The equity premium Puzzle :an evaluation of the french case.

Allais Olivier [*][†], Nicolas Nalpas [‡][§]

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Abstract

We examine the existence of an equity premium puzzle in France using both annual and quarterly time series data. We investigate the ability of a representative agent model with a CRRA time-separable utility function and a utility function that displays habit persistence to account for high equity premia. We employ the three main methodologies used in the literature: calibration, Hansen-Jagannathan volatility bounds and GMM estimation of the preference parameters of the representative consumer. The introduction of habit formation improves upon the results but does not resolve the equity premium puzzle in France.

Key word :Habit persistence, GMM, Volatility Bound, Calibration.

[*]EUREQua-Université de Paris 1.

[†]Maison des Sciences Economiques, 106–112 Bd. de l'Hôpital, 75647 Paris Cedex 13, France.TEL : (33) 1 55 43 42 13, FAX : (33) 1 55 43 42 31, E-mail : oac@univ-paris1.fr

[‡]TEAM Université de Paris I.

[§]Maison des Sciences Economiques, 106–112 Bd. de l'Hôpital, 75647 Paris Cedex 13, France. TEL (33) 1 55 43 42 71, E-mail : nalpas@univ-paris1.fr

1 Introduction

In the United States between 1889 and 1978, the real average return on equity was about 6% higher than the return on short-term Treasury bonds. This difference between the real return on equity and the real return on riskfree bonds is called the equity premium . This is the premium investors require for holding risky assets. The equity premium puzzle refers to the seeming inability of standard asset pricing models to explain the average equity premium in the US markets. Mehra and Prescott[13] showed by calibrating a Consumption-based Asset Pricing Model[12] that, to duplicate the historical equity premium, the model had to use an unrealistic high risk-aversion coefficient. In addition, accepting this high risk-aversion coefficient as a correct description of a representative consumer leads to another puzzle, namely, the riskfree rate puzzle identified by Weil[16].

The literature started with Lucas's CCAPM. This is a classic model for asset pricing in a dynamic framework. It enriches Sharpe's well-known CAPM[15], by taking the following observation into account : although financial markets exert an important effect on consumption, consumption retroactively influences financial markets. The degree of equity risk in the CCAPM model is no longer based on the covariance of its return with the market portfolio but on the covariance of its return with per capital consumption . If this covariance is high, the selling of the asset will greatly decrease the variance of the consumption process of the representative agent. In equilibrium, selling the asset to reduce risk or keeping it in the portfolio would be equivalent. This implies that the real average return on the equity must be "high".

Moreover, as the covariance between the return on equity and consumption growth rate is much higher than the covariance relating consumption growth rate to Treasury bond returns, equity, in the eyes of a representative agent, is a meager protection against consumption risk. The agent thus requires higher returns on equity, or a positive risk premium. However, consumption appears to fluctuate less than stock market returns, leading to a very low equity premium.

In the last 15 years, many economists have offered solutions to this puzzle (for an extensive review of the literature on this topic see Kocherlakota [11]). One of the most promising attempts generalizes the preferences of the representative consumer. Following this approach, some authors have proposed consumption functions with habit formation.

CRRA additive and separable utility functions (used in the Mehra and Prescott [13]) create an artificial link between the risk aversion coefficient and the inter-temporal elasticity of substitution. The underlying idea of the preference modifications is to introduce non-separability in the state space and/or in the time space. This kind of feature allows to break up this link.

Especially, utility functions with habit formation inspired by the work of Dusesenberry[5] introduce a time non-separability of preferences. In this framework, the satisfaction of an agent does not depend on its consumption, but on the surplus of its consumption as compared to a given level. An agent with such preferences will therefore be much more sensitive to variations of his consumption. In a framework closely related to Mehra and Prescott[13], Constantidines[4] was the first to establish that the introduction of this kind of preferences allows to solve the equity premium puzzle.

Two other methods have been proposed to account for the existence of an equity premium puzzle. The first method, inspired by the works of Hansen [8] and Hansen and Singleton[7], consists to estimate from Euler equations the parameters characterizing consumers' preferences, using the Generalized Method of Moments. The second method, developed by Hansen and Jagannathan[9], uses Euler equations to construct a mean-variance frontier from observed assets' returns, from which we can evaluate the consistency of a candidate stochastic discount factor to explain these observations. Since then, other economists (for instance, Campbell and Cochrane[1], Cochrane and Hansen[3], and Constantidines and Ferson[6]) have tried to confirm Constantidines's promising results[4] using these various analytical methods as well as with different periodicity data.

Up to now, most studies have concentrated on American data, and very few have looked at the validity of this puzzle in France. In particular, there has been no study using long-term French data. Our objective is to determine whether there is an equity premium puzzle in the French financial market. To do this, we use the three analytical methods described above, applied to two equally sized sample data of different periodicity.

The remaining of the paper is organized as follows. In the first section, we present the model, together with the data, that we employ in the remaining of the paper. The second section studies the ability of an additive and separable utility function to take account of the observed equity premia in France. In section 3, we examine the accuracy of habit formation to deal with the puzzle. Section 4 concludes the paper.

2 Model and Data

2.1 The Consumption-based Asset Pricing Model.

We use a model derived from Lucas[12]: The model considered is a frictionless pure exchange economy with a single representative agent and a single perishable consumption good. Two kinds of assets can be traded on this market: There is one risk free asset noticed by the subscript 0 and N risked assets labeled with the subscript n ($n = 1, ..., N$). $S_{n,t}$ will denote at time t the quantity of asset n holding. Each period, the equities yield a random dividend denoted $d_{n,t}$.

The representative agent seeks to maximize the discounted expected utility subject to his budget constraint:

$$E_0 \sum_{t>0} \beta^{t} U(c_t) \tag{1}$$

$$W_t = c_t + \sum_{n=0} p_{n,t}.S_{n,t}$$

$$W_{t+1} = \sum_{n=1} (p_{n;t+1} + d_{n;t}) S_{n;t} \quad (2)$$

Where c_t represents the per capita consumption, β the discount factor, E_0 is the expectation conditional to the information at time zero, $U(.)$ is an increasing and concave function, $p_{n;t}$ denotes the time t price of the security n.

Note that current wealth W_t is a state variable and that current asset holdings $S_{n;t}$ are the control variables of this program; we can thus write the following Bellman equation:

$$V(W_t) = \underset{\{S_t\}}{MAX} \left[U\left(W_t - \sum_{n=0} p_{n;t} S_{n;t} \right) + \beta E_t [V(W_{t+1})] \right] \quad (3)$$

The Euler equations of this problem are:

$$1 = E_t \left(m_{t+1} \frac{p_{n;t+1} + d_{n;t+1}}{p_{n;t}} \right), \quad n = 0; \ldots; N \quad (4)$$

Where m_{t+1} is the marginal rate of substitution of the representative consumer. m_{t+1} is also called stochastic discount factor in the financial literature. The analytic form of this term only depends on the form of the utility function chosen.

2.2 Data

In this paper, we use both quarterly and annual data.

All the rates of return are computed as in Mehra and Precott (1985). The following tables give summary statistics of these data:

Table 1
Summary statistics (percent)
French Annual Data, 1896-1996

	Average Rate of Return	Standard Error
Stock Index	4.5	22.45
riskfree Rate	-3.41	9.22
Consumption Growth Rate	1.83	4.64

Table 2
Summary statistics (percent)
French Quaterly Data, 73Q1-97Q4)

	Average Rate of Return	Standard Error
Stock Index	5.840	11.571
riskfree Rate	0.705	0.927
Consumption Growth Rate	0.568	0.720

3 The case of CRRA and separable utility function

To evaluate the consistency of each model, we have employed the three main methodologies that have been introduced in the literature to cope with the equity premium puzzle.

The first method, initiated by Mehra and Prescott[13], is a calibration exercise in which we assess the accuracy of a particular model in its capability to reproduce the first moment of assets' prices for given parameter values characterizing the endowment economy.

In the second, due to Hansen and Jagannathan[9], we examine whether the volatility of the intertemporal marginal rate of substitution induced by the consumer's preferences, is enough to reach the lower bound implied by asset returns data.

The third, implemented by Hansen and Singleton[7], consists in a GMM estimation of the representative agent preferences' parameters, together with a test of overidentifying restrictions given by the moment conditions.

The utility function considered in this section is the same as used by Mehra and Prescott, and can be written as follows:

$$U(c_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} \tag{5}$$

The corresponding stochastic discount factor is given by:

$$TMSI_{t;t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{6}$$

3.1 Calibration

We exactly follow the methodology initiated by Mehra and Prescott. Hence, we only consider two assets. One risky asset, which corresponds to the market portfolio. This equity share entitles its owner to a random dividend each period that exactly is the output of the single productive unit. The riskfree asset entitles its owner to one unit of the consumption good in the next period. The representative consumer have a constant relative risk aversion utility function which is given by 5. His maximization program is the same as 1.Then, by denoting with the superscript e the equity share, and the superscript f the riskfree asset, the first order conditions of this program are:

$$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{p_{t+1} + y_{t+1}}{p_t} \right) \right] \quad \text{with } y_{t+1} = d_{t+1} \tag{7}$$

$$p_t = E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \tag{8}$$

As the consumption good is perishable, at equilibrium the market clearing implies that the consumption is equal to the output (or dividend). Then $c_t = y_t$ for all t. On the other hand, Mehra and Prescott suppose that the growth rate of production follows a two states Markov chain, then we have:

$$y_{t+1} = x_{t+1} y_t \tag{9}$$

Where x_{t+1} is the growth rate of production defined in the set $\{\lambda_1; \lambda_2\}$ such as:

$$prob(x_{t+1} = \lambda_j | x_t = \lambda_i) = \phi_{ij} \text{ with } i; j = 1; 2: \tag{10}$$

This hypothesis allows us to evaluate without difficulty the conditional expectation of the equations 7, since we can predict the evolution of this economy by only knowing the level of x_t and y_t.Therefore, the equations 7 can be re-written by taking account of these hypotheses:

$$P^e(c; i) = \beta \sum_{j=1} \phi_{ij} (\lambda_j c)^{-\gamma} c^{\gamma} (P^e(\lambda_j c; j) + \lambda_j c) \tag{11}$$

$$P_i^f = P^f(c; i) = \beta \sum_{j=1} \phi_{ij} (\lambda_j)^{-\gamma} \tag{12}$$

From now, we are able to calculate the asset return by noticing that the current state is $(c; i)$ and the following state $(\lambda_j c; j)$:

$$r_{ij}^e = \frac{P^e(\lambda_j c; j) + \lambda_j c - P^e(c; i)}{P^e(c; i)} \tag{13}$$

$$R_i^f = \frac{1}{P_i^f} - 1 \tag{14}$$

Hence, the expected return on equity if the current state is i is:

$$R_i^e = \frac{\lambda_j (w_j + 1)}{w_i} - 1 \text{ with } w_i = \beta \sum_{j=1} \phi_{ij} (\lambda_j)^{1-\gamma} (w_j + 1) \tag{15}$$

The property of ergodicity allows us to write by noting π the stationary probabilities:

$$R^e = \sum_{i=1} \pi_i R_i^e \tag{16}$$

$$R^f = \sum_{i=1} \pi_i R_i^f \tag{17}$$

The equity risk premium can be easily computed as the di¤erence between these two returns.

The Mehra and Prescott' method is a calibration exercise. It concerns to verify if the model above constrained by a consumption process can produce couples (average riskfree rate, equity risk premium) close to these historically observed. Two kinds of parameters are considered here: on the one hand parameters de…ning the preferences of the representative consumer, and, on the other hand parameters de…ning the technology of our economy. The two state Markov chain is de…ned as follows:

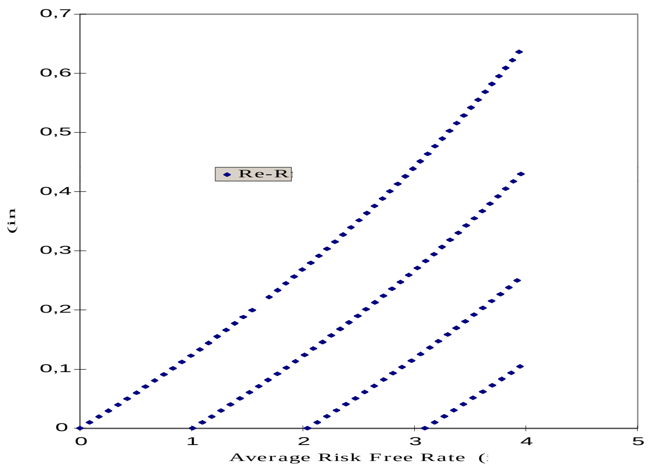
¸$_1$ = 1 + ¹ + ± and ¸$_2$ = 1 + ¹ ¡ ±

©$_{11}$ = ©$_{22}$ = © and ©$_{12}$ = ©$_{21}$ = 1 ¡ ©

Values attributed to parameters are chosen such that the average growth rate of the real per capita consumption ¹, the standard deviation of the average growth rate of the real per capita consumption ± , and the …rst-order serial correlation of this growth rate match the sample values for the France economy considering both samples described in the preceding section.

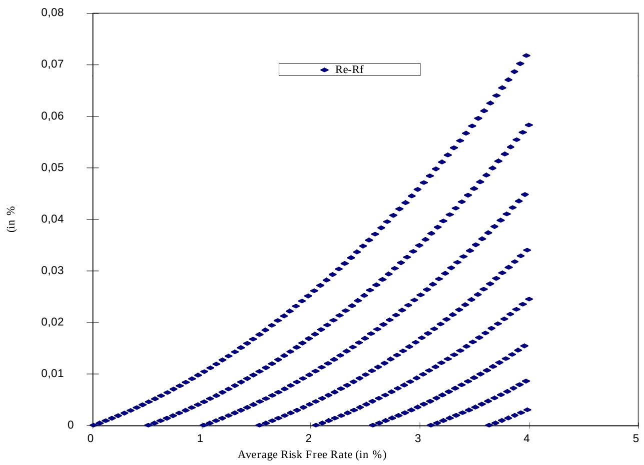
For the annual sample, we have ¹ = 0:0183, ± = 0:0464, and © = 0:47. The following graph allows to visualize the set of equity risk premia and real returns simulated by the model considered:

Figure 1: Equity Premia Simulated for France (1896-1996)



For the quarterly sample, we have ¹ = 0:0057, ± = 0:0072, and © = 0:41. The following graph allows to visualize the set of equity risk premia and real returns simulated by the model considered:

Figure 2: Equity Premia Simulated for France (73Q1-97Q4)



3.2 The Hansen and Jagannathan Volatility Bound

The Hansen and Jagannathan method compares the volatility of theoretical intertemporal marginal rates of substitution (thereafter IMRS), σ_m; to the volatility of the IRMS implied by asset returns, σ_x. For this kind of preference, the IRMS is:

$$TMSI_{t;t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \tag{18}$$

Thus, we can compute the mean, μ_m; and the standard deviation, σ_m of the IRMS for different values of γ: The Hansen Jagannathan frontier is constructed thanks to the Euler equations. To do this, we don't have to make any hypothesis about the preference of the representative agent[1]. This frontier gives σ_m in function of μ_m; by taking account of the sample means of asset returns denoted respectively μ_p and μ_x, and the variance-covariance matrix of returns[2], Σ_x. Then, we define σ_x as the minimum frontier of σ_m. We get the following equality[3]:

$$\sigma_m \geq \sigma_x \equiv \left[\left(\mu_p - \mu_m \mu_x\right)' \Sigma_x \left(\mu_p - \mu_m \mu_x\right) \right]^{\frac{1}{2}} \tag{19}$$

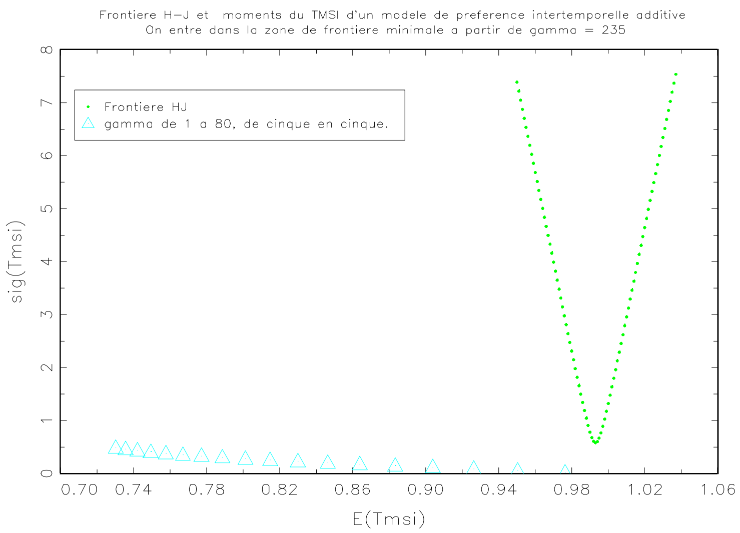
If data are consistent with theory, we have to verify on the graph that the empirical IMRS' volatility is higher than this implied by asset returns. We can

[1] We do not constraint the IMRS to take only positive values

[2] To increase the volatility of the IRMS, we construct theorical rates of return by taking the rate of return of the market portfolio, r_e; and the risk-free rate, r_f. The new price vectors are such as: $q'(t) = (1; 1; r_{e;t-1}; r_{f;t-1})$ and $x'(t) = (r_{e;t}; r_{f;t}; r_{e;t}.r_{e;t-1}; r_{e;t}.r_{f;t-1})$. We do not look for to give them any economic sense.

[3] See the corresponding Appendix for details.

Figure 3: HJ Frontier for the 73Q1-97Q4 sample (Δ from 1 to 80 by step of 5).



add a formal test to confirm this visualization. It tests if the difference between both of the standard deviations is not so large. We use Cecchetti, Lam, and Mark[2] methodology. We have to compute the difference $\Delta = \sigma_m - \sigma_x$ normed by the standard deviation of Δ. One rejects the null hypothesis if for a given γ, the point of coordinates (μ_m, σ_m) is below the frontier at a given level of significance[4].

The figure 3 presents the results for quarterly data. We observe the large difference between σ_m and σ_x. Indeed, The theoretical volatility of the IRMS is clearly lower than this of σ_x, for realistic values of the relative risk aversion coefficient. Considering annual data, we can not reject the model: we only need a relative risk aversion coefficient of ten. This leads to conclude that there is not an equity premium puzzle, whereas the related literature has always rejected this kind of model. If we consider the sub-sample 1956-1996, we observe on the graph 5 that the first and second order moments are clearly located below the Hansen Jagannathan curve. For the 1896-1939 sub-sample, the TMSI moments are in the Hansen Jagannathan curve from a CRRA of 7 (See Graph 4)[5]. This difference can be explained by the high volatility of the annual growth rate of per capita consumption during the 1896-1956 period, compared to that we observe during the 1956-1996 period. These remarks show the importance of the volatility concept, when we consider the empirical issues of the CCAPM.

[4] The Test statistics is asymptotically normal, and is equal to $\frac{\Delta}{\sigma_\Delta}$, under H_0 : $\Delta > 0$, such as $\sigma_\Delta = \left[\frac{\partial \Delta}{\partial \theta'} \Sigma_\theta \left(\frac{\partial \Delta}{\partial \theta}\right)\right]$ with $\theta' = (\mu_q, \mu_x, \Sigma_x, \mu_v, \sigma_v)$. Here, parameters of θ are estimated by their empirical moments. Thus, the test is one-sided,. Then for a significance level of 5%, we will accept H_0 if the critical value of the statistics is larger than -1.65.

[5] If we introduce in the sub-sample the second world war data, a CCRA of 5 is sufficient. Thus taking account high volatility allows having smaller CRRA.

Figure 4: HJ Frontier for the 1896-1996 sample (¢ from 1 to 40 by step of 5)

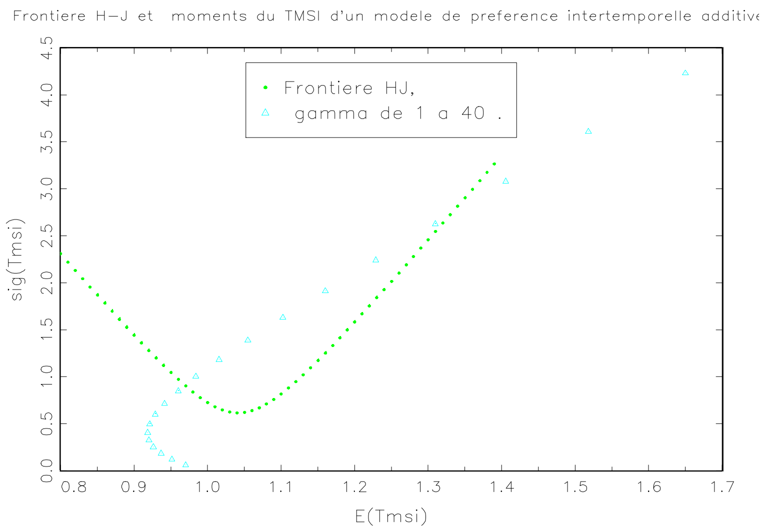
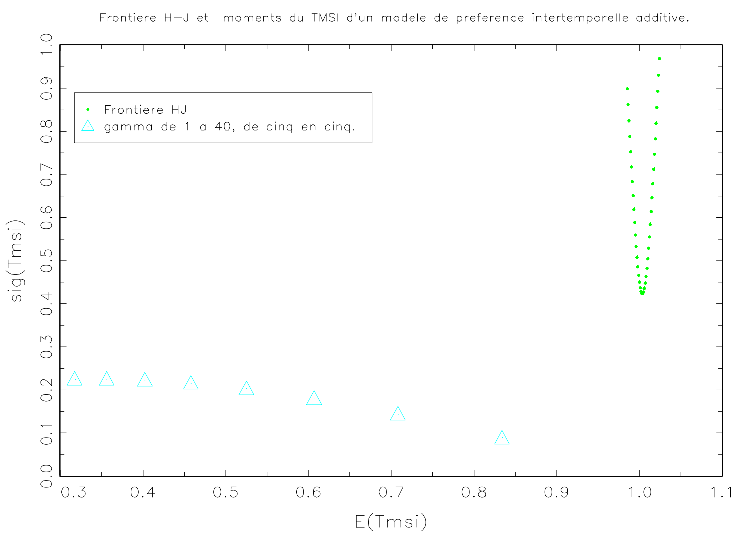


Figure 5: HJ Frontier for the 1956-1996 sub-sample (¢ from 1 to 40 by step of 5)



The table 2 confirms our results: we report the estimated values of μ_m, σ_m, σ_x and the test statistics τ for different values of γ. In table 2A, we present the quarterly data results. We notice that a CRRA of 30 is not sufficient to produce a test statistics larger than -1.65. Finally, we can only accept the consistency of the theory from a γ greater then 172.

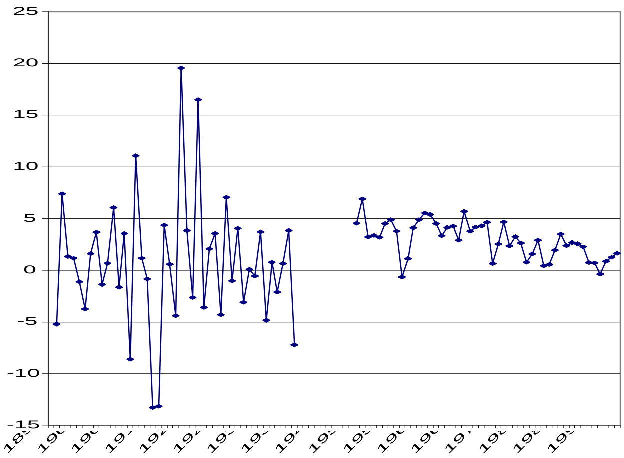
Tableau 2 : Test results for separable utility model

γ	$\hat{\mu}_m$	$\hat{\sigma}_m$	$\hat{\sigma}_x$	τ
Quarterly data				
0	0.9874	0.00716	-0.9815	-6.680
1	0.9820	0.01426	-1.856	-7.137
2	0.9766	0.02127	-2.749	-7.076
5	0.9608	0.04178	-5.379	-6.915
10	0.9362	0.07447	-9.525	-6.792
30	0.8546	0.1913	-23.24	-6.467
Annual data (1896-1996)				
0	0.9700	0.0612	0.8329	-5.015
1	0.9512	0.1228	0.9792	-4.5495
2	0.9367	0.1866	1.098	-3.8389
5	0.9184	0.4083	1.2523	-1.8913
6	0.9211	0.4988	1.229	-1.3543
10	0.9842	1.0053	0.7292	0.2609
Annual data (1956-1996)				
0	0.9595	0.0185	1.897	-5.591
1	0.9275	0.035	3.252	-5.315
2	0.8970	0.0516	4.553	-5.180
5	0.8129	0.092	8.137	-5.046
10	0.6947	0.1427	13.179	-5.021
30	0.4010	0.2180	25.708	-5.139
Annual data (1896-1996)				
0	0.9806	0.0660	0.667	-3.276
1	0.9728	0.1306	0.724	-2.698
2	0.9692	0.1960	0.7509	-2.020
3	0.9698	0.264	0.746	-1.417
5	0.9838	0.4167	0.646	-0.4835
10	1.1036	1.0028	0.8388	0.2195

The test results confirm the visualization of the Hansen Jagannathan volatility bounds. If we consider first the sub-sample (1896-1939), we notice that a CRRA only of 3 is sufficient to validate the theory. Nevertheless, we have to keep in mind that the consumption process during this period is very volatile, what introduces a bias in this kind of analysis. The following graph shows this feature:

On the contrary, if we consider the post war sub-sample, we conclude that the model can never be validate by data whatever values of γ taking into account. In this case, the equity premium puzzle appears more exacerbate. Nevertheless, the

Figure 6: Annual Growth Rate of the Per Capita Consumption Expenditures (1897-1996).



annual data contain durable goods, which have the property to bee less volatile than non-durable. That can explain the contrast with the quarterly data.

3.3 GMM Estimation

We now present the GMM estimation of our model. To conduct it, we consider the real return, of the market portfolio and the ninety-days government bonds. Table 3A presents the estimations of the Euler equations parameters and the results of the Hansen's overidentifying test in the case of quarterly data. We take into account for a second-order autocorrelation[6].

Tableau 3A : Estimation par GMM du modèle d'utilité séparable

Quarterly data

Instruments	β	γ	χ^2	Pval
Inst1	0.96 (0.003)	-4.53 (0.67)	31.62	0.00005
Inst2	1.35 (0.06)	100.87 (26.55)	32.31	0.002
Inst3	0.98 (0.002)	-2.37 (0.40)	32.51	0.143

We reject for both groups of instruments the model specification. For Inst2, we obtain a CRRA of 100 and a discount factor larger than one. Nevertheless, the CRRA is lower than this found in the sub-section above. This can be explained by the estimation of the discount factor, which is larger than one. When we consider high degree of relative risk aversion, a discount factor greater than one is not

[6] In order to obtain significant results

irregular, because of the small intertemporal elasticity of substitution induced. Since the riskfree rate is low, a discount factor less than one encourages the agent to borrow, what is in conflict with a small intertemporal elasticity of substitution. At equilibrium, for a given low risk free rate, we have to consider a discount factor larger than one, which tends to decrease his borrowing desire. This conclusion leads to the riskfree rate puzzle found by Weil[16]. One can also interpret this fact as the agent's will to delay its today consumption. In a habit formation framework, this will can be analyzed as a desire to reduce the negative effects of post purchase privations. Thus, the discount factor larger than one can justify the introduction of habit persistence in the consumption function.

For Inst1,the model has to be rejected because of the non-concavity of the utility function. For Inst3; the Hansen's test is not rejected, but the concavity hypothesis of the utility function is not verified. We then reject the model with separable utility when we consider quarterly data.

Concerning annual data, we have to reject the model too. Indeed, the good specification hypothesis is always rejected by the Hansen's test, whatever the group of instruments considered. We can't assess about the resolution of the puzzles.

Tableau 3B : GMM estimation of the separable utility model.
Annual data (1896-1996)

Instruments	β	γ	χ^2	Pval
Inst1	1.083 (0.038)	2.23 (0.67)	15.42	0.00391
Inst2	1.109 (0.038)	6.09 (1.34)	13.304	0.038
Inst3	1.15 (0.0035)	5.49 (1.092)	26.95	0.0079

If we constrain the sample to its most volatile part (table 3C), The good specification hypothesis is accepted for Inst1, and almost for the other groups of instruments. Nevertheless, the CRRA are not significantly different from zero. Hence, we can't again conclude about the puzzles resolution.

Tableau 3C : Estimation par GMM du modèle d'utilité séparable.
Annual data (1896-1939)

Instruments	β	γ	χ^2	Pval
Inst1	1.019 (0.02)	-0.44 (0.67)	9.12	0.059
Inst2	1.00 (0.01)	0.27 (0.77)	13.72	0.03
Inst3	1.011 (0.017)	-0.91 (0.26)	7.57	0.12

In the last sub-sample (table 3D) , the concavity constraint is not respected, what leads us to reject the model.

Tableau 3D : Estimation par GMM du modèle d'utilité séparable.

Annual data (1956-1996)				
Instruments	β	γ	χ^2	Pval
Inst1	0.96 (0.02)	-1.08 (0.5)	6.15	0.18
Inst2	0.92 (0.013)	-1.68 (0.39)	15.09	0.019
Inst3	0.91 (0.01)	-1.85 (0.35)	19.33	0.08

Finally, we note the perfect accordance of the conclusions given by the three different methods in the case of quaterly data. This is not surprising because most of studies on American data report the same results. The model with separable utility is not compatible with french data. If we consider it, we have shown there exists an equity premium puzzle and a riskfree rate puzzle. In contrary, the three main methodologies don't assess the same conclusions if we consider annual data. Nevertheless, the calibration techniques and the GMM estimations conclude to a bad specification of the classic CCAPM, while the Hansen Jagannathan volatility bounds does not allow it. Given the weakness of this specification for the stochastic discount factor, we are going to consider models with habit persistence in order to bring solutions to both enigma.

4 The Habit Formation Model

The main issue induced by the utilization of CRRA additive and separable utility functions is the result of the artificial link that it is created between two different notions: the relative risk aversion coefficient is the inverse of the intertemporal elasticity of substitution. While the first measures the agent's willingness to smooth his consumption over the states of nature, the second expresses his will to differ his consumption over the time space. To break this link, and then to incorporate time non-separability, some authors as Constantinides (1990) have introduce habit persistence in the agent's preferences. Thus, we now consider an utility function with habit formation property. That can be written as follows:

$$U(C_t; C_{t-1}) = \frac{(C_t - b_1 C_{t-1})^{1-\gamma} - 1}{1-\gamma} \tag{20}$$

Under this form, the satisfaction of an agent is no longer measured by the utility of his consumption, but by the utility of the surplus of his consumption as compared to a subsistence given level. An agent with such preferences will therefore be much more sensitive to variations of his consumption than an agent with CRRA and separable preferences.

We have now to give the analytic form of the intertemporal marginal rate of substitution:

$$TMSI_{t;t+1} = m_{t+1} = \beta\frac{(c_{t+1} - b_1 c_t)^{-\gamma} - \beta b_1 E_{t+1}[c_{t+2} - b_1 c_{t+1}]^{-\gamma}}{(c_t - b_1 c_{t-1})^{-\gamma} - \beta b_1 E_t[c_{t+1} - b_1 c_t]^{-\gamma}} \tag{21}$$

4.1 Calibration

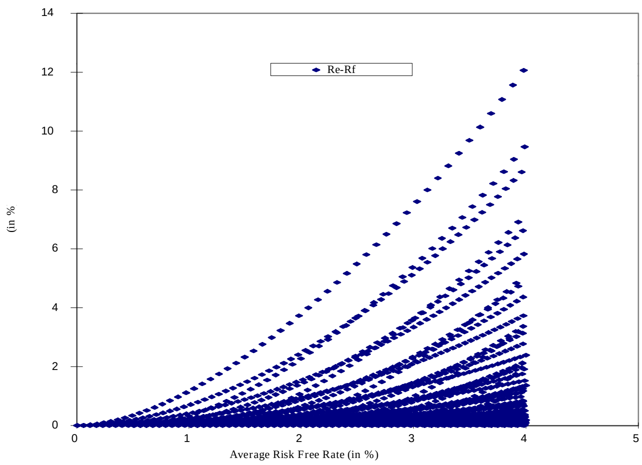
If we want to determine the asset prices in the Mehra and Prescott' world, we have to deal with two conditional expectation operators. this can be made thanks to the Markov property: by resuming the same notations as previously, if the current state of the economy is $(c; i)$, then the preceding state is $\left(\frac{c}{\lambda_i}; i\right)$, and the following state is $(\lambda_j c; j)$, and the state that succeeds to it is settled by $(\lambda_k \lambda_j c; k)$. The Euler equations[4] can therefore be re-written as follows:

$$P^e(c; i) = \beta \sum_{j=1}^{2} \phi_{ij} \frac{(\lambda_j c - b_1 c)^{-\gamma} - \beta b_1 \sum_{k=1}^{2} \phi_{jk} (\lambda_k \lambda_j c - b_1 \lambda_j c)^{-\gamma}}{\left(c - \frac{b_1 c}{\lambda_i}\right)^{-\gamma} - \beta b_1 \sum_{k=1}^{2} \phi_{ik} (\lambda_k c - b_1 c)^{-\gamma}} \left(P^e(\lambda_j c; j) + \lambda_j c\right) \tag{22}$$

$$P_i^f = P^f(c; i) = \beta \sum_{j=1}^{2} \phi_{ij} \frac{(\lambda_j c - b_1 c)^{-\gamma} - \beta b_1 \sum_{k=1}^{2} \phi_{jk} (\lambda_k \lambda_j c - b_1 \lambda_j c)^{-\gamma}}{\left(c - \frac{b_1 c}{\lambda_i}\right)^{-\gamma} - \beta b_1 \sum_{k=1}^{2} \phi_{ik} (\lambda_k c - b_1 c)^{-\gamma}} \tag{23}$$

One can then calculate the asset returns as well as the simulated risk premia by applying the equations 13, 15, and 16. The following graphs present the simulation results for both sample of data, using the same restrictions on the parameters as Mehra and Prescott: We impose the riskfree rate to be below 4%, the coefficient γ not to exceed 10.

Figure 7: Equity Premia Simulated for France (73Q1-97Q3)

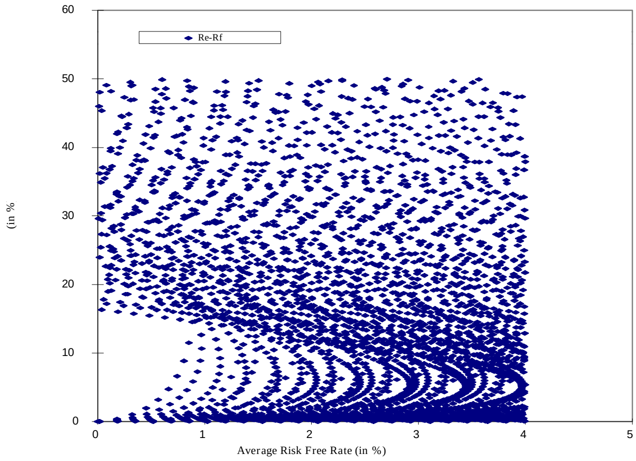


The introduction of habit persistence seems to resolve the equity premium puzzle whatever the data sample considered, since the model with habit formation can produce high equity premia and low riskfree rate. Nevertheless considering habit formation specification for preferences, γ is no longer the relative risk aversion coefficient. The consumption risk is now measured as follows:

$$RRA_c = c\frac{U''(c)}{U'(c)} = \frac{-\gamma\left(\left[1-b_1\lambda_i^{-1}\right]^{-\gamma-1} - \beta b_1^2 E_i\left[(\lambda_j - b_1)^{-\gamma-1}\right]\right)}{\left[1-b_1\lambda_i^{-1}\right]^{-\gamma} - \beta E_i\left[(\lambda_j - b_1)^{-\gamma}\right]}; i;j = 1;2 \quad (24)$$

We notice that the consumption relative risk aversion is an increasing function of the habit parameter b_1: Moreover, the high equity premia are simulated with high values of b_1 $(> 0{:}5)$. These values lead to consider consumption risk larger than 10. Thus, we are not anymore under the same specifications used by Mehra and Prescott (1985). To deeply assess this issue, we develop below the two other methodologies.

Figure 8: Equity Premia Simulated for France (1896-1996)



4.2 Hansen and Jagannathan Volatility Bound

As mentioned above, the IMRS now supposes the calculation of a conditional expectation. We use the Cecchetti, Lam, Mark[2] method to do this. We suppose that the growth rate of per capita consumption follows a first-order autoregressive process. If we denote it π. We have:

$$\pi_t = \mu_c(1 - \omega) + \omega\pi_{t-1} + \varepsilon_t \tag{25}$$

Such as π_t is defined by $\ln \frac{C_t}{C_{t-1}}$, μ_c is the mean of the process and ε_t an i.i.d. random variable with mean zero and variance σ_c^2: Given these hypotheses, we can determine the mean and the standard deviation of the IRMS

Equations21 and 25, give us the result :

$$TMSI_{t,t+1} = \Gamma(\pi_t, \pi_{t+1}) = \frac{\beta e^{-\gamma\pi_t}\left[(e^{\pi_{t+1}} + b)^{-\gamma} + \beta b e^{-\gamma\pi_{t+1}} E_{t+1}(e^{\pi_{t+2}} + b)^{-\gamma}\right]}{(e^{\pi_t} + b)^{-\gamma} + \beta b e^{-\gamma\pi_t} E_t(e^{\pi_{t+1}} + b)^{-\gamma}} \tag{26}$$

Where the conditional expectation of 26 is computed with the classic following formulae:

$$E_t\left[e^{\pi_{t+1}} + b\right]^{-\gamma} = \int_{-\infty}^{+\infty}\left[e^{(\mu_c(1-\omega) + \omega\pi_t + \varepsilon_t)} + b\right]^{-\gamma}\phi_c(\varepsilon_t)\,\partial\varepsilon_t \tag{27}$$

$\phi_c(\varepsilon_t)$ is the normal density with zero mean and variance σ_c^2: We deduce from the equation 25 that the conditional law of $(\pi_t, \pi_t + 1)$ is bi-normal such as:

$$\begin{pmatrix} \pi_t \\ \pi_{t+1} \end{pmatrix} \rightarrow N\left(\begin{pmatrix} \mu_c \\ \mu_c \end{pmatrix}; \frac{\sigma_c^2}{(1-\omega)^2} \begin{pmatrix} 1 & \omega \\ \omega & 1 \end{pmatrix} \right) \tag{28}$$

We obtain the mean and the variance of the IMRS.

$$\mu_v = \int_{-\infty}^{+\infty} -(\pi;\pi')\,\phi(\pi;\pi')\,\partial\pi\partial\pi' \tag{29}$$

and

$$\sigma_v = \int_{-\infty}^{+\infty} \left[-(\pi;\pi') - \mu_v\right]^2 \phi(\pi;\pi')\,\partial\pi\partial\pi'$$

The equation 27 calculation supposes that we are able to estimate the structural parameters of the consumption process: $\mu_c; \sigma_c; \omega$. Moreover, we must estimate $\mu_x; \mu_p; \Sigma_x$. If we consider the same returns as for the separable utility case, we must estimate 21 coefficients. The parameters' values are determined thanks to the GMM, which orthogonality conditions are given by the first and second order moments of asset returns, the first-order moments of asset prices, and the first and second order moments of growth rate of consumption and its first order autocorrelation[7]. Given these estimations, we are then able to compute the mean and the standard deviation of the IRMS in order to construct the Hansen Jagannathan frontier and the test statistics of $(\mu_x;\sigma_x)$ with $(\mu_v;\sigma_v)$. In the figure 9, we present the first two moments of different IRMS according to values of b and γ: The habit formation model seems to be consistent with French financial data. The more γ increases, closer to the frontier is the IRMS. Moreover, larger is the habit parameter b; smaller is the necessary value of γ to enter in the frontier. We then conclude that the habit formation model is better than the model analyzed in section II.

We now compute the Cecchetti, Lam, et Mark[2] test for different values of the habit parameters. We can see the results in the table 5a and 5b. They show that the habit formation is consistent with data. We can make the same remarks as previously concerning the symmetric evolution of the γ and b parameters. The volatility of the IRMS is as far as large that the habit parameter is high. This statement confirms the arguments developed in the previous section. Since larger is the habit parameter, higher is the consumption relative risk aversion, and lower is the necessary value γ to produce high volatility of the IRMS. Hence, γ is 6 for a habit coefficient equal to 0.7, while γ is 9 for a habit coefficient equal to 0.5.

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[7] The orthogonality conditions used are:

$$E\left[x_t - \mu_x\right] = 0;$$
$$E\left[q_t - \mu_q\right] = 0;$$
$$E\left[\mathrm{vec}\left(x_t x_t'\right) - \mathrm{vec}(\Sigma_x) + \mathrm{vec}\left(\mu_x \mu_x'\right)\right] = 0;$$
$$E\left[\pi_t - \mu_c\right] = 0;$$
$$E\left[\pi_t^2 - \left(\frac{\sigma_c^2}{(1-\omega^2)} + \mu_c^2\right)\right] = 0;$$
$$E\left[\pi_t \pi_{t-1} - \left(\frac{\omega}{(1-\omega^2)}\sigma_c^2 + \mu_c^2\right)\right] = 0.$$

Figure 9: HJ Bounds for di¤erent habit parameter values

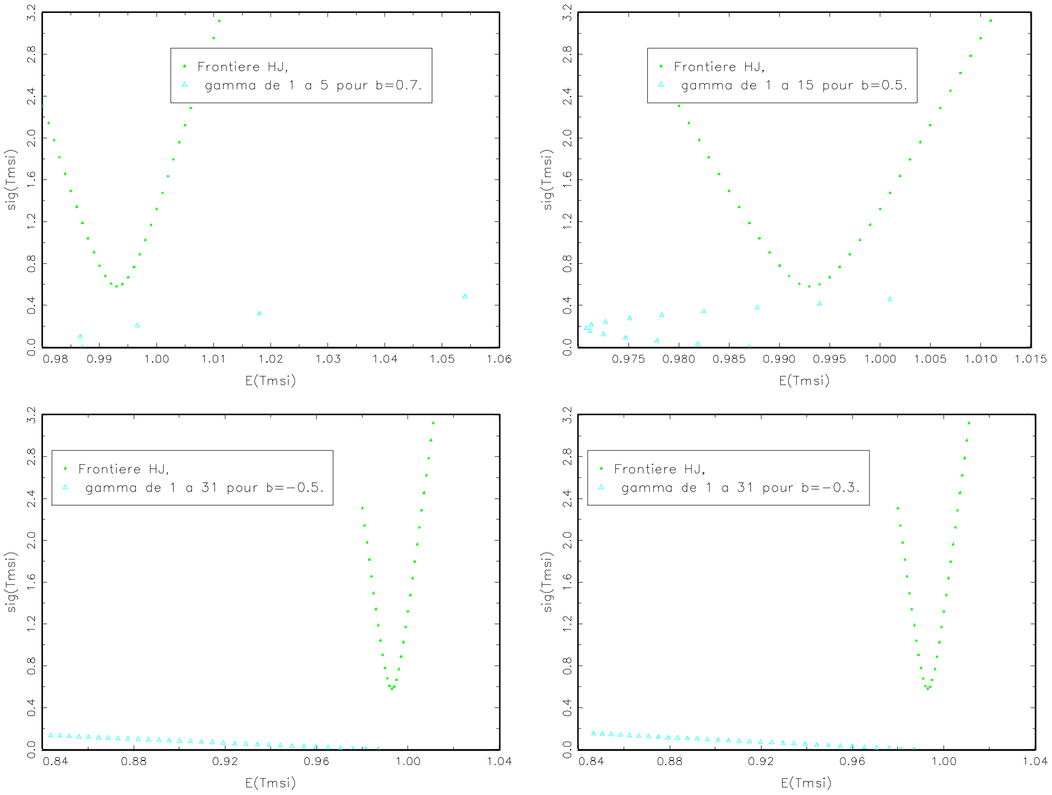

Tableau 5A : Test de la distance pour di¤érent paramètres b positif

Quarterly data

	b = 0:7					b = 0:5			
°	¹Ç m	¾Ç m	¾Ç x	¿	°	¹Ç m	¾Ç m	¾Ç x	¿
0	0.987	0	1.05	-2.93	0	0.987	0	1.05	-2.93
1	0.9866	0.102	1.102	-2.87	1	0.9819	0.031	1.858	-4.63
2	0.9966	0.201	0.7921	-0.83	2	0.9778	0.061	2.532	-5.77
3	1.018	0.324	4.264	-1.84	3	0.9747	0.092	3.053	-5.86
4	1.054	0.484	10.31	-2.07	5	0.9712	0.153	3.636	-4.30
5	1.117	0.822	20.94	-1.58	7	0.9713	0.214	3.616	-2.62
6	1.189	1.104	33.14	-0.41	9	0.9751	0.277	2.993	-1.38

Tableau 5B : résultat du test pour le modèle à fonction d'utilité séparable.

Annual data (1896-1939)

b = 0:5

°	¹Ç m	¾Ç m	¾Ç x	¿
0	1.000	1.843e-008	0.6309	-2.989
1	0.9939	0.1975	0.6690	-3.679
2	1.026	0.4202	0.5080	-0.4673
3	1.107	0.7245	0.7516	-0.0635
5	1.663	3.169	5.745	-0.7825
10	2.248	24.89	11.15	0.685

Annual data (1896-1939)

b = 0:5

°	¹Ç m	¾Ç m	¾Ç x	¿
0	0.9800	1.806e-008	0.7209	-3.178
1	1.0222	0.352	0.5128	-0.884
2	1.276	2.340	-0.0741	-0.0043
3	1.607	3.205	5.596	-0.218
5	0.4189	6.294	5.996	0.0118
10	-2.863	28.52	37.89	-0.3180

We …nd similar results when we consider the annual data sets (table 5B). As expected, we notice that the null hypothesis is accepted for lower values of ° over the 1896-1939 sub-sample. The explanation of this result has been given in the section II. If we consider the least volatile sub-sample (1959-1996), the habit formation property does not allow to produce enough volatility for the IRMS to accept the null hypothesis.

4.3 GMM Estimation

As previously, we use the GMM to estimate the parameters of interest[8]. The method supposes that we can perfectly identify the optimum. However we have

[8] See Allais[?]

noticed that the discount factor is di¢cult to estimate. We chose to balance the Euler equation by the discount factor to solve this estimation problem. Thus, we constrain beta to be di¤erent from zeros and we prevent the other parameters from exploding. Despite this constraint, we could not estimate the three parameters in one times. In fact, we chose to estimate the parameters beta and gamma by …xing the habit parameter[9]. Then, we retain the habit parameter b which minimizes the objective function. The results are presented in table 6. For inst1 and inst3, the concavity constraint is largely not checked. We do not present these results

Tableau 6A : estimation MMG du modèle d'utilité non séparable
Quaterly data

Instruments	β	γ	b	χ^2	Pval
Inst2	0:989 (0:001)	0:037 (0:017)	0:85	10:83	0:62
Inst2$'$	0:989 (0:001)	0:21 (0:088)	0:72	10:39	0:39

We choose to introduce inst2$'$. This vector of instruments gathers the di¤erent rates of returns only delayed of one period. The habit coe¢cient is 0.85 and 0.72 for Inst2 and inst2$'$ respectively. However, as Kocherlakota underlines, such habit coe¢cients are unrealistic. For, they suppose in particular that the representative consumer requires a high level of consumption to survive. Moreover, this implies, that the consumer (even for a relatively low gamma) is ready to pay much to maintain his level of consumption. Thus, the levels of the habit parameters can explain the very weak gamma estimation.

From table 6B and 6C, the habit parameters are less strong than in the case of the quarterly data. Compared to tables 3b and 3c, the gamma found are lower, but the assumption of good speci…cation of the model is checked for any vectors of instruments. Nevertheless, the coe¢cients gamma are not signi…cantly di¤erent from zeros.

Tableau 6B : estimation MMG du modèle d'utilité non séparable
Annual data (1896-1996)

Instruments	β	γ	b	χ^2	Pval
Inst2$'$	1:005 (0:019)	0:806 (0:52)	0:62	11:904	0:018
Inst2$''$	1:020 (0:016)	1:245 (0:08)	0:46	14:18	0:077

Tableau 6C : estimation MMG du modèle d'utilité non séparable
Annual data (1896-1936)

Instruments	β	γ	b	χ^2	Pval
Inst2$'$	0:98 (0:015)	0:459 (0:56)	0:39	7:45	0:113
Inst2$''$	0:989 (0:001)	0:64 (0:088)	0:53	9:53	0:151

[9] The habit paramerter is taken between [0; 1], with a stepsize of 0:01:

When we consider the less volatile part of the sample (table 6D), the concavity constraint is never checked and the coefficients gamma are strongly not significantly different from zeros.

Tableau 6D : estimation MMG du modèle d'utilité non séparable
Annual data (1956-1996)

Instruments	β	γ	b	χ^2	Pval
$Inst2'$	1.34 (0.39)	-5.87 (6.56)	0.85	1.93	0.89
$Inst2''$	1.36 (0.40)	-6.02 (0.088)	0.72	1.39	0.92

Let us interest in the equity premium and the riskfree rate puzzles: First of all, we must pay attention to the sense of the coefficient gamma. Indeed, contrary to the standard consumption-based asset pricing with power utility, the coefficient gamma do no longer correspond to the coefficient of relative risk aversion η. Yet, we need the expression of the relative risk aversion to know if the equity premium puzzle is solved. Ferson and Constantinides[6] derive the coefficient of relative risk aversion, in a deterministic economy, and they find the following expression :

$$\eta = \frac{\gamma}{1 - b\left[R(\beta R)^{\frac{-1}{\gamma}} - 1\right]/(R - b)}. \tag{30}$$

The calculation of η is carried out with the average of the Treasury bills returns. We remark that the RRA is increasing compared to the habit parameter. In other words, the representative agent becomes more and more averse to risk as its lagged consumption has a strong influence on its well being,or as its habits are strong. From the estimates of table 6A, for the vector of instruments $inst2'$, the η is 0.22. Thus, taking into account these estimates, it seem that the equity premium puzzle is solved. If we uses the results of table 5A, the RRA is 5.1[10].

Then, we calculate the average of the IMRS thanks to the equality26 and the estimates found previously, to see whether the riskfree rate puzzle is solved. The conditional expectation of the IMRS is 0.9882. Therefore the real interest rate is 1.011%. From the table 5A, we note that for greater coefficients gamma, the risk-free rate is approximately the same. Thus, for habit formation models, we solve the risk free rate puzzle. This result is not surprising. Indeed, in the time separable context, we know that for a positive growth rate, the future marginal utility of the individual is lower than that of the actual marginal utility. Moreover, increasing the coefficient gamma or reduce the elasticity of intertemporal substitution, increases the difference between the present marginal utility of consumption and the future marginal utility of consumption. So, we should increase the interest rate to prevent the representative consumer from reallocating his consumption from the future to the present. On the contrary, in the habit formation context, an increase in b causes a growth of future marginal utility and thus makes it possible to reduce the desire to reallocate future consumptions from the present

[10] The calculation is realised for a gamma equal to five and a habit coefficient équal to 0.7.

consumptions. Then, the level of the riskfree rate drops. Thus, intuitively, we understand why the riskfree rate puzzle is solved in the models of habit formation.

5 Conclusion.

In this empirical paper, we have analyzed the existence of an equity premium puzzle in the French stock market by using two samples of time series data. The …rst contains long period annual data (1896-1996), the second implements quarterly data (73Q2:97Q4). We have studied the ability of the representative agent model to produce high equity premia by considering two kinds of consumers' preferences:

- A time-separable utility function with a constant relative risk aversion coef…cient.

- A utility function with properties of habit persistence.

To evaluate the consistency of each model, we have employed the three main methodologies that have been introduced in the literature to cope with the equity premium puzzle.

The …rst method, initiated by Mehra and Prescott[13], is a calibration exercise in which we assess the accuracy of a particular model in its capability to reproduce the …rst moment of assets' prices for given parameter values characterizing the endowment economy.

In the second, due to Hansen and Jagannathan[9], we examine whether the volatility of the intertemporal marginal rate of substitution induced by the consumer's preferences, is enough to reach the lower bound implied by asset returns data.

The third, implemented by Hansen and Singleton[7], consists in a GMM estimation of the representative agent preferences' parameters, together with a test of overidentifying restrictions given by the moment conditions.

For the standard consumption-based asset pricing with power utility, we …nd the equity premium and the riskfree rate puzzles. On the contrary, in the context of habit formation, the equity premium and the riskfree rate puzzles are solved in the quarterly and annual data.

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