---

Neoclassical Kaldorian Case" (p. 295), and anticipated the 'Cambridge-Cambridge' terminology with their quip that their analysis "can encompass valid theories in Cambridge, Massachusetts, Cambridge, Wisconsin, or any other Cambridge." (P. 297).

and imply the trajectories of the real wage, return to capital, and rent of land. These factor prices can also be expressed as proportions of the average products of the inputs:

$$w = \phi_L \frac{Y}{L} \qquad (5)$$

$$i = \phi_K \frac{Y}{K} \qquad (6)$$

$$r = \phi_T \frac{Y}{T} \qquad (7)$$

Here w, i, and r are the real wage, profit rate, and rent of land. ϕ_L, ϕ_K, ϕ_T are the shares of labour, capital, and land in national income, as previously noted.

A production function must be specified to apply the model to historical data. The Cobb-Douglas is commonly used, and, indeed, I used a Cobb-Douglas for trial simulations and to determine a provisional trajectory for productivity growth. The function is:

$$Y = A_0(AL)^{\alpha}K^{\beta}T^{\gamma} \qquad (8)$$

where α, β, γ are positive fractions that sum to one when there is constant returns to scale, as will be assumed. A_0 is a scaling parameter. With a Cobb-Douglas technology, A can be factored out as A^{α} which is the conventional, Hicks neutral, total factory productivity index. In addition, in competitive equilibrium, the exponents α, β, and γ equal the shares of national income accruing to the factors (ϕ_L, ϕ_K, and ϕ_T). These shares are constants. They can be calculated from the national accounts of one year; in other words, the model can be calibrated from a single data point.[6]

Ultimately, however, the Cobb-Douglas is not satisfactory for understanding inequality since the essence of the matter is that the shares were not constant. Economists have proposed more general functions that relax that restriction. The simplest is the CES (constant elasticity of substitution). It is not general enough, however, for it requires that the elasticities of substitution between all pairs of inputs be equal (although not necessarily equal to one). Instead, I have used the translog production function.[7] It is the natural generalization of the Cobb-Douglas. With the translog, all shares can vary as can all of the pair-wise elasticities of substitution. The translog is usually written in logarithmic form:

$$\begin{aligned} LnY = {} & \alpha_0 + \alpha_K \ln K + \alpha_L \ln(AL) + \alpha_T \operatorname{Ln}T + \\ & \tfrac{1}{2}\beta_{KK}(\ln K)^2 + \beta_{KL}\ln K\ln(AL) + \beta_{KT}\ln K\ln T + \\ & \tfrac{1}{2}\beta_{LL}(\ln(AL))^2 + \beta_{LT}\ln(AL)\ln T + \tfrac{1}{2}\beta_{TT}(\ln T)^2 \qquad (9) \end{aligned}$$

---

[6] Van Zanden (2005) uses a Solow model with a Cobb-Douglas function to analyze early modern economic growth.

[7] Introduced by Christensen, Jorgenson, and Lau (1971) and Layard, Sargan, Ager, and Jones (1971).

subject to the adding up conditions $\alpha_K + \alpha_L + \alpha_T = 1$, $\beta_{KK} + \beta_{LK} + \beta_{TK} = 0$, $\beta_{KL} + \beta_{LL} + \beta_{TL} = 0$, and $\beta_{KT} + \beta_{LT} + \beta_{TT} = 0$. When all of the $\beta_{ij} = 0$, the translog function reduces to the Cobb-Douglas.

Logarithmic differentiation of the translog function gives share equations that imply trajectories of factor prices in accord with equations 5-7:

$$s_K = \alpha_K + \beta_{KK} \ln K + \beta_{KL} \ln(AL) + \beta_{KT} \ln T \qquad (10)$$

$$s_L = \alpha_L + \beta_{LK} \ln K + \beta_{LL} \ln(AL) + \beta_{LT} \ln T \qquad (11)$$

$$s_T = \alpha_T + \beta_{TK} \ln K + \beta_{TL} \ln(AL) + \beta_{TT} \ln T \qquad (12)$$

These equations are the basis for calibrating the model, as we will see.

Savings and Production Function Calibration

The savings and production functions are central to the growth model, and each must be estimated. Were there sufficient data, this could be done econometrically, but data are too limited for that. Instead they are calibrated.

There are two variants of the savings function. In the case of $I = sY$ (equation 2), s is determined by dividing real gross investment by real GDP. The ratio rises gradually from about 6% in 1760 to 11% in the 1830s and 1840s. It sags to about 10% in the 1850s.

The alternative savings function is the Kalecki function $I = (s_K\phi_K + s_T\phi_T)Y$ (equation 4). This function is preferred for two reasons. First, household budgets from the industrial revolution indicate that, on average, workers did not save. In some cases, income exceeded expenditure by a small amount; in other cases, the reverse was true. Overall, there was no net savings (Horrell and Humphries 1992, Horrell 1996). All of the savings, therefore, came from landlords and capitalists. Figure 3 shows the ratio of savings to their income. There is some suggestion that the savings rate out of property income rose in the 1760s and 1770s, but thereafter there was no trend. Regression of the savings rate on the shares of profits and rents in national income for the period 1770-1913 showed a small difference between landlords and capitalists:

$$I/Y = .138\phi_T + .196\phi_K \qquad (13)$$

The coefficients had estimated standards errors of .013 and .004 respectively. In this model, capitalists saved a higher proportion of income than landlords. I used this equation for most simulations except that I lowered the coefficient of savings by capitalists to .14 in the 1760s and .16 in the 1770s. This improved the simulations in those years and creates a small exogenous component to the rise in savings from 6.5% in 1760 to 7% in 1780. The increase in savings in later years remains dependent on changes in the distribution of income.

The parameters of the translog function must also be determined. While the parameters of the Cobb-Douglas function can be calculated from the factor shares at one point in time, the translog requires two sets of factor shares.[8] If the adding up conditions $\alpha_K + \alpha_L + \alpha_T = 1$, $\beta_{KL} + \beta_{LL} + \beta_{TL} = 0$ and $\beta_{KT} + \beta_{LT} + \beta_{TT} = 0$ are imposed[9] on equations 10-12, one gets:

$$\begin{vmatrix} s_K \\ s_L \\ s_T - 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & \ln K & \ln L & \ln T & 0 \\ 0 & 1 & 0 & \ln K - \ln AL & \ln AL - \ln T & -\ln AL - \ln T \\ -1 & -1 & 0 & 0 & \ln K - \ln AL & \ln T - \ln AL \end{vmatrix} \quad \begin{vmatrix} \alpha_K \\ \alpha_L \\ \beta_{KK} \\ \beta_{KL} \\ \beta_{KT} \\ \beta_{TT} \end{vmatrix}$$

If the values for the three shares and the corresponding K, T, L, and A are substituted into these three equations for two years, then one obtains six equations in the six unknown parameters α_K, α_L, β_{KK}, β_{KL}, β_{KT}, and β_{TT}. These can be solved by inverting the matrix and premultiplying the share vector with it. The remaining parameters can be calculated from the imposed conditions.

After some experimentation, I used $\phi_L = .53$, $\phi_K = .28$, and $\phi_T = .19$ in 1770 and .47, .43, and .1 in 1860. Values for A at these dates are also necessary. Indeed, the entire trajectory of A from 1760 to 1860 is necessary for later simulations. Trial rates for the periods 1760-1800, 1801-30, and 1831-60 have be obtained both from the growth rates of residual productivity in Table 1 (using the relationship that residual TFP equals the rate of labour augmenting technical progress raised to the power of labour's share) and by simulating a simplified version of the model with a Cobb-Douglas production function. The rates are refined by iterating between the translog production function parameters and the trajectory of labour augmenting technical change until a close fit between actual and simulated GDP is obtained. The resulting set of translog parameter values is shown in Table 2. The corresponding rate of labour augmenting technical change increased from .3% per year in the first period to 1.5% in the second and, finally, 1.7% in 1831-60.

How well does the model perform?

To see how well the model performs, we need to simulate it with historical values for the exogenous variables to check that the simulated values of the endogenous variables track their historical counterparts.

First, does the model track GDP? Figure 4 compares the actual and simulated series and shows that they are almost indistinguishable.

Second, does the model track the investment rate? Figure 5 compares the two, and the trends are similar. The saw toothed pattern in the historical series reflects Feinstein's presentation of his investment figures as ten year averages. The simulated investment series follows the upward trend of Feinstein's series. Some of the rise 1760-80 is due to the exogenous increase in the propensity to save out of profits. Otherwise, the growth in the investment rate is due to the shift of income to capitalists.

---

[8] This is suggested by Diewert's (1976) quadratic approximation lemma, which he used to prove that the Törnqvist-Divisia input index is exact for a translog production function.

[9] It is not necessary to explicitly impose the adding up condition $\beta_{KK} + \beta_{KL} + \beta_{KT} = 0$ since it is implied by the others.

Third, can the model explain the changes in factor shares? Figure 6 shows the simulation of labour's share, which is of key importance. The simulation follows the trend of the actual series closely. Figure 7 compares historical and simulated values of all three shares. The predicted share of land matches the actual share closely, including the flat phase of the late eighteenth century and the halving of the share from 1800 to 1860. The simulation also does a good job tracking capital's share.

Next consider factor prices. Figure 8 compares Feinstein's real wage with the real wage series implied by equation 5 evaluated for the translog production function. The translog simulation mimics the upward trend in the late eighteenth century, the stagnation from 1800 to 1840, and the subsequent ascent.

Figure 9 contrasts the simulated trajectories of the gross profit rate (profits divided by the capital stock) with both measures of the variable. The translog simulation certainly captures the rise in profits that began after 1800, although it shows a curious blip in the eighteenth century.

Figure 10 repeats the exercise for the real rent of land. The history of rent has been the subject of considerable controversy, but the Norton, Trist, and Gilbert (1889), Allen (1992), and Clark (2002) series agree reasonably well for this period, as does the Turner, Beckett, and Afton (1997) series after 1800. Figure 10 shows Clark's (2002, p. 303) series inclusive of taxes and rates. The simulated rent series closely follows this series.

The simulations show that the model reproduces the history of the important endogenous variables. This is important evidence for its validity and justifies the counterfactual simulations that will be discussed next.

The complementarity of technical progess and capital accumulation

Developing a model that replicates the trajectories of the principal endogenous variables is only the first stage in understanding the industrial revolution. Next we must vary the exogenous parameters–the savings rate and the rate of productivity growth–to discover how they affected the growth of GDP. Identifying the contributions of capital accumulation and productivity growth is usually done with growth accounting. With that procedure, the contributions are additive and independent. If there were less capital accumulation, for instance, only capital's contribution to growth would be affected. The contribution of technical progress would be unchanged. This is not a realistic description of industrializing Britain, however, for investment in equipment, factories, and, indeed, cities themselves was necessary in order to adopt the new methods. In that sense, capital accumulation and technical progress were complements, and that complementarity is a feature of the present model.

The complementarity can be seen by simulating economic growth with alternative savings rates and productivity growth rates (Figure 11). These simulations use the Keynesian savings function $I = sY$ in order to set economy-wide savings at particular levels. Three things should be noticed in Figure 11:

First, the trajectory of simulated actual income is close to the historical trajectory. The simulated actual curve is based on the historical series of $s = I/Y$ and the period rates of labour augmenting technical change that provided good fits between simulated and actual GDP (Figure 4). In this simulation, per capita GDP grows by 82% and reaches £61.8 per year. The question is how the rising investment rate and productivity growth contributed to this achievement.

Second, the bottom line is a counterfactual standard of comparison. It shows what would have happened to income per worker had there been no industrial revolution. This simulation embodies three assumptions: the savings rate remains constant at 6%, the level in 1760, technical progress occurs at 0.3% per year, the rate that characterized 1760-80 and presumably the pre-industrial period, and population grew as it did from 1760 to 1860. The last assumption is the most suspect since population growth accelerated after 1750, and the rise may well have been due to the expansion of the economy. However, the Solow model treats population growth as exogenous, and that assumption is maintained here.

Under these assumptions, per worker output not only fails to rise between 1760 and 1860; it actually falls–to £28.8 per year. The culprit is the relative fixity of land. While I allowed the cultivated acreage to increase at it actually did, neither that expansion nor the slow rate of productivity growth was enough to offset the negative effect of larger population on output per worker. The model implies that Britain would have experienced falling incomes in the early nineteenth century.

Third, the two middle lines show the separate impacts of rising investment and productivity on per capita income. One line assumes that the investment rate remained at 6%, while labour augmenting technical change accelerated at its historical rate.[10] With this simulation, per capita income reaches £33.0 per year in 1860. By itself, the acceleration of productivity growth would have increased final GDP by £4.2 (= £33 - £28.8), which is 13% of the difference between the simulated actual 1860 level (£60.8) and the no industrial revolution counterfactual level (£28.8). In contrast, when the savings rate is allowed to follow its historical trajectory but technical progress is kept at 0.3% per year, simulated GDP per head only reaches £35.8 in 1860. The gain over the no industrial revolution counterfactual is only £7.1 per person or 22% of the increase in 1860 GDP per head with respect to the non industrial revolution counterfactual.

Adding together the separate contributions of technical progress and capital accumulation accounts for only 35% of the gain in GDP with respect to the non-industrial revolution simulation. The remaining 65% was due to their interaction. The implication is that the industrial revolution depended on both productivity growth and capital accumulation occurring together. Both were critical, and the attempt to decompose the total into separate, additive contributions is impossible.

The complementarity between capital and labour is a feature of the translog production function and was fundamental to its ability to track the changing factor shares; namely, the elasticity of substitution between capital and labour. That elasticity was very low in the calibration used here. Figure 12 shows the labour-capital isoquant for the translog function in 1810 and, for comparison, a Cobb-Douglas isoquant through the same input combination. The Cobb-Douglas has an elasticity of substitution equal to one for all input pairs. In the figure, the Cobb-Douglas isoquant is quite flat, while the translog is close to a right angle. The translog is thus approximating a Leontief fixed proportions technology with an elasticity of substitution of zero. The low elasticity of substitution is the reason that the simulations do not detect separate influences of capital and technology on growth. With a fixed proportions technology, more capital produces no additional output if labour is fixed. In all of the specifications used here, technical change is treated as labour augmenting; in other words, an improvement in technology is like an increase in employment. With fixed proportions, more labour produces no additional output, so long as capital is fixed. Only if there is more of both–or in this case, greater efficiency and a higher accumulation rate–will output increase.

---

[10] The translog function is not necessarily concave for all parameter values and input levels. (The discerning reader may be able to see that the translog isoquant in Figure 12 turns up when capital increases from 600 to 800–in violation of the standard assumptions.) In the simulation using the historical rates of productivity growth and a counterfactual investment rate of 6%, the marginal production of labour becomes negative from 1841 onwards. For that reason, the simulated series is no longer reliable. However, it continues the preceding trend, so I have used the simulated value for 1861 in the calculations reported. A breakdown of the sources of growth for 1760-1841 would give the same kind of conclusion. This problem does not occur in any other simulations reported.

What is at issue is not just production function parameters but historical processes. Normally, we think of the production function as describing the input substitution possibilities in production itself, but more was involved during the industrial revolution. Textiles is the paradigm case. In most of Europe and in seventeenth century Britain, manufacturing was located in the countryside, and production took place in people's homes. The rural-cottage sector could expand by constructing more cottages including looms and other equipment. Arkwright's textile inventions resulted in factory production. Quickly, factories became concentrated in cites–Manchester is the paradigm–because the agglomeration of mills raised efficiency through external economies of scale. Brown (1988), for instance, showed that nominal wages were higher in large cities than in the country. Some of the premium accrued to landowners as higher house rents, and some to the workers themselves as compensation for the higher mortality rates in cities. Firms also had a choice between locating in the country and the city. That most located in the cities and paid a higher wage is a measure of the agglomeration economies from urban production.[11] The result was urbanization and a rise in capital intensity including investment in infrastructure. The industrial revolution was not just a question of erecting spinning machines–Manchester had to be built as well. Much more investment was devoted to housing and infrastructure than to equipment (Feinstein 1978, pp. 40-1, 1988b, p. 431).

Figure 13 diagrams the implications of the two modes of production. The rural-cottage (L, K) and urban-factory (L*, K*) modes are represented by fixed proportion technologies. The leftward shift of the factor price line shows the cost saving from urbanization under British conditions.[12] The diagram also indicates the convex hull of these technologies. This hull is the isoquant captured by the translog production function. Conceiving of it in terms of two underlying fixed proportions technologies explains the low elasticity of substitution between capital and labour.

---

[11] Crafts and Venables (2003) have emphasized the importance of agglomeration economies in explaining nineteenth century economic growth.

[12] While I have emphasized the stagnation of wages in industrializing Britain, it must also be noted that British wages were high by international standards (Allen 2001). High wages meant that it was Britain that innovated the urban-factory mode of production rather than lower wage countries on the continent.

Capital in this function is not simply industrial plant and equipment but includes housing, transportation, and other infrastructure as well. Indeed, the vertical extension of the isoquant is not just a mathematical artifact but represents more capital intensive technologies that did not increase output. These included cities with sewage and water supply systems rather than privies and wells. British industrialization was accomplished in the way that minimized the cost of producing GDP, so cities were built as inexpensively as possible and lacked water supply and sewage systems that would have improved life (as Engels noted) but would not have increased the production of cotton. It was not until the middle of the nineteenth century that water and sewage systems were constructed (Williamson 1990).

The model presented here provides a richer account of the industrial revolution than growth accounting. The model does not split up growth into contributions that are independent and additive. This is because the industrial revolution required investments to give effect to the new technologies of the period. Technical progress and capital accumulation went hand-in-hand.

Savings, Technology, and the Rate of Economic Growth

Delinking savings and productivity gains highlights the low elasticity of substitution between capital and labour in industrializing Britain and establishes the important point that growth cannot be decomposed into independent contributions that add up to the total in the manner posited in residual productivity decompositions. In reality, however, productivity and accumulation did not vary independently because they were connected by the distribution of income. Restoring the link and simulating the model with the Kalecki savings function (equation 4) is the best way to see how growth and distribution were affected by technical progress and the propensity to save.

I begin with growth. Between 1760 and 1860 GDP per worker rose by 82%. Why did that increase occur? Could Britain have done better? Williamson (1984) provocatively raised the question "Why was British Growth so Slow During the Industrial Revolution?" His answer emphasized the investment rate. "Britain tried to do two things at once–industrialize *and* fight expensive wars, and she simply did not have the resources to do both." (Williamson 1984, p. 689). During the Napoleonic Wars, government borrowing crowded out private investment cutting the rate of accumulation and income growth. In contrast, Crafts (1987, p. 247) emphasized the slow pace of technical progress: "as a pioneer industrializer Britain found it hard to achieve rapid rates of productivity growth on a wide front throughout the economy." Only if technology had advanced more rapidly could Britain have grown more rapidly.

The crowding out thesis has had a mixed reception. Heim and Mirowski (1987, 1991) have argued that British capital markets were too segmented for government borrowing to have crowded out private investment, a view that received some support from Buchinsky and Pollak (1993). Mokyr (1987) and Neal (1991, 1993) have argued that capital inflows offset government borrowing and precluded crowding out. Clark (nd) has found no impact of war finance on a variety of rates of return. On the other hand, Temin and Voth (2005) have inferred from the records of Hoare's bank that crowding out probably occurred, although to a smaller extent than Williamson thought. Early in this debate Crafts (1987, p. 248) established the important point that private investment declined very little during the French Wars–the government borrowing represented additional savings provided by an aristocracy eager to defeat revolution and protect its position.

Whatever one concludes about crowding out, the possibility that a low British savings rate retarded the industrial revolution remains a live issue: What government borrowing during the French Wars indicates is that the propertied classes had a great, untapped potential to save. Indeed, a savings rate of 17% out of property income is remarkably low by international standards. From a regression like equation 13, David (1978) deduced that 61% of American property income was saved in the nineteenth century. We can, therefore, ask what impact a higher savings propensity would have had on growth. The question can be considered over a longer time frame than simply the French Wars, although it includes them. This is doubly fortunate since Feinstein's real wage series shows that the stagnation in working class living standards was a much long run affair than Williamson thought when he analysed the impact of the wars on accumulation.

To investigate the impact of savings on growth, I have simulated the model varying the savings rate out of property income starting in 1801. The effect depends on the magnitude and direction of the change. Doubling the savings rate, for instance, would have had only a modest impact on GDP per head. The simulated value in 1860 rises to £708 million from its baseline value of £661 million. Since increasing the savings rate (in this model) has only a small impact on growth, one cannot say that British growth was significantly reduced by a low savings rate. The underlying reason is that capital ran into rapidly diminishing returns given the low elasticity of substitution of the production function. Conversely, however, a cut in the savings rate below the historical level significantly reduces GDP. This is shown in Figure 14, which graphs the increase in 1860 GDP with respect to the no industrial revolution counterfactual level discussed in the previous section. If the savings rate out of property income is cut to 40% of its actual value, then GDP growth is also cut to about 40% of the actual increment. Increases in the savings rate out of property income up to its historical level cause significant increases in the GDP gain. However, increases in the savings rate above the historical trajectory cause only minimal rises in GDP growth. While British capitalists and landowners had the capacity to save much more income, the implication of the simulations is that increments in GDP from more accumulation would not have justified the higher level of investment. The case that war related borrowing cut the growth rate by crowding out private investment does not receive support from these simulations.

The productivity growth rate had a much bigger impact on income growth *once allowance is made for the induced rise in savings implied by the Kalecki savings function.* If we set the rate of labour augmenting technical progress at 3% per year starting in 1801 (i.e. at twice the actual rate for 1801-30), then 1860 GDP is increased by 89%. This is much more than the 7% increase from doubling the savings rate out of property income. The response is much greater here than in the simulations of the last section because of the induced increase in savings.

Figure 14 also shows the results of a range of simulations in which the income gain in 1860 is related to productivity growth rates that vary from 40% to double their historical values. Increases in productivity growth imply increases in economic growth without evidence of diminishing returns. Once the induced savings response through changes in the distribution of income is included in the simulations, productivity emerges as the prime mover behind economic growth. It was responsible for the rise in GDP that actually occurred. Since a higher rate of productivity growth would have led to a higher rate of economic growth, we can conclude that it was productivity rather than the propensity to save that was responsible for the slow growth of the British economy during the industrial revolution.

Savings, Technology, and the Rise in Inequality

The more difficult question is why the rise in productivity growth, which is modelled as labour augmenting technical change, did not lead to a rise in the real wage. Why did inequality increase during the first decades of the nineteenth century? Was that inevitable or could Britain have achieved 'growth with equity'? We can answer the questions by exploring the impact of a higher savings rate out of property income and a higher productivity growth rate on the distribution of income.

While an increase in the savings rate would have had little impact on the rate of economic growth, it would have had a very large impact on inequality. Labour's share rises to 68% and capital's drops to 18% (Figures 15and 16) when s_K and s_T are doubled. The simulated rate of profit drops from 21% in 1860 to 8% (Figure 17), and real wage growth accelerates dramatically (Figure 18).

The model, thus, exhibits in exaggerated form the features of Keynes' ambiguous allusion to the widow's cruse. In his view, if capitalists reduced their savings, aggregate demand would rise, income would expand, and–ultimately–their profits would be restored. Conversely, raising the savings rate could not increase aggregate profits. In the simulations of this paper, an increase in the savings rate of capitalists increases the capital stock and cuts the rate of return. With an elasticity of substitution less than one, the share of profits declines and the share of wages rises. In Keynes's discussion of the widow's cruse, factor shares remain unchanged. In the model of this paper, increased savings by capitalists is counterproductive to their interests as a class since greater saving reduces total profits. The cause is the low elasticity of substitution rather than feedbacks via aggregate demand.

Because of the widow's cruse, a higher savings rate would have turned the British industrial revolution into an example of 'growth with equity.' However, as earlier simulations suggest, that was not a plausible scenario for capitalist Britain: Since a higher savings rate would have scarcely raised GDP, the private return to additional investment was small and so would not have occurred under a free enterprise system. Public investment was the only option, and the public sector did increase investment in the mid-nineteenth century through construction of public health systems. These investments increased welfare without raising conventionally measured GDP.

Productivity deserves more attention than savings since the increase in technical progress was the prime mover behind economic expansion and since greater productivity growth was the only way to speed up economic development. Productivity growth in conjunction with the Kalecki savings function was also the cause of the rise in inequality during the industrial revolution. If we simulate growth assuming a productivity growth rate of 0.3% per year, the average rate from 1760 to 1800, and use the Kalecki savings function, then very little changes in the first half of the nineteenth century: Income per head, the real wage, the profit rate, and factor shares show little movement. Increasing productivity growth after 1800 to its historical levels implies simulated trajectories for the endogenous variables that closely track their historical values, as we have seen (Figures 4-10). Thus, the rise in productivity growth was responsible for the rise in inequality. The reason is that productivity growth raised the demand for capital, that increased its rate of return, and the distribution of income shifted in favour of capital in view of the low elasticity of substitution in the production function. The shift of income to capitalists was necessary in order to provide the savings needed to implement the new factory methods. The rising portion of the Kuznets curve was, therefore, the result of the rise in productivity where capital and labour were poor substitutes in production and savings depended on property income.

This point is reinforced if we ask how faster productivity growth after 1800 would have affected inequality. Setting labour augmenting technical progress at 3% per year starting in 1801 would have increased 1860 GDP by 89%, as we have seen. The faster rate of productivity growth would have required more capital, its rate of return would have risen even higher, and its share would have risen even further at the expense of labour's. The consolation for workers would have been an earlier acceleration of the real wage. Growth, in other words, would have been so much faster, that the gains would have started 'trickling down' to the working class earlier than they actually did. Faster productivity growth was labour's best chance with Britain developing as a market economy.

Conclusion

The analysis of this paper changes the emphasis in our understanding of the industrial revolution. Three general revisions stand out. First, inequality rose substantially in the first four decades of the nineteenth century. The share of capital income expanded at the expense of both land and labour income. The average real wage stagnated, while the rate of profit doubled. Second, the explanation of growth cannot be separated from the discussion of inequality since each influenced the other. In the first instance, it was the acceleration of productivity growth that led to the rise in inequality. Reciprocally, it was the rising share of profits that induced the savings that met the demand for capital and allowed output to expand. Third, the sources of growth cannot be partitioned into separate, additive 'contributions' in the manner of growth accounting. This procedure has always been counterintuitive to economic historians, for how could the productivity gains of machine spinning or iron puddling have been realized without capital investment? The complementarity of investment and greater efficiency is very clear in the model of this paper. Moreover, these two general points are interconnected: the production function parameters that make capital accumulation and technical progress complements in the growth analysis are implied by the changes in the distribution of income.

With these general considerations in mind, we can outline the story of the industrial revolution as follows: The prime mover was technical progress beginning with the famous inventions of the eighteenth century including mechanical spinning, coke smelting, iron puddling, and the steam engine. It was only after 1800 that the revolutionized industries were large enough to affect the national economy. Their impact was reinforced by a supporting boost from rising agricultural productivity and further inventions like the power loom, the railroad, and the application of steam power more generally. The application of these inventions led to a rise in demand for capital--for cities, housing, and infrastructure as well as for plant and equipment. Consequently, the rate of return rose and pushed up the share of profits in national income. With more income, capitalists saved more, but the response was limited, the capital-labour ratio rose only modestly, the urban environment suffered as cities were built on the cheap, and the purchasing power of wages stagnated. Real wages rising in line with the growth of labour productivity was not a viable option since income had to shift in favour of property owners in order for their savings to rise enough to allow the economy to take advantage of the new productivity raising methods. Hence, the upward leap in inequality. The combination of productivity raising inventions and a sluggish supply of capital explain why Britain experienced the rising phase of the Kuznets curve during the first half of the nineteenth century.

Table 1

Growth Accounting

	Growth of		Due to growth in:			
	Y/L		K/L	T/L		A
1760-1800	.26%	=	.11	-.04	+	.19
1800-1830	.63	=	.13	-.19	+	.69
1830-1860	1.12	=	.37	-.19	+	.94

Note:
The table shows growth rates per year for Y/L and A. The entries for K/L and T/L are the contributions of their growth to the growth in Y/L, that is the growth rates per year of K/L and T/L multiplied by the factor shares of capital (.35) and land (.15), respectively.

Table 2

Translog coefficients

$\alpha_0 = 5.360991$

$\alpha_K = -2.72739$

$\alpha_L = 3.053638$

$\alpha_T = .673753$

$\beta_{KK} = -.98496$

$\beta_{KL} = .804883$

$\beta_{KT} = .180070$

$\beta_{LL} = -.62481$

$\beta_{LT} = -.18008$

$\beta_{TT} = -2.669036 \times 10^{-16}$

Figure 1

Historical Factor Shares, 1760-1860

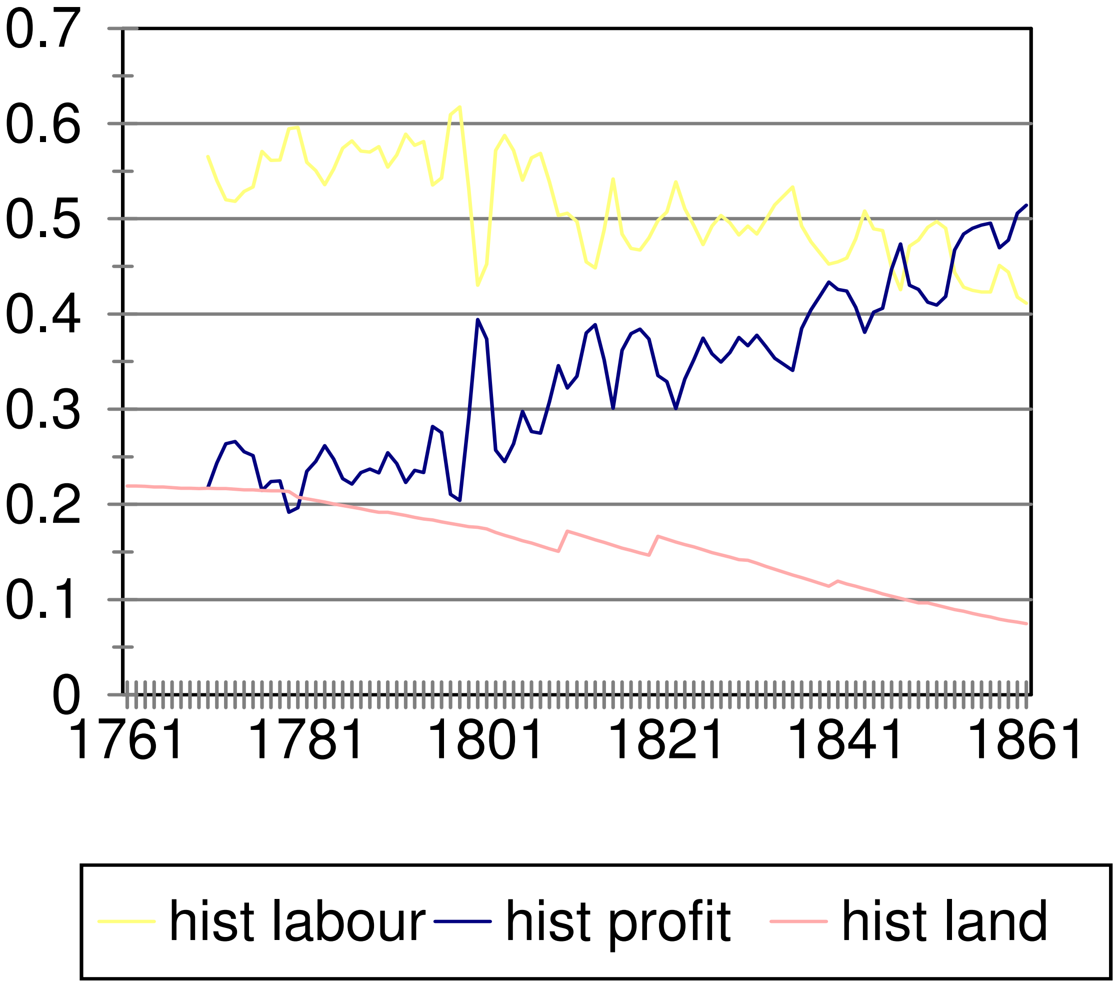


Figure 2

Historical Profit Rate, 1760-1860

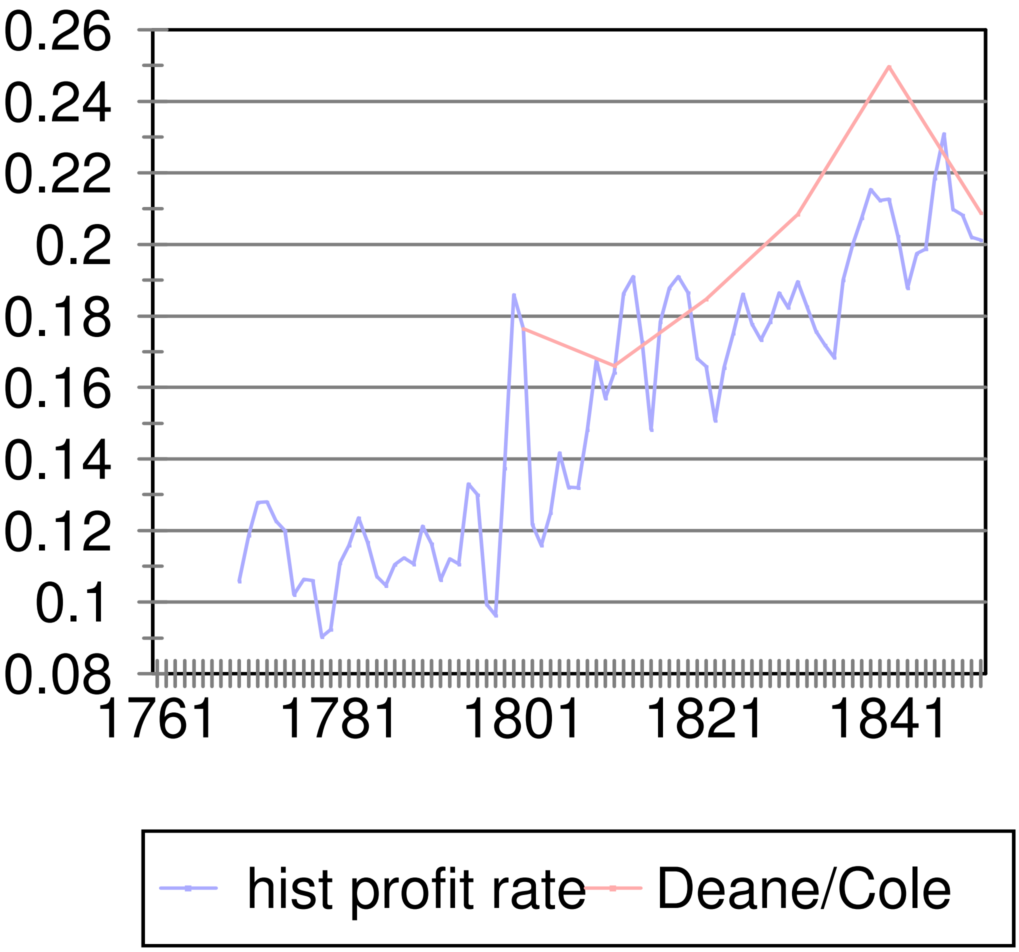


Figure 3

Savings Propensity out of Property Income

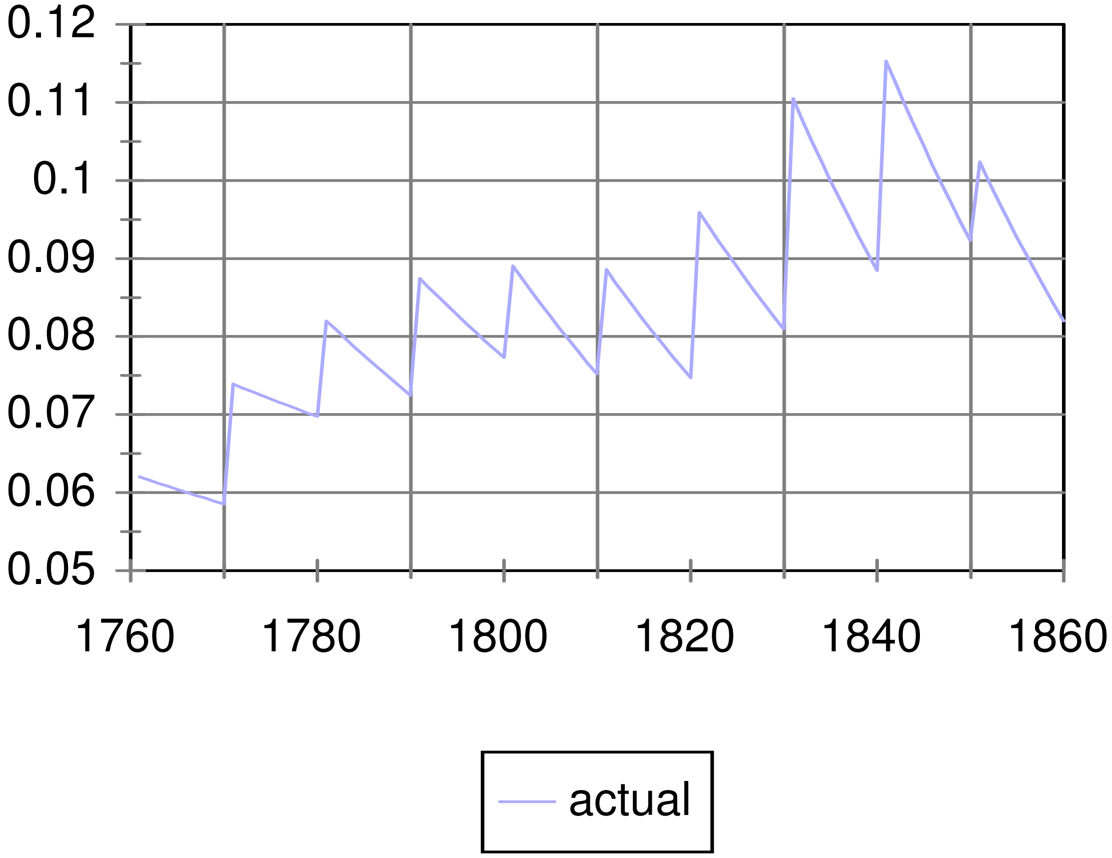


Figure 4

Actual and Simulated GDP

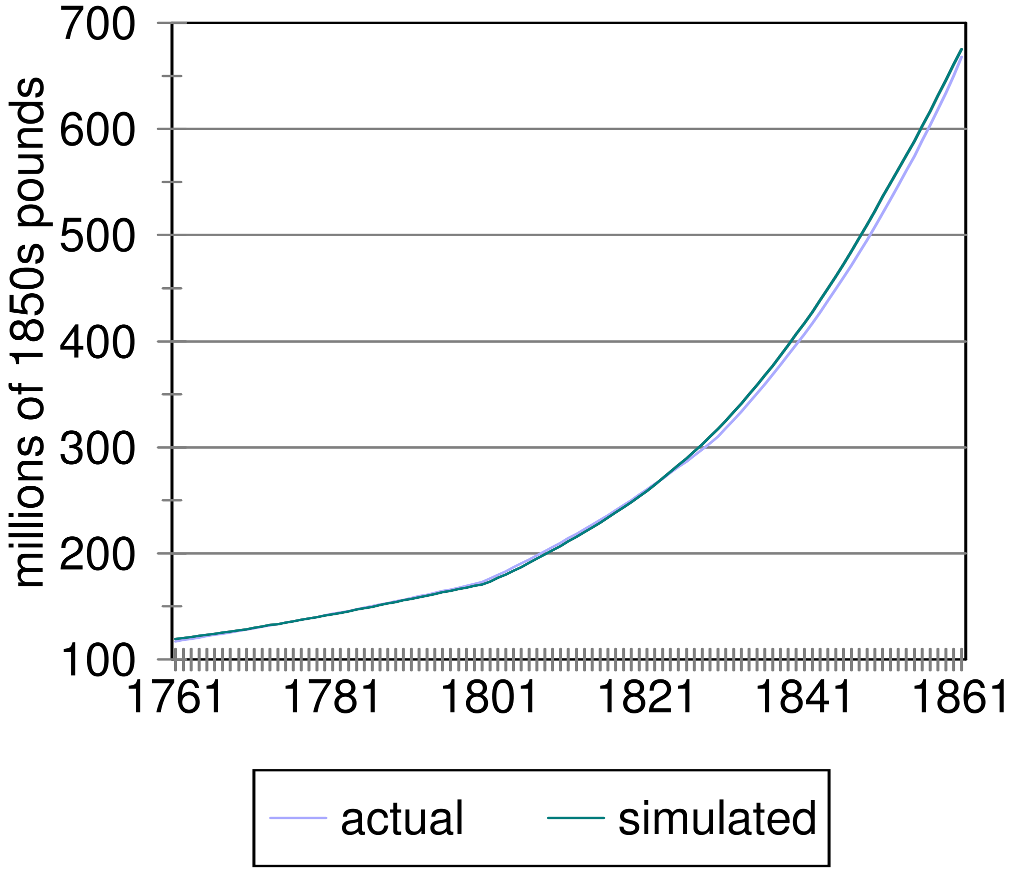


Figure 5

Actual and Simulated Investment Rate

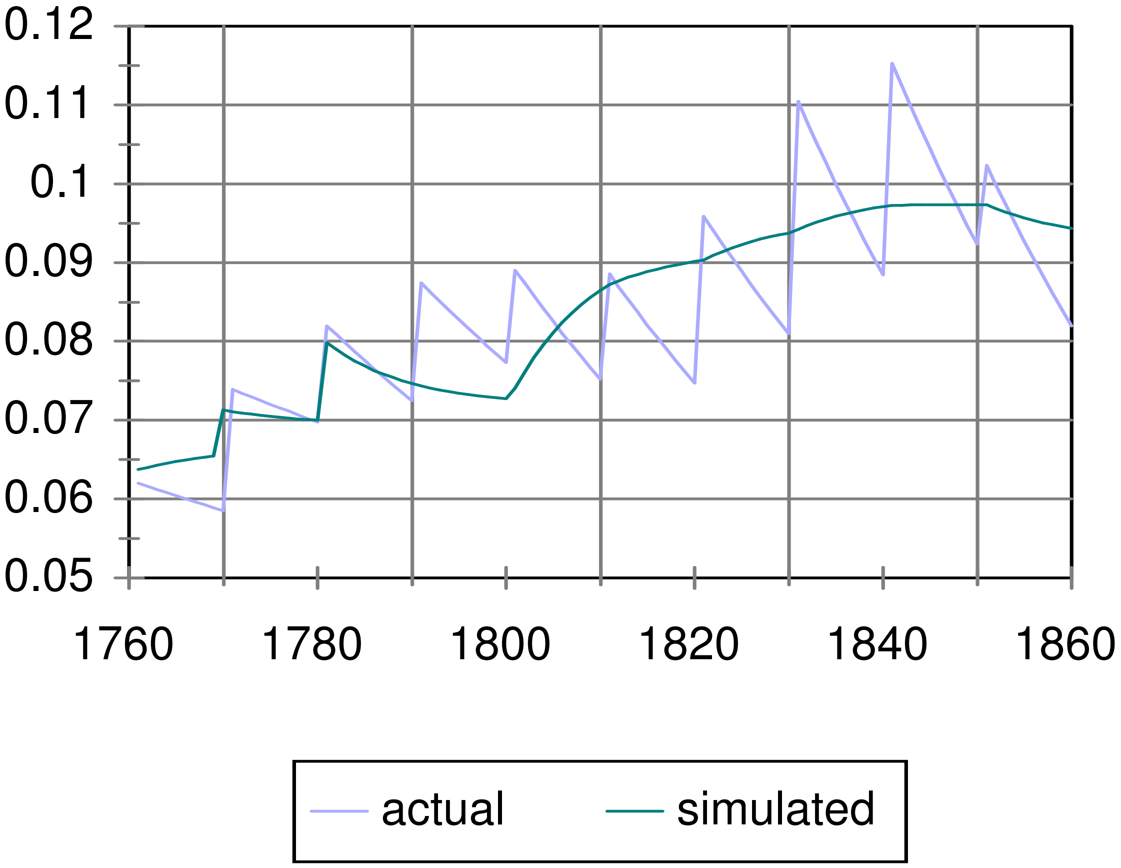


Figure 6

Labour's Share of GDP: Actual and Simulated

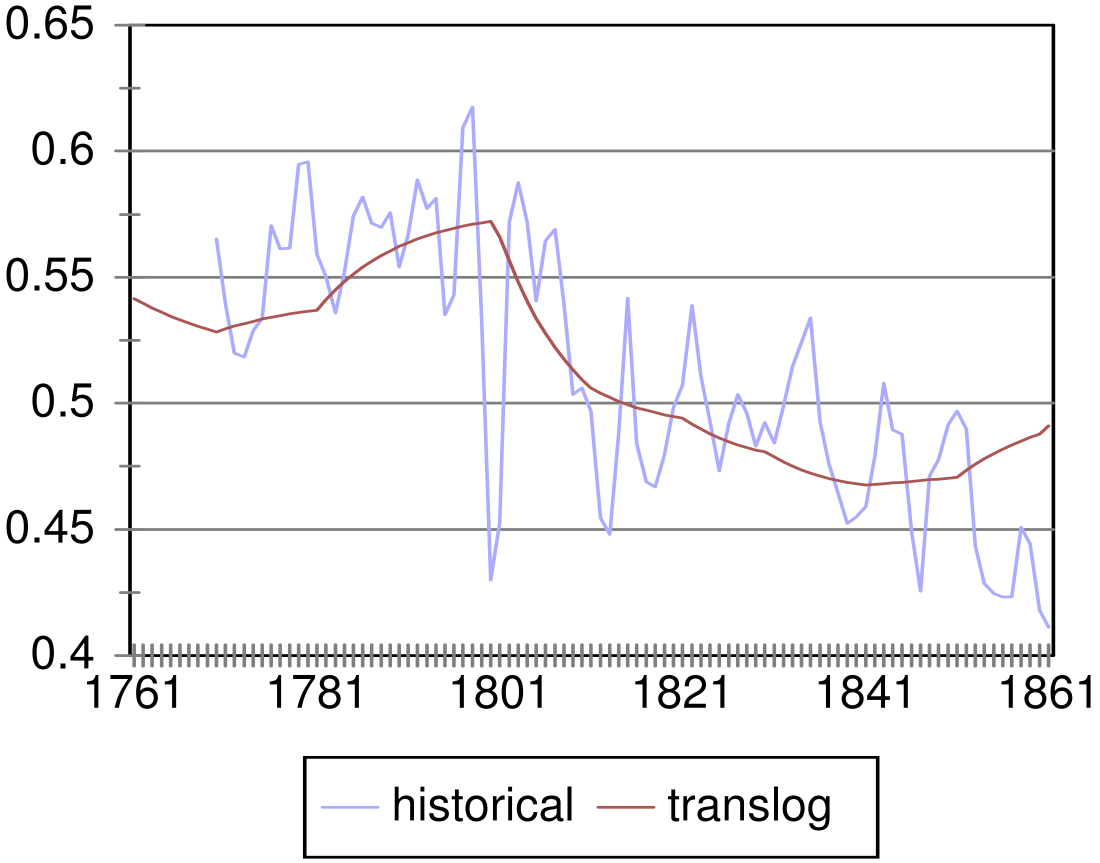


Figure 7

Factor Shares: Historical and Simulated

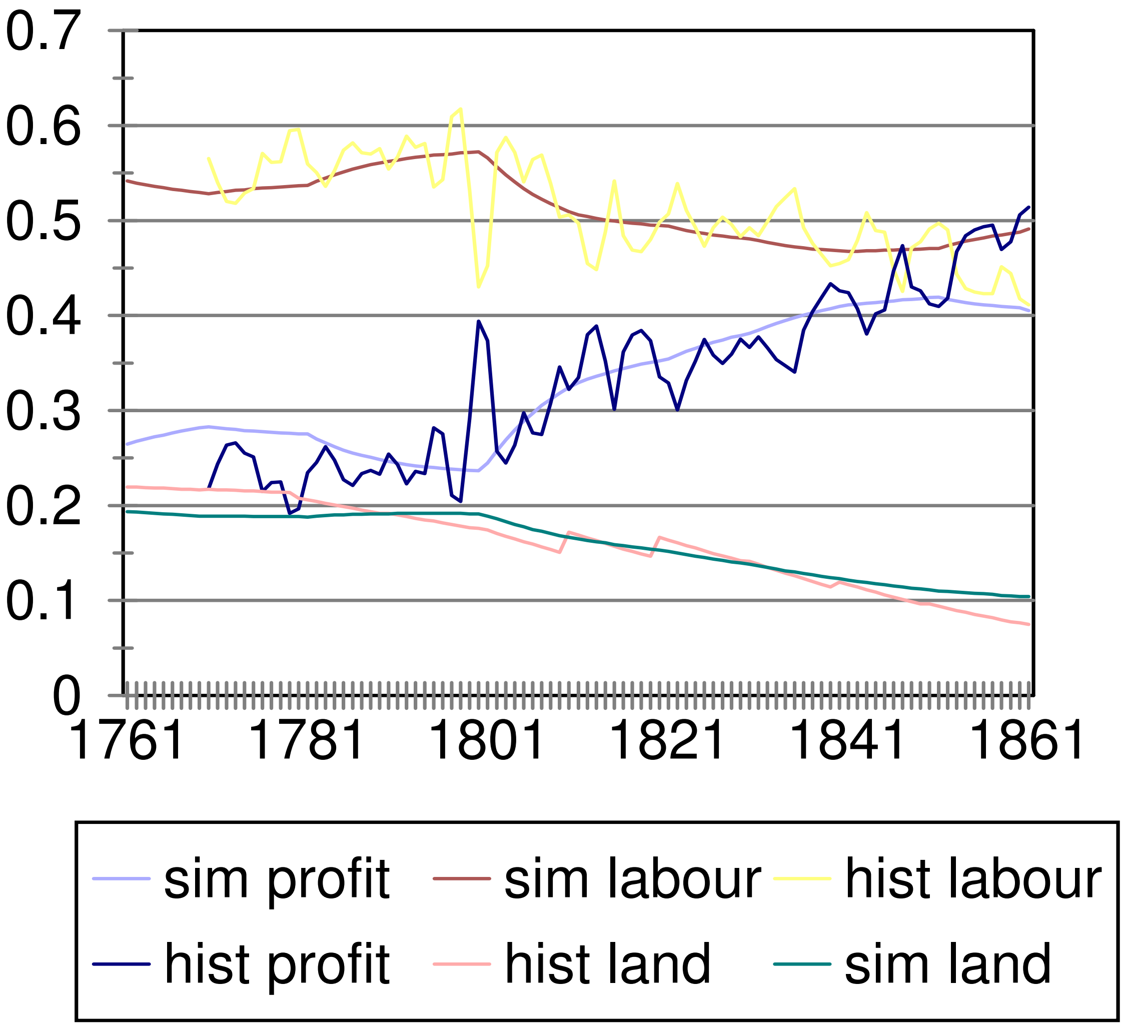


Figure 8

Real Wage: Actual and Simulated

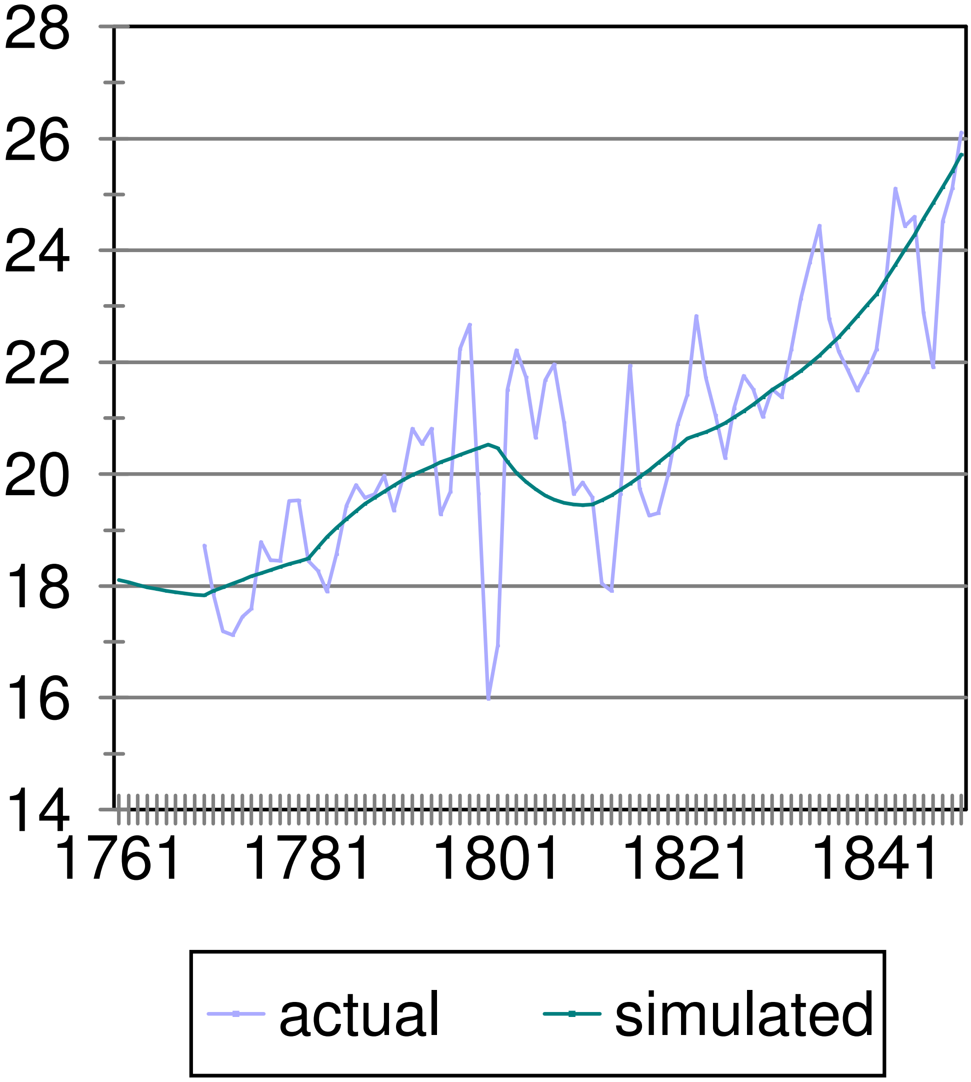


Figure 9

Profit Rate: Actual and Simulated

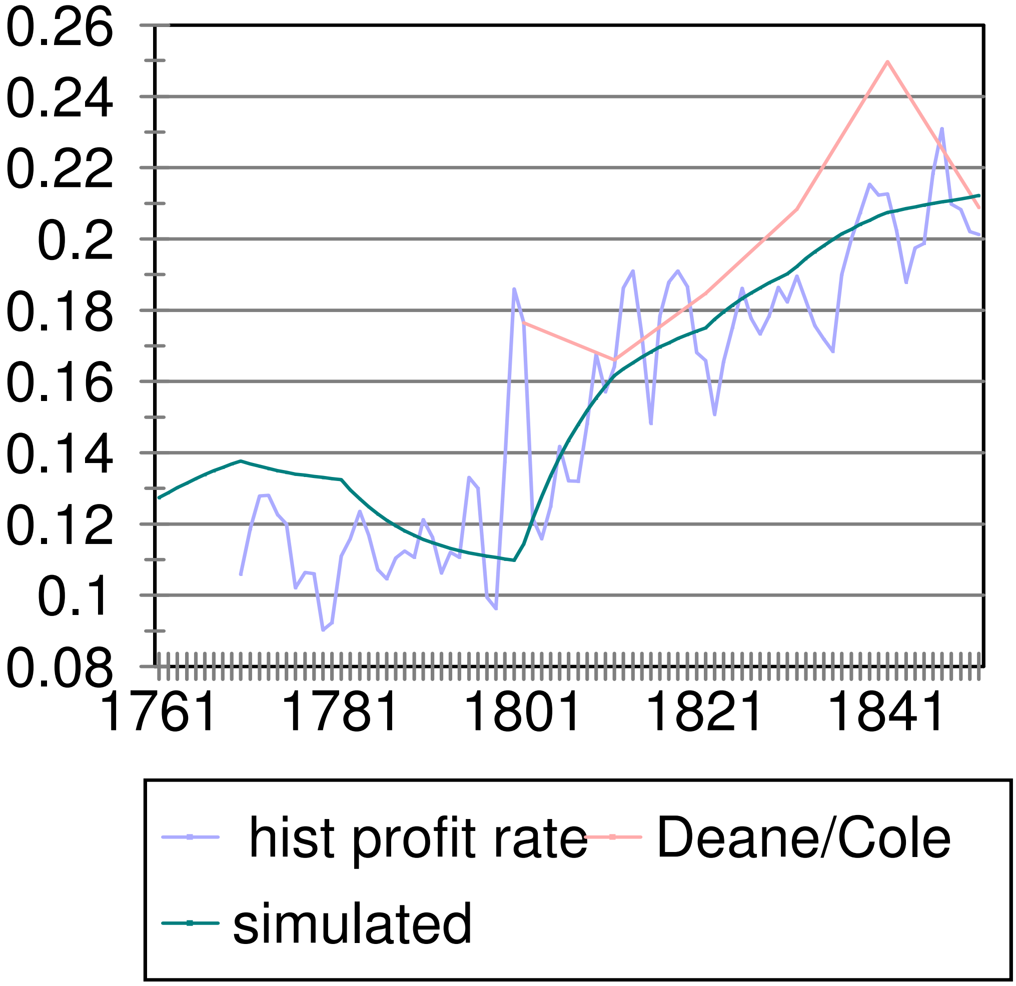


Figure 10

Land Rent: Actual and Simulated

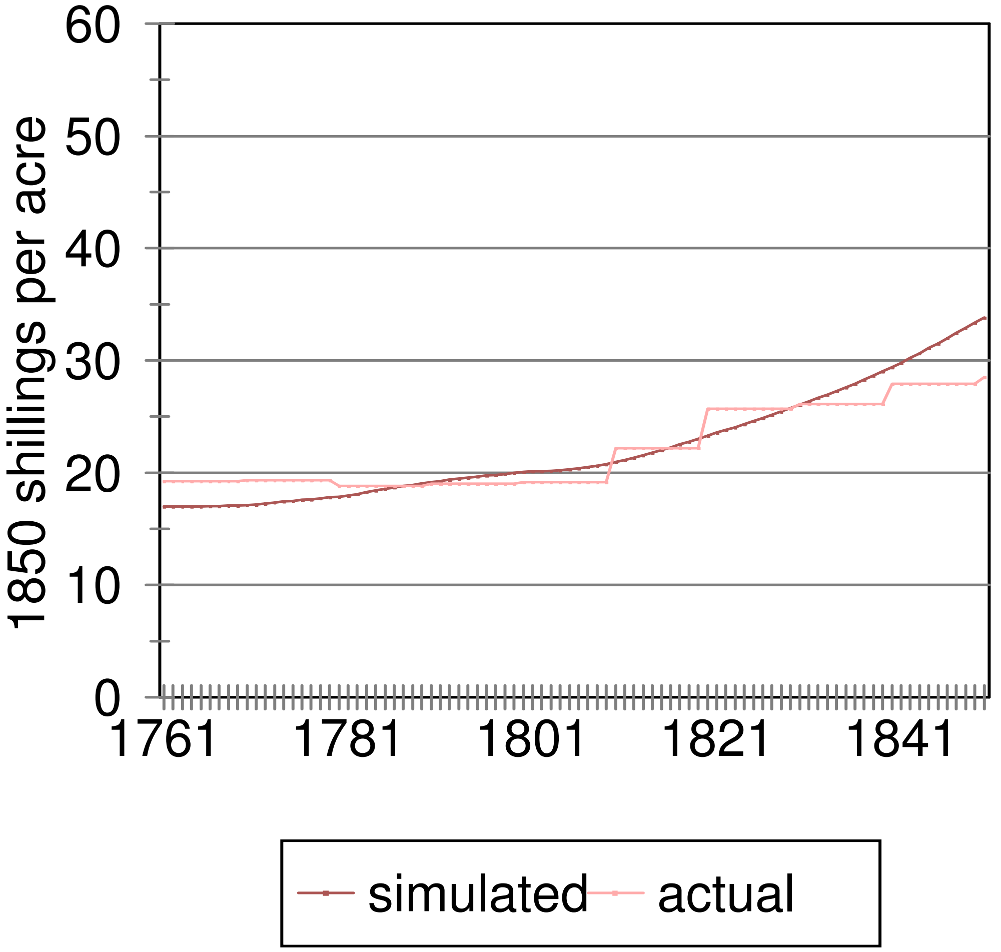


Figure 11

Alternative Growth Simulations

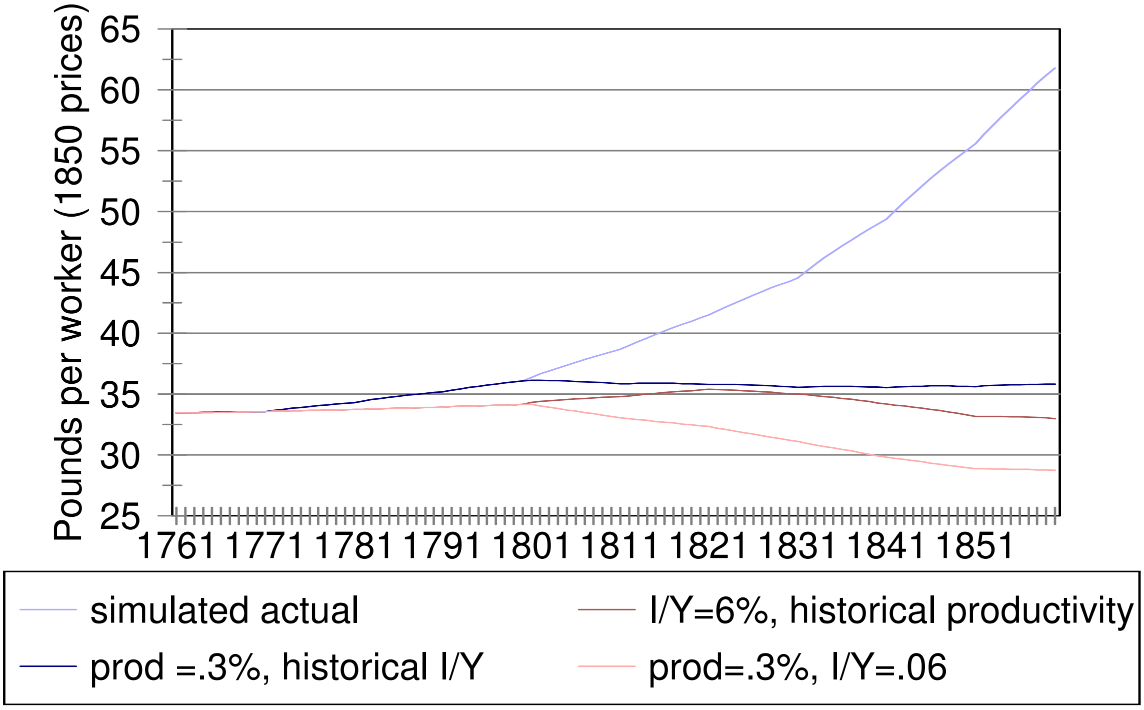


Figure 12

Cobb-Douglas and Translog Isoquants in 1810

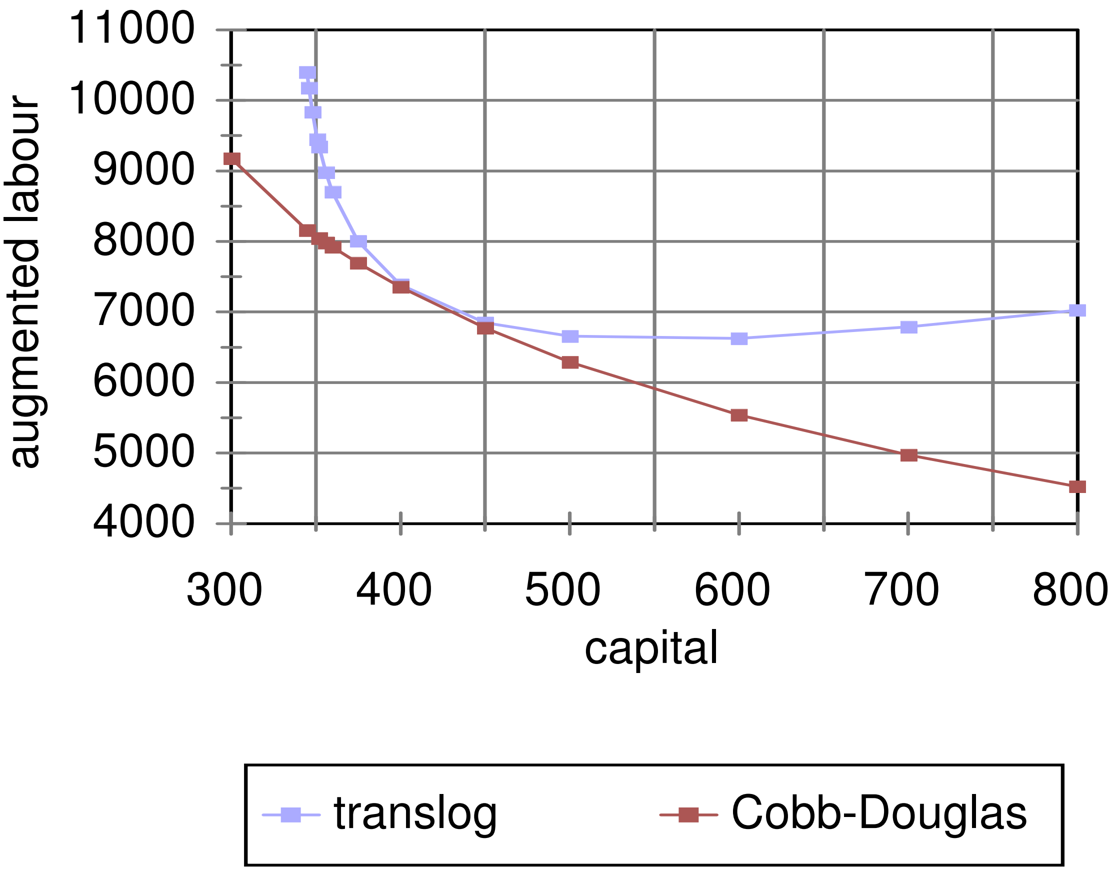


Figure 13

The urban-factory and rural-cottage modes of production

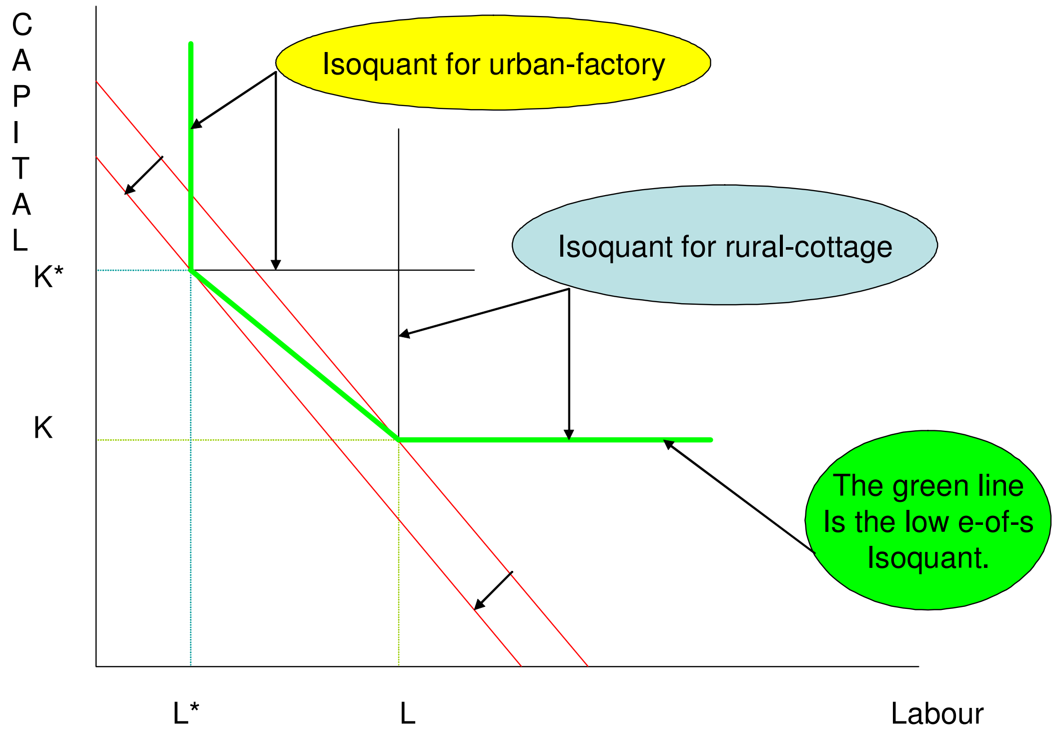
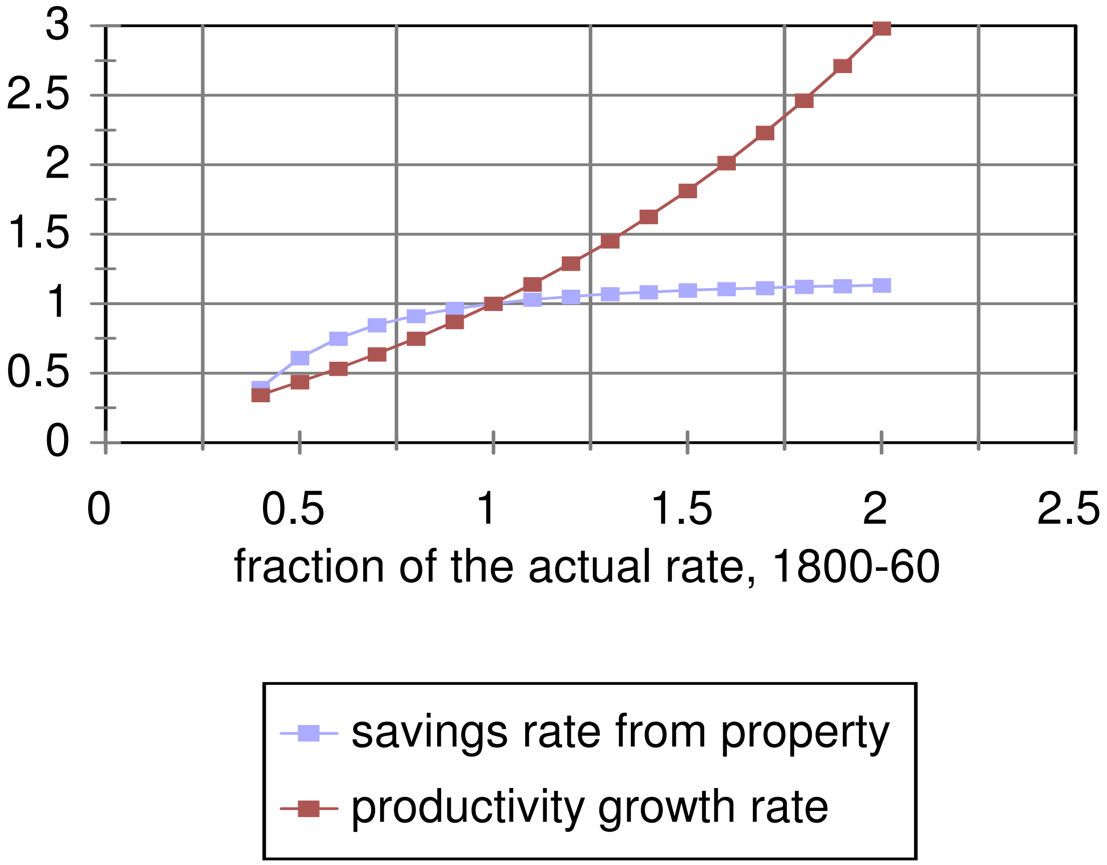

C
A
P
I
T
A
L
K*
K
Isoquant for urban-factory
Isoquant for rural-cottage
The green line
Is the low e-of-s
Isoquant.
L*
L
Labour

Figure 14

Simulated GDP gain

as a function of alternative savings and productivity



growth rates

The vertical scale is a/b where a is the simulated increase in 1860 GDP (implied by the fraction plotted on the horizontal axis) minus the simulated 1860 GDP with no industrial revolution and b is actual 1860 GDP minus simulated 1860 GDP with no industrial revolution.

Figure 15

Simulated Labour Shares

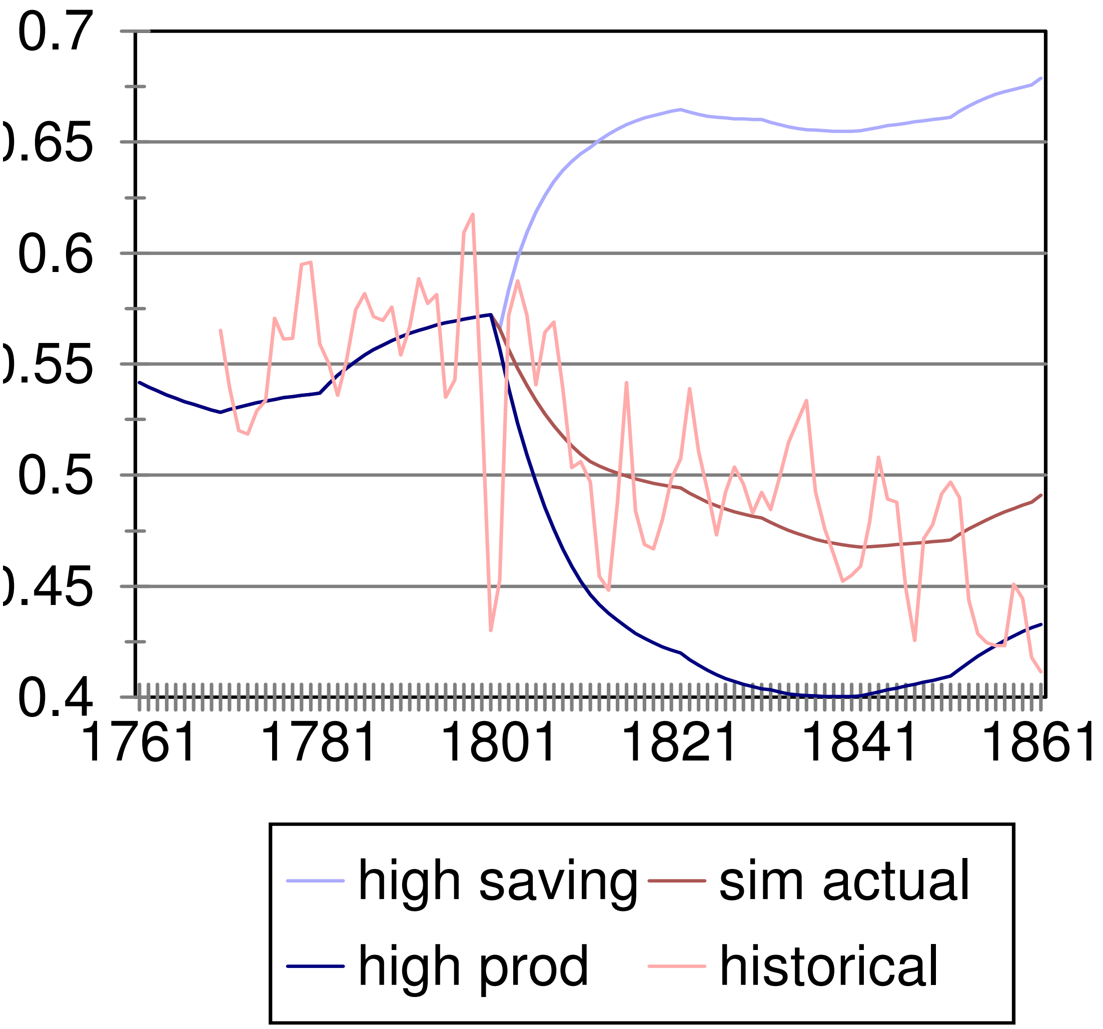


Figure 16

Simulated Capital Shares

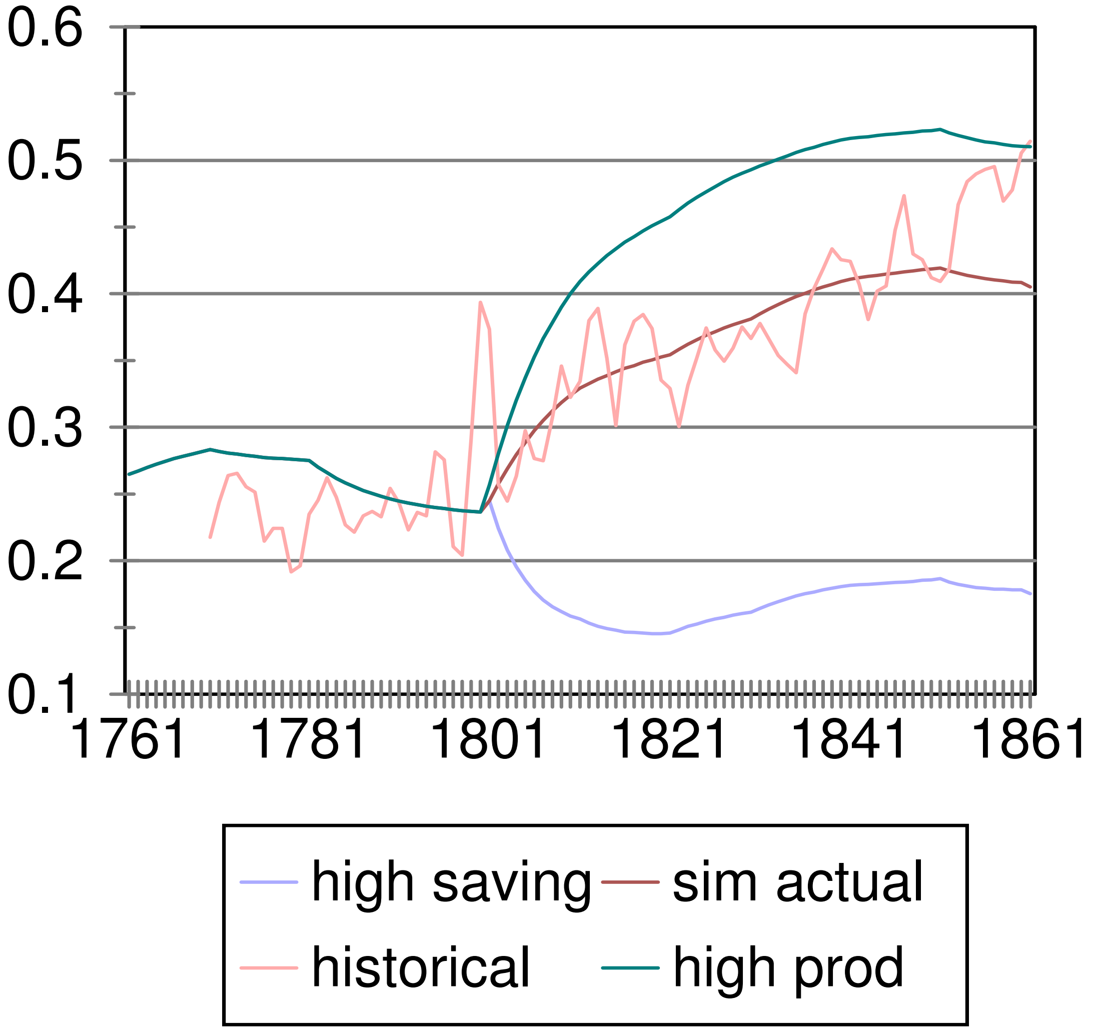


Figure 17

Simulated Profit Rate

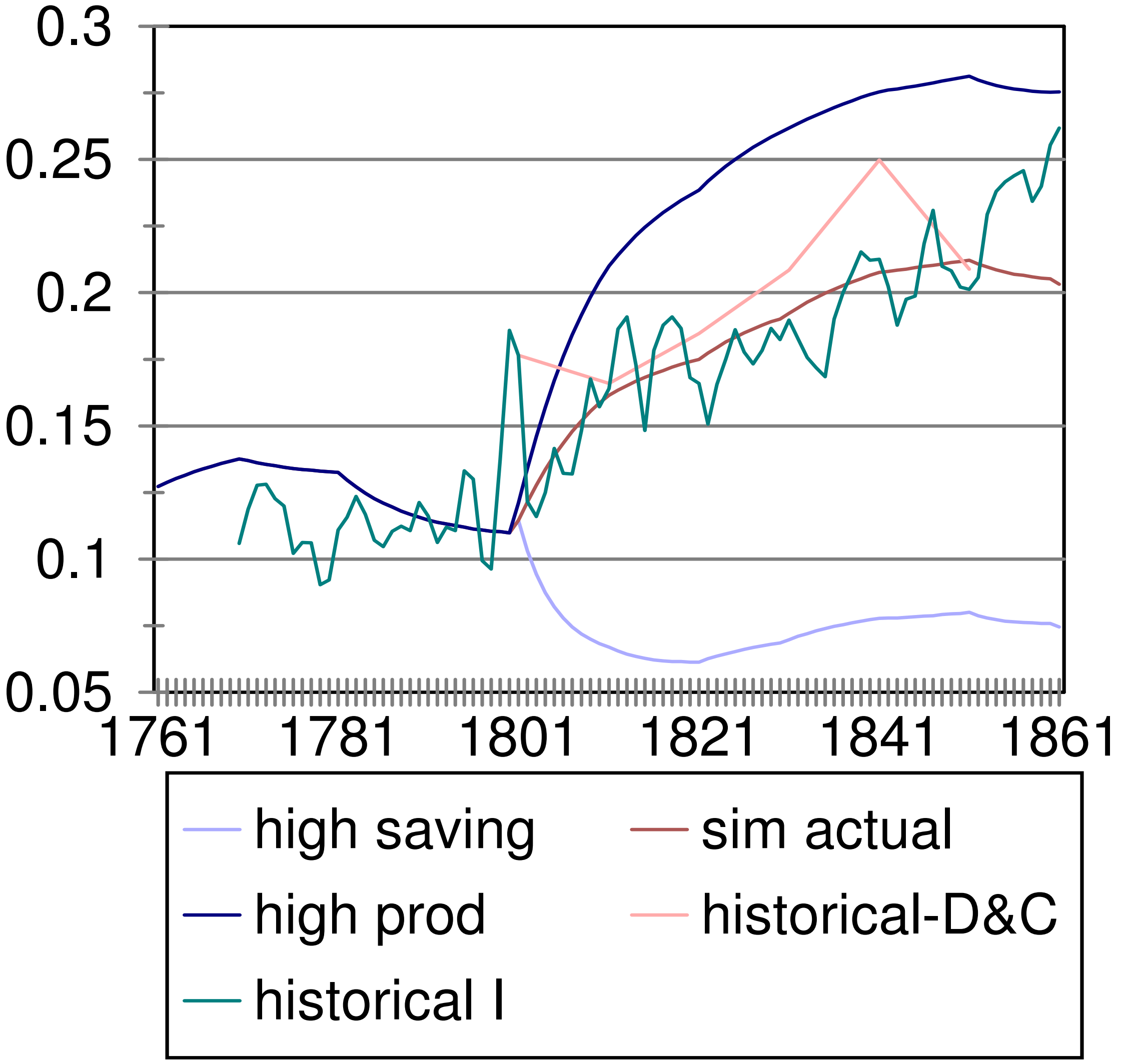
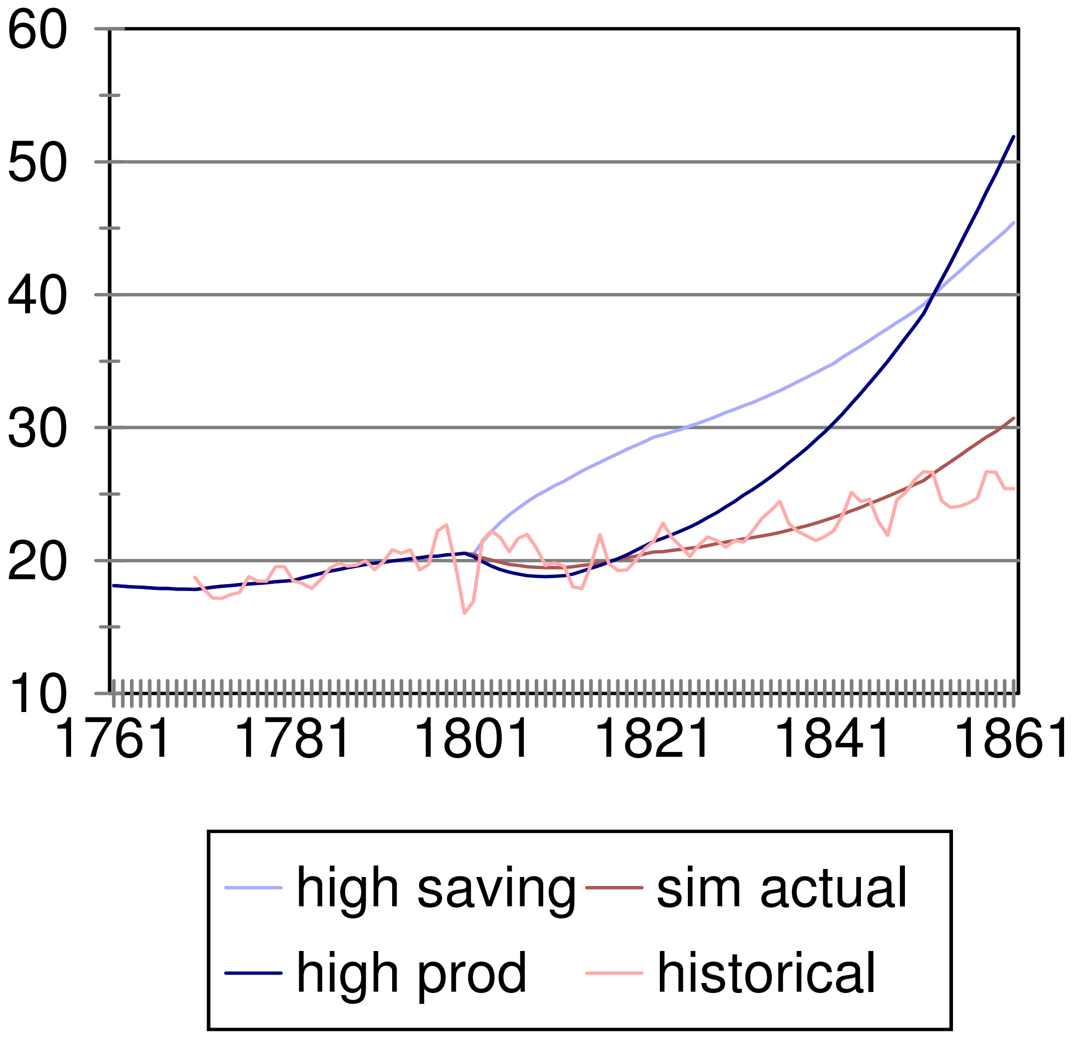


Figure 18

Simulated Real Wage



Appendix: Data Description

We know much more about economic growth during the industrial revolution than was known fifty years ago thanks to the efforts of several generations of economic historians. Key variables, however, have only been established for benchmark years–real national income, in particular, has been estimated only for 1760, 1780, 1801, 1831, and 1860. The small number of observations precludes the econometric estimation of important relationships and requires calibrating the model instead. Also different series use different benchmark years. To bring them into conformity and to simplify simulations, all series are annualized by interpolating missing values. As a result, the series are artificially smoothed but capture the main trends. Real values are measured in the prices of 1850-60 or particular years in the decade as available. The price level did not change greatly in this period. All values apply to Great Britain unless otherwise noted.

Crafts and Harley have been continuously improving the measurement of British GDP (Crafts 1985, Crafts and Harley 1992, Harley 1993), and I have relied on their work. Based on Deane's work, Feinstein (1978, p. 84) reckoned GDP in 1830 at £310 million and in 1860 at £650 million (both in 1851-60 prices), and I have extrapolated the 1830 estimate backwards using the Craft-Harley (1992, p. 715) real output index. This gives real GDP estimates for the benchmark years just noted.

The inputs were measured as follows:

land–acreage of arable, meadow, and improved pasture (commons are excluded). Allen (1994, p. 104, 2005) presents benchmark estimates for England and Wales. Following McCulloch (1847, Vol. I, pp. 554-5, 566-7), these have been increased by 12% to include Scotland.

Labour–for 1801, 1811, and continuing at ten year intervals, Deane and Cole's (1969, p. 143) estimates of the occupied population were used. The occupied population for 1760 was estimated by applying the 1801 ratio to the population. Voth (1991) has argued that the working year lengthened in this period. I have not tried to adjust the data for this change, so some of the rise in productivity that I report may be due to greater work intensity.

Capital (and real gross investment)–Feinstein (1988b, p. 441) presents average annual gross investment by decade from 1760 to 1860 for Great Britain. The magnitudes are expressed in the prices of the 1850s. He also estimated the capital stock in the same prices at decade intervals by equation 3. To annualize the data, I assumed that real gross investment in each year equalled the average for its decade. I reconstructed the capital stock year by year with equation 3. With the annualized data, a depreciation rate of $\delta = 2.4\%$ per year gives a capital stock series that matches Feinstein's almost exactly at decennial intervals. Therefore, 2.4% was used in subsequent simulations.

References

Abramovitz, Moses, and David, Paul (2001). "American Macroeconomic Growth: From Exploitation of Natural Resource Abundance to Knowledge-Driven Development," Stanford Institute of Economic Policy Research, SIEPR Discussion Paper 01-005.

Allen, Robert C. (1992). Enclosure and the Yeoman, Oxford, Clarendon Press.

Allen, Robert C. (1994), "Agriculture during the Industrial Revolution," in R. Floud and D. McCloskey, eds., The Economic History of Britain since 1700, Vol. I, 1700-1860, Cambridge, Cambridge University Press, pp. 96-122.

Allen, Robert C. (2001). "The Great Divergence in European Wages and Prices from the Middle Ages to the First World War," Explorations in Economic History, Vol. 38, pp. 411-447.

Allen, Robert C. (2005). "English and Welsh Agriculture, 1300-1850: Output, Inputs, and Income."

Antràs, Pol, and Voth, Joachim (2003). "Productivity Growth and Factor Prices during the British Industrial Revolution," Explorations in Economic History, Vol. 40, pp. 52-77.

Atkinson, A.B. (2005). "Top incomes in the UK over the 20th century," Journal of the Royal Statistical Society: Series A (Statistics in Society), Vol. 168, pp. 325-43.

Berg, Maxine, and Hudson, Pat (1992). "Rehabilitating the Industrial Revolution," Economic History Review, Vol. 45, pp. 24-50.

Brown, John C. (1988). "The Condition of England and the Standard of Living: Cotton Towns in the Northwest, 1806-1850," Journal of Economic History, Vol. 50, pp. 591-614.

Buchinsky, Moshe, and Pollack, Ben (1993). "The Emergence of a National Capital Market in England, 1710-1880," Journal of Economic History, Vol. 53, pp. 1-24.

Christensen, L. Jorgenson, D., and Lau, L. (1971). "Conjugate Duality and the Transcendantal Logarithmic Production Function," Econometrica, Vol. 39, No. 4-6, pp. 255-6.

Clark, Gregory (2002). "Land rental values and the agrarian economy: England and Wales, 1500-1914," European Review of Economic History, Vol. 6, pp. 281-308.

Clark, Gregory (nd). "Debts, Deficits, and Crowding Out: England, 1727-1840."

Crafts, N.F.R. (1976). "English Economic Growth in the Eighteenth Century: A Re-examination of Deane and Cole's Estimates," Economic History Review, Vol. 29, pp. 226-35.

Crafts, N.F.R. (1983). "British Economic Growth, 1700-1831: A Review of the Evidence," Economic History Review, Vol. 36, pp. 177-99.

Crafts, N.F.R. (1985a). British Economic Growth during the Industrial Revolution, Oxford, Clarendon Press.

Crafts, N.F.R. (1985b). "English Workers' Real Wages during the Industrial Revolution: Some Remaining Problems," Journal of Economic History, Vol. 45, pp. 139-44.

Crafts, N.F.R. (1987). "British Economic Growth, 1700-1850; Some Difficulties of Interpretation," Explorations in Economic History, Vol. 24, pp. 245-68.

Crafts, N.F.R. (2004). "Productivity Growth in the Industrial Revolution: A New Growth Accounting Perspective," Journal of Economic History, Vol. 64, pp. 521-35.

Crafts, N.F.R. and Harley, C. Knick (1992). "Output Growth and the Industrial Revolution: A Restatement of the Crafts-Harley View" Economic History Review, second series, Vol. 45, pp. 703-30.

Crafts, N.F.R. and Harley, C. Knick (2004). "Precocious British Industrialisation: A General-Equilibrium Perspective," in Leandro Prados de al Escosura, ed., Exceptionalism and Industrialisation: Britain and its European Rivals, 1688-1815, Cambridge, Cambridge University Press, pp. 86-107.

Crafts, N.F.R., and Venables, A.J. (2003). "Globalization in History: A Geographical Perspective," in M. Bordo, A. Taylor, and J. Williamson, eds., Globalization in Historical Perspective, National Bureau of Economic Research.

David, Paul (1977). "Invention and Accumulation in America's Economic Growth: A Nineteenth Century Parable," in International Organization, National Policies, and Economic Development, a supplement to the Journal of Monetary Economics, ed. by K. Brunner and A.H.Meltzer, vol. 6, pp. 179-228.

David, Paul (1978). "Lecture Notes on the U.S. Growth in the Nineteenth Century: Simulating 'The Grand Traverse' and 'The Cantabridgian Synthesis.' Delivered at The University of Cambridge, Lent Term,1978 [Available from: pad@stanford.edu.]."

Deane, P. And Cole, W.A. (1969). British Economic Growth, 1688-1959, Cambridge, Cambridge University Press, second edition.

Diewert, W. E. (1976). "Exact and Superlative Index Numbers," Journal of Econometrics, Vol. 4, pp. 115-45.

Engels, Frederick (1845). The Condition of the Working Class in England, translated and edited by W.O. Henderson and W.H. Chaloner, Oxford, Basil Blackwell, 1958.

Feinstein, Charles H. (1978). "Capital Formation in Great Britain," in P. Mathias and M.M. Postan, eds., Cambridge Economic History of Europe, Vol. VII, The Industrial Economies:

Capital, Labour, and Enterprise, Part I, Britain, France, Germany, and Scandinavia, Cambridge, Cambridge University Press.

Feinstein, Charles H. (1988a). "The Rise an Fall of the Williamson Curve," Journal of Economic History, Vol. 48, pp. 699-729.

Feinstein, Charles H. (1988b). "Sources and Methods of Estimation for Domestic Reproducible Fixed Assets, Stocks and Works in Progress, Overseas Assets, and Land," in Charles H. Feinstein and Sidney Pollard, eds., Studies in Capital Formation in the United Kingdom, 1750-1920, Oxford, Clarendon Press, pp. 258-471.

Feinstein, Charles H. (1998). "Pessimism Perpetuated: Real Wages and the Standard of Living in Britain during and after the Industrial Revolution," Journal of Economic History, Vol. 58, pp. 625-58.

Harley, C. Knick (1982). "British Industrialization before 1841: Evidence of Slower Growth during the Industrial Revolution," Journal of Economic History, Vol. 42, pp. 267-89.

Harley, C. Knick, (1993). "Reassessing the Industrial Revolution: A Macro View," in Joel Mokyr, ed., The British Industrial Revolution: An Economic Perspective, Boulder, Westview Press, pp. 171-226.

Heim, Carol, and Mirowski, Philip (1987). "Interest Rates and Crowding Out during Britain's Industrial Revolution," Journal of Economic History, Vol. 47, pp. 117-40.

Heim, Carol, and Mirowski, Philip (1991). "Crowding Out: A Response to Black and Gilmore," Journal of Economic History, Vol. 51, pp. 701-6.

Hoffmann, W.G. (1955). British Industry, 1700-1950, Oxford, Blackwell.

Horrell, Sara (1996). "Home Demand and British Industrialisation," Journal of Economic History, Vol. 56, pp. 561-604.

Horrell, Sara, and Humphries, Jane (1992). "Old Questions, New Data, and Alternative Perspectives: Families' Living Standards in the Industrial Revolution," Journal of Economic History, Vol. 52, pp. 849-880.

Jackson, R.V. (1987). "The Structure of Pay in Nineteenth-Century Britain," Economic History Review, second series, Vol. 40, pp. 561-70.

Kuznets, Simon (1955). "Economic Growth and Income Inequality," American Economic Review, Vol. 45, pp. 1-28.

Layard, P.R.G., Sargan, J.D., Ager, M..E., and Jones, D.J. (1971). Qualified Manpower and Economic Performance, London, Allen Lane.

Lewis, William Arthur (1954). "Economic Development with Unlimited Supplies of

Labour," Manchester School of Economics and Social Studies, Vol. 22, pp. 139-91.

Lindert, Peter H. (2000). "Three Centuries of Inequality in Britain and American," in Handbook of Income Distribution, Vol. I, ed. by A. Atkinson and F. Bourguignon, Elsevier Science, pp. 167-216.

Lindert, Peter H., and Williamson, Jeffrey G. (1983a). "English workers' living standards during the industrial revolution: a new look," Economic History Review, second series, vol. 36, pp. 1-25.

Lindert, Peter H., and Williamson, Jeffrey G. (1983b). "Reinterpreting Britain's Social Tables, 1688-1913," Explorations in Economic History, Vol. 20, pp. 94-109.

Mankiw, N. Gregory, Romer, David, and Weil, David N. (1992). "A Contribution to the Empirics of Economic Growth," Quarterly Journal of Economics, Vol. 107, pp. 407-37.

Matthews, R.C.O., Feinstein, C.H., and Odling-Smee, J.C. (1982). British Economic Growth, 1856-1973, Stanford, Stanford University Press.

McCloskey, D.N. (1981). "The Industrial Revolution: A Survey," in R.C. Floud and D.N. McCloskey, eds., The Economic History of Britain since 1700, Cambridge, Cambridge University Press, Vol. I, pp. 103-27.

McCulloch, J.R. (1847). A Descriptive and Statistical Account of the British Empire, London, Longman, Brown, Green, and Longmans.

Mokyr, Joel (1987). "Has the Industrial Revolution been Crowded Out? Some Reflections on Crafts and Williamson," Explorations in Economic History, Vol. 24, pp. 293-319.

Mokyr, Joel (2002). The Gifts of Athena: Historical Origins of the Knowledge Economy, Princeton, Princeton University Press.

Neal, Larry (1990). The Rise of Financial Capitalism: International Capital Markets in the Age of Reason, Cambridge, Cambridge University Press.

Neal, Larry (1991). "A Tale of Two Revolutions: International Capital Flows, 1789-1819," Bulletin of Economic Research, Vol. 43, pp. 57-92.

Norton, Trist, and Gilbert (1889). "A Century of Land Values: England and Wales," reprinted in E.M. Carus-Wilson, ed., Essays in Economic History, New York, St. Martin's Press, Vol. III, pp. 128-31.

O'Rourke, Kevin H. and Williamson, Jeffrey G. (2005). "From Malthus to Ohlin: Trade, Industrialisation and Distribution since 1500," Journal of Economic Growth, Vol. 10, pp. 5-34.

Samuelson, Paul A., and Modigliani, Franco (1966). "The Pasinetti Paradox in Neoclassical and More General Models," Review of Economic Studies, Vol. 33, pp. 269-301.

Solow, Robert M. (1956). "A Contribution to the Theory of Economic Growth," Quarterly Journal of Economics, Vol. 70, pp. 65-94.

Solow, Robert M. (1957). "Technical Change and the Aggregate Production Function," Review of Economics and Statistics, Vol. 39, pp. 312-20.

Soltow, L. (1968). "Long-run changes in British income inequality," Economic History Review, Vol. 21, pp. 17-29.

Temin, Peter (1997). "Two Views of the British Industrial Revolution," Journal of Economic History, Vol. 57, pp. 63-82

Temin, Peter, and Voth Hans-Joachim (2005). "Credit Rationing and Crowding Out during the Industrial Revolution: Evidence from Hoare's Bank, 1702-1862," Explorations in Economic History, Vol. 42, pp. 325-48.

Turner, M.E., Beckett, J.V., and Afton, B. (1997). Agricultural Rent in England, 1690-1914, Cambridge, Cambridge University Press.

Van Zanden, Jan Luiten (2005). "Cobb-Douglas in Pre-Modern Europe: Simulating Early Modern Growth," International Institute of Social History, Working Paper (www.iisg.nl/staff/jvz.html).

Voth, Hans-Joachim (2001). "The Longest Years: New Estimates of Labor Input in England, 1760-1830," Journal of Economic History, Vol. 61, p. 1065-82.

Williamson, Jeffrey G. (1984). "Why was British Growth so Slow during the Industrial Revolution?" Journal of Economic History, Vol. 44, pp. 687-712.

Williamson, Jeffrey G. (1985). Did British Capitalism Breed Inequality?, Boston, Allen & Unwin.

Williamson, Jeffrey G. (1990). Coping with City Growth during the British Industrial Revolution, Cambridge, Cambridge University Press.

Wrigley, A.E. and Schofield, R.S. (1981). The Population History of England, 1541-1871: A Reconstruction, London, Edward Arnold (Publishers) Ltd.